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Evaluation of Matrix Factorisation Approaches for Muscle Synergy Extraction

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Abstract

The muscle synergy concept provides a widely-accepted paradigm to break down the complexity of motor control. In order to identify the synergies, different matrix factorisation techniques have been used in a repertoire of fields such as prosthesis control and biomechanical and clinical studies. However, the relevance of these matrix factorisation techniques is still open for discussion since there is no ground truth for the underlying synergies. Here, we evaluate factorisation techniques and investigate the factors that affect the quality of estimated synergies. We compared commonly used matrix factorisation methods: Principal component analysis (PCA), Independent component analysis (ICA), Non-negative matrix factorization (NMF) and second-order blind identification (SOBI). Publicly available real data were used to assess the synergies extracted by each factorisation method in the classification of wrist movements. Synthetic datasets were utilised to explore the effect of muscle synergy sparsity, level of noise and number of channels on the extracted synergies. Results suggest that the sparse synergy model and a higher number of channels would result in better estimated synergies. Without dimensionality reduction, SOBI showed better results than other factorisation methods. This suggests that SOBI would be an alternative when a limited number of electrodes is available but its performance was still poor in that case. Otherwise, NMF had the best performance when

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the number of channels was higher than the number of synergies. Therefore, NMF would be the best method for muscle synergy extraction. *Keywords:* Muscle synergy, Matrix factorisation, Surface electromyogram, Non-negative matrix factorisation, second-order blind identification, Principal component analysis, Independent component analysis.

1 1. Introduction

² 1.1. Muscle synergy

"How does the central nervous system (CNS) control body movements and posture?" This question has been discussed for over a century with no conclusive answer. The coordination of muscles and joints that accompanies movement requires multiple degree of freedoms (DoFs). This results a high level of com-6 plexity and dimensionality [1]. A possible explanation to this problem considers the notion that the CNS constructs a movement as a combination of small groups of muscles (synergies) that act in harmony with each other, thus reducing the dimensionality of the problem. This idea could be traced to the first decades 10 of the twentieth century [2] and has been formulated and developed through 11 the years [3, 4, 5] to reach the Muscle Synergy hypothesis [6, 7, 8]. The mus-12 cle synergy concept posits that the CNS achieves any motor control task using 13 a few synergies combined together, rather than controlling individual muscles. 14 Although the muscle synergy hypothesis is criticized for being very hard to be 15 falsified [9], a repertoire of studies have provided evidence and support for it. 16 Those pieces of research could be categorized into two main categories: direct 17 stimulation and behavioural studies. 18

The stimulation approaches were conducted by exciting the CNS at different locations to study the resulting activation pattern. Earlier studies focused on the organization of motor responses evoked by micro-stimulation of the spinal cord of different vertebral species, such as frogs [3, 4, 5, 10, 11], rats [12] and cats [13]. They revealed that the responses induced by simultaneous stimulation of different loci in the spinal cord are linear combinations of those induced by

separate stimulation of the individual locus. Those findings were supported 25 by another direct stimulation studies where a relatively long period of electric 26 stimulation applied to different sites in the motor cortex resulted in complex 27 movements in rats [14], prosimians [15] and macaques [16, 17]. The chemical 28 micro-stimulation has been used through N-methyl-D-aspartate iontophoresis 29 injected into the spinal cord of frogs which evoked an electromyographic (EMG) 30 patterns that could be constructed as a linear combination of a smaller group 31 of muscle synergies [7]. 32

Similarly, the behavioural studies rely on recording the electrical activity of 33 the muscles (electromyogram, EMG) during a specific task (or tasks) or natural 34 behaviour. Then, a number of synergies is extracted from the signals using com-35 putational techniques. The identified synergies should be able to describe the 36 recorded signal for the related task or behaviour. Studies have been carried out 37 on cats where four muscle synergies were sufficient to reproduce 95% of postural 38 hind-limb muscles response data [18] and five synergies accounted for 80% of 39 total variability in the data [19]. Similar research on monkeys during grasping 40 activity showed that three muscle synergies accounted for 81% of variability [16]. 41 In humans, muscle synergies were identified from a range of motor behaviours 42 [20, 21] with the ability to describe most of the variability in EMG signals. In 43 addition, other studies show that complex motor outputs such as upper limb 44 reaching movements [22], cycling [23, 24] and human postural control [25] are a 45 result of the combination of few muscle synergies. 46

In the recent years, many studies applied the muscle synergy concept to analyse and study body movements and muscle coordination in diverse applications. For instance, it has been used to establish the neuromuscular system model [26]. Moreover, the hypothesis has been used in many clinical applications [27] in addition to several biomechanical studies such as walking and cycling [28, 29]. The extracted synergies are utilised in prosthesis control through classification [30, 31] and regression [32].

⁵⁴ 1.2. Mathematical models for muscle synergy

In all studies, the muscle synergies are estimated from the recorded electrical activity of the muscle. Signals are either collected using surface EMG or invasively using needle EMG. Then, the EMG recordings needs to be modelled in order to compute the muscle synergies.

Two main muscle synergy models have been proposed: the time invariant or synchronous model [6, 7] and the time-varying or asynchronous model [33, 8]. The electrical activity for single muscle or channel $\mathbf{m}(t)$ is a vector that could be expressed according to the time-invariant model as a combination of synchronous synergies \mathbf{s} (scalar values activated at the same time) multiplied by a set of time-varying coefficients or weighting functions \mathbf{w} as shown in equation 1

$$\mathbf{m}(t) = \sum_{i=1}^{i=r} s_i \mathbf{w}_i(t) \tag{1}$$

where r is the number of synchronous synergies. Since synergies contribute to each muscle activity pattern with the same weighting function $\mathbf{w}_i(t)$, the synergy model is synchronous without any time variation.

⁶⁹ On the other hand, the time-varying synergies are asynchronous as they ⁷⁰ are compromised by a collection of scaled and shifted waveforms, each one of ⁷¹ them specific for a muscle or channel. Thus, the muscle activity $\mathbf{m}(t)$ can be ⁷² described according to the asynchronous model with a group of time-varying ⁷³ synergy vectors scaled and shifted in time by c and τ , respectively, as shown in ⁷⁴ equation 2.

$$\mathbf{m}(t) = \sum_{i=1}^{i=r} c_i \mathbf{s}_i (t - \tau_i)$$
(2)

In this case, the model is capable of capturing fixed relationships among the
muscle activation waveforms across muscles and time. By means of comparison,
time-invariant synergies can acquire the spatial structure in the patterns but any
fixed temporal relationship can be recovered only indirectly from the weighting
functions associated with its synchronous synergy.

Although the time-varying model provides a more parsimonious representation of the muscle activity compared to the time-invariant model, some studies have shown evidence that the muscle synergies are synchronised in time [34, 10].
Therefore, most recent muscle synergies studies apply the time-invariant model
for synergy extraction. This is done by using matrix factorization techniques on
multichannel EMG activity to estimate the muscle synergies and their weighting
functions.

⁸⁷ 1.3. Comparison of Matrix factorization techniques

According to the time-invariant model, the estimation of muscle synergies 88 (spatial profile) and their weighting functions (temporal profile) from a multi-89 channel EMG signal is a blind source separation (BSS) problem. This problem 90 is approached by matrix factorisation techniques to estimate the set of basis 91 vectors (synergies). Various matrix factorisation algorithms have been applied 92 based on different constraints. The most commonly used factorisation tech-93 niques to extract synergies for myoelectric control and clinical purposes are 94 principal component analysis (PCA) [35] which was applied in [36], indepen-95 dent component analysis (ICA) [37] that was used in [30] and [38], in addition 96 to non-negative matrix factorization (NMF) [39] which have been used in [40, 32] 97 and [41]. 98

In this paper, these three techniques are compared among themselves and to 99 second-order blind identification (SOBI) [42], a technique which has not been 100 used for muscle synergy estimation previously. A first evaluation of the matrix 101 factorisation algorithms for muscle synergy extraction was reported in 2006 [43] 102 where the algorithms were tested with simulated data under different levels and 103 kinds of noise and they were applied on real data to show the similarities be-104 tween their estimated synergies. A more recent study [44] used joint motion 105 data to evaluate kinematics and muscle synergies estimated by PCA, ICA and 106 NMF using the quality of reconstructing the data by synergies as a metric for 107 evaluation. Here, we are concerned with nature and number of muscle synergies 108 and the factors that affect their quality which have not been discussed by those 109 studies. The sparsity of synergies is investigated where synthetic sparse and 110 non-sparse synergies are compared to study their effect on the matrix factorisa-111

tions. Moreover, the ratio between number of channels and synergies (dimension 112 reduction ratio) is studied. Those comparisons are carried out under different 113 noise levels to show the robustness of factorisation methods to noise. In addi-114 tion, synergies extracted from a real dataset by the four matrix factorisation 115 techniques were used to classify between wrist movements. The classification 116 accuracy was used as a metric in the factorisation methods comparison. We 117 aim to compare current matrix factorisation techniques in addition to SOBI 118 and investigate the factors that affect the quality of their extracted synergies 119 such as sparsity and channel/synergy ratio. 120

121 2. Methods

122 2.1. Real dataset

We used the Ninapro first dataset [45, 46] which consists of recordings for 53 wrist, hand and finger movements. Each movement/task has 10 repetitions from 27 healthy subjects. The dataset contains 10-channel signals rectified by root mean square and sampled at 100 Hz as shown in Figure 1. The real dataset is used in the comparison between matrix factorisation techniques. Moreover, it is used as a part of the synthetic data creation as discussed in 2.2.

For the real data comparison, the three main degree of freedoms (DoF) investigated for the wrist motion are wrist flexion and extension (DoF1), wrist radial and ulnar deviation (DoF2), and wrist supination and pronation (DoF3). Wrist movement through these three degrees of freedom are essential for prosthetic control [47]. Thus, they may highlight the application of muscle synergies in myoelectric control.

135 2.2. Synthetic data

The performance of each matrix factorisation algorithm was tested using synthetic datasets as ground truth. Since the studies [34, 10] showed an evidence that the muscle synergies are synchronised in time, the data was generated according to the time-invariant model [6] in which EMG activity for j^{th} -channel



Figure 1: Example of 10-channel EMG envelopes recorded during wrist extension movement for 5 s of Subject 4/repetition 1 (the amplitude is normalised only in figure to highlight the differences between channels).

is the summation of its coefficients in each synergy (s_{ij}) , weighted by the respective weighting function $(\mathbf{w}_i(t))$, as the following:

$$\mathbf{m}_{j}(t) = \sum_{i=1}^{i=r} s_{ij} \mathbf{w}_{i}(t) + g(\epsilon)$$
(3)

where $\mathbf{m}_{j}(t)$ is the simulated EMG data over channel j, while ϵ is a Gaussian noise vector and g(x) is the Heaviside function used to enforce non-negativity. For *m*-channel data, this model could be expanded into its matrix form. In this case, the synthetic EMG data \mathbf{M} is a matrix with dimensions (*m* channels×*n* samples) as

$$\mathbf{M}_{(m \times n)} = \mathbf{S}_{(m \times r)} \times \mathbf{W}_{(r \times n)} + g(\mathbf{E})$$
(4)

where r is the number of synergies (r < m) and \mathbf{E} is the matrix form of the Gaussian noise vector ϵ for all channels. \mathbf{S} $(m \times r)$ and \mathbf{W} $(r \times n)$ are the synergy matrix and weighting function matrix form, respectively.

In order to generate a synthetic EMG signal that mimics the real EMG data and carries the synergistic information, the three elements in equation 4

should be designed so that they reflect real activities under diverse assumptions. 152 The synergy matrix $\mathbf{S}_{(m \times r)}$ was assigned a non-negative random values between 153 [0,1] to retain the additive nature of synergies, while each weighting (activation) 154 function $\mathbf{W}_{(r \times n)}$ is a real EMG envelope randomly assigned from the Ninapro 155 dataset from different subjects and movements. This approach based on real 156 data was chosen to ensure that the generated signal retains the statistical prop-157 erties of the EMG signal rather than assigning randomly generated signals for 158 the weighting function as done in the past [43]. Finally, the non-negative part 159 of the Gaussian noise is applied to the mixture by the Heaviside function $q(\mathbf{E})$. 160 An example of the generated synthetic EMG signal is shown in Figure 2. 161

The synthetic signals were generated with different settings to compare the factorisation methods under various conditions. In all settings, the number of synergies (r) was fixed to four synergies. This choice was based on the fact that the number of synergies used in previous studies varied from one or two synergies [32] to six synergies [48], for example.

Three criteria were investigated: the sparsity of synergy matrix, the num-167 ber of channels, and the added noise level. The sparsity of the synergy matrix 168 $\mathbf{S}_{(m \times r)}$ is investigated since all muscles (channels) may be not activated during 169 a specific movement at the same time. The sparse synergies were created by 170 constraining each channel by 40% sparsity level (i.e., a maximum of for chan-171 nels being active in each synergy) to ensure that each channel has at least one 172 non-zero value in the four synergies. This approach would typically avoid hav-173 ing channels that are inactive in all 4 synergies as shown in Figure 2a as an 174 example of sparse synthetic synergies. In comparison, the non-sparse synergies 175 are non-negative random values between [0,1]. Secondly, the effect of dimension 176 reduction between the generated signal and synergies (basis vectors) is exam-177 ined. The number of synergies is fixed to 4 in all settings while the number of 178 channels are 4 (no dimension reduction), 8 or 12 channels. Finally, the effect of 179 additive signal to noise ratio (SNR) is compared at three levels: 10, 15 and 20 180 dB. In total, 10 synthetic datasets are generated, each containing 1000 separate 181 trials for each setting. 182



(a) Synthetic sparse synergies





(c) The resulting synthetic EMG dataset (after adding the noise).

Figure 2: An example of 8-channel synthetic EMG signal (Panel 2c) creation using four sparse synergies (Panel2a) and their respecting weighting functions (Panel 2b) which is a randomly selected real EMG segments with 15 dB SNR.

183 2.3. Matrix factorisation algorithms

The muscle synergy time-invariant model is approached as a blind source separation problem, where a multichannel EMG signal matrix $\mathbf{M}(t)$ is modelled as a linear mixture of synergies and "source signals". Therefore, according to equation 1, $\mathbf{M}(t)$ will follow the linear matrix factorisation model as follows

$$\mathbf{M}(t) = \mathbf{SW}(t) \tag{5}$$

In this context, **S** is the mixing (synergy) matrix while $\mathbf{W}(t)$ contains the source vectors (weighting functions) with dimensions number of synergies \times time. The noise is disregarded in equation 5. In order to estimate unique solutions, additional constraints are needed.

PCA constrains the components of the model in equation 5 to be orthogonal,

where the first component holds the largest variance and the variance progressively decreases for each component [49]. Here, PCA has been performed using the "pca" Matlab function (version 2016a).

For ICA, the fixed-point algorithm introduced in [50] has been used. Unlike PCA, ICA attempts to extract independent components by whitening the data to remove any correlation. Then, it rotates the pre-whitened data to extract the non-Gaussian components.

NMF imposes a non-negative constraint on the extracted factors. The algorithm relies on a cost function to quantify the quality of approximation between the data matrix \mathbf{M} and its factorised non-negative matrices \mathbf{S} and \mathbf{W} where $\mathbf{M} \approx \mathbf{SW}$. Values of \mathbf{S} and \mathbf{W} are updated and optimised to find the local minima numerically. The Matlab function "nnmf" (version 2016a) was used to perform the NMF based on [51].

SOBI [42] has not been applied to extract muscle synergies before. However, 206 it is included in this comparison because SOBI utilises the joined diagonalisa-207 tion of time delayed covariance matrices to estimate the unknown components. 208 Therefore, it could reveal more information about the temporal profile of the 209 EMG activity. Thus, SOBI leads to components that are uncorrelated at those 210 time delays and, therefore, it is sometimes considered an alternative to ICA, 211 which is based on higher order statistics. Here, SOBI was performed using 212 the default 4 diagonalised covariance matrices with the function "sobi" in the 213 ICALAB package [52]. 214

As an illustration, the real 10-channel EMG epoch shown in Figure 1 is factorised with the four matrix factorisation methods (PCA, ICA, SOBI and NMF) into two synergy model as shown in Figure 3.

218 2.4. Factorisation performance comparison using synthetic data

The synthetic data was used to compare the ability of the four matrix factorisation techniques to estimate the muscle synergies in three different settings (SNR, number of channels and synergies sparsity). The comparison relies on the similarity between estimated and true synergies using the correlation coefficient



Figure 3: Two-component muscle synergy extracted via the four matrix factorisation methods for the 10-channel EMG signal recorded during wrist extension movement for 5 seconds (Subject 4/repetition 1)

on the basis of full identification of true synergies and similarity level between
them. The sequence of this process is shown in Figure 4.

The first step is to match each of the extracted synergies with the true 225 ones by calculating Pearson's correlation coefficients between them. True and 226 estimated synergies with the highest correlation value are matched together. 227 This matching is done freely and unconstrained. In other words, without forcing 228 a full match (all four estimated synergies matched with all four true synergies) 229 because in some cases two or more estimated synergies have the maximum 230 correlation with the same true synergy. In those cases, the factorisation is not 231 successful since the extracted synergies failed to fully represent all true synergies. 232 Hence, the "fully matched" criterion is the ability of the factorisation method 233 to estimate fully distinctive synergies that match all true synergies without 234 duplication. The success rate for a "fully matched" is computed across the 235 10 generated datasets. It is used as a metric to judge the ability of extracted 236 synergies to fully represent all the true synergies, since a good factorisation 237



Figure 4: Block diagram for the comparison between matrix factorisation techniques.

238 would represent all of them.

In order to account to the chance that synergies may be randomly paired, the correlation coefficients between the true synergies and a set of randomly generated synergies are computed and the pairing rates are compared against for each factorisation method using a two-sample *t*-test with significance level set up at (p < 0.05).

Secondly, the correlation coefficient values for fully identified synergies are averaged for each trial. The grand average is computed for 10000 trials (1000 epochs \times 10 datasets) of each setting combination. Then, it is normalised by the random synergy's correlation coefficients (chance grand average) as baseline removal as the following:

$$Normalised \ grand \ average = \frac{(grand \ average - chance \ grand \ average)}{(1 - chance \ grand \ average)}$$

The normalised grand average of the correlation coefficients between estimated and true synergies is computed for each matrix factorisation method with all different combination of the 3 settings (SNR levels, number of channels and sparsity). This criterion is an indicator of general factorisation quality. Therefore, we statistically analysed it to compare the factorisation techniques and the effect of all 3 settings using the 2-way ANOVA method with the significance level at (p < 0.05).

251 2.5. Factorisation performance comparison using Real data

Since there is no ground truth to compare each technique with for the real data, we compared the techniques regarding their application for prosthesis control. In several studies [31, 53], muscle synergy is used as a feature to classify different hand and wrist movements. Therefore, the factorisation techniques are assessed according to their classification accuracy for the 3 main wrists DoF.

To this end, the Ninapro real dataset is divided into training and testing 257 sets with 60% (6 repetitions of each task) of the data assigned to training for 258 each subject. For each factorisation technique, synergies are estimated from 259 training repetitions for each task. Those synergies are used to train k-nearest 260 neighbours (k-NN) classifier (k=3 for simplicity). Four classifiers are trained 261 using the training synergies, three of them to classify between 2 tasks of each 262 wrist DoF while the 4^{th} classifier is trained to classify between all 6 tasks. The 263 number of synergies extracted was one for each repetition (two for each DoF) 264 as in [32] to avoid permutation issues. The testing dataset - which contains 265 the other four repetitions of each task - is used to test those classifiers. One 266 synergy is estimated directly from each task repetition in the test set using the 267 four factorisation methods and used to predict the task through the trained 268 classifiers. The classification error count for each DoF is used to evaluate the 260 factorisation techniques. 270

271 2.6. Number of synergies

For the classification accuracy comparison using real datasets, the functional approach to determine number of synergies were chosen. A one-synergy model was applied for EMG activity of each movement. On the other hand, for the synthetic dataset comparison, the number of underlying synergies was known to be four.

The generated synthetic dataset can also be used to test the mathematical methods to determine the number of synergies. The minimum description length (MDL) was chosen as an alternative to the explained variance methods as the latter is biased towards PCA since this relies on maximising the explained variance on the first components. The MDL method determine the number of
synergies that could minimise the MDL. For more details please see Appendix
Appendix A.

In this study we use the synthetic dataset to test the ability of MDL method 284 to estimate the required number of synergies across various settings (Sparsity, 285 noise and channel to synergy ratio). Since four true synergies are used, only 286 the 8 and 12 channels datasets were investigated as the MDL boundary cannot 287 estimate number of synergies when it is equal to channels. This is not a prob-288 lem in practical applications since the muscle synergy hypothesis implies the 280 concept of dimension reduction. In addition, three level of SNR (10, 15 and 20 290 dB) of sparse and non-sparse datasets were explored with 1000 trials for each 291 combination. The result for correct estimation of synergies number is analysed 292 via analysis of variance (ANOVA) and multiple comparison of population. 293

294 3. Results

295 3.1. Number of synergies

The model selection method based on MDL was examined with the synthetic EMG data where the number of synergies are known (four synergies). The MDL method was tested on 1000 trials for each combination of sparsity, three levels of noise and two number of channels (8 and 12 channels).

The ANOVA shows that sparsity has no significant effect on the estimation 300 of the correct number of synergies p > 0.05, while number of channels has a 301 significant effect with [F(1,11) = 19.94, p = 0.003] as 12-channels datasets 302 performs better than 8-channel signals (shown in Figure 5). As for the level 303 of noise, the 10 dB SNR had a significantly worse performance than 15 and 304 20 dB SNR with the effect of noise significant at [F(2, 11) = 24.22, p = 0.007]305 by 1-way ANOVA. This indicates that, the MDL method for estimating the 306 correct number of synergies performs better with lower noise and more available 307 channels, as expected. 308



Figure 5: Percentage of correct synergy number estimation using the MDL method across the three settings (noise, number of channels and sparsity).

309 3.2. Factorisation performance comparison using synthetic data

The four matrix factorisation methods were compared on the basis of two 310 criteria: synergy full identification success rate and the normalised grand aver-311 age of correlation coefficients for the fully identified synergies. The comparison 312 was done on 10000 trials (10 datasets of 1000 trails) for each combination of 313 the three settings (sparsity, SNR and number of channels). An example of one 314 setting of non-sparse, 12-channel with 15 dB SNR is shown in Figure 6. All the 315 four factorisation techniques had converged for all trails except for ICA which 316 failed to converge in 1.48% of the trails. 317

The four factorisation methods were assessed by their ability to fully identify 318 all 4 true synergies by matching them according to their Pearson's correlation 319 coefficients values. In order to rule out any statistical chance from it, a two-320 sample t-test was conducted to compare the success rate of each technique 321 and the randomly generated synergies. All the techniques succeeded to reject 322 the null hypothesis (p < 0.05) for all the settings. Hence, there is a significant 323 difference between the matching success rate for each of the matrix factorisation 324 methods and the randomly generated synergies. An example of the success rate 325 for one of the settings is shown in Figure 6a, while the average success rate to 326 fully identify the true synergies for all settings is represented in Figure 7. NMF 327



Figure 6: The results for non-sparse, 12 channels dataset with 15dB SNR. Panel 6a, the success ratio for the factorisation techniques to fully match the true synergies is shown. Panel 6b, the normalised similarity values for each technique single trial with the same settings. Error bars indicate standard deviation.

³²⁸ and PCA are has the highest success rates to fully identify synergies.

The correlation coefficients of the matched synergies were normalised by 329 the random synergy correlation coefficients as shown in Figure 6b. Then the 330 normalised correlation coefficient of synergies (synergy matrix) were averaged 331 across trials. The grand average for each factorisation method was normalised 332 by the chance's grand average. In Figure 8, the normalised grand average (simi-333 larity metric) for the four matrix factorisation methods is plotted for all different 334 settings (sparsity, number of channels and noise level). It is worth mentioning 335 that although NMF have the highest similarity for all settings except for the 336 four channel case (the results for the sparse, four-channel setting for NMF are 337 mostly negative). On the other hand, all four algorithms perform worse with 338 four channels (no dimension reduction) with SOBI being the best algorithm 339 among them in this case. 340

In order to explore the significance of those settings the two-way ANOVA was performed with post-hoc multiple comparison test. The result shows that



Figure 7: Violin graph for the success rate of full synergy identification for each method across all settings. The mean and median are represented in the Figure as red crosses and green squares respectively.

number of channels and sparsity had a significant effect on the grand normalised 343 average at $[F(2,688)=1364.5, p \le 0.05]$ and [F(1,400)=7.35, p=0.007] respec-344 tively. The multiple comparison test shows that sparse synergies and the higher 345 number of channels show better similarity levels. On the other hand, the noise 346 level fails to reject the null hypothesis. This means that the level of noise used in 347 these experiments did not affect the quality of estimated synergies significantly 348 unlike the sparsity or number of channels. In addition, this was supported by 349 the interaction results, where factorisation methods and number of channels in-350 teraction showed a significant effect on the grand normalised average, as well 351 as factorisation method and sparsity interaction. On the contrary, the noise 352 level and factorisation techniques interaction have no significance on the grand 353 normalised average. 354

The computational efficiency was compared after each technique ran for 100 times on Matlab 9 with Intel core i7 processor(2.4 GHz, 12 GB RAM) and the median value for the running time were computed. PCA and SOBI were the fastest with (0.0012 s and 0.0015 s) respectively followed by NMF with 0.0063 s while ICA was significantly slower by 0.6419 s.



Figure 8: The normalised grand average of correlation coefficients for the fully identified synergies compared across all 3 settings (sparsity, SNR and number of channels) for the 4 matrix factorisation methods. Error bars indicate standard deviation.

360 3.3. Factorisation performance comparison using Real data

An example of the four matrix factorisation methods is shown in Figure 3 by applying them on 10-channel EMG data. In order to show the similarities and differences in the estimated synergies and their weightings functions of each technique. For example, synergies extracted by PCA and SOBI have similarities in this example since both techniques are based on covariance matrices. The number of synergies needed in this example was chosen to be two according to the MDL method.

In addition, to compare between the matrix factorisation techniques, a onecomponent synergy was used to train a k-NN classifier (k=3) in order to classify between two antagonistic movements (one DoF) for each technique. This was calculated for the three wrist DoFs separately as shown in Table 1. In addition, the same synergies were used to classify between all six movements (three DoFs). The average classification error rate and its standard deviation for the 27 subjects is also represented in Table 1.

³⁷⁵ 4. Discussion and Conclusion

In this paper, we compared the most common matrix factorisation techniques (PCA, ICA and NMF) for muscle synergy estimation alongside SOBI,

	PCA	ICA	SOBI	NMF
DoF1	1	28	8	1
(wrist flexion	(0.46%)	(12.06%)	(3.70%)	(0.46%)
and extension)	(0.4070)	(12.9070)	(3.7070)	(0.4070)
DoF2	19	20	10	1
(wrist radial and	12	$\frac{29}{(12,4907)}$	19	(0.46%)
ulnar deviation)	(0.0070)	(10.4370)	(0.0070)	(0.4070)
DoF3	7	21	18	5
(wrist supination	(2.9407)	(14.95%)	10 (0 2207)	(9.2107)
and pronation)	(0.24/0)	(14.00/0)	(0.0070)	(2.3170)
All 3 DoFs	43	122	65	41
(all 6 movements)	(6.64%)	(18.83%)	(10.03%)	(6.33%)

Table 1: The classification error count and (error percentage) for each wrist's DoF (Sample size=216) and all 3 DoFs (sample size=648) across 27 subjects

a BSS method that had not been applied for synergy extraction yet. Many studies rely on muscle synergy concept such as myoelectric control and biomechanical research. However, only two studies [43, 44] compared various factorisation methods (excluding SOBI) for synergy estimation without investigating the factors that affect the factorisation quality - except for noise.

Herein, the comparison was held on real data and synthetic signals generated 383 with known synergies and under different settings. Using the synthetic data we 384 studied the effect of those settings on the muscle synergy extraction for each 385 technique. The sparsity nature of synergies and level of noise was investigated 386 in addition to the number of channels needed to extract the four synthetic 387 synergies. The ability of the four factorisation methods to extract synergies 388 from synthetic data was judged according to two metrics: success rate to fully 389 identify synergies (Figure 7) and the correlation coefficients between true and 390 estimated synergies (Figure 8). Moreover, the synthetic data was used to assess 391 the MDL method to determine number of synergies needed under those three 392

393 settings.

For the real datasets, since there is no ground truth to compare synergies estimated, we compared the factorisation methods according to the ability of their extracted synergies to classify wrist movements (Table 1) as a proof of concept for prosthesis control [30, 40]. PCA and NMF had the best classification accuracy followed by SOBI, while ICA had the lowest accuracy.

On the other hand, the synthetic datasets results showed that NMF and 399 PCA had better success rate to fully identify the four true synergies than SOBI 400 and ICA. However, NMF and SOBI had the best normalised grand average of 401 correlation coefficients (similarity level) between estimated and true synergies 402 followed by PCA then ICA. Notably, NMF performed poorly with four-channel 403 datasets when there was not any dimension reduction. In general, all algorithms 404 perform better with higher number of channels compared to synergies, where 405 SOBI was the best algorithm when there is no dimension reduction. There-406 fore, SOBI would be a relevant algorithm in situations with limited number of 407 electrodes as it is preferable to minimise the number of electrodes for practical 408 prosthesis control [54, 55]. 409

The two-way ANOVA showed that the tested range of SNR has no signifi-410 cance effect on the factorisation performance, although it is noticed that ICA 411 was the most unaffected method to noise according to the multiple compari-412 son test. On the other hand, sparsity had a significant effect (p < 0.05) on the 413 correlation between true and estimated synergies. According to the multiple 414 comparison test, the sparse synergies are easier to estimate by all factorisation 415 methods. Moreover, number of channels shows a significant effect (p < 0.05) on 416 the correlation between estimated synergies and true ones. In addition, higher 417 number of channels to number of synergies ratio provides better synergy extrac-418 tion. 419

Regarding the estimation of the number of synergies, the multichannel EMG signal is reduced into a lower subspace for the purpose of synergy extraction. The estimation of this subspace's dimension or, in other words, the number of synergies is crucial for the factorisation process. In the literature, there are

two main approaches to determine the appropriate number of synergies: the 424 functional and the mathematical ones. The functional approach determines the 425 number of synergies according to the myoelectric control requirements such as 426 assigning two [56, 57] synergies for each DoF. On the other hand, the math-427 ematical approach relies on explained variance (using tests such as scree plot 428 and Bart test) or the likelihood criteria (such as Akaike information criteria and 429 MDL) [58]. Here, we explored the MDL as an alternative for variance explained 430 methods. The results show that MDL performs better with higher channel to 431 synergy ratio. This supports the current challenges for effective synergy iden-432 tification with limited number of electrodes. However, further investigation is 433 needed to compare between different number of synergies estimation methods 434 using synthetic datasets with various settings. 435

Other limitations are worth noting. The results may be biased towards NMF 436 due to the non-negative nature of the simulated synergies. However, this choice 437 is supported by previous studies [40] which suggested the usefulness of NMF 438 due to the additive nature of the synergies. In addition, further examination is 439 needed if the setting of EMG acquisition changes dramatically (really bad SNR, 440 much higher number of channels, etc.) to evaluate the validity of our conclusions 441 in those settings. Finally, since various studies employ the muscle synergy in 442 prosthesis control, a simple approach (k-NN classifier) was used in this paper as 443 an example to guide synergy application and to support the synthetic results. 444 We treated this part of the study as a proof of concept. Additional work is 445 needed with more advanced techniques and variety of tasks and movements. 446

In conclusion, this paper compared matrix factorisation algorithms for mus-447 cle synergy extraction and the factors that affect the quality of estimated syn-448 ergies. Our findings suggest that the presence of sparse synergies and higher 449 number of channels would improve the quality of extracted synergies. When 450 the number of channels equal to synergies (no dimension reduction), SOBI per-451 formed better than other methods although the performance was still poor in 452 this case. Otherwise, NMF is the best solution for robust synergy extraction 453 when number of channels/muscles is higher than the required muscle synergies. 454

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⁶⁷² Appendix A. Minimum description length (MDL)

The MDL method for determining the number of synergies is performed by calculating the maximum likelihood estimates of factor loading matrix \mathbf{A} and the unique variances diagonal matrix $\boldsymbol{\Psi}$ according to the factor analysis model

$$\mathbf{C} = \mathbf{A}\mathbf{A}^T + \boldsymbol{\Psi} \tag{A.1}$$

where **C** is the covariance matrix of $\mathbf{M}_{m \times n}$ the multi-channel EMG signal matrix with *m* channels and *n* samples.

This is done for different number of synergies (r) between $1 \le r \le \frac{1}{2}(2m+1-\sqrt{8m+1})$ in order to minimise the MDL. The boundary for r is set by comparing the number of equations with unknowns in order to have an algebraic solution for equation A.2.

$$L(\mathbf{A}, \boldsymbol{\Psi}) = -\frac{1}{2} \left\{ \operatorname{tr}(\mathbf{C}(\boldsymbol{\Psi} + \mathbf{A}\mathbf{A}^T)^{-1}) + \log(\det(\boldsymbol{\Psi} + \mathbf{A}\mathbf{A}^T)) + m\log 2\pi \right\}$$
(A.2)

$$MDL = -L(\mathbf{A}, \boldsymbol{\Psi}) + \frac{\log n}{n} \left(m(r+1) - \frac{r(r-1)}{2} \right)$$
(A.3)

The number of synergies r are selected to minimise the MDL value in equation A.3.