

Manuscript version: Author's Accepted Manuscript

The version presented in WRAP is the author's accepted manuscript and may differ from the published version or Version of Record.

Persistent WRAP URL:

<http://wrap.warwick.ac.uk/116632>

How to cite:

Please refer to published version for the most recent bibliographic citation information. If a published version is known of, the repository item page linked to above, will contain details on accessing it.

Copyright and reuse:

The Warwick Research Archive Portal (WRAP) makes this work by researchers of the University of Warwick available open access under the following conditions.

© 2019 Elsevier. Licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International <http://creativecommons.org/licenses/by-nc-nd/4.0/>.



Publisher's statement:

Please refer to the repository item page, publisher's statement section, for further information.

For more information, please contact the WRAP Team at: wrap@warwick.ac.uk.

1 **A practical consolidation solution based on** 2 **the time-dependent discharge rate around** 3 **PVDs**

4 Yang Yu¹, Gang Wu^{1,2}, Hongyue Sun¹, Xueyu Geng^{2*}

5 (1. Ocean College, Zhejiang University, Zhoushan, Zhejiang 316021, China)

6 (2. School of Engineering, The University of Warwick, Coventry CV4 7AL, UK)

7 (*Corresponding author Xueyu Geng, Xueyu.Geng@warwick.ac.uk, School of Engineering, The
8 University of Warwick, Coventry CV4 7AL, UK)

9
10 **Abstract:** This study presents a practical consolidation solution for ground
11 improvement used prefabricated vertical drains (PVDs) by incorporating the available
12 time-dependent discharge rate around PVDs, which can be easily obtained by
13 laboratory test and field monitoring. Only radial consolidation is taken into account in
14 the derivation to significantly simplify the final expression as the vertical
15 consolidation can be neglected in a typical soft ground improvement project. The
16 proposed solution is verified by the finite element method (FEM) and two case
17 histories, including vacuum preloading and surcharge loading. The verification results
18 show that the proposed solution can predict the development of excess pore water
19 pressure (EPWP) and the degree of consolidation (DOC) effectively and accurately.
20 Design charts and framework are developed to assist geotechnical engineers in using

21 this solution for field construction and performance prediction.

22 **Keywords:** axisymmetric consolidation; time-dependent **discharge rate**; equal-strain
23 assumption; numerical simulation

24

25 **Nomenclature**

26 r, z radial and vertical coordinates

27 u, \bar{u}_r, u_r, u_0 excess pore water pressure (EPWP), average radial EPWP, virtual
28 EPWP in the radial direction and initial average EPWP

29 C_v, C_h vertical and horizontal coefficients of consolidation

30 m_v volumetric compressibility

31 k_v, k_h, k_s vertical hydraulic conductivity and horizontal hydraulic conductivity within
32 undisturbed zone, and horizontal hydraulic conductivity of smear soil

33 γ_w unit weight of water

34 t elapsed time

35 h, d_e, d thickness of soft soil, effective influence diameter of PVD and PVD spacing

36 E_s constrained modulus of the soil

37 ε_v volumetric strain

38 r_d, r_s, r_e radius of the equivalent cylinder of PVD, the smeared zone, and the effective
39 influence zone of PVD

40 $q(t)$ time-dependent **discharge rate** around PVDs

41 A wetted cross-section area

42 $g(t)$ time-dependent EPWP gradient at $r = r_d$

43 c_s ratio between the horizontal hydraulic conductivity of soil within smear zone and
44 undisturbed zone
45 n, s ratio between the radii of effective influenced zone and PVD and ratio between
46 the radii of smear zone and PVD
47 U, U_t degree of consolidation (DOC) and modified DOC induced by multi-stage
48 loading
49 s, s_c time-dependent settlement at any given time and the ultimate settlement
50 a, b width and thickness of band-shaped drainage board
51 t_{n-1}, t_n starting and ending time of each stage under a constant-speed loading process
52 $\Delta p', \Delta p''$ loading increments that correspond to the first and second stages of loading
53 q_{w0} initial value or short-term value of the **discharge rate** around PVDs
54 A_0 coefficient with respect to the time-dependent **discharge rate**

55

56 **Introduction**

57 Civil engineering infrastructures are commonly constructed on weak soils that require
58 improvement to resist applied loads [27, 31, 38, 48]. Improving soils in a
59 time-efficient and cost-effective manner can not only ensure a safer, more reliable,
60 efficient, sustainable, and resilient infrastructure but also offer great value to investors,
61 contractors, and users [1, 4, 47, 34]. Prefabricated vertical drains (PVDs) are widely
62 used in practice by creating short horizontal drainage paths to accelerate the process
63 of soil consolidation [5, 8, 12, 22, 23, 39, 43, 45].

64 As a valuable factor to provide rational guidance on the effectiveness of soft ground

65 improvement, the degree of consolidation (DOC) evaluated by field settlement
66 measurement is widely used in the field. Nonetheless, the use of settlement
67 measurement to estimate DOC could be problematic [17, 33, 35, 41]. Chu and Yan [16]
68 indicated it is more suitable to predict DOC by utilizing pore pressure dissipation
69 instead of settlement measurement due to the existing post-construction settlement
70 during the primary consolidation. Currently, the pore pressure within the subsoil is
71 measured by piezometers or pore pressure transducers which can cause some
72 discrepancy [19].

73 During the PVD-assisted consolidation process, pore water can be squeezed out of
74 soil under surcharge loading or sucked out through vacuum preloading, flows into and
75 through PVDs, and then removed by pumps. Generally, the more water squeezed out
76 from soil, the more water removed from PVDs by pumps due to the high discharge
77 capacity of pumps. Thus, field and laboratory tests showed that soil consolidation
78 behavior is associated with the variation of time-dependent discharge rate within
79 PVDs [3, 14, 42]. According to aforementioned description of the relationship
80 between time-dependent discharge and consolidation behavior, the prediction of DOC
81 by adopting the time-dependent discharge rate around PVDs can be a potential
82 complementary approach to predict the effectiveness of soft ground improvement by
83 using settlement and pore pressure measurement.

84 A simplified solution is derived in this study to predict soil consolidation behavior
85 with respect to the discharge rate around PVDs. Considering the fact that an accurate,
86 time-dependent discharge rate of PVDs can be determined by laboratory experiment

87 and field monitoring, the proposed solution can be used to predict actual soil
88 consolidation behavior, then to instruct the geotechnical design and predict the
89 consolidation behavior during the construction. The proposed analytical solution is
90 verified by a comparison with a finite element model (FEM). At the end, this study
91 presents two case histories to demonstrate its practical prediction to the field
92 consolidation behavior. Furthermore, some actual design steps by using the proposed
93 method and design charts are presented for its practical uses.

94

95 **Consolidation Model**

96 **Governing equations**

97 Governing equations are established based on an axisymmetric model for the soil
98 consolidation with a single PVD penetrating the soil stratum completely shown in
99 Figure 1. The axisymmetric model is based on the following assumptions:

- 100 (1) The soil unit cell has a constant thickness and is assumed to be homogeneous and
101 saturated during the entire consolidation process.
- 102 (2) The seepage of pore water within the subsoil yields Darcy's law.
- 103 (3) The soil is subjected to a constant surcharge loading, which is applied
104 instantaneously. The initial excess pore water pressure (EPWP) is uniform along the
105 depth of the soil.
- 106 (4) The stress-strain relationship of the soil deposit is assumed to be linearly elastic
107 during the entire consolidation process.
- 108 (5) The small strain assumption is valid on the unit cell.

109 (6) Deformation occurs on the vertical direction only, and the horizontal displacement
110 is neglected. And the vertical deformation is caused by pore water pressure
111 dissipation.

112 (7) One-way drainage unit cell is assumed herein, namely pervious boundary for top
113 and impervious boundary for bottom of the unit cell.

114 (8) The soil deposit is fully saturated during the entire consolidation process with two
115 phases, namely liquid and solid phases.

116 Based on these assumptions, the governing equations of consolidation of soil are
117 presented as follows [2]:

$$118 \quad \frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} + C_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (1)$$

$$119 \quad C_v = \frac{k_v}{m_v \gamma_w}, C_h = \frac{k_h}{m_v \gamma_w} \quad (2)$$

120 where r and z are the radial and vertical coordinates, respectively; u represents
121 time-dependent EPWP; C_v and C_h are the vertical and horizontal coefficients of
122 consolidation, respectively; m_v denotes the volumetric compressibility, and k_v and k_h
123 are the vertical and horizontal hydraulic conductivity, respectively, **which can be**
124 **measured by laboratory or field tests [7, 40]**; γ_w is the unit weight of water; and t is the
125 elapsed time.

126 Orleach [36] concluded that vertical consolidation of soil can be neglected in most of
127 the cases with PVDs, for example, when the time factor $T_v/T_h < 0.02$ (i.e., $T_v = C_v t/h^2$,
128 $T_h = C_h t/d_e^2$, h and d_e are the thickness of soft soil and the effective influence diameter
129 of PVD, respectively. $d_e = 1.05d$ when PVDs are arranged in a triangular pattern or d_e

130 = $1.13d$ when PVDs are arranged in a square pattern. d denotes the spacing between
131 two adjacent PVDs). These parameters can be readily obtained in practice. For
132 example, if PVDs are arranged in a triangular pattern with a typical spacing of 1.0 m
133 ($d_e \approx 1.05$ m) and the thickness of soil deposit is larger than 10 m, and the C_h and
134 C_v values are assumed to be the same, then T_v/T_h would be less than 0.02. In most
135 practices, the PVD-assisted ground technique is used to improve thick soft soil ($h >$
136 10 m). Moreover the C_v/C_h value is normally less than 1.0 [37]. Hence, the scenario of
137 $T_v/T_h < 0.02$ is very common in actual practice and the vertical consolidation behavior
138 is neglected in this study.

139 In general, the strain condition is assumed to be free-strain or equal-strain condition to
140 solve a consolidation problem [6, 32, 44]. When flexible surcharge is applied on the
141 ground surface, it will result in differential settlement at the surface. For this scenario,
142 free-strain condition can be assumed. However, when rigid surcharge is applied on the
143 ground surface, the surface settlement will be uniform. For this scenario, equal-strain
144 condition can be assumed. To better stabilize the upper structure, the surface
145 settlement should remain uniform [46]. As a result, the equal settlement is a basic
146 requirement for PVD-assisted ground within the design scheme. According to
147 Orleach's conclusion and equal-strain assumption, Eq. (1) can be simplified as [24,
148 25]:

$$149 \quad \frac{\partial \bar{u}_r}{\partial t} = C_h \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} \right) \quad (3)$$

150 where \bar{u}_r and u_r are the average radial EPWP and virtual EPWP in the radial
151 direction, respectively.

152 Based on the equal-strain assumption, the vertical strain along the radial direction of
153 the soil is equal to its average value:

$$154 \quad \frac{\partial \varepsilon_v}{\partial t} = -\frac{1}{E_s} \frac{\partial \bar{u}_r}{\partial t} \quad (4)$$

155 where E_s denotes the constrained modulus of the soil; ε_v represents the volumetric
156 strain.

157 Eq. (4) is substituted into Eq. (3) after considering the smear effect to obtain the
158 following governing equations:

$$159 \quad -\frac{k_s}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) = \frac{\partial \varepsilon_v}{\partial t}, r_d \leq r \leq r_s \quad (5a)$$

$$160 \quad -\frac{k_h}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) = \frac{\partial \varepsilon_v}{\partial t}, r_s \leq r \leq r_e \quad (5b)$$

161 where k_s is the horizontal hydraulic conductivity of the smeared soil; r_d , r_s , r_e
162 represent the radii of the equivalent cylinder of PVD, the smeared zone, and the
163 effective influence zone of the PVD, respectively, as shown in Figure 1.

164

165 **Boundary and initial conditions**

166 Three boundary conditions are established at $r = r_d$, $r = r_s$, and $r = r_e$. $r = r_d$ stands for
167 the interface between the equivalent cylinder of PVD and the smeared soil. Note that
168 as vertical consolidation is not taken into account in the present work, the effect of
169 gravity on the discharge is ignored in the soil deposit and is taken account in the
170 PVDs when well resistance exists. The discharge develops consistently along the
171 length of the PVD during the consolidation process. Introducing the Dupuit
172 assumption [21], the EPWP gradient at $r = r_d$ can be obtained:

173
$$\left. \frac{\partial u_r}{\partial r} \right|_{r=r_d} = \frac{q(t)\gamma_w}{k_s A} = g(t) \quad (6)$$

174 where $q(t)$ is the time-dependent **discharge rate** around PVDs; $A = 2\pi r_d h$ is the
 175 wetted cross-section area; and $g(t)$ represents the time-dependent EPWP gradient at r
 176 $= r_d$.

177 $r = r_s$ stands for the interface between the smeared soil and the undisturbed soil. The
 178 continuity of the flow rate at $r = r_s$ can be described by:

179
$$k_s \left. \frac{\partial u_r}{\partial r} \right|_{r=r_s^-} = k_h \left. \frac{\partial u_r}{\partial r} \right|_{r=r_s^+} \quad (7)$$

180 $r = r_e$ stands for an impervious boundary, beyond which the EPWP is not influenced
 181 by the PVD. The impervious boundary condition at $r = r_e$ is described by:

182
$$\left. \frac{\partial u_r}{\partial r} \right|_{r=r_e} = 0 \quad (8)$$

183 Under the initial condition, it is assumed that the radial EPWP is equal to the constant
 184 initial average EPWP u_0 , which equals to the magnitude of instantaneous surface
 185 surcharge loading. This initial condition can be expressed as follows:

186
$$\bar{u}_r|_{t=0} = u_0 \quad (9)$$

187

188 **Solutions**

189 Double integrating Eq. (4) about r and introducing the boundary conditions expressed
 190 by Eqs. (6) to (8), the radial EPWP can be obtained as follows by assuming $c_s = k_s/k_h$:

191
$$u_r = u_r|_{r=r_s} - \frac{g(t)r_d}{r_e^2 - r_d^2} \left(r_e^2 \ln \frac{r_s}{r} - \frac{r_s^2 - r^2}{2} \right), r_d \leq r \leq r_s \quad (10a)$$

192
$$u_r = u_r|_{r=r_s} + \frac{g(t)r_d}{r_e^2 - r_d^2} c_s \left(r_e^2 \ln \frac{r}{r_s} - \frac{r^2 - r_s^2}{2} \right), r_s \leq r \leq r_e \quad (10b)$$

193 It can be seen that the EPWP at $r = r_s$ is the only unknown variable in Eq. (10). To
 194 obtain its expression, the average radial EPWP is introduced, which is defined as:

$$195 \quad \bar{u}_r = \frac{1}{\pi(r_e^2 - r_d^2)} \int_{r_d}^{r_e} 2\pi r u_r dr \quad (11)$$

196 Substituting Eq. (10) into Eq. (11), the average radial EPWP is obtained by integration,
 197 as presented in detail in Appendix A:

$$198 \quad \bar{u}_r = u_r|_{r=r_s} + \frac{2g(t)r_d s^2}{n^2 - 1} F_a \quad (12)$$

199 where

$$200 \quad F_a = f_n + f_s + f_0 \quad (13)$$

$$201 \quad f_n = \left[c_s \left(\frac{n^2}{2s^2} \ln \frac{n}{s} + \frac{4s^2 - 3n^2}{8s^2} \right) + \frac{\ln s}{2s^2} + \frac{1 - s^2}{4s^2} \right] \frac{n^2}{n^2 - 1} \quad (14)$$

$$202 \quad f_s = \frac{1}{8} (1 - c_s) \frac{s^2}{n^2 - 1} \quad (15)$$

$$203 \quad f_0 = \left(\frac{1}{8s^2} - \frac{1}{4} \right) \frac{1}{n^2 - 1} \quad (16)$$

204 where $n = r_e/r_d$, $s = r_s/r_d$.

205 Eq. (17) is obtained by integrating Eq. (5a) and (5b) on r .

$$206 \quad \frac{\partial u_r}{\partial r} = \frac{\gamma_w}{2k_s} \left(\frac{r_e^2}{r} - r \right) \frac{\partial \varepsilon_v}{\partial t}, r_d \leq r \leq r_s \quad (17a)$$

$$207 \quad \frac{\partial u_r}{\partial r} = \frac{\gamma_w}{2k_h} \left(\frac{r_e^2}{r} - r \right) \frac{\partial \varepsilon_v}{\partial t}, r_s \leq r \leq r_e \quad (17b)$$

208 By substituting $r = r_d$ into Eq. (17), the following equation is obtained based on Eqs.
 209 (4) and (6):

$$210 \quad \frac{\partial \bar{u}_r}{\partial t} = -E_s \frac{2g(t)k_s}{\gamma_w r_d (n^2 - 1)} \quad (18)$$

211 Integrating Eq. (18) on t and introducing the initial condition of Eq. (9), the average

212 EPWP can be expressed as:

$$213 \quad \bar{u}_r = u_0 - E_s \int_0^t \frac{2g(t)k_s}{\gamma_w r_d (n^2 - 1)} dt \quad (19)$$

214 Substituting Eq. (19) into Eq. (12), the EPWP at $r = r_s$ is obtained:

$$215 \quad u_r|_{r=r_s} = u_0 - E_s \int_0^t \frac{2g(t)k_s}{\gamma_w r_d (n^2 - 1)} dt - \frac{2g(t)r_d s^2}{n^2 - 1} F_a \quad (20)$$

216 Substituting Eq. (20) to Eq. (10), the solutions for the radial EPWP at any arbitrary
217 radial distance are obtained:

$$218 \quad u_r = u_0 - F_c k_s \int_0^t g(t) dt - g(t) r_d [F_s + F(R)], r_d \leq r \leq r_s \quad (21a)$$

$$219 \quad u_r = u_0 - F_c k_s \int_0^t g(t) dt - g(t) r_d [F_s + c_s F(R)], r_s \leq r \leq r_e \quad (21b)$$

$$220 \quad F_c = \frac{2E_s}{\gamma_w r_d (n^2 - 1)} \quad (22)$$

$$221 \quad F_s = \frac{2s^2 F_a}{n^2 - 1} \quad (23)$$

$$222 \quad R = \frac{r}{r_d} \quad (24)$$

$$223 \quad F(R) = \frac{n^2}{n^2 - 1} \ln \frac{s}{R} - \frac{s^2 - R^2}{2(n^2 - 1)} \quad (25)$$

224 Based on the definition of DOC, Eq. (19) can be used to calculate the DOC:

$$225 \quad U = 1 - \frac{\bar{u}_r}{u_0} = \frac{F_c k_s}{u_0} \int_0^t g(t) dt \quad (26)$$

226 where U is the DOC.

227 The solution for the EPWP without considering the smear effect can be obtained by
228 assuming $r_s = r_e$ in Eq. (21a) or assuming $r_s = r_d$ in Eq. (21b), and $k_s = k_h$ for both
229 solutions. The DOC without a smear effect can be obtained by the aforementioned
230 process.

231 As shown in the above procedure of derivation, the final solution has been
232 significantly simplified; therefore, it can be used conveniently. In addition to typical
233 parameters required for conventional consolidation theories, the time-dependent
234 **discharge rate** around PVDs is required to estimate the consolidation behavior. As
235 expected, the time-dependent **discharge rate** in PVDs is affected by the lateral stress
236 around PVDs. There are various equipments reported in previous literature [3, 10]
237 used to simulate the field stress state and then measure the **discharge rate** in PVDs in
238 laboratory scale. According to the proposed solution incorporating the available
239 measurement of time-dependent **discharge rate** in laboratory, the consolidation curve
240 can be determined to instruct the geotechnical design of associated projects. The
241 detailed procedure is presented in the section of discussion. Additionally, the proposed
242 solution can also be used to predict the real-time development of EPWP and
243 settlement by measuring directly and accurately with the assistance of some gauges
244 (e.g., Groundwater Flowmeter) installed in the PVDs.

245

246 **Verification**

247 **Comparison with numerical simulation**

248 Figure 2 shows the numerical model established with ABAQUS. The yellow area is
249 the undisturbed soil, and the gray area is the smeared zone. The radius of the smeared
250 zone is assumed as $2r_d$ [26]. An elastic constitutive model is used in the numerical
251 analysis. Table 1 shows the properties of the soft soil according to Rixner et al [37].
252 As the vertical consolidation is not considered in this study, the gravity is not applied

253 on the numerical model. Thus, the flow rate in the numerical simulation around the
254 nodes of the PVD remains consistent along the length of the PVD. To better display
255 the numerical model, the thickness of the soil is set to be 50 cm ($h = 50$ cm). The
256 influence radius r_e equals to 20 times of the equivalent radius of the PVD (i.e., r_d) [13]
257 which is defined according to one of the most commonly-used band-shaped drainage
258 boards as [24, 25]:

$$259 \quad r_d = \frac{a + b}{\pi} \quad (27)$$

260 where a and b are the width and thickness of the band-shaped drainage board and the
261 geometric dimension of the most commonly-used band-shaped drainage board is 100
262 mm \times 4 mm.

263 Impervious boundaries are created on the outer surfaces of the numerical model as
264 shown in Figure 2. A 100 kPa instantaneous load is applied on the upper surface of the
265 numerical model. Note that the discharge rate around PVDs is unknown during the
266 consolidation process. If a specific time-dependent discharge rate is applied on the
267 surface of the PVD, it is difficult for the numerical model to converge due to the
268 specific time-dependent discharge rate is related to many factors, such as lateral
269 pressure, hydraulic conductivity in smeared zone and the permeability at the interface
270 between PVDs and soil, and normally the development of discharge rate may not be
271 applied on numerical model directly. Therefore, to obtain the time-dependent
272 discharge rate and verify the proposed solution, the EPWP around PVDs is assumed
273 to be 0 kPa during the simulation of the consolidation process and the flow rate at the
274 node of PVD can be recorded in calculation.

275 Figure 3 shows the development of the settlement versus the radial distance of the
276 measuring point from the center of the PVD. It is shown that the settlements at 30, 60,
277 and 100 days are approximately uniform at different distances from the center of the
278 PVD. This result implies that the equal-strain assumption is suitable for the
279 consolidation of soil with PVDs.

280 Figure 4 shows the variation of flow rate around PVDs with time. The flow rate
281 decreases exponentially with time. Substituting these rates into Eq. (21) yields the
282 development of EPWP in the radial direction. Figure 5 shows the comparison between
283 the simulated and calculated normalized EPWP in the radial direction by FEM and the
284 solution proposed in this study, respectively. Good agreements are obtained between
285 the simulated and calculated results for the model. Figure 5 shows that the EPWP
286 increases with the increase of distance from the center of the PVD. With an increase
287 of the time, the difference in the EPWP from the center to a certain distance becomes
288 negligible. Also, the difference between the simulated and calculated results decreases.
289 From Figure 5, it can be concluded that the proposed solution can well predict the
290 variation of EPWP at different distances from the center of the PVD during the entire
291 process of consolidation.

292 To obtain the DOC by the numerical simulation, the settlement at 100 years is first
293 simulated by the numerical model and considered as the ultimate settlement of the
294 model. The simulated DOC of the model at any time can be calculated by:

295
$$U = \frac{s}{s_c} \quad (28)$$

296 where s and s_c are the settlement at any time and the ultimate settlement, respectively.

297 Figure 6 compares the simulated and calculated DOC, which are in good agreement. It
298 can be observed that the proposed solution slightly underestimated the DOC as
299 compared with the simulated result at the initial time. Their difference becomes
300 smaller with an increase of the time. The overall relative error does not exceed 10%.
301 This comparison also proves that the proposed solution can well predict the
302 consolidation behavior of the soft soil as that in the FEM model.

303

304 **Comparison with field tests**

305 Case A

306 A well-documented case history involving a fill embankment at the Saga Airport in
307 Japan was reported by Chai et al [9]. This airport was constructed on a reclaimed land
308 close to the Ariake Bay. The deposit consists of a weathered crust, a sand layer and
309 soft and highly-compressible clay. Table 2 presents the thicknesses and properties of
310 the soil strata.

311 PVDs were installed at a depth of approximately 25 m in a square pattern with a
312 spacing of 1.5 m. The cross-sectional dimension of each PVD is 100 mm × 4 mm. The
313 fill was placed at a rate of approximately 0.03 m/day for multiple stages, which can be
314 modeled as multi-ramp loading. The first filling lasted 18 days. After 72-day
315 suspension, the second filling took 116 days. When the fill height reached 3.5 m, the
316 filling stopped for 194 days. The unit weight of the fill material was 20 kN/m³, and
317 the final applied pressure on the ground surface was 70 kPa. Figure 7 shows the cross
318 section of the embankment and the instrumentation locations in the field. As shown in

319 Table 2, AC2 was the thickest soil layer in this cross section and would dominate the
 320 consolidation process. Moreover, the PVDs penetrated the AC2 layer completely. To
 321 simplify the calculation, only the AC2 layer was considered in this study.

322 Figure 8 shows the flow rate around the PVD with time. **The time-dependent flow rate**
 323 **was obtained by physical modeling tests in the laboratory to simulate the field**
 324 **condition.** This figure illustrates that the time-dependent flow rate decreased
 325 exponentially with time. **The fitting relationship between the flow rate of the PVD and**
 326 **time was obtained by the regression method as $v = 0.0282\exp(-0.0159 \times t)$ ($R^2 =$**
 327 **0.9777).** The DOC under instantaneous loading can be obtained by substituting the
 328 **fitting curve expression into Eq. (26).**

329 The construction of the embankment could be divided into two stages, which were
 330 modeled as surcharge loading on the ground surface in two stages. The Terzaghi
 331 equation can be modified to calculate the DOC in each stage as follows [18]:

$$332 \quad U_t = \sum_1^n U_{(t - \frac{t_n + t_{n+1}}{2})} \frac{\Delta p_n}{\sum \Delta p} \quad (29)$$

333 where U_t is the modified DOC; t_{n-1} and t_n are the starting and ending time of each
 334 stage under a constant-speed loading process, respectively. Since two stages of
 335 surcharge loading were used in this case history, the modified DOC can be calculated
 336 as follows:

$$337 \quad \text{when } 0 < t < t_1 \quad U_t = U_{(\frac{t}{2})} \frac{\Delta p'}{\sum \Delta p} \quad (30)$$

$$338 \quad \text{when } t_1 < t < t_2 \quad U_t = U_{(t - \frac{t_1}{2})} \frac{\Delta p_1}{\sum \Delta p} \quad (31)$$

339 when $t_2 < t < t_3$
$$U_t = U_{\left(t-\frac{t_1}{2}\right)} \frac{\Delta p_1}{\sum \Delta p} + U_{\left(\frac{t-t_1}{2}\right)} \frac{\Delta p''}{\sum \Delta p} \quad (32)$$

340 when $t_3 < t < t_4$
$$U_t = U_{\left(t-\frac{t_1}{2}\right)} \frac{\Delta p_1}{\sum \Delta p} + U_{\left(\frac{t-t_2+t_3}{2}\right)} \frac{\Delta p''}{\sum \Delta p} \quad (33)$$

341 where $\Delta p'$ and $\Delta p''$ are the loading increments that correspond to the first and second
 342 stages of loading, respectively.

343 Figure 9 illustrates the comparison on settlement and EPWP calculated by proposed
 344 solution with field measurement and the prediction using other solutions. The
 345 Hansbo's consolidation theory [25] is one of the most commonly-used solutions in the
 346 practice, which is simplified with reasonable assumptions. Deng et al [20] proposed a
 347 rigorous analytical solution for consolidation of soil with PVDs at a changing drain
 348 resistance. The calculated settlement was computed using Eq. (28), in which the
 349 ultimate settlement was determined according to the measured data from the field.
 350 Table 2 and Figure 9 show the associated soil parameters for the AC2 layer which is
 351 required in Eqs. (21) to (26). Both the Hansbo's solution and Deng et al's solution
 352 were applied herein by using the same modified Terzaghi method as used in proposed
 353 solution for stage loading. Chai and Miura [10] used FEM to simulate the entire
 354 process of surcharge loading for this project and the EPWP was obtained. It can be
 355 illustrated from Figure 9 that the settlement increased rapidly during the surcharge
 356 loading. The increase rate of the settlement became slow after the completion of fill
 357 loading. It can be seen from Figure 9 that the settlement obtained by proposed
 358 solution is consistent with field measurement. Thus it can be concluded that the
 359 proposed solution can accurately predict the settlement.

360 By comparing the settlement results with other solutions, it illustrated that the
361 proposed solution is identical with Deng et al's solution and numerical simulation
362 results by Chai and Miura [10]. Due to high simplification and limitation of Hansbo's
363 consolidation solution, the settlement results obtained by Hansbo's consolidation
364 solution exhibit some discrepancy compared with field measurement and other
365 solutions. Deng et al's solution assumes the varying well resistance in PVDs which
366 can predict the development of settlement with sufficient accuracy, whereas the
367 varying well resistance is expressed with an approximated formula by regression
368 summarized by laboratory experimental results. As a result, the overall development
369 of settlement deviates from the field measurement with acceptable error. Moreover,
370 similar to other rigorous analytical solutions, Deng et al's solution is difficult to apply
371 by geotechnical engineers due to its complexity of the equations and required
372 parameters. The numerical simulation results by FEM are close to the proposed
373 solution and field measurement. However, according to the complexity of field project,
374 the modeling process is complicated and time-consuming. By comparing with other
375 solutions, it can be concluded that the proposed solution can be an alternative with
376 acceptable accuracy and simplification in predicting the development of settlement.

377 Figure 9 also shows the comparison of EPWP with other solutions. The EPWP was
378 measured at the location of 5 m away from the center line of PVDs and 6 m below the
379 ground surface. Although the calculated EPWP by proposed solution was lower than
380 that the measured one, the distributions of the EPWP obtained by the proposed
381 solution and measurement are similar. This difference may result from the

382 inhomogeneous soil layer and possible measurement errors in the field.
383 Even though it is quite different from the measured value, the EPWP calculated by the
384 proposed solution is in good agreement with those by the rigorous solution and the
385 FEM before the surcharge load reaches the maximum. The above discussion
386 demonstrates that the proposed solution, even though is highly simplified, can be
387 conveniently used to predict the consolidation behavior of soft soil with PVDs for
388 field applications with good accuracy.

389

390 Case B

391 Chu and Yan [16] reported a road embankment built on soft soil deposit at Tianjin
392 Port in China. The thickness of the soil deposit was approximately 20 m. The soft clay
393 at the depth of 5 to 6 m was reclaimed recently using clay slurry dredged from seabed.
394 The clay below the reclaimed layer is original seabed clay. To rapidly increase the
395 strength of the soil layer, the vacuum preloading technique was utilized. Field
396 implementation was used in two sections (namely Section I and Section II). The
397 details of the soil properties and construction procedure are well documented in the
398 literature [15]. Figure 10 shows the schematic diagram of this project, which consists
399 of a road section of 364.5 m long and 51 m wide. A vacuum pressure of 80 kPa was
400 applied continuously for 90 days to compress the soft soil deposit.

401 The vacuum pressure increasing from 0 to 80 kPa in a very short period could be
402 considered as instantaneous and uniform loading. The Hansbo's solution could not be
403 applied in this case history as Hansbo's consolidation theory is based on the

404 assumption of instantaneous and uniform fill loading subjected on the subsoil.
405 However, the time-dependent **discharge rate** around PVDs increases with the
406 existence of vacuum pressure. In the absence of time-dependent measurement of
407 discharge in PVDs, back analysis was carried out to evaluate the development of
408 discharge. According to the EPWP measurement, the development of **discharge rate**
409 was obtained by utilizing Eq. (21). Then the obtained time-dependent **discharge rate**
410 was substituted into Eq. (26) to calculate the DOC with time. The real-time settlement
411 was estimated based on the ultimate settlement calculated by the Asaoka method
412 using the monitored settlement data and the DOC with time.

413 Figure 11 shows the settlement and the EPWP reduction calculated by the proposed
414 solution as compared with the measured results in Section I and II. Even though there
415 are some differences, the proposed solution reasonably predicted the settlement and
416 the EPWP reduction in this project. As the ultimate settlements measured in different
417 sections are different (the ultimate settlements are 1.0 m in Section I and 1.2 m in
418 Section II, respectively), the development of settlement calculated by proposed
419 solution is different. And the soil profile is basically the same in both sections, and
420 thus the calculated EPWP in different sections is on the same curve. Due to some
421 novel ground treatment methods are proposed during the recent decades, some
422 conventional analytical solutions may not be adaptive in these new techniques.

423 However, when the time-dependent **discharge rate** is available and accurate, the
424 precise prediction can be achieved by utilizing the proposed solution. As the
425 prediction by proposed solution is consistent with field measurement, it can be

426 concluded that the proposed solution is a potential alternative in geotechnical design
427 of PVD-assisted ground.

428

429 **Discussion**

430 To apply the proposed solution to field implementation and monitoring, a parametric
431 analysis should be conducted to obtain the DOC and time factor curve. Then the
432 design charts are prepared to make reasonable design and instruct field construction.

433 According to Deng et al. [19], the **discharge rate** around PVDs can be expressed as
434 follows:

$$435 \quad q(t) = q_{w0} \exp(-A_0 t) \quad (34)$$

436 where q_{w0} is the initial value or short-term value of the **discharge rate** around PVDs;
437 A_0 is the coefficient with respect to the time-dependent **discharge rate**.

438 DOC is one of the critical parameters for design of an entire consolidation process.

439 The DOC can be expressed as follows by incorporating Eqs. (6) (26) (34) and then
440 integrating the incorporated formula.

$$441 \quad U = \frac{F_c}{u_0} \frac{q_{w0} \gamma_w}{A} \frac{1}{A_0} [1 - \exp(-A_0 t)] \quad (35)$$

442 Generally, the expression for DOC is comprised with the time factor T_h ($T_h = C_h t / d_e^2$).

443 Thus to analyze the influence of coefficient A_0 , a dimensionless factor α is introduced
444 herein and expressed as follows:

$$445 \quad \alpha = A_0 \frac{d_e^2}{C_h} \quad (36)$$

446 When time becomes infinite, the DOC is 100%. Thus, the DOC can be expressed as a

447 well-known formula, which is show as follows:

$$448 \quad U = 1 - \exp(-\alpha T_h) \quad (37)$$

449 Figure 12 illustrates the development of DOC with the time factor. To obtain the time
450 needed for a specific DOC, the dimensionless factor α is required to be determined.

451 Based on the fact that the DOC is equal to 100% when time is infinite, the
452 dimensionless factor α can be derived as follows:

$$453 \quad \alpha = \frac{4n^2 q_{w0} \gamma_w}{\pi k_h (n^2 - 1) u_0 h} \approx \frac{4q_{w0} \gamma_w}{\pi k_h u_0 h} \quad (38)$$

454 Under field conditions, the parameters q_{w0} and k_h are often variable due to the nature
455 of soil deposit and measurement errors. Their variability should be considered in
456 design. The mean horizontal hydraulic conductivity is assumed to be 2×10^{-8} m/s here
457 according to the literature [14], and the coefficient of variation is assumed to be 0.5.
458 The mean q_{w0} is assumed to be 3×10^{-8} m³/s, and its coefficient of variation is
459 assumed to be 0.5 [28–30, 49]. These two parameters are assumed to have Gamma
460 distributions. Figure 13 and Figure 14 show the ranges of the dimensionless factor α .

461 Obviously, if there are more detailed measurements of aforementioned parameters
462 determined by field and laboratory tests, the distribution of the dimensionless factor α
463 may be more rational to instruct the geotechnical design. In practical process without
464 available parameters, an arrangement of PVDs is first assumed and the uncertain
465 parameters are assumed to be distributed in a typical statistical model based on the
466 mean value and standard deviation summarized in previous references. Then a most
467 possible factor α can be determined according to the density of scatter points(e.g.
468 Figure 13 and 14). Simultaneously, the consolidation curve can be determined when

469 the factor α is obtained. According the obtained consolidation curve, the target of
470 consolidation can be estimated to optimize the arrangement of PVDs by repeating
471 aforementioned process If the curve for the deterministic dimensionless factor is not
472 available in Figure 12, a linear interpolation could be conducted to determine the
473 curve, which will be used to determine the time factor T_h required for the target DOC.
474 According the obtained consolidation curve, the target of consolidation can be
475 estimated to optimize the arrangement of PVDs by repeating aforementioned process.

476 In summary, to apply the proposed solution in this study, the following framework is
477 suggested:

478 **Step 1:** Define the target DOC at a certain time period based on project requirements.

479 **Step 2:** Select a PVD pattern and improvement approach (e.g., fill preloading or
480 vacuum preloading) and determine a loading procedure (e.g., single or staged loading).

481 **Step 3:** Measure relevant parameters by field surveying and laboratory tests,
482 including the geometric features of soil and PVD properties, for instance, the initial or
483 short-term value of the discharge rate around PVDs, the geometric dimension of PVD,
484 the hydraulic conductivity of soil, and the thickness of the soil stratum.

485 **Step 4:** Based on Eq. (38), the factor α is obtained to determine the associated DOC –
486 time factor curve according to Figure 12. If some properties cannot be determined
487 accurately, such as the hydraulic conductivity of soil, a specific range of α can be
488 determined according to the obtained initial or short-term discharge rate around PVDs
489 from Figure 13 and Figure 14, and then assume a most possible factor α according to
490 the density of scatter points.

491 **Step 5:** Based on the obtained specific factor α , the associated DOC – time factor
492 curve is used to calculate the dimensionless time factor T_h for the soil deposit to reach
493 the target DOC. If the obtained factor α is not available in Figure 12, linear
494 interpolation should be carried out to obtain the required DOC – time factor curve.
495 Find the corresponding time factor under the target DOC and then determine PVD
496 spacing.

497 **Step 6:** In the field, the real-time **discharge rate** around PVDs, EPWP within the soil
498 deposit and settlement are monitored to assess the actual consolidation behavior. The
499 development of the consolidation process can be predicted by substituting the
500 time-dependent **discharge rate** into the proposed solution. The proposed solution
501 based on the **discharge rate** around PVDs can be a complementary approach to better
502 describe the consolidation behavior. If the trend of the predicted consolidation process
503 cannot reach the target, modify the loading process to redesign the consolidation
504 process.

505

506 **Conclusions**

507 This study presents a practical solution for the consolidation of soft soil deposit with a
508 single PVD based on the discharge boundary condition around PVDs under
509 equal-strain assumption. The solution can predict EPWP and DOC of soft soil
510 improved with PVDs. The comparisons of the proposed solutions with numerical
511 simulation and measured data in two case histories verified the applicability and
512 accuracy of the proposed solutions. Following conclusions can be made from this

513 study:

514 (1) An axisymmetric consolidation solution based on the discharge boundary
515 condition around PVDs under equal-strain assumption is proposed in this study. The
516 prediction of the EPWP and the DOC can be determined by substituting the
517 time-dependent **discharge rate** around PVDs into the proposed analytical solution.

518 (2) Numerical simulation is performed by FEM to simulate the consolidation behavior
519 of a soft soil deposit with a single PVD. As the discharge around PVDs remains
520 unknown during the entire consolidation process, the idealized consolidation behavior
521 without well resistance was simulated to determine the time-dependent **discharge rate**
522 around PVDs. The relationship between the **discharge rate** around PVDs and the
523 consolidation time obtained by FEM simulation is substituted into the proposed
524 analytical solution. The results show that the DOC and EPWP calculated by the
525 proposed solution are in good agreement with the simulated results.

526 (3) Two case histories were used to evaluate the proposed analytical solution. The
527 comparisons show that the settlement-time curve obtained by the proposed solution is
528 in good agreement with the measured data. The proposed solution can be applied in
529 various kinds of soft ground improvement techniques, including surcharging loading
530 and vacuum preloading, with available time-dependent **discharge rate** around PVDs.

531 (4) Design charts for the relationship between the initial short-time **discharge rate** and
532 the dimensionless factor α are developed to help choose the range of the
533 dimensionless factor α and the most possible dimensionless factor α , which can be
534 used to determine the consolidation curve. Based on the target DOC and obtained the

535 consolidation curve, the required design parameters can be determined. On the other
 536 hand, the real-time consolidation behavior can be predicted based on proposed
 537 solution by utilizing the field discharge measurement monitored by gauges. The
 538 proposed solution can be a potential complementary approach to the prediction of
 539 consolidation behavior by utilizing settlement and pore pressure measurement in field.

540

541 **Acknowledgments**

542 This study was sponsored by the Key R&D project of Zhejiang Province
 543 (2017C03006) and Zhejiang Provincial National Science Foundation of China
 544 (LQ17D020001).

545

546 **Appendix A**

547 Assume $n = r_e/r_d$, $s = r_s/r_d$ and $R = r/r_d$. The average radial EPWP can be obtained by
 548 combining Eqs. (10) and (11) as follows:

$$549 \quad \bar{u}_r = u_r|_{r=s} + 2r_d \left[\int_1^s -\frac{g(t)R}{(n^2-1)^2} \left(n^2 \ln \frac{s}{R} - \frac{s^2-R^2}{2} \right) dR + \int_s^n \frac{g(t)R}{(n^2-1)^2} c_s \left(n^2 \ln \frac{R}{s} - \frac{R^2-s^2}{2} \right) dR \right] \quad (A1)$$

550 The second part of Eq. (A1) can be rewritten as Eqs. (A2) and (A3) as follows:

$$551 \quad 2r_d \int_1^s -\frac{g(t)R}{(n^2-1)^2} \left(n^2 \ln \frac{s}{R} - \frac{s^2-R^2}{2} \right) dR \quad (A2)$$

$$552 \quad 2r_d \int_s^n \frac{g(t)R}{(n^2-1)^2} c_s \left(n^2 \ln \frac{R}{s} - \frac{R^2-s^2}{2} \right) dR \quad (A3)$$

553 Eq. (A4) can be obtained by integrating Eq. (A2) as follows:

$$554 \quad -\frac{2g(t)r_d}{(n^2-1)^2} \int_1^s R \left(n^2 \ln \frac{s}{R} - \frac{s^2-R^2}{2} \right) dR \quad (A4)$$

$$555 \quad -\frac{2g(t)r_d}{(n^2-1)^2} \int_1^s [n^2(\ln s - \ln R)R - \frac{s^2 - R^2}{2} R] dR \quad (A5)$$

$$556 \quad -\frac{2g(t)r_d}{(n^2-1)^2} \left[\frac{n^2(s^2-1)}{2} \ln s - n^2 \left(\frac{s^2}{2} \ln s - \frac{s^2}{4} + \frac{1}{4} \right) - \frac{s^2}{2} \frac{s^2-1}{2} + \frac{s^4-1}{8} \right] \quad (A6)$$

$$557 \quad -\frac{2g(t)r_d}{(n^2-1)^2} \left(-\frac{n^2}{2} \ln s + n^2 \frac{s^2-1}{4} - \frac{s^4-s^2}{4} + \frac{s^4-1}{8} \right) \quad (A7)$$

$$558 \quad \frac{g(t)r_d}{(n^2-1)^2} \left[n^2 \ln s - \frac{n^2}{2} (s^2-1) + \frac{(s^2-1)^2}{4} \right] \quad (A8)$$

559 Eq. (A9) can be obtained by integrating Eq. (A3) as follows:

$$560 \quad \frac{2g(t)r_d}{(n^2-1)^2} c_s \int_s^n R \left(n^2 \ln \frac{R}{s} - \frac{R^2 - s^2}{2} \right) dR \quad (A9)$$

$$561 \quad \frac{2g(t)r_d}{(n^2-1)^2} c_s \int_s^n \left[n^2(\ln R - \ln s)R - \frac{R^2 - s^2}{2} R \right] dR \quad (A10)$$

$$562 \quad \frac{2g(t)r_d}{(n^2-1)^2} c_s \left[n^2 \left(\frac{n^2}{2} \ln n - \frac{n^2}{4} - \frac{s^2}{2} \ln s + \frac{s^2}{4} \right) - \frac{n^2(n^2 - s^2)}{2} \ln s - \frac{n^4 - s^4}{8} + \frac{s^2}{2} \frac{n^2 - s^2}{2} \right] \quad (A11)$$

$$563 \quad \frac{2g(t)r_d}{(n^2-1)^2} c_s \left(\frac{n^2 s^2}{2} + \frac{n^4}{2} \ln \frac{n}{s} - \frac{3n^4 + s^4}{8} \right) \quad (A12)$$

$$564 \quad \frac{g(t)r_d}{(n^2-1)^2} c_s \left(n^4 \ln \frac{n}{s} - \frac{3n^4 + s^4 - 4n^2 s^2}{4} \right) \quad (A13)$$

565 The average radial EPWP can be obtained by combining Eqs. (A1), (A8), and (A13)

566 as follows:

$$567 \quad \bar{u}_r = u_r \Big|_{r=r_s} + \frac{2g(t)r_d s^2}{n^2-1} F_a \quad (A14)$$

568 where

$$569 \quad F_a = f_n + f_s + f_0 \quad (A15)$$

$$570 \quad f_n = \left[c_s \left(\frac{n^2}{2s^2} \ln \frac{n}{s} + \frac{4s^2 - 3n^2}{8s^2} \right) + \frac{\ln s}{2s^2} + \frac{1-s^2}{4s^2} \right] \frac{n^2}{n^2-1} \quad (A16)$$

571
$$f_s = \frac{1}{8}(1 - c_s) \frac{s^2}{n^2 - 1} \quad (\text{A17})$$

572
$$f_0 = \left(\frac{1}{8s^2} - \frac{1}{4} \right) \frac{1}{n^2 - 1} \quad (\text{A18})$$

573

574 **References**

575

576 [1] Aoyama K, Yoshida T, Kamitani T. An alternative of soil scarification treatment
577 for forest restoration: effects of soil replacement. J Forest Res-Jpn
578 2017;14(1):58-62.

579 [2] Barron RA. Consolidation of fine-grained soils by drain wells. Transactions of the
580 American Society of Civil Engineers 1948;113(1):718-742.

581 [3] Bo MW. Discharge capacity of prefabricated vertical drain and their field
582 measurements. Geotext Geomembranes 2004;22(1-2):37-48.

583 [4] Cai Y, Qiao H, Wang J, Geng X, Wang P, Cai Y. Experimental tests on effect of
584 deformed prefabricated vertical drains in dredged soil on consolidation via
585 vacuum preloading. Eng Geol 2017;222:10-19.

586 [5] Cai Y, Qiao H, Wang J, Geng X, Wang P, Cai Y. Experimental tests on effect of
587 deformed prefabricated vertical drains in dredged soil on consolidation via
588 vacuum preloading. Eng Geol 2017;222:10-19.

589 [6] Cargill KW. Prediction of consolidation of very soft soil. Journal of Geotechnical
590 Engineering 1984;110(6):775-795.

591 [7] Chai J, Agung PMA, Hino T, Igaya Y, Carter, JP. Estimating hydraulic
592 conductivity from piezocone soundings. Géotechnique 2011;61(8):699-708.

- 593 [8] Chai J, Bergado DT, Shen S. Modelling prefabricated vertical drain improved
594 ground in plane strain analysis. *Ground Improvement, ICE Proceedings*
595 2013;166(GI2):65-77.
- 596 [9] Chai J, Miura N, Sakajo S, Bergado D. Behavior of vertical drain improved
597 subsoil under embankment loading. *Soils Found* 1995;35(4):49-61.
- 598 [10]Chai J, Miura N. Investigation of factors affecting vertical drain behavior. *J*
599 *Geotech Geoenviron* 1999;125(3):216-226.
- 600 [11]Chai J, Shen S, Miura N, Bergado DT. Simple method of modeling
601 PVD-improved subsoil. *J Geotech Geoenviron* 2001;127(11):965-972.
- 602 [12]Chen J, Shen S, Yin Z, Xu Y, Horpibulsuk S. Evaluation of effective depth of
603 PVD improvement in soft clay deposit: a field case study. *Marine Georesources*
604 *and Geotechnolgy* 2016;34(5):420-430.
- 605 [13]China Academy of Building Research. Chinese technical code for ground
606 improvement for buildings (JGJ 79-2002). China Architecture and Building Press
607 Beijing; 2002.
- 608 [14]Chu J, Bo MW, Chang MF, Choa V. Consolidation and permeability properties of
609 Singapore marine clay. *J Geotech Geoenviron* 2002;128(9):724-732.
- 610 [15]Chu J, Yan S, Yang H. Soil improvement by the vacuum preloading method for an
611 oil storage station. *Géotechnique* 2000;50(6):625-632.
- 612 [16]Chu J, Yan SW. Estimation of degree of consolidation for vacuum preloading
613 projects. *Int J Geomech* 2005;5(2):158-165.
- 614 [17]Cosenza P, Korošak D. Secondary consolidation of clay as an anomalous

615 diffusion process. *Int J Numer Anal Met* 2014;38(12):1231-1246.

616 [18]Das BM. *Advanced soil mechanics*. New York: CRC Press; 2013.

617 [19]Deng Y, Liu G, Lu M, Xie K. Consolidation behavior of soft deposits considering
618 the variation of prefabricated vertical drain discharge capacity. *Comput Geotech*
619 2014;62:310-316.

620 [20]Deng Y, Xie K, Lu M. Consolidation by vertical drains when the discharge
621 capacity varies with depth and time. *Comput Geotech* 2013;48:1-8.

622 [21]Dupuit J. *Etudes Théoriques et Pratiques sur le mouvement des Eaux dans les*
623 *canaux découverts et à travers les terrains perméables*. Paris: Dunod; 1983.

624 [22]Fu H, Fang Z, Wang J, Chai J, Cai Y, Geng X, et al. Experimental comparison of
625 electroosmotic consolidation of Wenzhou dredged clay sediment using
626 intermittent current and polarity reversal. *Mar Georesour Geotec*
627 2018;36(1):131-138.

628 [23]Geng X, Yu HS. A large-strain radial consolidation theory for soft clays improved
629 by vertical drains. *Géotechnique* 2017;67(11):1020-1028.

630 [24]Hansbo S, Jamiolkowski M, Kok L. Consolidation by vertical drains.
631 *Géotechnique* 1981;31(1):45-66.

632 [25]Hansbo S. Consolidation of clay by band-shaped prefabricated drains. *Ground*
633 *Engineering* 1979;12(5):16-25.

634 [26]Hansbo S. *Foundation engineering*: Newnes; 1994.

635 [27]Hegde A, Sitharam TG. Experiment and 3D-numerical studies on soft clay bed
636 reinforced with different types of cellular confinement systems. *Transportation*

- 637 Geotechnics 2017;10:73-84.
- 638 [28]Hong HP, Shang JQ. Probabilistic analysis of consolidation with prefabricated
639 vertical drains for soil improvement. *Can Geotech J* 1998;35(4):666-677.
- 640 [29]Hong HP. An efficient point estimate method for probabilistic analysis. *Reliability
641 Engineering and System Safety* 1998;59(3):261-267.
- 642 [30]Hong HP. One-dimensional consolidation with uncertain properties. *Can Geotech
643 J* 1992;29(1):161-165.
- 644 [31]Khemissa M, Mekki L, Mahamedi A. Laboratory investigation on the behaviour
645 of an overconsolidated expansive clay in intact and compacted states.
646 *Transportation Geotechnics* 2018;14:157-168.
- 647 [32]Leo CJ. Equal strain consolidation by vertical drains. *J Geotech Geoenviron
648* 2004;130(3):316-327.
- 649 [33]Li AL, Rowe K. Effects of viscous behavior of geosynthetic reinforcement and
650 foundation soils on the performance of reinforced embankments. *Geotext
651 Geomembranes* 2008;26(4):317-334.
- 652 [34]Ma L, Shen S, Luo C, Xu Y. Field evaluation on strength increase of soft
653 structured clay under stage-constructed embankment. *Marine Georesources and
654 Geotechnology* 2011;29(4):317-332.
- 655 [35]Ni P, Xu K, Mei G, Zhao Y. Effect of vacuum removal on consolidation
656 settlement under a combined vacuum and surcharge preloading. *Geotext
657 Geomembranes* 2019;47(1):12-22.
- 658 [36]Orleach P. Techniques to evaluate the field performance of vertical drains:

659 Massachusetts Institute of Technology; 1983.

660 [37]Rixner JJ, Kraemer SR, Smith AD. Prefabricated vertical drains: Engineering
661 guidelines. Washington, D.C.: Federal Highway Administration; 1986.

662 [38]Rondonuwu SG, Chai J, Cai Y, Wang J. Prediction of the stress state and
663 deformation of soil deposit under vacuum pressure. *Transportation Geotechnics*
664 2016;6:75-83.

665 [39]Shen S, Chai J, Hong Z, Cai F. Analysis of field performance of embankments on
666 soft clay deposit with and without PVD-improvement. *Geotextiles and*
667 *Geomembranes* 2005;23(6):463-485.

668 [40]Shen S, Wang J, Wu H, Xu Y, Ye G, Yin Z. Evaluation of hydraulic conductivity
669 for both marine and deltaic deposits based on piezocone testing. *Ocean*
670 *Engineering* 2015;110:174-182.

671 [41]Shi J, Qian S, Zeng LL, Bian X. Influence of anisotropic consolidation stress
672 paths on compression behaviour of reconstituted Wenzhou clay. *Géotechnique*
673 *Letters* 2015;5(4):275-280.

674 [42]Tran-Nguyen HH, Edil TB, Schneider JA. Effect of deformation of prefabricated
675 vertical drains on discharge capacity. *Geosynth Int* 2010;17(6):431-442.

676 [43]Wang J, Ma J, Liu F, Mi W, Cai Y, Fu H, et al. Experimental study on the
677 improvement of marine clay slurry by electroosmosis-vacuum preloading.
678 *Geotext Geomembranes* 2016;44(4):615-622.

679 [44]Wang XS, Jiao JJ. Analysis of soil consolidation by vertical drains with double
680 porosity model. *Int J Numer Anal Met* 2004;28(14):1385-1400.

- 681 [45] Wu H, Shen S, Ma L, Yin Z, Horpibulsuk S. Evaluation of the strength increase
682 of marine clay under staged embankment loading: a case study. *Marine*
683 *Georesources and Geotechnology* 2015;33(6):532-541.
- 684 [46] Xie KH, Lu MM, Liu GB. Equal strain consolidation for stone columns
685 reinforced foundation. *Int J Numer Anal Met* 2009;33(15):1721-1735.
- 686 [47] Yuan J, Hicks MA. Large deformation elastic electro-osmosis consolidation of
687 clays. *Comput Geotech* 2013;54:60-68.
- 688 [48] Zhang N, Shen S, Wu H, Chai J, Xu Y, Yin Z. Evaluation of effect of basal
689 reinforcement under embankment loading on soft marine deposits. *Geotextiles*
690 *and Geomembranes* 2015;43(6):506-514.
- 691 [49] Zhou W, Hong HP, Shang JQ. Probabilistic design method of prefabricated
692 vertical drains for soil improvement. *J Geotech Geoenviron* 1999;125(8):659-664.
693

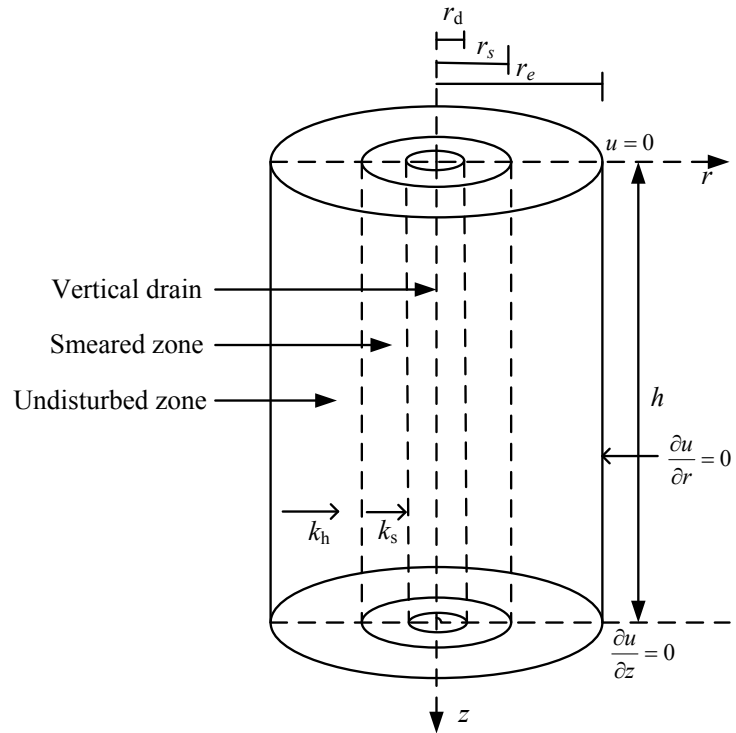
Table 1 Parameters for the numerical simulation followed by Rixner et al. 1986

Undisturbed zone	Density (g/cm ³)	2
	Modulus of elasticity (kPa)	1000
	Poisson ratio	0.35
	Permeability (m/s)	2×10^{-9}
	Void ratio	1.5
Smearred zone	Density (g/cm ³)	2
	Modulus of elasticity (kPa)	1000
	Poisson ratio	0.35
	Permeability (m/s)	1×10^{-9}
	Void ratio	1.5

Table 2 Parameters for the subsoil followed by Chai et al. 1995

Layer	H (m)	γ (kN/m ³)	e_0	C_c	OCR	k_h (10 ⁻⁸ m/s)	k_v (10 ⁻⁸ m/s)	C_h (m ² /d)	C_v (m ² /d)
B	1.0	15.0	2.0	0.58	5	11.45	7.6	0.1	0.067
AC1	2.8	14.5	2.0	1.0	2	5.7	3.8	0.08	0.053
AS1	1.3	15.5	1.8	0.1	1.2	290	290	54	54
AC2	15.0	14.5	2.5	2.0	1.2	2.64	1.76	0.045~0.087	0.03~0.058
AS2	2.5	16.0	1.7	0.1	1.2	290	290	178	178
AC3	1.5	16.0	1.75	0.7	1.2	2.64	1.76	0.26	0.173

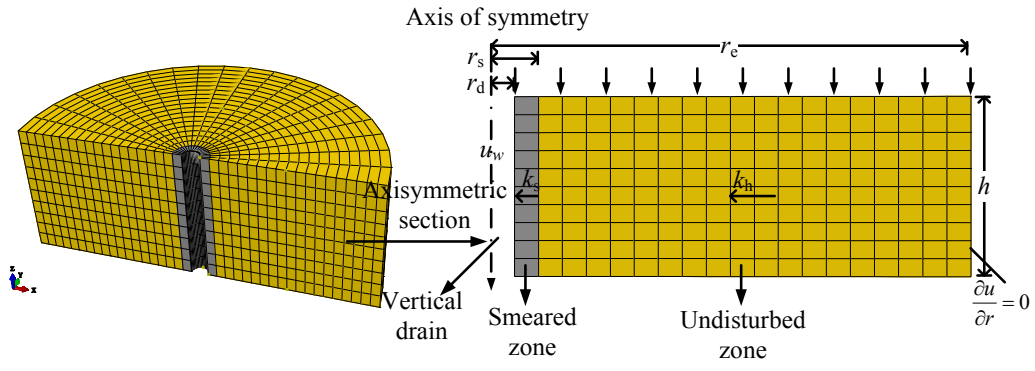
Note: Layer B is the top weathered crust. Layers AC1, AC2, and AC3 are soft clay layers. Layers AS1 and AS2 are sand layers. H is the thickness of the layer. γ is unit weight. e_0 is initial void ratio.



696

697

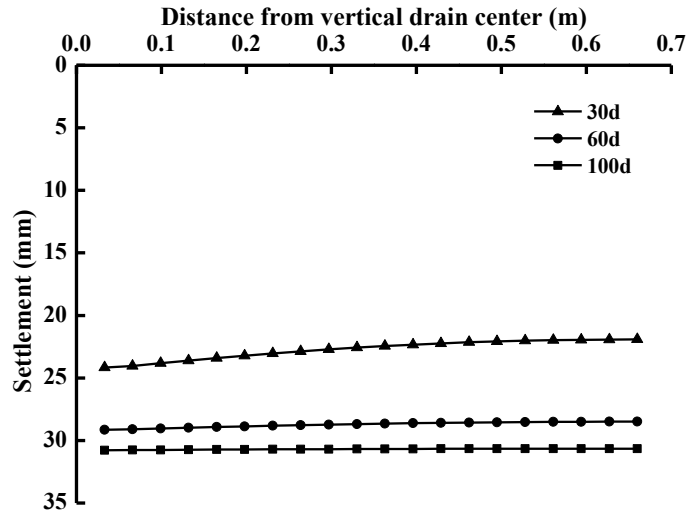
Figure 1 Schematic diagram of axisymmetric consolidation model



698

699

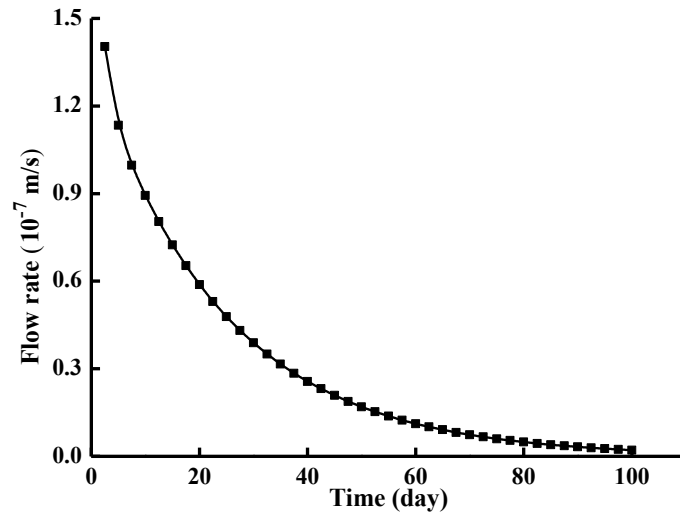
Figure 2 Schematic diagram of the axisymmetric consolidation model for numerical simulation



700

701

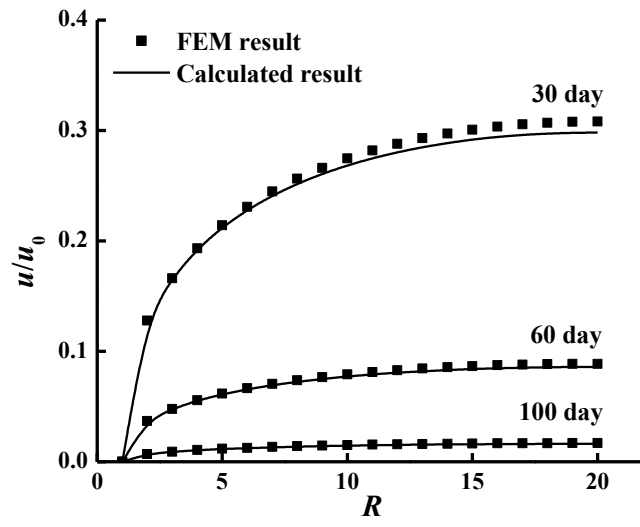
Figure 3 Settlement versus distance r from the center of the vertical drain



702

703

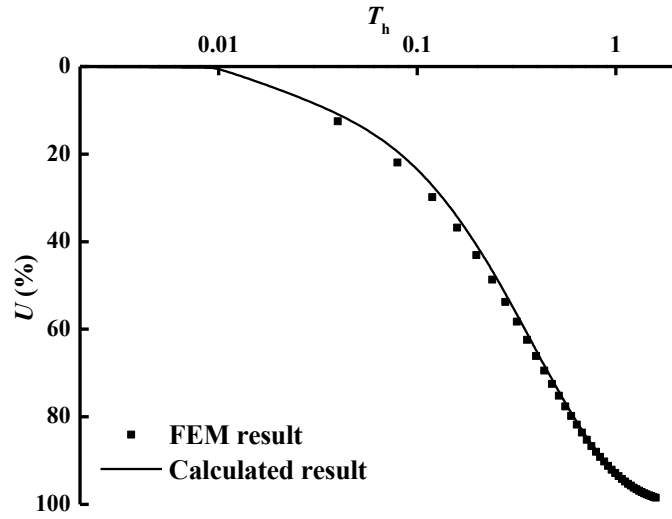
Figure 4 Flow rate versus time



704

705

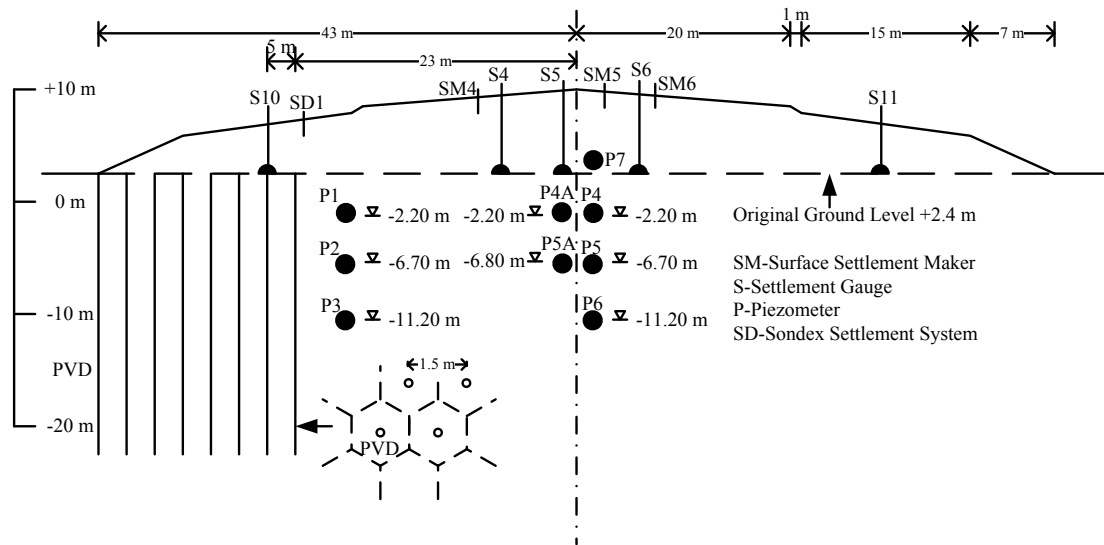
Figure 5 Comparison of u_0/u_0 between the FEM and calculated result



706

707

Figure 6 Comparison of DOC between simulation and calculated results

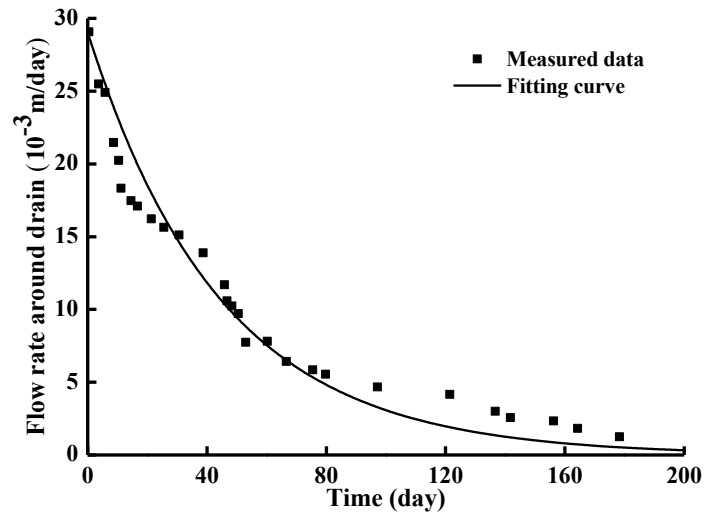


708

709

Figure 7 Cross-section of embankment and field instrumentation locations followed by Chai et al.

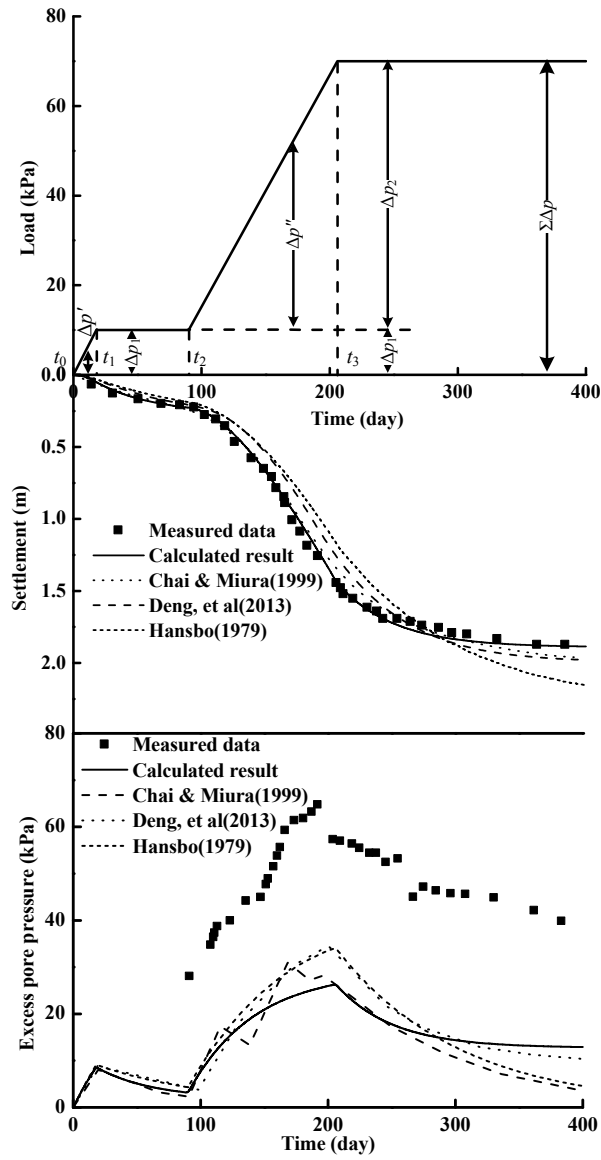
710



711

712

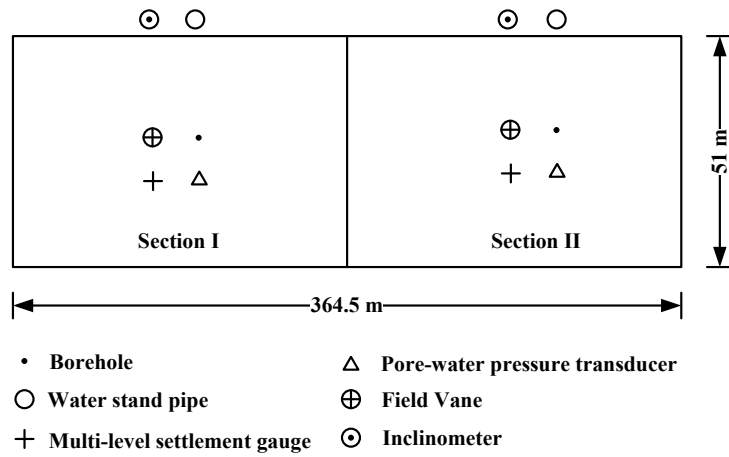
Figure 8 Flow rate measured in the field



713

714

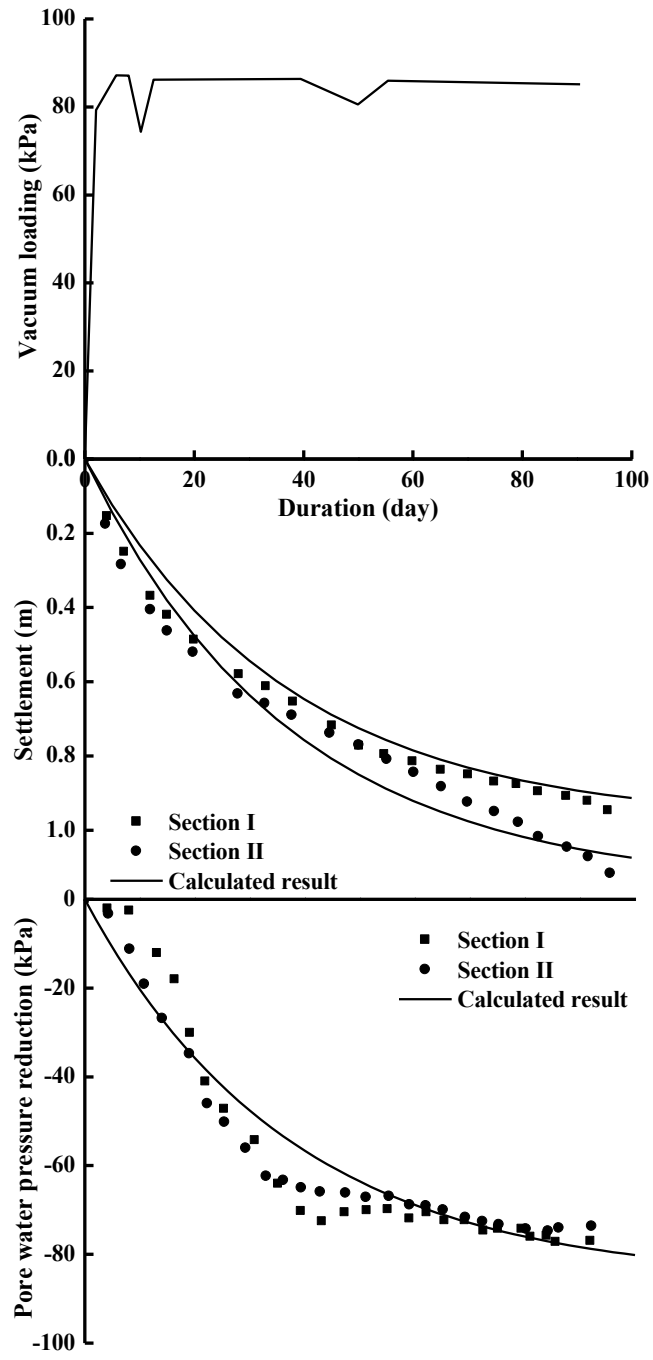
Figure 9 A comparison of settlement and EPWP



715

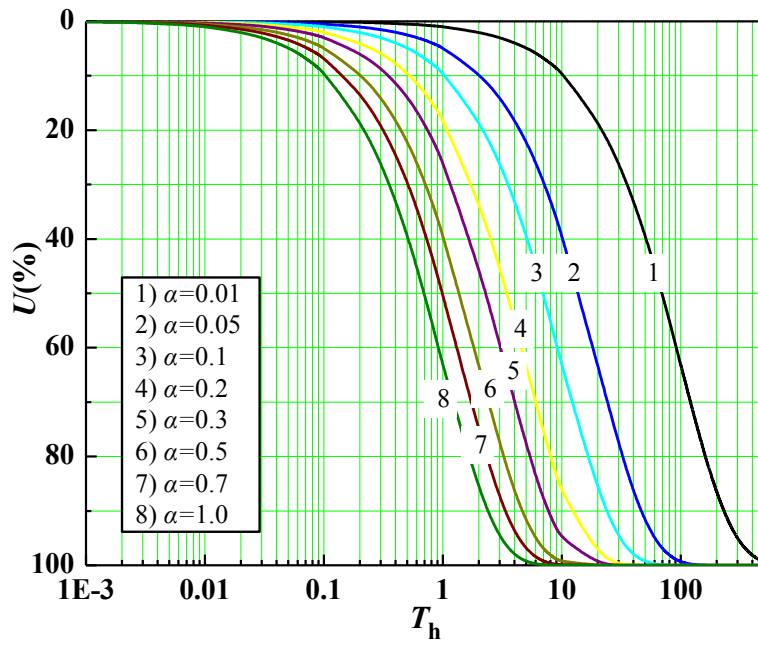
716

Figure 10 Project site and plan view of instrumentation followed by Chu and Yan 2005



717
718

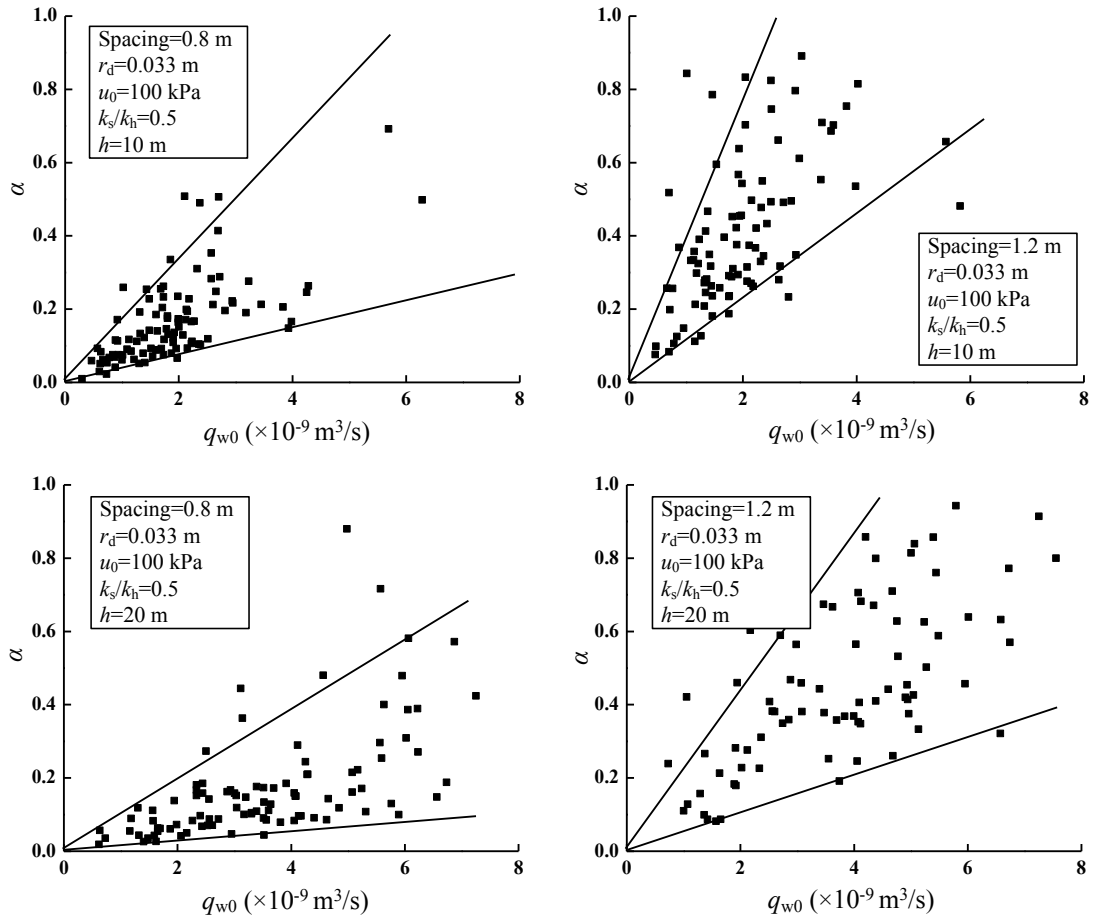
Figure 11 Comparison of settlement and EPWP reduction



719

720

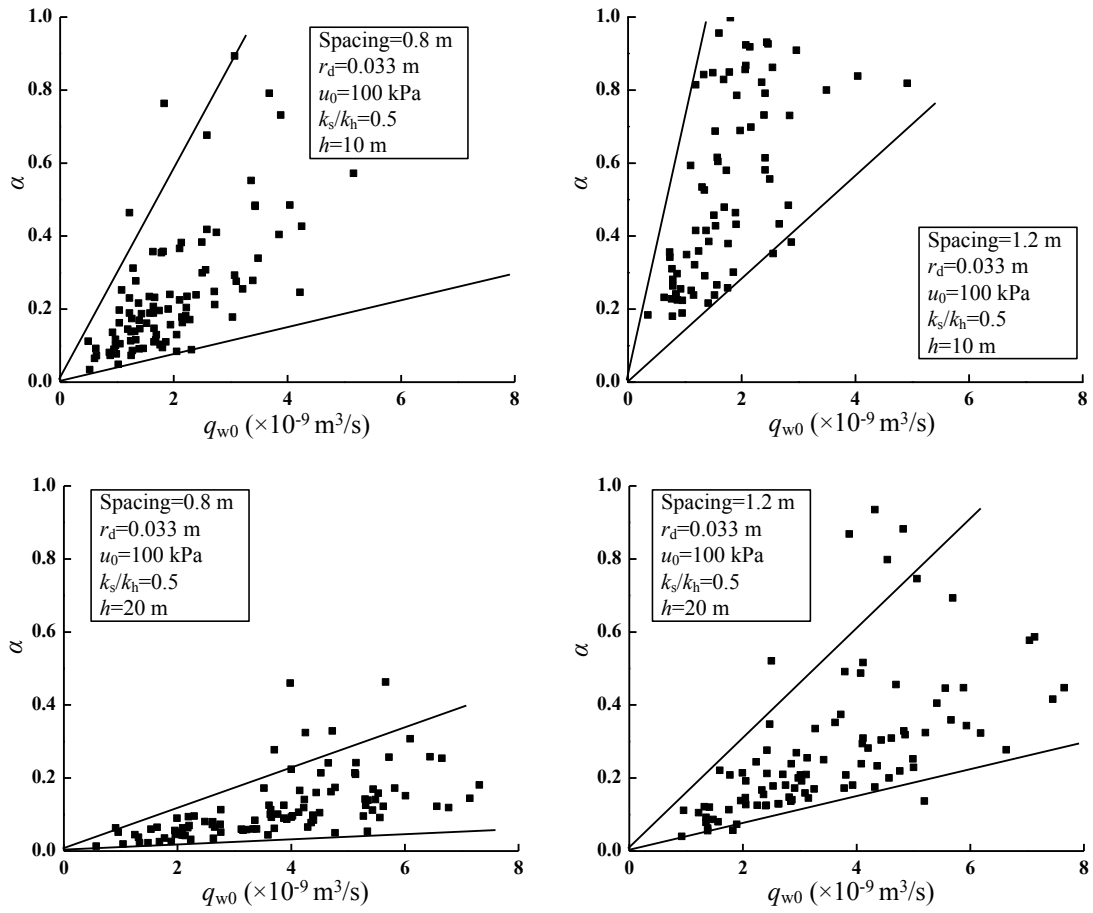
Figure 12 DOC – time factor curve



721

722

Figure 13 Distribution of α for PVDs arranged in a triangular pattern



723

724

Figure 14 Distribution of α for PVDs arranged in a square pattern