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A Review of the Analytical Methods used for Seaplanes Performance Prediction

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A Review of the Analytical Methods used for Seaplanes Performance Prediction

Abstract

Purpose – this paper aims to investigate the different analytical methods used to predict the performance of seaplanes in the wing-in-ground effect region. This was achieved by comparing between the analytical methods available in the literature. The paper also addresses the weaknesses in each method and states which of them can be expanded to include the nonlinear effects.

Design/methodology/approach – first of all, the elemental hydrodynamic characteristics of seaplanes are discussed. Secondly, five different analytical methods are reviewed. The advantages and disadvantages of each method are stated. After that, the heave and pitch equations of seaplane motion are illustrated. The procedure of obtaining the solution of the heave and pitch equations of seaplane motion is explained. Finally, the results obtained from the most common methods are compared.

Findings – the results show that the current analytical methods available are based on different assumptions and considerations. As a result, no method is optimal for all types of seaplanes. Moreover, some of the analytical methods do not study the stability of the seaplane which is a major issue in the design stage. Also, no method takes in consideration the nonlinear effects of motion of seaplanes in heave and pitch axes.

Practical Implications – the previous work has many limitations and only applicable under some assumptions. There was insufficient work to define the motion of the craft in the in-ground effect region where the craft experiences nonlinear characteristics. In order to be able to define the motion in this region, the analytical methods available have to be investigated and compared.

Originality/value – the information provided in the research paper can be used by seaplane designers to distinguish between the analytical methods available and gives them valuable insight into the dynamic stability of seaplanes. The work can also be extended to provide better understanding of the wing-in-ground effect phenomenon.

Keywords: Seaplane, Planing, Analytical, Savitsky, Performance, Prediction, Ekranoplan.

Paper Type: General review.

I.   Introduction

The seaplane concept was initially developed in the Soviet Union by the Central Hydrofoil Design Bureau under the guidance of the soviet engineer R.E. Alekseev. It is also known as Ekranoplan. The first seaplanes produced were the Orlyonok and Lun types shown in figures (1) and (2) respectively (Rozhdestvensky, 2006).

Figure 1 Orlyonok Ekranoplan (Collu,   **Figure 2** Lun Ekranoplan (Collu, 2008).

2008).
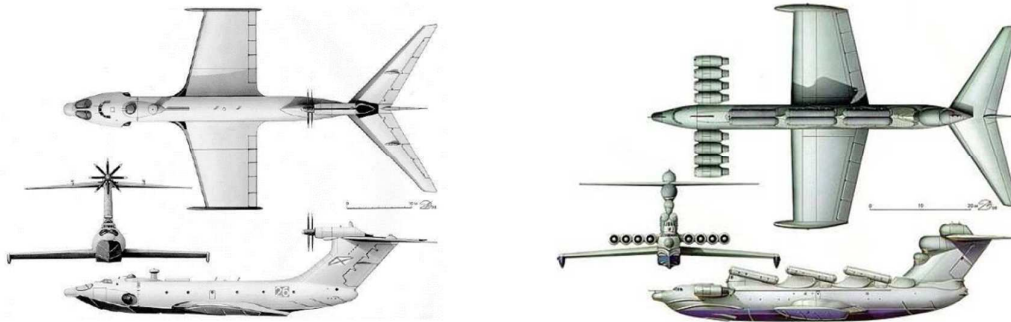
The performance of seaplanes has been widely investigated in the past century. The first studies in the development of seaplanes were taken on high speed planing hulls which have similar performance characteristics as seaplanes as they are designed to glide on top of water and take advantage of the positive dynamic lift produced by their motion. Seaplanes have the ability to fly close to water surface and use the wing-in-ground effect phenomenon to create more lift force and use less power to fly. According to Yun, Bliault and Doo (2010) wing-in-ground (WIG) effect can be defined as the enhanced lift force acting on a craft that is flying close to water or ground surface. The enhanced lift force is produced by the higher pressure increase on the under-surface of the craft due to higher deceleration of the air trapped between the surface and the craft. Figure (3) shows the airflow lines around the hull and wings of a seaplane and explains how the ground effect phenomenon is experienced.

Figure 3 WIG effect on a seaplane (Yun, Bliault and Doo, 2010).
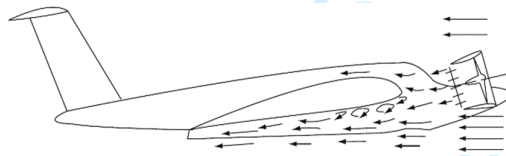
In the recent years, the need for a fast watercraft has increased sharply in the different areas of civil or military applications. One of the prerequisites of a successful seaplane design is the appropriate hydrodynamic stability prediction (Dala, 2015). Hydro-planing hulls have a unique instability phenomenon known as porpoising defined as a periodic, bounded, vertical motion that a craft might show at certain speeds (Faltinsen, 2010). This behaviour is a function of craft speed and can happen even in calm water. Porpoising can lead to structural damage or diving when the motions are very severe that the craft hull is thrown out of water and subsequently impacts on the water surface (Faltinsen, 2010). In 1964, Savitsky published a research on the hydrodynamics of prismatic planing hulls and presented a mathematical approach to study the dynamics of planing surfaces. Savitsky's work suggested a set of empirical equations that allow the performance of prismatic planing hulls to be studied in the design stage. This analytical approach is still being used as the main analytical approach in speedboat design. Prismatic bodies have constant cross section and straight buttocks through the length of the craft. Figure (4) shows a typical high speed planing hull. The parameters commonly used in the analytical performance prediction are speed, weight, length, beam,

dead-rise angle (β) and longitudinal centre of gravity. These parameters define the basic geometry of the craft (Almeter, 1993).

**Figure 4** Typical planing hull (Faltinsen, 2010).
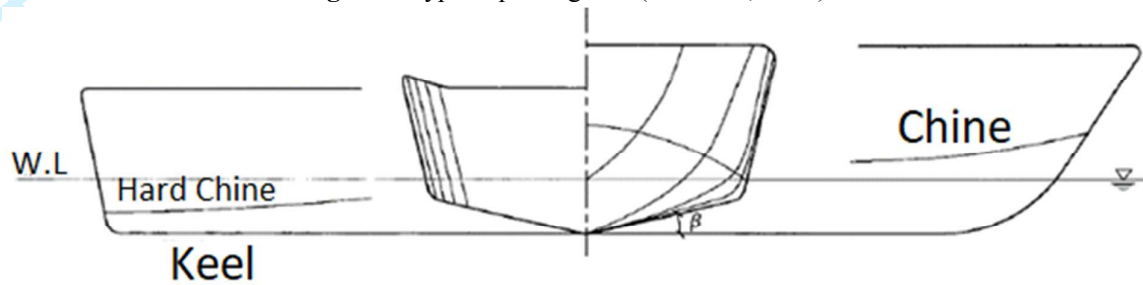
The purpose of this paper is to review the analytical methods used in the prediction of the performance of seaplanes. As the seaplane is a WIG craft that has intermediate configuration between ships and aircraft, the main issue in the design of the seaplane is the stability during take-off and landing. In the region, the hydrodynamic and aerodynamic forces are coupled and very important to consider otherwise the craft cannot take-off. First of all, the hydrodynamic characteristics of seaplanes will be illustrated. Secondly, the performance prediction methods will be briefly discussed. Thirdly, the analytical methods available in the open literature will be explained in details and compared to each other. After that, seaplanes motion will be reviewed in which the linear equations of seaplane motion will be presented. Furthermore, the results obtained by Savitsky will be compared to results obtained by other methods.

II.   Hydrodynamic Characteristics of Prismatic Planing Surfaces

It is critical to study the hydrodynamic characteristics of planing surfaces before undertaking the design of a seaplane. Planing starts when the centre of gravity of the hull is lifted above its normal still-floatation height. A planing surface is designed to be supported by the dynamic reaction between the body and the water. There are two different types of pressure forces acting on the hull of a WIG craft. The first one is the hydrostatic force (buoyancy force). According to Archimedes principle, the hydrostatic force acting on a body that is fully or partially submerged in water equals the weight of the water that the body displaces. The buoyancy force is always in the upward direction and passes through the centre of mass of the body. The second force is the hydrodynamic force which depends on the fluid flow around the hull and proportional to the speed square. The total hydrodynamic pressure drag of seaplanes is composed of two different types. The first one is the pressure drag developed by water pressure acting normal to the inclined hull. The second one is the viscous drag acting tangential to the bottom of the hull and is the result of fluid friction (Murray, 1950). Figure (5) shows the different forces acting on a planing surface in viscous water.

Figure 5 Forces acting on a planing surface (Murray, 1950).
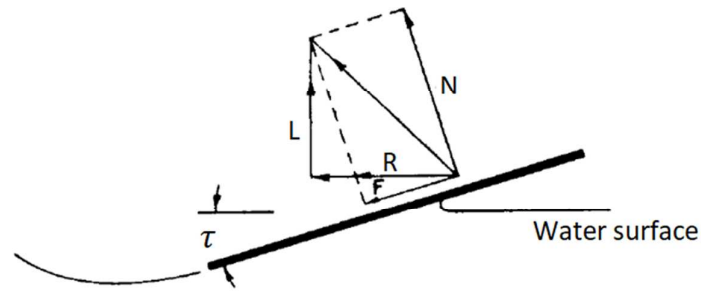
Seaplanes have different operating modes depending on speed and position. Figure (6) explains the different operating modes of a seaplane.

Figure 6 Seaplane operating regions.
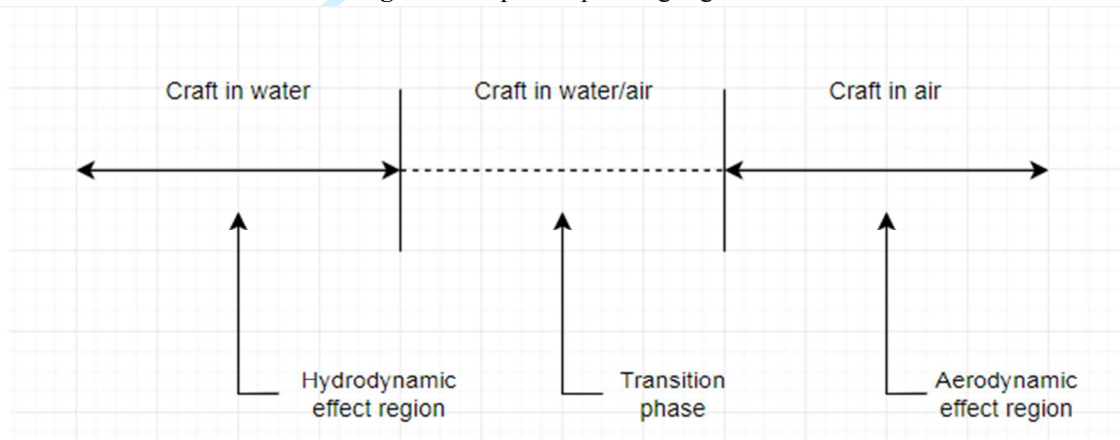
The operating modes can be explained as follows (Yun, Bliault and Doo, 2010):

- At low Froude number $F_n < 0.4$, the seaplane travels in water and can be considered as a displacement hull (moving through water by pushing the water aside). In this region the craft is affected by hydrodynamic and hydrostatic forces. The hydrostatic force (restoring force) is dominant in this region relative to the hydrodynamic forces (added mass and damping).
- At higher Froude number ($0.4 < F_n < 1.0$), the seaplane enters the planing mode where it starts to rise up and glide on the top of water surface. In this case, the craft is affected by both hydrodynamic and hydrostatic forces in which the hydrostatic force is less dominant. Moreover, the craft is also affected by aerodynamic forces.
- When Froude number $F_n > 1.0$, the seaplane gets out of water and becomes completely in air in which it only encounters aerodynamic forces.

According to Almeter (1993), the basic speed regimes that the planing hull can operate in can be defined with respect to the volumetric Froude number ($F_{n\nabla}$) as follows:

- Pre-planing: it is also called displacement mode. It is the hydrodynamic effect region and can be experienced up to $F_{n\nabla} = 2.5$. Most of the weight of the hull is supported by hydrostatic forces (buoyancy).
- Semi-planing: it is also known as semi-displacement mode. It is the transition phase and can be experienced in the range of $2.5 < F_{n\nabla} < 4.0$. In this case, the weight of the hull is supported by both hydrostatic (buoyancy) and hydrodynamic forces. As the speed increases the contribution of hydrodynamic forces in lifting the weight of the craft increases while the hydrostatic forces contribution decreases.
- Fully-planing: it is the aerodynamic effect region. It can be experienced when $F_{n\nabla} \geq 4.0$. At higher speeds the weight of the hull is supported by aerodynamic forces only.

It can be understood from Almeter's study that when the seaplane is hydroplaning, the pressure forces acting on the surface of the hull are buoyancy and dynamic pressure. Each of the forces has a different centre of pressure. The buoyancy force has a centre of hydrostatic pressure, while dynamic forces have a centre of hydrodynamic pressure as shown in figure (7).

Figure 7 The centre of hydrodynamic and hydrostatic pressures (Ibrahim and Grace, 2010).

Instantaneous centre of buoyancy
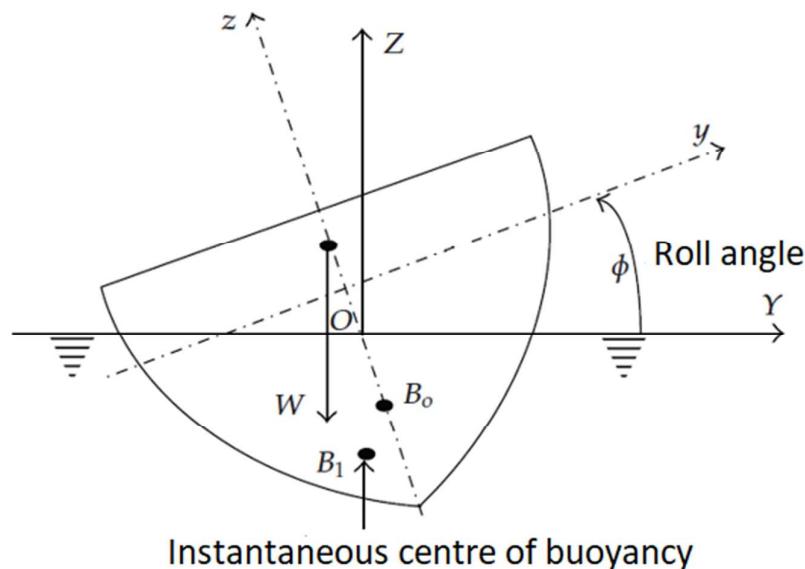
Savitsky (1964) claims that the horizontal centre of buoyancy is 33% of the wetted length forward of the transom. The latter goes on to claim that the horizontal centre of dynamic pressure is 75% forward of the transom in case of a small angle of attack. The pressure distribution on a planing surface is presented in figure (8). The figure shows that the centre of dynamic pressure is approximately at a point 75% forward of the transom. As the speed increases, the forces start to change from hydrostatic to hydrodynamic. This means that at higher speeds the buoyancy force can be neglected and the centre of pressure moves from the centre of buoyancy to the centre of dynamic pressure (Savitsky, 1964).

Figure 8 Pressure distribution on a planing surface (Almeter, 1993).
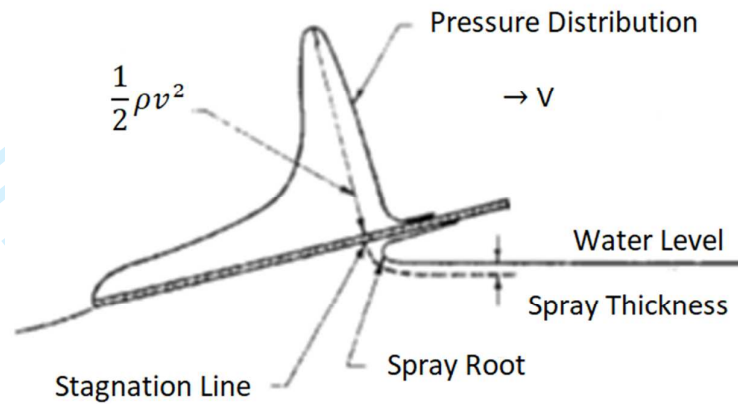
The basic hull design of seaplanes demonstrates a hull that assists in lifting off the craft in the water. Priyanto et al. (2012) suggests that when a hull is in planing mode, there is a tendency that it trims at a certain angle. This means that the front of the hull will lift out of water and the rear part of the hull will immerse partially in water. Figure (9) explains the difference between a hull in the planing and pre-planing (displacement) modes. The hydrodynamic lift and resistance will be encountered at the rear part of the hull where the front will be affected by aerodynamic forces (Priyanto et al, 2012).

Figure 9 (A) Displacement hull. (B) Planing hull (Priyanto et al, 2012).
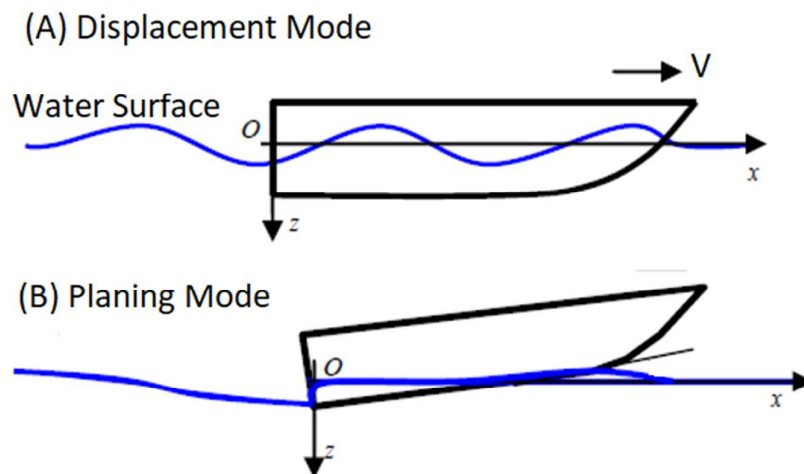
III.   Performance Prediction Methods

In the last century, fundamental research on the hydrodynamics of water-based aircraft has been carried out. The first experimental research on planing surfaces was conducted by Baker in 1912. This is followed by wider investigations carried by Sottorf in 1932. After that, more examinations on the topic were carried out by Shoemaker (1934), Sambraus (1938), Sedov

(1947), Locke (1948), Korvin-Kroukovsky et al. (1949) and Murray (1950). Subsequently, in 1964, Savitsky discussed the hydrodynamic characteristics of planing surfaces and presented a method to predict the performance of prismatic planing surfaces (Almeter, 1993).

As previously stated, the performance of planing hulls is predicted by studying the relations between different variables such as speed, displacement, longitudinal length, beam length, trim angle, dead-rise angle and longitudinal centre of gravity. These variables are called the basic dimensions (geometry) and loading of the planing hull. The shape of the hull can be concave, convex or straight, and can have high warp or high beam taper. Resistance prediction methods can generally be classified into the following categories (Almeter, 1993):

1. Analytical methods (Also called empirical prediction methods).
2. Graphical prediction methods.
3. Planing hull series prediction methods.
4. Numerical methods.
5. Statistical methods.
6. Experimental methods.

It is important in the design stage to choose the most applicable performance prediction method that conforms with the shape and geometry of the planing hull. This is because if the method is not applicable to the examined hull, it might over or under-predict the performance of the hull (Almeter, 1993). The hydrodynamic analysis techniques for seaplanes available in the open literature are summarised in figure (10).

Figure 10 Performance prediction methods (Yousefi, Shafaghat and Shakeri, 2013).
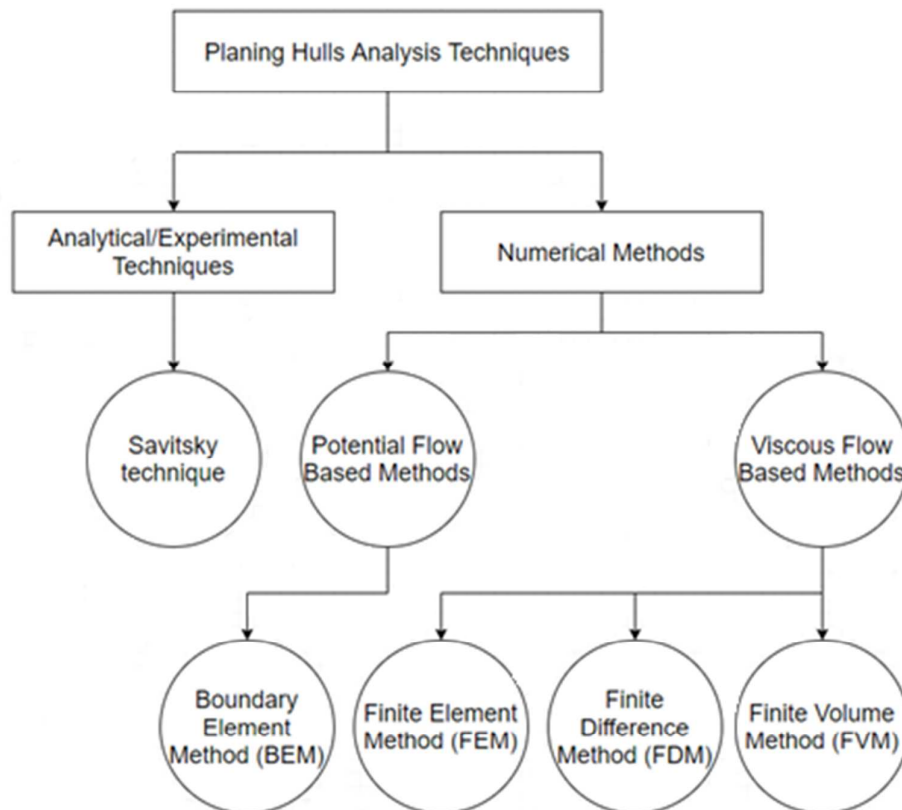
In this paper, attention will be given only to the analytical methods especially Savitsky's method. In the next sub-sections, the analytical methods available in the open literature will be discussed.

A. Savitsky's Method

The equations developed by Savitsky (1964) describe the wetted area, lift force, drag force, centre of pressure and the porpoising stability limits of hard chine prismatic planing plate in terms of its dead-rise angle, trim angle, speed and weight. This method is based on the dynamic lift equations first developed by Sedov (1947). Once the shape and geometry of the hull are defined, it becomes easier to predict its performance. Figure (11) shows the basic terms that describe a planing hull according to Savitsky (1964).

Figure 11 Planing hull characteristics (Savitsky, 1964).
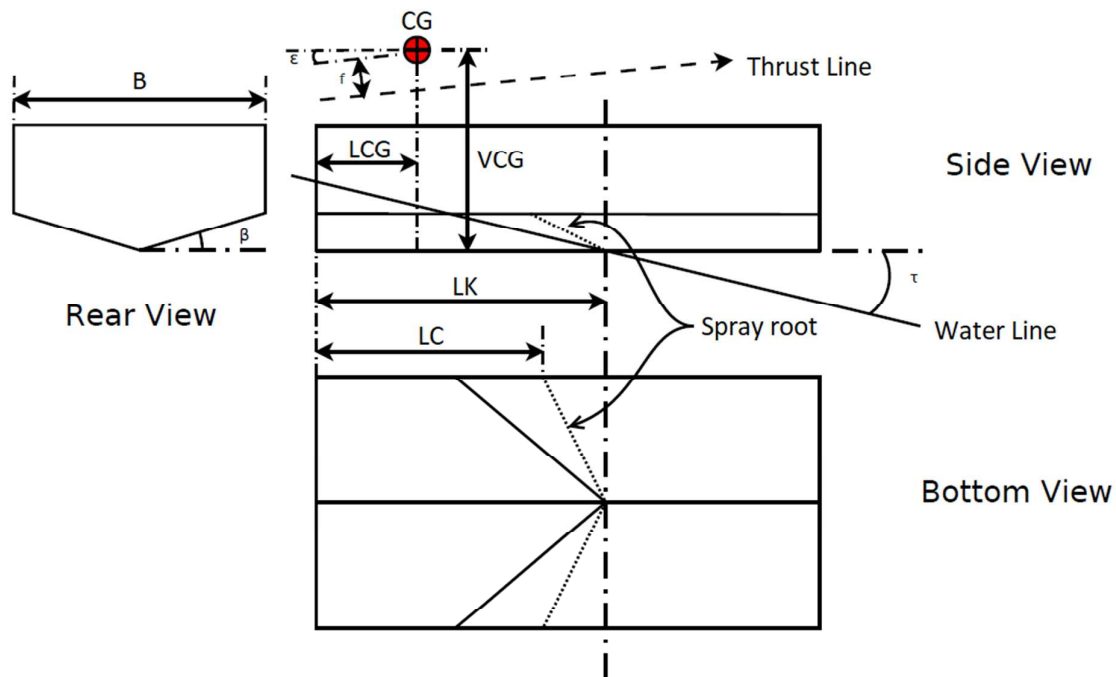
The figure demonstrates that the intersection of the bottom surface with the undisturbed water surface is along the two sloping lines (O-C) between the keel and chines. It can be observed from figure (11) that for a V-shaped planing hull, there is no noticeable evidence of water pile-up at the keel line. When the hull starts to rise and have a larger trim angle, the water will pile-up at the keel. Also, along the spray root line (O-B) there is a tendency of the water surface to rise before the initial point of contact with water O. Savitsky (1964) argues that the spray root line is slightly convex. However, as the curvature is relatively small, it is considered straight. As a result, the mean wetted length of a dead-rise planing surface can be defined as the average of the keel length and chine length calculated from the back of the hull (transom) to the point of intersection with spray root line (O-B).

As shown in figure (12), the total hydrodynamic drag on a planing hull has two components:

- The fluid friction drag D_f.

- The pressure drag $\dfrac{D_f}{cos\tau}$.

Figure 12 Hydrodynamic drag components (Savitsky, 1964).

$$D = \Delta\,tan\tau + \frac{D_f}{cos\tau}$$
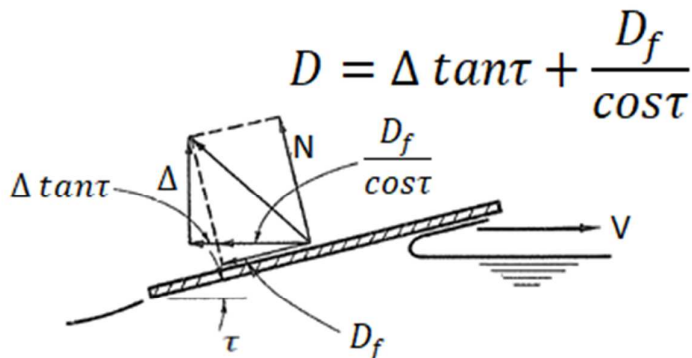
In order to develop his equations, Savitsky (1964) studied the equilibrium of the planing craft. First of all, he assumed that the planing hull is moving in a constant speed with no acceleration in any direction. Secondly, the planing hull is considered to have a constant dead-rise angle (β), a constant equilibrium trim angle (τ_e) and a constant beam length (b) for the whole wetted planing area. Nevertheless, Savitsky's theory only investigates the hydrodynamic conditions. This means that the weight of the hull is balanced only by the hydrodynamic lift forces. According to Savitsky (1964), equilibrium is achieved when the following conditions apply:

- The summation of forces in the vertical direction is zero.
- The summation of forces in the horizontal direction is zero.
- The summation of moments about the centre of gravity CG is zero (pitching moment equilibrium).

Figure (13) shows the different forces and parameters Savitsky (1964) has used in the development of his method.

Figure 13 Analysed planing hull (Savitsky, 1964).

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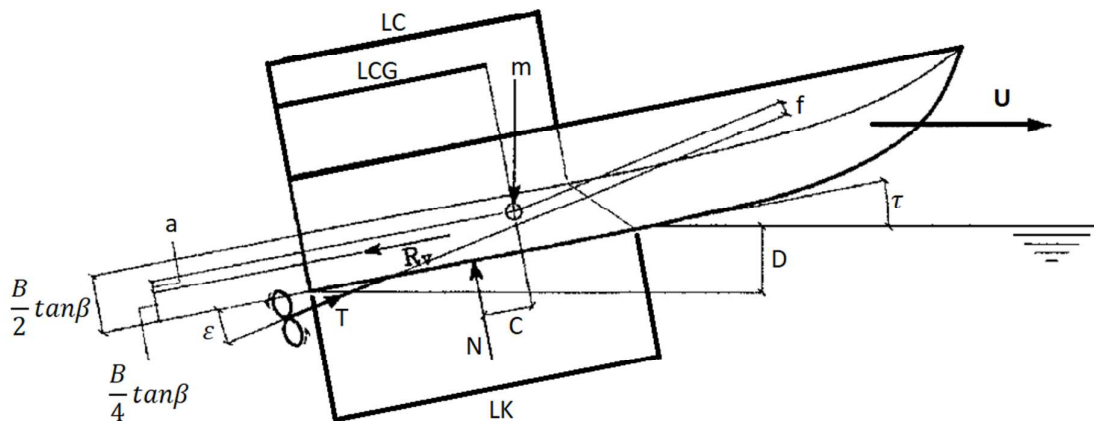
It is worth mentioning that in his analysis, Savitsky (1964) considered the beam to be more important than the length of the hull because the wetted length of the hull does not remain constant. It varies with trim angle, loading and speed while the wetted beam generally remains constant. Moreover, the latter points out that at high speeds, it is possible to change the wetted length of the planing hull without changing its hydrodynamic characteristics. In addition, Savitsky (1964) used Froude law of similitude to produce the planing coefficients and symbols in his analysis. It can be noted that these analysis can be applied to study the performance of water-based aircraft.

By applying the equilibrium principle, the equilibrium trim angle (τ_e) can be calculated. After that, the performance characteristics of the planing hull can be predicted. The procedure of Savitsky's method is explained as follows:

1) The geometry of the hull is defined in which the following variable are specified:
    a) The total mass of the boat m (or can be expressed as Δ).
    b) The beam length b.
    c) The longitudinal distance of centre of gravity measured from the transom LCG.
    d) The vertical distance of centre of gravity measured from the keel VCG.
    e) The dead-rise angle β.
    f) The trim angle τ.
    g) The velocity of the craft V.
    h) The inclination of thrust line relative to keel line ε.

2) Then a few variables are calculated in the same order as follows:
    a) The speed coefficient (which is the beam Froude number):
    $$C_v = \frac{V}{\sqrt{gb}}$$
    (1)

    b) The lift coefficient of dead-rise planing surface:
    $$C_{L\beta} = \frac{mg}{\frac{1}{2}V^2 b^2 \rho}$$
    (2)

    c) The lift coefficient of an equivalent flat plate C_{Lo} is calculated from the following equation:
    $$C_{Lo} = C_{L\beta} + 0.0065\beta C_{Lo}^{0.6}$$
    (3)

    d) The wetted length-beam ratio λ is calculated from the following equation:
    $$C_{Lo} = \tau^{1.1}\left[0.012\lambda^{0.5} + \frac{0.0055\lambda^{2.5}}{c_v^2}\right]$$
    (4)

    Then the wetted length is calculated as: $L_w = \lambda b$

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e) The mean velocity over bottom of planing surface:

$$V_m = V \left[1 - \frac{0.012\lambda^{0.5}\tau^{1.1} - 0.0065\beta(0.012\lambda^{0.5}\tau^{1.1})^{0.6}}{\lambda\cos(\tau)} \right]^{0.5}$$

(5)

f) The friction drag coefficient:

$$C_f = \frac{0.075}{(log_{10}(R_e) - 2)^2}$$

(6)

Where R_e is Reynold's number and can be calculated as:

$$R_e = \frac{V_m \lambda b}{v}$$

(7)

g) The water friction drag D_f:

$$D_f = \frac{1}{2}\frac{\rho V_m^2 \lambda b^2}{\cos(\beta)}\left(C_f + \Delta C_f\right)$$

(8)

Where ΔC_f is ATTC standard roughness = 0.0004

h) Then, the total hydrodynamic drag can be calculated as follows:

$$D = mg\, tan(\tau) + \frac{D_f}{\cos(\tau)}$$

(9)

i) The centre of dynamic pressure is found from:

$$C_p = 0.75 - \frac{1}{\frac{5.21C_v^2}{\lambda^2} + 2.39}$$

(10)

j) Then the two distances a and c shown in figure (13) are calculated from:

$$c = LCG - C_p\lambda b$$

(11)

$$a = VCG - \frac{b}{4}\tan(\beta)$$

(12)

k) The equation of equilibrium of pitching moment is then solved:

$$M_{tot} = mg\left[\frac{c}{\cos(\tau)}(1 - \sin(\tau)\sin(\tau + \varepsilon)) - fsin(\tau)\right] + D_f(a - f)$$

(13)

If the equation satisfies the equilibrium (sum of moments = 0) then the wetted length of keel L_k and the vertical depth of trailing edge of craft below level of water d are found from:

$$L_k = \lambda_e b + \frac{b \tan(\beta)}{2\pi \tan(\tau_e)} \tag{14}$$

$$d = L_k \sin(\tau_e) \tag{15}$$

If the equation of equilibrium does not equal to zero, a different trim angle (τ) must be assumed and the procedure repeated till two different values of moment are found (negative and positive) and then by interpolation the equilibrium trim angle (τ_e), D_f and λ can be found (Savitsky, 1964).

B. Morabito's Method

This method claims that the pressure at the stagnation point is far greater than the pressure at the other parts of the hull. Therefore, the problem becomes very complex and direct calculation methods cannot be applied to calculate the pressure distribution along the hull surface. As a result, the pressure can be calculated in length-wise and breadth-wise directions independently. It could then be extended to a three-dimensional distribution over the hull. Figure (14) shows the three-dimensional pressure distribution over the bottom of a planing surface (Morabito, 2010).

Figure 14 3D pressure distribution over the bottom of a planing hull (Iacono, 2015).
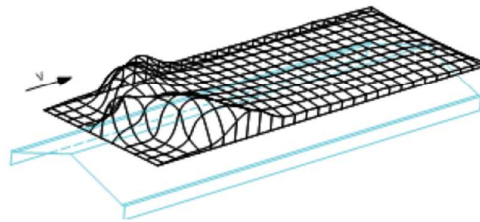
Iacono (2015) studied Morabito's method and states that the dynamic pressure along the planing hull exhibits a maximum at the stagnation point. Eventually, the pressure deteriorates and reaches atmospheric pressure at the end of the hull. As explained in figure (15), Morabito's method focuses on the pressure distribution along the longitudinal keel line at the bottom of the hull. Also, it calculates the pressure at the transom and the longitudinal pressure distribution over other sections (Iacono, 2015).

In the case of the keel line, Morabito (2010) introduced the following equation to calculate the maximum pressure at the stagnation point:

$$\frac{P_{max}}{q} = \sin^2\alpha$$

(16)

Where:

α is the angle between the stagnation line and keel line shown in the next figure.

q is the pressure along the line $= \frac{1}{2}\rho V^2$

Figure 15 Components of planing hull explained by Morabito (Iacono, 2015).
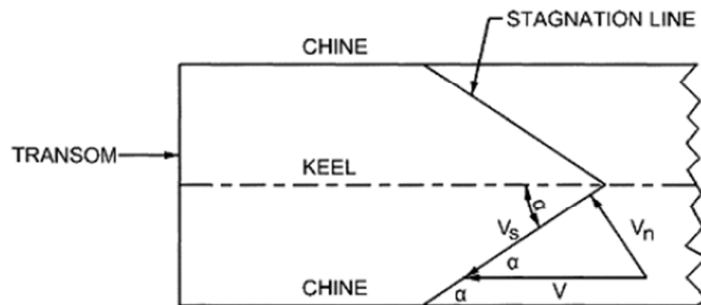
The pressure gradually decreases along the keel line till it becomes almost zero at the transom. The pressure reduction along the line can be calculated from the following equation:

$$\frac{P_L}{q} = 0.006 \frac{\tau^{1/3}}{X^{2/3}}$$

(17)

P_L is the pressure behind the stagnation point and X is the dimensionless distance from the stagnation and can be calculated from:

$$X = \frac{x}{b}$$

(18)

Where b is the breadth of the hull.

Then, Morabito modified the equation of reduced pressure along the keel line as:

$$\frac{P}{q} = \frac{0.006\tau^{1/3}X^{1/3}}{(X + \frac{(0.006\tau^{\frac{1}{3}})^{1.5}}{2.588(\frac{Pmax}{q})^{1.5}})}$$

(19)

Morabito calculated the pressure at the transom by introducing the following equation:

$$P_T = \frac{(\lambda_y - X)^{1.4}}{(\lambda_y - X)^{1.4} + 0.05}$$

(20)

Where λ_y is the dimensionless distance between the transom and the stagnation line as each longitudinal section and can be calculated from:

$$\lambda_y = \lambda - \frac{(Y - 0.25)}{\tan(\alpha)}$$

(21)

13

Where $Y = \frac{y}{b}$ is the dimensionless transverse distance from the longitudinal symmetry (keel) line (the same as the previously defined X but in the transverse direction).

The previous equations of Morabito only measure the pressure distribution at the transom, at the stagnation point and along the symmetry line in between them. Morabito claims that the pressure declines along the stagnation line and consequently, at each longitudinal section the maximum pressure is less than that on the longitudinal symmetry (keel) line. The latter has used the Swept Wing Theory to calculate the pressure reduction along the other sections (Morabito, 2010).

As previously presented in figure (15), Morabito (2010) suggests that the fluid velocity is a combination of two components, velocity along the stagnation line and velocity normal to it. Using the normal component of velocity and resulting pressure, the ratio of transverse pressure along the stagnation line is found as follows:

$$\frac{P_{YStag}}{P_R} = [1.02 - 0.25Y^{1.4}]\frac{0.5-Y}{0.51-Y}$$
(22)

By multiplying the previous equation by the maximum pressure, the pressure over the stagnation line at a desired longitudinal section is found as:

$$\frac{P_{max}}{q} = \frac{P_{YStag}}{P_R}sin^2(\alpha)$$
(23)

Morabito's method is not able to define many terms needed in predicting the hydrodynamic performance of planing hulls. For example, it cannot define the porpoising stability limit. As a result, it cannot be used as the staple method for boat design.

C. CAHI Method

The CAHI method was proposed by Almeter (1993). This method is used to predict the performance of prismatic planing hulls. It is also known as Lyubomirov method or TSAGI method from the Central Aero-hydrodynamic Institute in Moscow. The CAHI method was initially developed by Perelmuter (1938) who investigated the take-off characteristics of seaplanes (Alourdas, 2016).

Almeter (1993) developed this method based on the same dynamic lift equations prepared by Sedov (1947) that Savitsky (1964) used to develop his method. In Savitsky's method, the trim angle is corrected based on the constant dead-rise while in the CAHI method, the wetted area increases with dead-rise.

CAHI method supports the claim of Chambliss and Boyd (1953) who investigated the planing characteristics of two v-shaped hulls of different dead-rise angles. CAHI method agree with Chambliss and Boyd (1953) that in theory for a given lift coefficient, any increase in the dead-rise angle will increase the trim angle and wetted length of a planing hull. This means that the hydrodynamic resistance will increase (Chambliss and Boyd, 1953). The procedure of CAHI method can be summarised as follows:

1) The same variables as Savitsky's method should be defined and then the equation of moment should be solved to obtain the mean wetted length-beam ratio λ. Once an acceptable λ is obtained (almost 0.75*LCG) the trim angle τ and the dead-rise lift coefficient can be calculated. The equations for the mentioned variables are as follows:

$$M = \frac{\frac{0.7\pi\lambda}{1+1.4\lambda}\left[0.75 + 0.08\frac{\lambda^{0.865}}{\sqrt{C_v}}\right] + \frac{(\lambda-0.8)\lambda^2}{(3\lambda+1.2)C_v^2}}{\frac{0.7\pi}{1+1.4\lambda} + \frac{(\lambda-0.4)\lambda}{(\lambda+0.4)C_v^2}}$$

(24)

$$C_{L\beta} = \frac{\Delta}{0.5\rho V^2 b^2}$$

(25)

$$\frac{C_{L\beta}}{\tau} = \frac{0.7\pi\lambda}{1+1.4\lambda} + \frac{(\lambda-0.4)\lambda^2}{(\lambda+0.4)C_v^2}$$

(26)

2) The mean wetted length-beam ratio and the trim angle can now be calculated for a dead-rise planing hull from the following:

$$\lambda_\beta = \frac{\lambda^{0.8}}{\cos(\beta)}[1 - 0.29(\sin(\beta))^{0.28}].\left[1 + 1.35(\sin(\beta))^{0.44}.\frac{M}{\sqrt{C_v}}\right]$$

(27)

$$\tau_\beta = \tau + \frac{0.15(sine(\beta))^{0.8}}{C_v^{0.3}}.\frac{1 - 0.17\sqrt{\lambda_\beta\cos(\beta)}}{\sqrt{\lambda_\beta\cos(\beta)}}$$

(28)

3) After that, the wetted surface S, the average bottom velocity V_m and the drag of prismatic hull are calculated as follows:

$$S = \frac{b^2\lambda_\beta}{\cos(\beta)}$$

(29)

$$V_m = V\left[1 - \frac{\tau}{1+\lambda}\right]$$

(30)

$$D = \Delta\tan(\tau_\beta) + \frac{0.5C_f\rho SV_m^2}{\cos(\tau_\beta)}$$

(31)

C_f can be calculated from the same equation proposed by Savitsky:

$$C_f = \frac{0.075}{(logR_e-2)^2}$$

(32)

4) Finally, the wetted keel length and the wetted chine length are calculated as follows:

$$\lambda_\beta = \frac{L_m}{b}$$

(33)

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$$L_m = \frac{L_k + L_c}{2}$$

(34)

$$L_k - L_c = \frac{b \tan(\beta)}{\pi \tan(\tau)}$$

(35)

D. Payne's Method

In 1995, Payne studied the planing theory. The latter has discussed the difference empirical equations used to predict the performance of flat and v-shaped planing hulls available at that time. As a result, a method to predict the resistance of planing hulls was proposed.

In his study, Payne (1995) points out that Savitsky's equations are the most accurate equations developed in the last century for describing the total hydrodynamic drag and lift forces acting on a planing hull. Therefore, he compared his method to Savitsky's method. Figure (16) presents a comparison between Payne's and Savitsky's results. The figure shows the lift produced by a planing hull versus the wetted length/beam ratio. It can be observed that when the wetted length/beam ratio is low, Payne's method overestimated the lift force. As the length/beam ratio increases, Payne's method gives lower lift force estimations.

Figure 16 Comparison between Payne and Savitsky Methods (Payne, 1995).
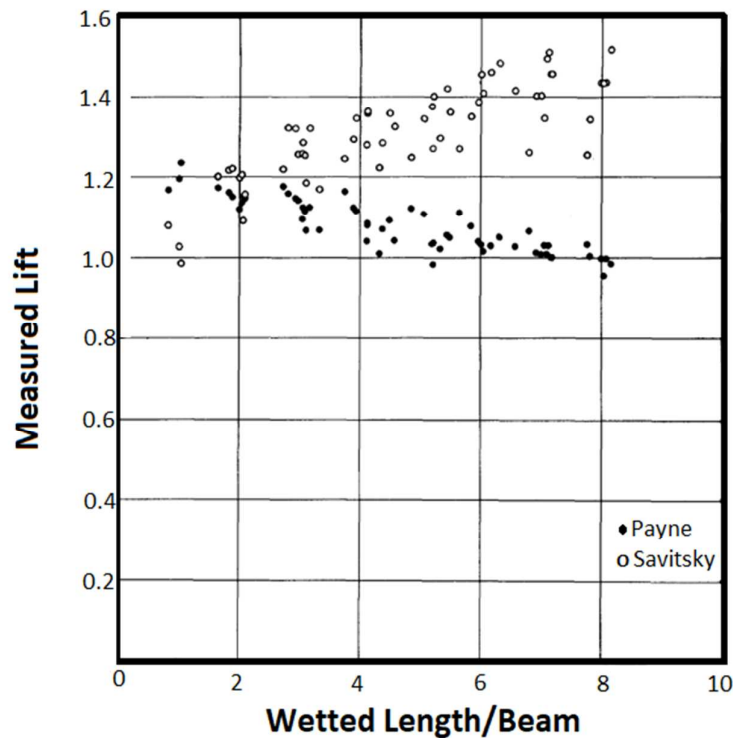
It is worth mentioning that Payne (1995) claims that the hydrostatic pressure acting on a planing hull is less than Archimedes force.

Table (1) summarises the different empirical equations of hydrodynamic lift of planing plates developed previously as provided by Payne (1995).

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Table 1 Equations of hydrodynamic lift of planing plates.

Author	Year	Equation	Geometrical specifications
Perring and Johnson	1935	$C_L = 0.9\tau A^{0.42}$	$\beta = 0°$
Sottorf	1937	$C_L = 0.845\tau A^{0.5}$	$\tau \leq 10°$
Perelmuter	1938	$C_L = \dfrac{2A\tau}{(1+A)}$	$5° \leq \tau \leq 8°$
Sedov	1939	$C_L = \dfrac{0.7\pi A\tau}{(1.4+A)}$	$\tau \leq 4°$
Siler	1949	$C_L = \dfrac{\pi A \sin\tau \cos\tau}{(4+A)} + 0.88 \sin^2\tau \cos\tau$	$\beta = 0°$
Korvin-Kroukovsky et al.	1949	$C_L = 0.012\tau^{1.1}A^{0.5}$	$\tau \leq 4°$ $\beta = 0°$
Locke	1949	$C_L = \dfrac{k}{2}\tau^n$ k and n are given in the reference as functions of the aspect ratio A	$\beta = 0°$
Korvin-Kroukovsky	1950	$C_L = \dfrac{0.73\pi A\tau}{(2+A)} + 0.88\tau^2$	$0.25° \leq \tau \leq 10°$
Schnitzer	1953	$C_L = \varphi\left(\dfrac{\pi^3 A}{16} \sin\tau \cos^2\tau \right.$ $\left. + 0.88\sin^2\tau\cos\tau\right)$ $\varphi = \dfrac{1}{\sqrt{1+\lambda^2}}\left(1 - \dfrac{0.485}{1+\dfrac{1}{\lambda}}\right)$	$0° \leq \tau \leq 45°$ $\beta = 0°$ $\beta = 30°$
Shuford	1954	$C_L = \dfrac{\frac{\pi}{2}A\tau}{(1+A)}\cos^2\tau + \sin^2\tau\cos\tau$	$\tau \leq 16°$ $\beta = 0°$
Brown	1954	$C_L = \dfrac{2\pi}{\cot\frac{\tau}{2} + \pi + (2\cot\frac{\tau}{2} - \pi)\frac{1}{A}}$	$A > 1$
Brown	1954	$C_L = (1.67\sin\tau + 0.09).(1-A)\sin\tau \cos\tau$ $+ \dfrac{2\pi A}{3\cot\frac{\tau}{2}}$	$A < 1$
Farshing	1955	$C^3 + [2.293 - 1.571A)\tau - 2.379 - A]C^3$ $+ [2A + 4 + (6.283A - 4.584)\tau]C - 6.283A\tau = 0$ $C_L = \xi C$ $\xi = 1.359 - \tanh\left(\dfrac{1+A}{8A}\right)$ $+ \left(\dfrac{\tau° - 18°}{90.53}\right)\tanh\dfrac{1}{A^2}$	$18° \leq \tau \leq 30°$

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Farshing	1955	$C^3 + [2.293 - 1.571A)\tau - 2.379 - A]C^3 + [2A + 4 + (6.283A - 4.584)\tau]C - 6.283A\tau = 0$ $C_L = \xi C$ $\xi = 1.359 - \tanh\left(\dfrac{1+A}{8A}\right)$	$2° \leq \tau \leq 18°$
Shuford	1958	$C_L = \dfrac{\frac{\pi}{2}A\tau}{(1+A)}cos^2\tau + \dfrac{4}{3}sin^2\tau\,cos^3\tau$	$8° \leq \tau \leq 18°$ $\beta = 0°$ $\beta = 20°$ $\beta = 40°$

Payne's theory is based on two-dimensional flow analyses of a flat plate. It can be seen as an improved version of the resistance prediction methods available at its time. The latter modified the coefficients developed previously. Furthermore, Payne (1995) made different assumptions based on the revision of the experimental data available. He states that the modifications are made to the coefficients used in the "added mass" equations for planing forces predicted formerly.

E. Shuford's Method

This method was developed to predict the performance of deep-V planing hulls operating at high-speed regime where the buoyancy force is negligible. It does not discuss the effects of spray drag. It discusses the effects of the vertical spray rails on the performance of planing hulls. It has been modified several times to produce improved performance prediction methods. Brown (1971) produced a version of this method that takes in consideration the buoyancy force which makes his method applicable to lower speeds (lower Froude number). This modified version is based on the same basis as Savitsky's method (Brown, 1971). The equations and procedure of this method is explained in reference (Shuford, 1958).

F. Summary of the Methods Discussed

The specifications of each analytical method discussed previously are listed in the table (2). The advantages and disadvantages of each method along with its validation method are summarised in the table.

Table 2 Methods specifications.

Method/Author	Advantages	Disadvantages	Validated with
Savitsky	• It can predict the porpoising stability limit. • It can predict the performance of hulls with pure planing conditions which have similar performance characteristics	• Applicable to steady state conditions only. • Only hydrodynamic investigations. No other forces are considered. • Only applicable to trim angle $\tau < 4°$. At higher trim angle, the results starts to deviate from the results of the experiments. • The centre of dynamic pressure is assumed to	Previous analytical methods

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|			
	as seaplanes. • It is the most common method used in speedboat design.	be at 75% of the mean wetted length forward of the transom which is not accurate when analysing seaplanes. • It assumes that the thrust is always parallel to the axis thruster (prime mover axis) which may not be always true. • Spray drag (whisker spray) is not included or taken into account. • It start to behave irrationally when the dead-rise angle (β) is higher than 50° or when the dead-rise angle is not constant along the hull.	
Morabito	• It can be used to predict the performance of displacement and planing hulls. • Very simple and easy to use.	• It does not define the porpoising stability limit of planing hulls. • It is not applicable for high coefficient of speed C_v. • It only investigates the pressure distribution along the keel line and stagnation line of the planing hull. • It does not explain the relations between the different design variables of the planing hull (dead-rise and trim angles). • It cannot be mathematical combined with the aerodynamic effect because it only explains the hydrodynamic pressure on the hull. • It does not investigate the contribution of the hydrostatic force (Buoyancy). • Spray drag (whisker spray) is also not included or taken into account.	CFD and experiments
CAHI	• Was initially developed to predict the characteristics of seaplanes. Thus, it can be modified to give more accurate results under different conditions.	• This method is based on Savitsky's method. As a result, it has the same limitations. • It does not define the porpoising stability limit of planing hulls. • Only applicable to a certain hull geometry. • Only applicable under the same conditions and assumptions it is based on.	Experiments
Payne	• It can be used to predict the performance of displacement hulls. • Very simple and easy to use.	• It does not define the porpoising stability limit of planing hulls. • It is not applicable for high coefficient of speed C_v. • It only discusses the hydrodynamics of flat plates with no dead-rise angle. • It lacks the investigations of the aerodynamic forces acting on planing hulls.	Experiments and previous analytical methods
Shuford	• It can be applied to high speed-regime ($F_n > 1.0$). • Applicable to high trim angle	• It is based on the same basis as Savitsky's method. • Pure hydrodynamic conditions.	Experiments

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	$8° \leq \tau \leq 18°$. • Different dead-rise angles were tested in the development of this method.		

IV.   Heave and Pitch Equations of Seaplane Motion

The equations of motion of a seaplane advancing at a constant forward velocity with arbitrary heading in regular sinusoidal sea waves are presented in this section. In order to compare the results of Savitsky's method with the results of these equations, the oscillatory motions are assumed to be linear and harmonic.

A seaplane can experience motions in 6 directions. Hence, the performance of seaplanes is presented by a 6 degrees of freedom (DOF) system. The 6 motions are a set of independent displacements and rotations that completely define the displaced position and orientation of the seaplane (Fossen, 2011). Therefore, seaplanes motion can be considered to be made of three translational (linear) components (surge, sway and heave), and three rotational (angular) components (roll, pitch and yaw). Figure (17) shows the sign convention of the 6 motions of a planing hull (Lewis, 1989).

Figure 17 The 6 motions of a planing hull (Salvesen, Tuck and Faltinsen, 1970).

η_1 = surge η_3 = heave η_5 = pitch
η_2 = sway η_4 = roll η_6 = yaw
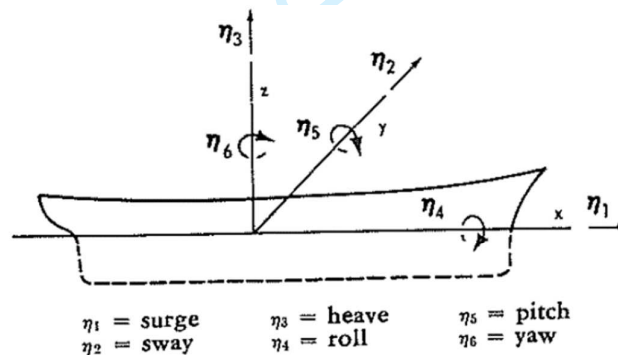
It can be observed from figure (17) that the linear displacements about the x,y and z axes are η_1 (*surge*), η_2 (*sway*) and η_3 (*heave*) respectively. In addition, the angular displacements about the x,y and z axes are η_4 (*roll*), η_5 (*pitch*) and η_6 (*yaw*) respectively (Lewis, 1989).

By taking into consideration that the responses are linear and harmonic, the six linear equations of motion can be written using subscript notation as follows (Ogilvie, 1969):

$$\sum_{k=1}^{6}\left[(M_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k\right] = F_j e^{i\omega t}$$
(36)

Where:

$j = 1 - 6$

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M_{jk} is the component of the generalised mass matrix of the craft in the jth direction due to k^{th} motion.

A_{jk} is the added-mass coefficient in the jth direction due to k^{th} motion.

B_{jk} is the damping coefficient in the jth direction due to k^{th} motion.

C_{jk} is the hydrostatic restoring force coefficient in the jth direction due to k^{th} motion.

F_j are the complex amplitudes of the exciting forces and moments in the jth direction. ($F_j e^{i\omega t}$ are forces and moments given by the real part).

For a planing hull with lateral symmetry, the 6 coupled equations of motion are reduced to two sets of equations, connecting respectively, the heave, pitch and surge, and the sway, roll and yaw. This means that the linear equations are not coupled with the angular equations. Moreover, as long as the planing hull is assumed to be a slender body, the hydrodynamic forces associated with the surge motion are much smaller than the forces associated with the other 5 equations of motion. As a result, the motion of the craft can be described by the two coupled equations of heave and pitch motions (Salvesen, Tuck and Faltinsen, 1970).

A heaving and pitching system of seaplane motions behaves like a two degree of freedom spring-mass system. According to Ogilvie (1969), this assumption is clear when a craft model is given heave or pitch displacements from its equilibrium position. It will rapidly oscillate several times before it comes to rest. Therefore, the resulting equations of heave and pitch of seaplanes are expressed as follows:

$$(m + A_{33})\ddot{\eta}_3 + A_{35}\ddot{\eta}_5 + B_{33}\dot{\eta}_3 + B_{35}\dot{\eta}_5 + C_{33}\eta_3 + C_{35}\eta_5 = F_3 e^{i\omega t} \tag{37}$$

$$A_{53}\ddot{\eta}_3 + (A_{55} + I_{55})\ddot{\eta}_5 + B_{53}\dot{\eta}_3 + B_{55}\dot{\eta}_5 + C_{53}\eta_3 + C_{55}\eta_5 = F_5 e^{i\omega t} \tag{38}$$

The determination of the coefficients and exciting force and moment is a major problem in the motion prediction. In order to simply this problem, the craft can be divided into transverse strips or segments. The coefficients are then calculated by applying a strip theory (Brown, 1971). The added mass and damping coefficients are calculated using a two-dimensional hydrodynamic theory.

The general solution for each of the two previous equations has two components. The homogenous solution and the particular integral. The homogenous solution is obtained when the system is considered under no external excitation forces or moments. On the other hand, the particular integral is obtained when the external excitation forces and moments are considered.

In order to obtain the homogenous solution, the equations of heave and pitch can be written as:

$$(m + A_{33})\ddot{\eta}_3 + A_{35}\ddot{\eta}_5 + B_{33}\dot{\eta}_3 + B_{35}\dot{\eta}_5 + C_{33}\eta_3 + C_{35}\eta_5 = 0 \tag{39}$$

$$A_{53}\ddot{\eta}_3 + (A_{55} + I_{55})\ddot{\eta}_5 + B_{53}\dot{\eta}_3 + B_{55}\dot{\eta}_5 + C_{53}\eta_3 + C_{55}\eta_5 = 0$$
$$\text{(40)}$$

If a steady-state solution is assumed then heave and pitch can have the following form:

$$\eta_3 = Z_0 e^{\lambda t}$$
(41)

$$\eta_5 = \theta_0 e^{\lambda t}$$
(42)

If equations (41) and (42) and substituted in equations (39) and (40), the following equations will be obtained:

$$(m + A_{33})\lambda^2 Z_0 + A_{35}\theta_0\lambda^2 + B_{33}\lambda Z_0 + B_{35}\lambda \theta_0 + C_{33}Z_0 + C_{35}\theta_0 = 0$$
(43)

$$A_{53}\lambda^2 Z_0 + (A_{55} + I_{55})\lambda^2 \theta_0 + B_{53}\lambda Z_0 + B_{55}\lambda \theta_0 + C_{53}Z_0 + C_{55}\theta_0 = 0$$
(44)

These two equations can be written in the form of a matrix as follows:

$$\begin{bmatrix} (m + A_{33})\lambda^2 + B_{33}\lambda + C_{33} & A_{35}\lambda^2 + B_{35}\lambda + C_{35} \\ A_{53}\lambda^2 + B_{53}\lambda + C_{53} & (A_{55} + I_{55})\lambda^2 + B_{55}\lambda + C_{55} \end{bmatrix} \begin{bmatrix} Z_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
(45)

For non-trivial solutions of Z_0 and θ_0, the determinant of equation (45) is set to be zero. As a result, the characteristic equation can be written in the following form:

$$a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0$$
(46)

Where:

$$a = [(m + A_{33})(A_{55} + I_{55}) - A_{35}A_{53}]$$
(47)

$$b = [(m + A_{33})B_{55} + B_{33}(A_{55} + I_{55}) - A_{35}B_{53} - B_{35}A_{53}]$$
(48)

$$c = [(m + A_{33})C_{55} + B_{33}B_{55} + C_{33}(A_{55} + I_{55}) - A_{35}C_{53} - B_{35}B_{53} - C_{35}A_{53}]$$
(49)

$$d = [B_{33}C_{55} + C_{33}B_{55} - B_{35}C_{53} - C_{35}B_{53}]$$
(50)

$$e = [C_{33}C_{55} - C_{35}C_{53}]$$
(51)

Equation (46) is a fourth order characteristic equation of the system. This equation is solved to obtain four roots. The characteristics of motion of the seaplane will depend on the nature of the roots of this equation. Assuming that all roots are pairwise distinct, the solution has three possible cases, described as follows:

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- If the equation has 4 real roots then the general solution of the two equations of heave and pitch (Equations (43) & (44)) can be given by:

$$\eta_{3h} = Z_{01}e^{\lambda_1 t} + Z_{02}e^{\lambda_2 t} + Z_{03}e^{\lambda_3 t} + Z_{04}e^{\lambda_4 t}$$
(52)

$$\eta_{5h} = \theta_{01}e^{\lambda_1 t} + \theta_{02}e^{\lambda_2 t} + \theta_{03}e^{\lambda_3 t} + \theta_{04}e^{\lambda_4 t}$$
(53)

Where Z_{0i} and θ_{0i} are constants and can be determined from the initial conditions by substituting the values of λ_i $(i = 1,2,3,4)$ into equation (45).

- In the case of two real roots and two complex conjugate roots, the general solution of the two equations is in the following form:

$$\eta_{3h} = Z_{01}e^{\lambda_1 t} + Z_{02}e^{\lambda_2 t} + e^{\alpha t}(Z_{03}e^{i\omega_1 t} + Z_{04}e^{-i\omega_1 t})$$
(54)

$$\eta_{5h} = \theta_{01}e^{\lambda_1 t} + \theta_{02}e^{\lambda_2 t} + e^{\alpha t}(\theta_{03}e^{i\omega_1 t} + \theta_{04}e^{-i\omega_1 t})$$
(55)

The constants in this case can also be determined from equation (45) by substituting the values of λ_1, λ_2 and $\alpha \pm i\omega_1$. The system in this case will oscillate with only one natural frequency ω_1 of damped oscillations.

- If the solution gives a pair of two complex conjugate roots. Then the homogenous solution can be expressed as follows:

$$\eta_{3h} = e^{\alpha t}(Z_{01}e^{i\omega_1 t} + Z_{02}e^{-i\omega_1 t}) + e^{\varphi t}(Z_{03}e^{i\omega_2 t} + Z_{04}e^{-i\omega_2 t})$$
(56)

$$\eta_{5h} = e^{\alpha t}(\theta_{01}e^{i\omega_1 t} + \theta_{02}e^{-i\omega_1 t}) + e^{\varphi t}(\theta_{03}e^{i\omega_2 t} + \theta_{04}e^{-i\omega_2 t})$$
(57)

Where ω_1 and ω_2 are the two natural frequencies of damped oscillations.

These three possible cases of damped vibrations discussed above depend on the values of the constants a, b, c, d and e. In the case of repeated roots, the corresponding Eigen-modes must be multiplied by t^k, where k is the algebraic multiplicity of the root.

When the excitation forces and moments of equations (37) & (38) are considered then the particular integral of the two equations have the following form:

$$\eta_{3p} = P_1 \cos(\omega_1 t) + P_2 \sin(\omega_1 t)$$
(58)

$$\eta_{5p} = P_3 \cos(\omega_1 t) + P_4 \sin(\omega_1 t)$$
(59)

Then the general solution of the heave and pitch equations of motion will be the summation of the homogenous solution and the particular integral. It can be written in the following form:

$$\begin{bmatrix} \eta_3 \\ \eta_5 \end{bmatrix} = \begin{bmatrix} \eta_{3h} \\ \eta_{5h} \end{bmatrix} + \begin{bmatrix} P_1 \\ P_3 \end{bmatrix} \cos(\omega_1 t) + \begin{bmatrix} P_2 \\ P_4 \end{bmatrix} \sin(\omega_1 t)$$

(60)

V.   Comparison of Results

In this section, Savitsky's method results will be compared to results obtained from CAHI method. Moreover the porpoising stability limit obtained by the heave and pitch equations of motion will be compared to the limit obtained by Savitsky.

As discussed before, CAHI method is based on the same basis as Savitsky's method. The results obtained by both methods are in good agreement as shown in the next two figures.

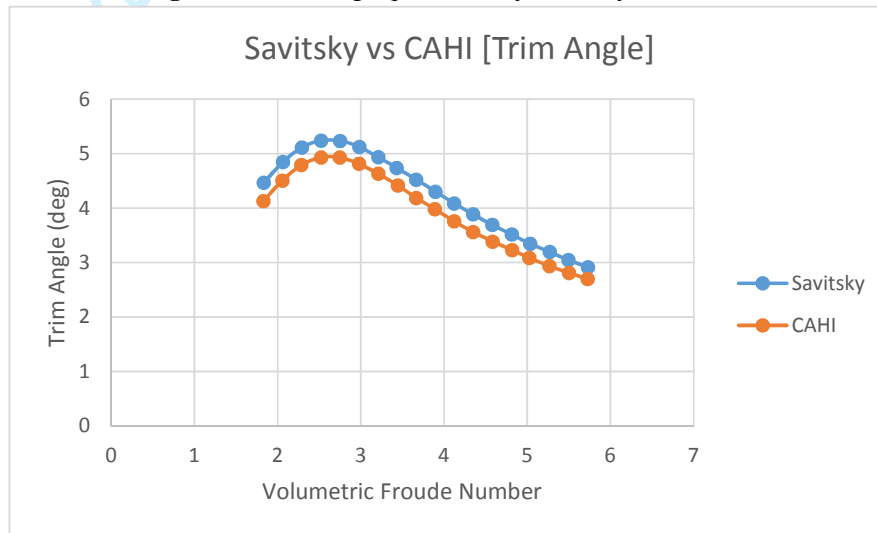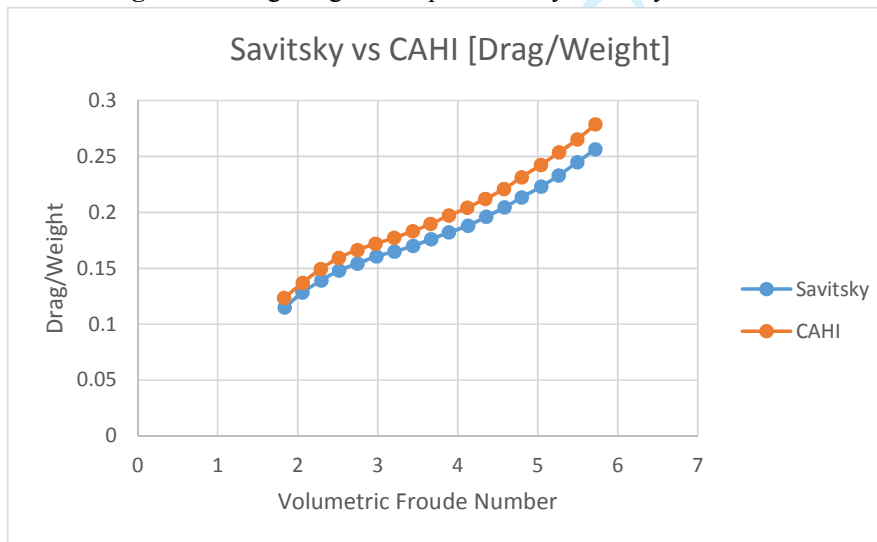**Figure 18** Trim angle predicted by Savitsky and CAHI.

**Figure 19** Drag/weight ratio predicted by Savitsky and CAHI.

The higher drag obtained by CAHI method is because this method assumes that the wetted area increases with the increase of the dead-rise angle. As a result, higher drag force will be obtained by the CAHI method. In addition, Savitsky assumes that the trim angle is corrected

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in constant dead-rise angle hulls. While on the other hand, CAHI assumes that the trim angle increases with the dead-rise angle. These assumptions justify the higher trim angle obtained by Savitsky.

Figure (20) shows the porpoising stability line obtained by heave and pitch equations of motion and Savitsky method for a constant dead-rise angle of 10°. For a given dead-rise angle, if the combination of the lift coefficient and the trim angle are above the line, then the hull will tend to porpoise. It can be noted that the higher the dead-rise angle the more stable the planing hull. Higher dead-rise angle will allow a higher trim angle to be reached without inducing porpoising behaviour.

Figure 20 Porpoising stability limit obtained by heave and pitch equations and Savitsky.
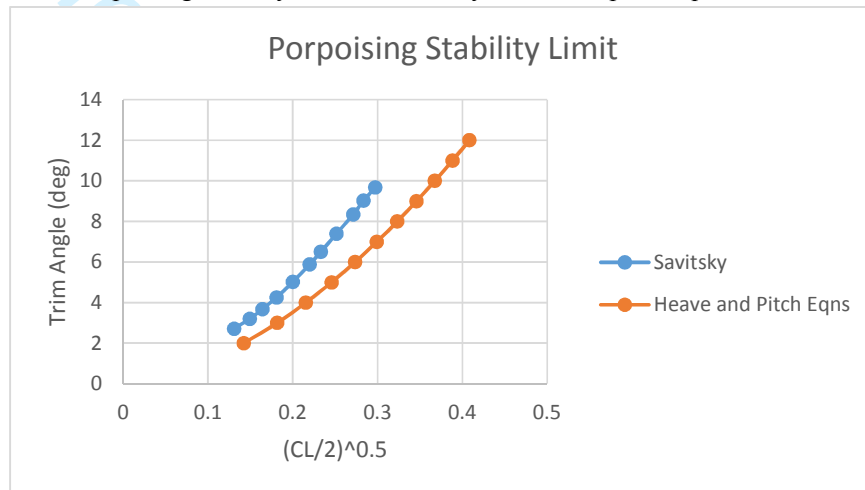
The porpoising line obtained by heave and pitch equations allows for higher speed characteristic prediction. The results obtained are for higher coefficient of lift as well. Savitsky's method can obtain results for lower speed regime and lower coefficient of lift. It can be concluded that this method can be extended to allow for more accurate prediction than Savitsky under various conditions.

VI.　　Conclusion

The prediction of the performance of seaplanes is very important especially in the design stage. It allows the designer to produce enhanced seaplanes that could fly comfortably under different conditions. Moreover, seaplanes can reduce the environmental harm of air transport as they take advantage of the high lift force in the wing-in-ground effect region. The performance characteristics of seaplanes have been discussed. In addition, several analytical methods used for seaplane performance prediction have been explained. The methods have been compared with each other along with their validation methods. It can be noted that the main issue with seaplanes is the take-off and landing stability in which the craft experiences nonlinear and unsteady hydrodynamic and sea water wave characteristics. The available analytical methods lack the ability of predicting the stability limits of the craft. Moreover, most of the methods discussed are valid under certain geometry and conditions. For example, Savitsky's method can predict the performance up to a trim angle of 4°. Also, it can be used for a dead-rise angle of 50° or less. Nevertheless, it is valid for steady state conditions only.

Therefore, no analytical method is good for all types of seaplanes. The prediction depends on the geometry of the hull and the operation conditions.

Heave and pitch equations of motion have the ability to predict the performance of seaplanes for higher speed-regime than the other methods. Also, this approach can be modified to produce results for a wider geometry range than Savitsky's method. It also can be expanded to allow for nonlinear performance prediction.

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Nomenclature

g	Acceleration of gravity (m/s²)	ρ	Density of fluid (kg/m³)
V	Velocity (m/s)	F_n	Froude Number $= \dfrac{V}{\sqrt{gb}}$
V_m	Mean velocity (m/s)	$F_{n\nabla}$	Volumetric Froude Number $= \dfrac{V}{\sqrt{g\sqrt[3]{\nabla}}}$
b	Seaplane beam (m)	R_e	Reynold's number $= \dfrac{V_m \lambda b}{v}$
S	Wetted surface area (m²)	$\emptyset$	Roll angle (deg)
A	Aspect ratio $= \dfrac{b^2}{S}$	β	Dead-rise angle (deg)
t	Time (s)	τ	Trim angle (deg)
Δ or m	Seaplane mass (kg)	D_f	Fluid friction drag (N)
∇	Static volume of displacement (m³)	v	Kinematic viscosity of fluid (m²/s)
C_v	Speed coefficient $= \dfrac{V}{\sqrt{gb}}$	P_{max}	Maximum pressure at stagnation point (Pa)
$C_{L\beta}$	Lift coefficient, dead-rise surface	q	Pressure along the keel line (Pa)
C_{Lo}	Lift coefficient, zero dead-rise	P_L	Pressure behind the stagnation point (Pa)
λ	Wetted length-beam ratio	P_T	Pressure at the transom (Pa)
λ_y	Dimensionless distance between stagnation point and transom	M	Moment (N.m)
L_w	Wetted length (m)	η	Displacement coordinate vector
L_k	Keel length (m)	η̇	Velocity coordinate vector
L_m	Mean wetted length (m)	η̈	Acceleration coordinate vector
L_c	Chine length (m)	F_3	Amplitude of the exciting force (heave)
C_f	Friction-drag coefficient	F_5	Amplitude of the exciting moment (pitch)
C_p	Centre of dynamic pressure (m)	I_{55}	Mass moment of inertia (kg.m²)
ε	Inclination of thrust line relative to keel line (deg)	ω	Circular frequency of the encounter (rad/sec)