



Technische Universität Ilmenau

Fakultät für Maschinenbau

Master Thesis

Modeling and track planning for the automation of BMW model car

To achieve the degree of:

Master of Science (M. Sc.)

in Mechatronische Systeme

Submitted by: Rubén Toshiharu Tabuchi Fukuhara Date and Place of Birth: 24 April 1989, Lima - Perú

Supervisor (TU Ilmenau): M.Sc. Shih-Jan Lin Supervisor (PUCP): Phd. Julio Tafur Date and Place: 15 May 2017, Ilmenau - Germany



Abstract

In recent years, autonomous driving technologies have become a topic of growing interest due to the promise of safer and more convenient mode of transportation. An essential element in every autonomous driving system is the control algorithm. Classical control schemes, like PID, are not able to manage Multiple Inputs-Multiple Outputs, complex, non-linear systems. A more recent control strategy is Model predictive control (MPC), a modern control method that has shown promising results in systems with complex dynamics. In MPC, a sequence of optimal control inputs are predicted within a short time horizon based on the car dynamics, and soft or hard restriction of the system.

In this work, three different nonlinear-MPC (NMPC) controllers were formulated based on a kinematic, and two dynamic models (double-track and single-track). The steering system's dynamics were additionally identified using experimental data. Each MPC was solved applying direct methods, by transforming the optimal control problem to a Nonlinear programming (NLP) problem using the Multiple shooting scheme with a Runge-Kutta 4 integrator. The NLPs were solved using the state-of-the-art optimization solver IpOpt. Before the real-time implementation, all the NMPC controllers were simulated in different scenarios and multiple configurations. The results allowed to select the most suitable controllers to be implemented in a 1:5 scale robotic car.

Finally, two NMPC controllers based on the kinematic, and the single-track dynamic model were implemented in the robotic car. The algorithms were tested in two different scenarios at the maximum possible speed. The obtained results from the tests were very promising, and provide compelling evidence that MPC could be implemented as the core of future autonomous driving algorithms, since it computes the optimal control inputs, taking in consideration the restrictions inherent to the system.



Zusammenfassung

In den letzten Jahren sind autonome Fahrtechnologien durch das Versprechen eines sicheren und bequemen Transportmittels immer meht in den wissenschaftlichen Fokus gerückt. Ein essenzielles Element in jedem autonomen Antriebssystem ist der Steueralgorithmus. Klassische Kontrollsysteme, wie PID, sind nicht in der Lage, Multiple Inputs-Multiple Outputs, sowie komplexe, nichtlineare Systeme zu regeln. Eine neuere Kontrollstrategie ist die Modellprädiktive Regelung (engl. Model Predictive Control), eine moderne Steuerungsmethode, die vielversprechende Ergebnisse in Systemen mit komplexer Dynamik gezeigt hat. In MPC wird eine Sequenz von optimalen Steuereingängen innerhalb eines kurzen Zeithorizonts unter Berücksichtigung der Dynamik und weicher oder harter Systemebeschränkung prädiktiert.

In dieser Masterarbeit wurden drei verschiedene nichtlineare-MPC (NMPC) Regler unter Verwendung eines kinematischen, und zwei Dynamische (Einspurmodelle und Zweispurmodelle) Fahrzeugmodells formuliert. Die Dynamik des Lenksystems wurde auch anhand von experimentellen Daten identifiziert. Jeder MPC wurde durch direkte Methoden gelöst, indem die optimalen Kontrollsysteme unter Verwendung des Mehrfachschießverfahren mit einem Runge-Kutta 4 Integrator zu nichtlinearen Optimierungen (NLP) Problem umgewandelt wurden. Die NLPs wurden mit dem hochmodernen Optimierungssolver IpOpt gelöst. Vor der Echtzeit-Implementierung, wurden alle NMPC-Regler in verschiedenen Szenarien und mehreren Konfigurationen simuliert. Die Ergebnisse erlaubten es, die besten Steuerungen auszuwählen, die in einem Modellfahrzeug im Maßstab 1:5 implementiert werden sollen.

Schließlich wurden die kinematischen, und das Einspurmodelle NMPC-Regler im Modellfahrzeug implementiert. Die Algorithmen wurden in zwei verschiedenen Szenarien mit der maximal möglichen Geschwindigkeit getestet. Die erzielten Ergebnisse des Tests waren sehr vielversprechend und lieferten überzeugende Hinweise darauf, dass MPC als Kern der zukünftigen autonomen Fahralgorithmen implementiert werden kann, da die optimalen Steuereingänge unter Berücksichtigung der Systemebeschränkungen berechnet.



Acknowledgments

I would first like to thank my practical advisor M.Sc. Shih-Jan Lin, for his support during the implementation of the developed NMPC algorithm in the robotic car. The testbed vehicle and image processing algorithm he developed were important parts to complete the autonomous driving algorithm implementation. I also thank Prof. Pu Li, for accepting me into his group. The door to Prof. Pu Li office was always open whenever I had a question about my research. My sincere thanks to Dr. Julio Tafur, for being my co-advisor and for providing worthy comments and advice.

I would also like to thank my friends Max, Felipe, and Sihela for their helpful and inspiring comments.

My deeply thanks to CONCYTEC for the financial support during my stay in Germany.

Last, but not least, I want to thank Alexandra Elbakyan and her brave initiative. Without the website she started, it wouldn't have been possible to gather enough information to make this work.



Contents

Ab	ostrac	TENFRA	3				
Zu	Zusammenfassung 5						
Ac	know	ledgments	7				
-			•				
I	Intro	oduction	3				
	1.1	Motivation	3				
	1.2	Problem Statement	5				
	1.3	Overview	6				
2	Des	cription of the 1:5 scale robotic car	7				
	2.1	General description	7				
	2.2	Physical properties	9				
	2.3	Drivetrain and steering components description	10				
	2.4	Sensors and peripherals for data acquisition	11				
	2.5	On-board computer	13				
	2.6	Data communication interface	13				
3	Con	uputational methods for real-time					
	opti	mal trajectory tracking	15				
	3.1	Motivation for the use of Model predictive control (MPC) $\ldots \ldots \ldots$	16				
	3.2	Principle of MPC	17				
	3.3	Nonlinear model predictive control (NMPC)	18				
	3.4	Direct methods for solving optimal control problems	19				
		a. Direct single shooting	20				
		b. Orthogonal collocation on finite elements	20				
		c. Direct multiple shooting	21				

Contents

	3.5	ion of the Nonlinear programming (NLP) solver	23						
		a.	IpOpt C++	24					
		b.	CasADI	24					
4	Mat	hemati	ical models for vehicles	25					
	4.1	Steerin	ng system's dynamics	26					
		a.	System identification	26					
		b.	Identified model for the steering system	27					
	4.2	Kinem	natic model	28					
	4.3	3 Dynamic models for lateral dynamics							
		a.	Tire models	29					
			a.1. Magic formula tire model	30					
			a.2. Linear tire model	31					
		b.	Double-track model for lateral dynamics	32					
		c.	Single-track model for lateral vehicle dynamics	34					
	4.4	Paran	neters for the vehicle models	36					
	4.5	Mecha	anical and operational bounds	37					
5	NM	PC cor	troller design for autonomous driving	39					
	5.1	Kinen	natic model based NMPC controller	40					
		a.	Cost function	41					
		b.	Variables bounds	41					
	5.2	.2 Dynamic models based NMPC controllers							
		a.	Cost function	42					
		b.	Variables bounds	43					
	5.3	b. Obsta	Variables bounds	43 43					
	5.3	b. Obsta a.	Variables bounds cle avoidance Obstacle as a closed region	43 43 43					
	5.3	b. Obsta a. b.	Variables bounds	43 43 43 43 44					
	5.3 5.4	b. Obsta a. b. NMPO	Variables bounds	43 43 43 43 44 45					
	5.3 5.4	b. Obsta a. b. NMPC a.	Variables bounds	43 43 43 43 44 45 45					
	5.3 5.4	b. Obsta a. b. NMPC a. b.	Variables bounds	 43 43 43 43 44 45 45 47 					
6	5.3 5.4 Sim	b. Obsta a. b. NMPC a. b.	Variables bounds	43 43 43 44 45 45 45 47					
6	5.3 5.4 Sim	b. Obsta a. b. NMP(a. b. ulations	Variables bounds	43 43 43 44 45 45 45 47 49					
6	5.3 5.4 Sim algo 6.1	b. Obsta a. b. NMP(a. b. ulations rithms	Variables bounds	43 43 43 44 45 45 45 47 49 50					
6	5.3 5.4 Sim algo 6.1	b. Obsta a. b. NMP(a. b. ulations rithms Vehicl a.	Variables bounds	43 43 43 44 45 45 47 49 50 50					
6	5.3 5.4 Sim algo 6.1	b. Obsta a. b. NMP(a. b. ulations rithms Vehicl a. b.	Variables bounds	43 43 43 43 44 45 45 45 47 49 50 50 50					
6	5.3 5.4 Sim algo 6.1	b. Obsta a. b. NMP(a. b. ulations rithms Vehicl a. b. c.	Variables bounds	43 43 43 44 45 45 45 45 47 49 50 50 50 51 52					

Contents

	6.3	Path tracking simulations	54
		a. Kinematic model results	54
		b. Double-track model results	56
		c. Single-track model results	58
		d. Performance comparison between vehicle models	59
		e. Phase difference of the steering angle and the input signal due to	
		delay	62
	6.4	Obstacle avoidance simulations	63
		a. Kinematic model results	63
		b. Single-track model results	65
7	Exp	erimental evaluation of autonomous driving algorithms	67
	7.1	Command center software	69
	7.2	Path planning	70
		a. Reference points and look-ahead distance	70
		a.1. Path curve identification applying image processing	70
		a.2. Parameters and limits for the path curve parametrization	72
	7.3	Experimental results in the double lane change maneuver test track $\ . \ .$	73
		a. Kinematic model experimental results	73
		b. Single-track model experimental results	75
	7.4	Experimental results in the circular test track	77
		a. Kinematic model experimental results	77
		b. Single-track model experimental results	79
	7.5	Influence of the steering's dynamics in the real-time performance of the	
		vehicle	81
8	Con	clusions and future work	85
•	8.1	Conclusions	85
	8.2	Future work	88
	0.2		00
Ri	bliogi	raphy	91



List of Figures

1.1	Autonomous driving representation [1]	3
2.1 2.2	1:5 scale BMW model car	8
	computer	8
2.3	1:5 scale robotic car dimensions	9
2.4	Electronic Speed controller (ESC) EZRUN-150A-PRO	10
2.5	Hitec HS-5805MG servomotor	11
2.6	CMOS Camera MT-HDM108	11
2.7	Hokuyo UTM - 30LX	12
$3.1 \\ 3.2$	Model predictive control working principle [1]	17
	shooting method. Extracted from [2]	22
4.1	Experiment to identify the steering system's model	26
4.2	Servo response to step input	27
4.3	Identified model vs measured data. Step input response	27
4.4	Kinematic model for the car	28
4.5	Lateral tire forces in pure cornering (s=0). Graph extracted from $[3]$.	30
4.6	Comparison of Magic formula tire model (Pacejka 2006) vs linear tire	
	model. Graph extracted from [3]	31
4.7	Double-track dynamic model	32
4.8	Slip angle scheme [4]	33
4.9	Single-track dynamic model	35
5.1	NMPC controller based on the Kinematic model. Block diagram for	
	Inputs-Outputs	40

5.2	NMPC controller based on the Dynamic model (Double-track or Single-	
	track). Block diagram for Inputs-Outputs	42
5.3	Simulation program block diagram	46
5.4	Real-time experimental software block diagram	48
6.1	Circular track curve	50
6.2	Double lane change maneuver curve. Based on [5]	51
6.3	Obstacles testing track.	52
6.4	Simulation results for path tracking control with the Kinematic model in two different trajectories. $v_r = 2.22 \ m/s \ [8 \ km/h]$. Prediction horizons	
	length $N = 5$, 10, 15,,,,,,,	54
6.5	Simulation results for the reference tracking control with the four wheels	-
	model in two different trajectories. Different prediction horizons N were	
	tested	56
6.6	Simulation results for the reference tracking control with the bicycle	
	model in two different trajectories. Different prediction horizons N were	
	tested	58
6.7	Average computational time per iteration for the simulations of the car	
	models with different prediction horizon lengths N $\ldots \ldots \ldots \ldots$	59
6.8	Trajectory and control inputs comparison between NMPC controllers with	
	different vehicle models. Prediction horizon length $N = 15$. Sampling	
	rate $\Delta t = 100 \ ms.$	61
6.9	Servomotor phase difference between input signal and actual position.	
	Simulated with the kinematic model $(N = 10)$ at $v_x = 5m/s$	62
6.10	Simulation results for obstacle avoidance with the Kinematic model.	
	$v_x = 2.22 \ m/s \ [8 \ km/h].$ Different prediction horizons N were tested	63
6.11	Simulation results for obstacle avoidance with the Bicycle model. $v_{ref} =$	
	2.22 m/s [8 km/h]. Different prediction horizons N were tested	65
7.1	1:5 vehicle test tracks	68
7.2	User interface for vehicle control	69
7.3	Camera's raw stream (top) and geometric features extraction (bottom) $\ $.	71
7.4	Reference points obtained from the camera	72
7.5	Experimental results for path tracking control with the Kinematic model	
	(N=10) . $v_x = 3.33 \ m/s \ [12 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$	74
7.6	Experimental results for path tracking control with the Dynamic model	
	(N=10) . $v_{ref}=3.33~m/s$ [12 km/h]. Sampling rate $\Delta t=100~ms$ $~$	76

7.7	Experimental results with the Kinematic model $(N=10)$ in the circum-		
	ference test . v_x = 5.55 m/s [20 km/h]. Sampling rate Δt = 100 ms		
		78	
7.8	Experimental results for path tracking control with the Single-track model		
	(N=10) . $v_{ref} = 5.55 \ m/s \ [20 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$	80	
7.9	Steering's dynamics influence in the performance of the NMPC controllers.		
	Vehicle model: Kinematic (N=10) . $v_x=3.33\ m/s$ [12 km/h]. Sampling		
	rate $\Delta t = 100 \ ms$	82	
7.10	Steering's dynamics influence in the performance of the NMPC controllers.		
	Vehicle model: Kinematic (N=10) . $v_x = 5.55 \ m/s \ [20 \ km/h]$	83	
7.11	Steering's dynamics influence in the performance of the NMPC controllers.		
	Vehicle model: Single-track model (N=10).	83	





List of Acronyms

AD	Automatic differentiation
BEC	Battery elimination circuit
BFGS	Broyden–Fletcher–Goldfarb–Shanno
BVP	Boundary value Problem
CAS	Computer algebraic system
COG	center of gravity
ESC	Electronic Speed controller
IPM Interior point methods	
IPM	Interior Point methods
IpOpt	Interior Point Optimizer
IVP	Initial value problem
LTV-MPC	Linear time variant MPC
MIMO	Multiple Inputs - Multiple Outputs
MPC	Model predictive control
NLP	Nonlinear programming
NMPC	Nonlinear model predictive control

Master thesis Toshiharu Tabuchi

1

List of Figures

OCP	Optimal contro	ol problem
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RC Radio-controlled

RHC Receding horizon control

RK4 Runge-Kutta 4

SQP Sequential quadratic programming



Chapter 1

Introduction

1.1 Motivation



Figure 1.1 – Autonomous driving representation [1]

In recent years, autonomous driving technologies have become a topic of growing interest due to the promise of safer and more convenient mode of transportation. According to the Association for Safe International Road Travel (ASIRT), nearly 1.3 million people die in road accidents every year, making an average of 3287 deaths per day. Additionally, 20-50 million end injured or disabled [6]. Of these crashes, an estimated of 94% are attributed to driver's errors. From table (1.1), it can be seen that "Recognition error", which included driver's inattention, and internal or external distractions, was the most frequently (41%) reason for accidents. Decision error such as miscalculation of speed for weather conditions, entry curve speed, or false assumption of other drivers maneuvers was the second most common cause (33 %) [7]. Autonomous vehicles can potentially reduce the number of accidents and stress-related-to-driving illness by using partial assistance or fully autonomous driving. Moreover, seniors and people with disabilities may also be benefited, as a result of the increase in freedom and independence. In the future, centralized systems could reduce commuting time by efficiently choosing the timing in every intersection and reduce the stress related to driving.

Critical Reason	Percentage
Recognition error	41%
Decision error	33%
Performance error	11%
Non-performance error (sleep,etc)	7%
Other	8%
Total	100%

Table 1.1 – Driver-related critical reasons. Extracted from [7]

An increasing number of competitions to test the capabilities of autonomous vehicle algorithms show the great interest to this type of technologies in the recent years. One of the most recent milestone for fully autonomous ground vehicles was the DARPA Grand challenge 2005, a 212 km off-road course with different number of difficulties, in which 5 of the 23 teams completed successfully the course. [8]. Later, the DARPA urban challenge was created to address more challenging task like avoiding moving obstacles and navigating in confined environments, following more complex rules. Since then, other competitions have been carried out. The most notables are the Intelligent Vehicle Future Challenge, the Hyundai Autonomous Challenge, the Vislab Intercontinental Autonomous Challenge, the Public Road Urban driverless-car test, and the autonomous drive of the Bertha-Benz historic route [9].

An essential element in every autonomous driving system is the control algorithm. Classical control schemes, like PID, have already been tested in vehicles for realistic scenarios like the DARPA Grand challenge. However, without heuristic functions, this type of controllers exhibit poor stability and disturbance rejection, even at moderate speeds [9, 10]. A more recent control strategy is Model predictive control (MPC), a modern control method that has shown promising results in systems with complex dynamics. In MPC, a sequence of optimal control inputs are predicted within a short time horizon using the dynamics, and soft or hard restriction of the system. Improvements in computational hardware and numerical algorithms have allowed the implementation of MPC for real time applications like driverless vehicles.

Another major difficulty for many researchers are the high costs associated with experimentation on real-size vehicles. These costs may include the vehicle itself, its maintenance and the required facilities for testing. In this thesis, in order to explore the capabilities of MPC in autonomous driving within an acceptable budget, an algorithm will be implemented in an 1:5 scale robotic car. Other works that also use model scale cars as testbeds for autonomous driving are [11–14], but only Liniger [13] and Verschueren [14] implement optimal controllers. Model scale cars allow realistic testing scenarios compared to virtual simulations, but without all the inconveniences of full sized cars. This work will implement both simulation and experimental tests to evaluate the performance of the proposed Nonlinear model predictive control (NMPC) controller.

1.2 Problem Statement

The main objective of this thesis assignment is the implementation of a control algorithm in an 1:5 scale robotic car that provides lane keeping and trajectory tracking capabilities. A controller using the principle of non-linear MPC (NMPC) with a suitable vehicle model is proposed. A monocular CMOS camera and an image processing algorithm provide the information to compute the reference in real time. In order to solve this problem, the following tasks will be properly accomplished in the present work:

- Review of the direct methods approach for optimal control theory and state-ofthe-art nonlinear numerical optimization solvers.
- Formulation of different mathematical models that describe the movement law of the 1:5 scale robotic car.
- Design, simulation, and evaluation of different NMPC controllers based on the proposed vehicle models.
- Selection and implementation of real-time NMPC controllers to be implemented in the 1:5 scale robotic car, based on simulation results.
- Evaluation of the real-time NMPC controllers in preselected driving situations.

1.3 Overview

The content presented in this thesis is organized as follows:

- *Chapter 2* describes the 1:5 scale robotic car in which our NMPC controllers are implemented and tested. The physical parameters required for the vehicle models are listed. Also, a brief description of the sensors, actuators, and the on-board computer are given.
- *Chapter 3* introduces concisely the principle of MPC control scheme, as well as the direct methods used for solving optimal control problems. Finally, the software tools used to program our NMPC controllers are presented.
- *Chapter 4* presents the mathematical models used to describe the movement law of the robotic car. Three models are proposed. A Kinematic model is deduced from geometrical properties, without considering inertial effects and forces in the wheels. Second, a Double-track dynamic model is presented, in which inertial effects and forces are introduced to increase the accuracy. Third and final, a Single-track dynamic model is deduced as a simplification from the previous one.
- *Chapter 5* formulates the NMPC controllers using the Kinematic and dynamic models presented in the last chapter. The cost function, constraints, and boundaries for trajectory tracking and obstacle avoidance are presented.
- *Chapter 6* simulates the NMPC controllers on three scenarios and different prediction horizon lengths. The results are presented and analyzed in order to select the controllers to be implemented in the robotic car.
- *Chapter* 7 implements two selected NMPC controllers, the Kinematic and Singletrack model, in the robotic car. The autonomous algorithms are tested in two different test tracks. Also, the influence of the steering's dynamics in the system stability are investigated.
- *Chapter 8* summarizes the obtained achievements and results in this work, as well as the problems encountered. Finally, some future remarks are given, as suggestions for improvement and problem solving.

Chapter 2

Description of the 1:5 scale robotic car

In this chapter, general descriptions of the 1:5 scale robotic car physical characteristics, and its components such as actuators, sensors, and computational hardware are presented. As mentioned in the chapter 3, the mathematical model of the system is a key element in an MPC controller. Then, in order for this model to be complete, the correct parameters has to given, so that it can match, as close as possible, the real dynamics of the system. These parameters were carefully obtained by measuring the robotic car.

Four main groups represent the vehicle car components. The first main group is the rolling chassis, which includes the vehicle chassis, wheels, suspensions, and whole powertrain. Second is the on-board computer, which combines a mini-ITX motherboard with a desktop size CPU. Third comes the sensors group, which consists of a CMOS camera, LIDAR sensor, tachometer, 6-axis IMU, and GPS unit. The last group is the power system, which gathers the LiPo batteries, power regulators and monitoring sensors.

2.1 General description

The vehicle used in the present work is a self-contained robotic system, which has an on-board computer and multiple sensors, that allows it to drive autonomously without relying on external systems, like in [11–14]. The robot was built upon an 1:5 scale Radio-controlled (RC) car, model "4WD 530E" from the manufacturer FG Modellsport [15]. A high-torque brushless motor provides power to the four-wheels, and allows the car



(a) BMW model car with painted body

Figure 2.1 – 1:5 scale BMW model car

to reach speeds above 100 km/h, according to the manufacturer. The rolling chassis, shown in figure (2.1b) is composed by a high rigidity aluminum frame, suspension system, driveshaft, brushless motor and wheels. The frame allocates two LiPo batteries, with capacities of 6S - 16000 mAh, and 4S - 16000 mAh. The first battery powers the actuators of the car, while the second the on-board computer and sensors. The power sources are isolated to reduce electrical noise interference. The computer and peripherals are mounted over a PMMA (acrylic glass) platform, on top of chassis, as shown in figure (2.2).



Figure 2.2 – Internals of the robotic car. 1) CMOS camera. 2) LIDAR. 3) On-board computer

2.2 Physical properties

The physical properties of the vehicle are summarized in table (2.1) and the complementary schematics in figure (2.3). All the properties were carefully measured using standard tools.



Figure 2.3 – 1:5 scale robotic car dimensions

Table 2.1 – Physical properties of the 1:5 scale robotic car

Property	Symbol	Value
Total mass	m	$15.6\mathrm{kg}$
Rotational Inertia around CG in Z axis	I_z	$0.4734\mathrm{kg}\mathrm{m}^2$
Distance between COG and frontal wheels	L_{f}	$0.271\mathrm{m}$
Distance between COG and rear wheels	L_r	$0.255\mathrm{m}$
Maximum length	L_{max}	$0.900\mathrm{m}$
Lateral distance between wheels midpoints	W	$0.325\mathrm{m}$
Maximum width	W_{max}	$0.400\mathrm{m}$
Wheel diameter	D	$0.115\mathrm{m}$
Maximum steering angle	δ_{max}	$\pm 21 \mathrm{rad}$

2.3 Drivetrain and steering components description

The vehicle is powered by a sensorless brushless motor, which distributes torque to the four wheels.

The brushless motor revolutions are controlled using an ESC, which also provides energy to the steering servo through an integrated Battery elimination circuit (BEC). The ESC is shown in figure (2.4), and its characteristics summarized in table (2.2).



Figure 2.4 – ESC EZRUN-150A-PRO

Table 2.2 –	ESC EZ	RUN-150A-	PRO: Te	echnical c	characteristics

Characteristic	Value
Continous/Burst Current	150A/1080 A
Operating Voltage	7.4 - 22.2 V (2-6 Lipo cells)
BEC Output	3 A at 5.75 V
Motor type	Sensorless brushless motor

For the steering, a Hitec high-torque servomotor is used (see fig. (2.5)), which can provide up to 2.42 N-m of torque and has a maximum speed of 6.5 rad/s at no load. Its technical data is summarized in table (2.3).



Figure 2.5 – Hitec HS-5805MG servomotor

Table 2.3 – Steering servo: Technical charact	teristics
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Characteristic	Value
Control signal	PWM 1500 µs neutral
Operating Voltage	4.8 - 6.0V
Operating speed	$6.5\mathrm{rad/s}$ (6 V - no load)
Stall torque	$2.42\mathrm{Nm}$

2.4 Sensors and peripherals for data acquisition

The car uses a series of sensors in order to obtain data of its surroundings and actual state. A monocular CMOS camera and a LIDAR scanner are used to capture road and obstacles information (see fig. (2.6), and (2.7)). These two sensors are directly connected to the on-board computer, and their technical data are summarized in tables (2.4), and (2.5)



Figure 2.6 – CMOS Camera MT-HDM108

Characteristic	Value
Model	MT-HDM108 HD-SDI 1080P
Image sensor	2.0 MP 1/3" Sony CMOS
Effective pixels	1944 x 1092 @ $30/25$ FPS
Actual resolution used	$320 \ge 240$ pixels
Lens	$3.6~\mathrm{mm}$ with ICR

 Table 2.4 – CMOS Camera MT-HDM108: Technical characteristics



Figure 2.7 – Hokuyo UTM - 30LX

Table 2.5 – Hol	kuvo UTM	- 30LX Laser	range finder:	Technical	characteristics
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Characteristic	Value
Manufacturer	Hokuyo Automatic Co LTD
Model	UTM - 30LX
Detection range	0.1 - 60 m
Minimum detection width	$130~\mathrm{mm}$ at $10~\mathrm{m}$
Total scan range	270°
Angular resolution	0.25°
Scan speed	$25\mathrm{ms}$
Interface	USB ver 2.0 Full speed (12 Mbps)

2.5 On-board computer

The on-board computer runs the main program, which is in charge of the calculations required for the whole autonomous operation of the vehicle. A mini-ITX mainboard was chosen, due to the small form factor required. This board has installed 4 GB of DDR3 RAM, and a low-power consumption CPU to increase the runtime operation with batteries. The storage is supplied by an 120 GB SDD hardrive, which allows fast operation, with reduced power consumption and low weight; the SDD stores a Windows 10 x64 - education ver, which supports the software framework in which the autonomous driving algorithm runs. All the components are summarized in the table (2.6).

Table 2.6 -	On-board	computer	components
	· · · · · · · · · ·		

Component	Detail
CPU	Intel Core i5-4570s
Mainboard	Gigabyte GA-H97N-WIFI
Memory RAM	4 GB DDR3
Hard-drive	SSD 120 GB
Operating system	Windows 10 x64 Education ver.

2.6 Data communication interface

The on-board computer requires an interface with the sensors and actuators of the car. In the center of the communication system rest a single-chip microcontroller based on a high performance 1-T architecture 80C51. This microcontroller receives, buffer, filter, code and send the sensors data to the computer; also, it receives, decode and send the instructions from the computer to the car's actuators. The computer and microcontroller communicate through a RS-232 serial standard interface at 115200 bits/second. The minimum sampling rate achievable to receive and transmit data to all the car's devices is 50 ms.



Chapter 3

Computational methods for real-time optimal trajectory tracking

Nowadays, automation has become an inherent component of most daily use machines. There is multiple automatic control strategies to suit broad range of applications, being PID the most extensively used, due to its low cost implementation and easiness of tuning. However, a complex problem like autonomous driving, requires a controller that can deal with states and control constraints imposed by the dynamics, mechanical configuration of the car, and man-imposed restrictions like environmental regulations, fuel consumption, and traffic laws. MPC, due to its inherent capability to deal with constraints, has become an attractive method to solve this kind of problems. Moreover, MPC uses a complete multivariable system framework with on-line process optimization [16]. Many researches in autonomous driving have adopted MPC, since it provides a simple design scheme that integrates the model of the vehicle and its constraints, while performing the tracking of a planned trajectory as an optimization task [3, 5, 14, 17, 18]. In this chapter, an introduction to MPC, NMPC, and their solution methods through numerical solvers are presented.

3.1 Motivation for the use of Model predictive control (MPC)

Model predictive control (MPC) is an advanced control technique able to control systems with states and control inputs constraints. In this work, all the problems will be implemented in a digital computer. Then, the formulation will be given in discrete way. There is three crucial aspects that makes MPC an attractive methodology [16,19]. The first point is that the design framework works around Multiple Inputs - Multiple Outputs (MIMO), time-domain dynamic models; thus, the weight tuning parameters can be directly related to physical values, which are more meaningful (compared to PID parameters, for example). Second, its inherent capability to deal with soft and hard constraints, in both states and control inputs. The third aspect is the ability to optimize on-line a predicted trajectory based on the model given to the controller; hence, an early control action allows a superior tracking.

MPC has already been proposed as a controller for autonomous vehicle applications. In [3], Gao proposed an hierarchical scheme. A high level algorithm plans the trajectory with a simplified point-mass model, in a predictive horizon formulation; then, a low level MPC controller computes the vehicle inputs using a higher fidelity model. In [5], an on-line MPC controller is implemented and the stability is compared at different speed and prediction horizon lengths. MPC controllers have also been used in larger vehicles. For example, in [20], an Linear time variant MPC (LTV-MPC) is simulated for reference tracking of a truck in different scenarios. Other representative works of MPC for autonomous driving can be found in [17, 18, 21–34].

As in this work, some researches have already seen the advantages of using scale model cars for testing. In [13], an NMPC controller for 1:43 scale cars is proposed, using FORCES [35] for the solver generation. The feedback was given by an external, fixed camera above the testing tracking. In [14], also 1:43 scale cars are used, but a time-optimal formulation with NMPC is proposed, using ACADO as the optimization solver [36, 37].

3.2 Principle of MPC

In the MPC scheme, an open loop Optimal control problem (OCP) is solved on-line within a given finite "prediction horizon". In figure (3.1), this time horizon is defined as p^1 . Some other definitions may define a second horizon for the control variables, know in the literature as "control horizon"². The measurement or observed states at the step k are taken as the initial states x(0). The controller predicts the future dynamic response based on the system model, the constraints, and the initial states; then computes the optimal control input by minimizing an objective function. Using the Receding horizon control (RHC) principle [16], only the first control sample sequence is given to the system, while ignoring the rest. In the next sampling period, the prediction horizon p is receded one step and the whole process is repeated again. All these steps are summarized in Algorithm 1.



Figure 3.1 – Model predictive control working principle [1]

The MPC framework has some primary elements that directly relates to its final performance. The first one is the objective function, frequently named "J", that determines which parameters will be optimized in every iteration. Generally, a quadratic function is adopted, since it is convex, fast to compute, and provides an smooth behavior. Also, some weights are applied, in order to define the hierarchy of each parameter. The next key element is the prediction horizon length "p". There is no exact definition on how to determine its length, but it is recommended that it goes beyond the transient dynamics of the system, or at least, large enough to anticipate and avoid any critical constraint that may appear [2]. Nonetheless, it can not be too large, or the computational time to

¹Since all the solution will be implemented in a digital computer, a discrete MPC notation will be used along the text.

²In this work, the prediction horizon and the control horizon will have the same length, and thus, the name "prediction horizon" will be used for both of them interchangeably.

3 Computational methods for real-time

optimal trajectory tracking

Algorithm 1: MPC Strategy. Adapted from [2]
Input : Prediction horizon p, sampling time ΔT , objective function weights
Output : Optimal control inputs, optimized predicted states
1 repeat
2 Measure or observe the process state x at time t_k ;
3 Formulate the MPC problem for the optimization time $t \in [t_k, t_{k+N}]$;
4 Compute the optimal control sequence $\bar{u}^*(t), t \in [t_k, t_{k+N}]$ by solving the MPC
problem;
5 Apply optimal control $\bar{u}(t) = \bar{u}^*(t_k), t \in [t_k, t_{k+1}]$ to the system until $t = t_{k+1}$;
6 Set $k = k + 1;$
7 until interrupt flag is set;

solve the problem may exceed the sampling time of the system, forcing the algorithm to decrease the sampling rate, and thus, deteriorating the general performance. Finally, the last key element is the dynamic model of the system. A highly detailed model may give accurate results in theory; however, the high computational cost may turn the controller impractical for real-time applications. Thus, a balance between performance and accuracy has to be made. Some model designers may simplify some equations based on empirical experience of the system's behavior.

Nonlinear model predictive control (NMPC) 3.3

Most system models in real life are inherently non-linear, although many linearized models are used, because of their simplicity. However, complex systems like cars present many challenges, like non-holonomic kinematic constraints, highly coupled control states [17], and nonlinear behavior [38]. In this situation, a linear model is clearly not adequate to describe the process.

As stated in the previous section, the MPC is a particular type OCP, which can be defined as (3.1).

$$\min_{u(t)} \quad \Psi(\mathbf{x}(t_f), t_f) + \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t), t) dt \quad (3.1a)$$
et to $\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), t), \quad t \in [t_0, t_f] \quad (3.1b)$

subject to

 $g(\mathbf{x}(t), \mathbf{u}(t), t) \leq \mathbf{0}$ (3.1c)

(3.1b)

 $\mathbf{x}(t_0) = \mathbf{x}_0$ (3.1d) where x(t) and u(t) are the state and control variables, x_0 in (3.1d) are the initial state values, and the interval $[t_0, t_f]$ is the prediction horizon in the MPC scheme (normally fixed). In this problem, (3.1a) is the objective function which consist of an integral term $L(\mathbf{x}(t), \mathbf{u}(t), t)$, called the *Lagrange term*, and a terminal term $\Psi(\mathbf{x}(t_f), t_f)$, called the *Mayer term*. The differential equations in (3.1b) represent the vehicle model in the interval $[t_0, t_f]$. States and control constraints are described in (3.1c).

The solution approaches for OCPs are normally divided in three major groups: dynamic programming, indirect, and direct methods [39]. Also, a comparison between methods can be found in [40].

- **Dynamic Programming** turns the OCP in a Hamilton-Jacobi-Bellman equation by recursive computation of the feedback control. This approach is restricted to small state dimensions due to the Bellman's "curse of dimensionality".
- Indirect methods makes use of concepts from calculus of variations like the the Pontryagin Maximum Principle and the Euler-Lagrange differential equations to transform the OCP into a Boundary value Problem (BVP). Then, this BVP is numerically solved. For the reasons explained above, this method is also known as *first optimize, then discretize*. The principal drawback is the difficulty to derive the BVP, which becomes even more complex when constraints are introduced.
- **Direct methods** converts the continuous time OCP into a finite dimensional NLP problem by using a discretization technique [41]. This method is also portrayed as *first discretize, then optimize*. Nowadays, direct methods are the most widely used for solving constrained OCPs; even though the resulting NLP is large, there are state of the art optimization solvers that can solve this type of setups very efficiently [40, 42]. In this thesis, multiple shooting, which is a direct method, was used to discretize the NMPC problem into a NLP.

3.4 Direct methods for solving optimal control problems

In the last section, it was explained how the direct methods converts dynamic optimization problem into a finite dimensional Nonlinear programming (NLP), and why they are preferred over the other methodologies. The reformulation is achieved through discretization of the optimal control problem, which is represented as a Initial value problem (IVP). All direct methods parametrize first the control trajectory (generally

3 Computational methods for real-time optimal trajectory tracking

in a piece-wise pattern); however, they vary in the way the states parametrization are managed. In the literature, two different approaches are mentioned for the direct methods.

The first one is the sequential approach. A sequential integration of states (simulation), and optimization controls trajectory is executed in every iteration. The state trajectory x(t) is defined as an implicit function of the controls u(t), which means that the solver only optimize the controls trajectory. The *direct single shooting methods* belong to this category. On the other hand is the simultaneous approach, in which the states and controls are introduced as optimization variables. The most popular variants of the simultaneous approach are the global collocation and multiple shooting. Both, sequential and simultaneous approaches are briefly described in the following lines.

a. Direct single shooting

In this method, a grid of time points t_k , for k = 0, ..., N are initially generated. The controls u(t) are discretized along the generated time intervals, usually as piecewiese constants values. Then, the problem is solved, by sequentially integrating the states (simulation) and optimizing for the controls trajectory u(t). The solution defines a trajectory $\bar{x} = x(t; \bar{u})$ for each control sequence \bar{u} . The benefits of this method are its simplicity, and the few degrees of freedom generated, even for large ODE or DAE systems [39]. However, this approach presents problems with unstable systems, due to the lack of information for the initial states.

b. Orthogonal collocation on finite elements

Also called pseudo-spectral methods [43], collocation on finite elements is particular case of the implicit Runge-Kutta equations. For a given time interval ΔT (which in the specific case of MPC, is the sampling rate of the system), the states trajectory are approximated using *collocation polynomials*, as seen in equations (3.2). Generally, controls trajectory are assumed piece-wise constant. The orthogonal collocation points, denominated as t_k , for $k = 0, \ldots, M$ can be obtained from the quadratures of Legendre,
Radau or Lobatto [2]. To ensure continuity of the ODE, equality restrictions are given between final and initial values of each subsequent intervals.

where,
$$X_{i}^{(l)}(t) = \sum_{k=0}^{N} x_{ik}^{(l)} L_{lk}(t), \qquad i = 1 \dots n$$

 $\sum_{\substack{\nu=0\\\nu \neq kl}}^{N} \left[\frac{t - t_{\nu}}{t_{lk} - t_{\nu}} \right]$ (3.2)

After the discretization, a very large, but sparse NLP is obtained. One advantage of this method is that sparse NLP can be solved efficiently by numerical solvers that exploit this particular structures. Also, collocation methods can deal with unstable systems, and has fast local convergence. Some works have successfully used collocation for solving NMPC problems [44, 45]

c. Direct multiple shooting

This numerical method for BVP was developed by Bock and Plitt around the 80s [46]. The working principle in multiple shooting is similar to single shooting. A grid of time points t_k , for k = 0, ..., N are initially generated, and the controls u(t) are discretized along the time intervals. The difference with the single shooting relies in how the integration is made in each subinterval. In the multiple shooting, the ODE is integrated (simulated) independently in each sub-interval with different initial values, designated by a new variable \hat{x}_k (see equation (3.3) and figure (3.2) (left)).

$$\dot{x}_k(t) = f(x_k(t), u_k), \quad t \in [t_k, t_{k+1}],$$
(3.3)

$$x_k(t_k) = \hat{x}_k, \qquad (3.4)$$

Thus the designation of "multiple" shooting (and simultaneous approach), because each interval "shoots" independently. The continuity of the dynamics are satisfied by the residual constraints in the equality (3.5), that connects the discretized intervals [44].

$$\hat{x}_{k+1} = x_k(t_{k+1}, \hat{x}_k, u_k) \tag{3.5}$$

3 Computational methods for real-time optimal trajectory tracking

After the convergence of the optimization solver, the states trajectories may look like in figure (3.2)(right).



Figure 3.2 – (left) Single trajectories obtained through the solution of the ODEs. (right) Convergence of state and control profiles for the direct multiple shooting method. Extracted from [2]

The multiple shooting technique generates an sparse NLP as a function of the states and control trajectories. The advantages of this method are the resulting sparse NLP, and that the initial states information can be used. Thus, this method can handle unstable systems with path and terminal constraints. In terms of speed, multiple shooting competes, and can even surpass the collocation approach [47]. In this work, multiple shooting will be applied for solving the BVP of the dynamic models.

Finally, each of the shooting intervals is approximated using numerical techniques; Runge-Kutta methods are commonly used for this task. More sophisticated techniques like orthogonal collocation had also been used to approximate the shooting intervals [48]. In this work, the well-known **Runge-Kutta 4 (RK4)** is used, since it provides a good balance between performance and precision. Moreover, each time interval Δt can be further divided during the integration to increase the accuracy of the results. From the software development point of view, this allows a more modular and flexible code, in which the precision can be adjusted just by changing the grid size.

3.5 Selection of the NLP solver

As explained in the last section, an optimization problem can be transformed to a NLP in order to use the efficient solvers available. This solvers use different algorithms to approach the problem, which can be categorized in three [40,49,50]: Sequential quadratic programming (SQP), Interior Point methods (IPM), and heuristic methods. SQP and IPM are found in most of the applications, while the heuristic methods are used for more particular cases³. Depending of the particular characteristics of the problem, one approach may be more suitable than rest. The principal considerations for the selection of a solver are: size of the problem, number of constraints, degree of non-linearity, and CPU time [49]. Also, the availability and quality of documentation, as well as its scalability without major changes should be taken in consideration. Finally, a particular circumstance in this project was that the solver had to be compatible with the Windows OS environment, so that it could interface with a previous developed software.

After reviewing surveys and comparative tests [44,51,52], in this work, the solver Interior Point Optimizer (IpOpt) [53] was chosen. IpOpt is a well-known NLP solver based on the IPM approach. It can solve efficiently large-scale, sparse non-linear optimization problems. It is also well-documented and can be integrated with different linear solvers to improve its capabilities in future works. Furthermore, it is distributed as open source under the Eclipse Public Licencse(EPL) free of charge, even for commercial applications. Finally, pre-compiled libraries for Windows are available, an important requirement for this specific work.

In order to use IpOpt, the required inputs had to be analyzed. IpOpt request from the user an objective function, variables bounds, constraints, gradient of the objective function, and the Jacobian of the constraints. The Hessian of the Lagrangian can also be provided, or can be approximated by the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm integrated in IpOpt. In this work, the gradient and Jacobian were generated using CasADI [54], which is a symbolic framework for automatic differentiation and optimal control.

 $^{^{3}\}mathrm{heuristics}$ methods involve non-conventional approaches like neural networks, and evolutionary or genetic algorithms

a. IpOpt C++

IpOpt is software package for nonlinear optimization [53], with great benefits at large scale problems. The solver was initially developer by Andreas Wächter as part of his doctoral dissertation research, but it has been improved along the years, thanks to its open-source license. At the core of the solver, a primal-dual interior point algorithm coupled with a line-search filter method is used. IpOpt has been written in C++ and has shown good performance for online optimization problems [2,14]. As have been mentioned earlier, the advantages of IpOpt are its compatibility with multiple linear solvers, good documentation, free of charge distribution, integration with platforms like MATLAB[®] and Modelica[®], and its availability for different operating systems. IpOpt has also been used by other researchers for autonomous driving applications, with fairly good results [2,14,44,55,56].

b. CasADI

CasADi is a symbolic framework for algorithmic differentiation and numeric optimization, developed by Joel Andersson and Joris Gillis [54,57]. The user can easily define symbolic expressions using Computer algebraic system (CAS); then their derivatives can be obtained using state of the art algorithmic differentiation in forward, reverse modes, and graph coloring techniques as sparse Jacobians and Hessians. Furthermore, the derivatives can be exported into C++ functions and used with other applications, like in this work. CasASI also provides front ends in either Python and MATLAB[®]. It can also be integrated with state of the art solvers like Sundials (CVODES, IDAS and KINSOL), IpOpt, WORHP, SNOPT and KNITRO. It is also worth mentioning that CasADI is open-source, written in self-contained C++ code⁴, and it is well-documented.

⁴depends only in the C++ Standard Library

Chapter 4

Mathematical models for vehicles

As mentioned in the previous chapter, MPC controllers require a system model in order to predict its future outputs. In this chapter, three vehicle models, and a steering systems dynamics are used to characterized our robotic car's movement law.

The steering system model is introduced first, since it is a core component along all the vehicle models presented in this chapter. This system was identified by fitting a second order, black-box model using experimental data.

For the vehicle mathematical representation, one kinematic and two dynamic models were formulated; they were based on previous works from other researchers, in which some modifications were thoughtfully made in order to meet the particular setup of the vehicle used in this work. A kinematic model was deduced from geometrical properties, without considering inertial effects and forces in the wheels. This model is accurate at reduced speeds, in which inertial effects like drifting and slip are not noticeable.

At higher speeds, inertial effects and forces were introduced to generate a more accurate model. The most common tire models are introduced at the beginning of the dynamic vehicle models section, since they provide the forces on which the movement equations are based. Next, a double-track dynamic model is presented; then, a single-track dynamic model, also known as the "bicycle model", was deduced as a simplification from the previous one.

Finally, the parameters used in all the models are summarized, taking in consideration the information from chapter (2).

Master thesis Toshiharu Tabuchi

4.1 Steering system's dynamics

The steering system is a critical component for the control of the car. However, little attention is paid to the this system, and only few researchers address their dynamic response. When the delay of the steering is comparable, or even higher than the sampling's rate of the system, problems could present during the execution on real-time, especially when the vehicle moves at higher speeds. To overcome this problem, in this work, the steering dynamics were modeled as a second order, linear system. The methodology and results are presented next.

a. System identification

First, the dynamic response of the steering's servomotor was measured, by sending an step impulse to the actuator, and measuring the output with a rotational encoder. The test was made with the car positioned on the floor in a stationary position, to take in account the friction effects between the wheels and road surface (see fig. (4.1)). The sampling rate used was $\Delta t = 20 \ ms$. The measured data is shown in figure (4.2).



Figure 4.1 – Experiment to identify the steering system's model

After the measurement, the model was identified using the System Identification Toolbox from MATLAB®. Initially, the steering system was modeled as a generic, black-box model, in which one of the states was intentionally defined as the output steering angle δ to avoid expressing it as a linear combination of the states. Different system orders were tried, to find the best balance between complexity, and fit index. Finally, a second order system was selected, with a fit index of 81.07%. Higher system orders provided no greater improvement.



Figure 4.2 – Servo response to step input

b. Identified model for the steering system

The second order identified model is presented in the next lines.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -5.5844 & 5.1870 \\ -6.0771 & -7.9005 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 9.0813 \\ 0.7431 \end{bmatrix} \delta_{in}$$

$$\delta = x_1$$
(4.1)

where δ is the output angle of the servo, and δ_{in} is the input signal to the servomotor. The measured data vs the identified model are presented in figure (4.3).



Figure 4.3 – Identified model vs measured data. Step input response

4.2 Kinematic model

A first approach to estimate the trajectory of a vehicle is through a kinematic analysis, in which inertial effects are neglected. The kinematics of the vehicle's center of gravity can be modeled using just geometric relations. From figure (4.4), the differential equations (4.2) can be derived.

The states x, y, ψ, β represent the yaw angle, the Cartesian coordinates, and the body slip angle of the car. L_f and L_r represent the longitudinal distances from the center of gravity (COG) of the car to the frontal and rear wheels, respectively. Also, it should be noticed that in this model, the speed V_x is a parameter (written in uppercase), not a control input.

The steering angle δ may be defined directly as the control input. However, as explained in a previous section, the steering angle δ presents an inherent delay due the inertial effects. Then, δ is defined as function of the servomotor input signal δ_{in} . Notice that addition of the steering dynamics increases the order of the system by two. In summary, the final kinematic model presented here has actually five states, and one control input.



Figure 4.4 – Kinematic model for the car

$$\dot{x} = V_x \cos(\psi + \beta)$$

$$\dot{y} = V_x \sin(\psi + \beta)$$

$$\dot{\psi} = \frac{V_x}{L_f + L_r} \tan(\delta)$$
(4.2)

where
$$\beta \approx \frac{L_f}{L_f + L_r} \delta$$
,
 $\delta = f(\delta_{in}), \, \delta_{in}$ is the servomotor input signal.

4.3 Dynamic models for lateral dynamics

The next approach considers the inertial effects and forces to build a more realistic model of the vehicle movement. The models presented here focus on the lateral dynamic control, meaning that longitudinal forces are neglected. This approximation is adequate, since in steady-state normal driving conditions, no sudden longitudinal accelerations occur. Another characteristic that is derived from the previous premises is that normal forces are considered constants. This means that inertial effects like "load transfer" were neglected due to the absent of sudden braking or acceleration. Similar to the Kinematic model case, the steering was defined as the servomotor input signal function δ_{in} , which increased the total model order system by two.

An introduction to the most common tire models is presented first, since they provide the interaction forces in which all the dynamic vehicle models are based. Then, two vehicle models are presented in this section. The former is the double-track model, which considers the forces exerted in all the four wheels of the vehicle. The latter, the single-track model, which is a simplification derived from the former model; nevertheless, the essential dynamic effects are maintained, while gaining appreciable reduction in computational time.

a. Tire models

Many tire models have been formulated and tested; however, two models stand out in most of the applications in autonomous driving. The first one is the linear model, which is by far the most used one, because of its simplicity and good approximation of the tire response in normal driving conditions. The second most used is the Magic formula tire model, formulated by Pacejka, which considers the saturation at higher slip angles, a behavior seen in more challenging driving conditions. Other models that are also mentioned in the literature, but are not further explained in this section are the Dugoff analytical tire model, LuGre dynamic friction model, the Brush tire model, the SWIFT tire model, and even neural-network based models [58].

a.1. Magic formula tire model

The Magic formula tire model is a semi-empirical, non-linear tire model, which is able to describe the tire forces under wide range of operation. This model has shown good fitting indexes compared to other more analytical, complexer ones [13, 14, 59, 60]. The following function is used as grey-box model in which the parameters C, B, S_v, E , and S_h are identified using experimental data.

$$F_{tire} = D \sin \left(C \arctan(B\Theta) \right) + S_v$$
where $\Theta = (1 - E)(X + S_h) + \frac{E}{B} \arctan \left(B(x + S_h) \right)$

$$(4.3)$$

If F_{tire} represent the lateral forces, then X is the slip angle. Else, if F_{tire} act as the longitudinal forces, X is the slip ratio. Figure (4.5) shows the lateral forces generated as a function of the slip angle for different friction coefficients. Longitudinal forces present similar behavior, thus their graphs are not presented.



Figure 4.5 – Lateral tire forces in pure cornering (s=0). Graph extracted from [3]

a.2. Linear tire model

As it can be seen in figure (4.5) and (4.6), for small slip angles ($\alpha \leq 5^{\circ}$), the lateral forces generated present a linear behavior. This linear model has demonstrated to work well under normal driving conditions [3, 12, 14, 32, 59, 61–71], and even in off-road conditions like in the DARPA Challenge 2005 with good results [10]. Thus, under the assumptions mentioned before, the following function approximates well the forces generated in the tires.

$$F_{tire} = C_{\alpha} \alpha \tag{4.4}$$

where C_{α} is the cornering stiffness for either the frontal or the rear wheels, since they may vary depending of the weight distribution of the vehicle.



Figure 4.6 – Comparison of Magic formula tire model (Pacejka 2006) vs linear tire model. Graph extracted from [3]

b. Double-track model for lateral dynamics

In this model, all four wheel lateral forces are considered in the dynamic equations. A graphical representation of the model is presented in figure (4.7). The states $\dot{\psi}$, ψ , β , x, y represent the yaw's rate, yaw angle, body slip angle, and the Cartesian coordinates of the vehicle, respectively. The control inputs are speed v_x , and servomotor input δ_{in} . In summary, since the steering dynamics were considered here, this model has seven states, and two control inputs.



Figure 4.7 – Double-track dynamic model

The differential equations of the model in (4.5) were modified from [72] using the considerations described at the beginning of the chapter.

$$\ddot{\psi} = \frac{1}{I_z} (L_f(F_y^{fL}\cos(\delta_L) + F_y^{fR}\cos(\delta_R)) + \frac{W}{2} (F_y^{fR}\sin(\delta_R) - F_y^{fL}\sin(\delta_L)) - L_r(F_y^{rL} + F_y^{rR}))$$
(4.5a)

$$\dot{\psi} = \dot{\psi}$$
 (4.5b)

$$\dot{\beta} = \frac{1}{mv_x} (F_y^{fL} \cos(\beta - \delta_L) + F_y^{fR} \cos(\beta - \delta_R)$$
(4.5c)

$$+\left(F_y^{rL}+F_y^{rR})cos(\beta)\right)-\dot{\psi}$$

$$\dot{x} = v_x \cos(\psi) - v_x \tan(\beta) \sin(\psi) \tag{4.5d}$$

$$\dot{y} = v_x \sin(\psi) + v_x \tan(\beta) \cos(\psi) \tag{4.5e}$$

where $\delta = f(\delta_{in})$, δ_{in} is the servomotor input signal.

The linear tire model was employed to describe the lateral forces, represented in equations (4.6). This tire model is generally used for normal driving conditions, and provides a good approximation of the forces generated due to frictional effects. F_{yf} and F_{yr} have both a linear relationship with the slip angle α^5 (see figure (4.8)). C_f and C_r are the frontal, and rear tire stiffness coefficients, which depend of the road and tires condition.



Wheel Turning Left

Figure 4.8 – Slip angle scheme [4]

$$F_{y}^{fL} \approx C^{f} \alpha^{fL}(t)$$

$$F_{y}^{fR} \approx C^{f} \alpha^{fR}(t)$$

$$F_{y}^{rL} \approx C^{r} \alpha^{rL}(t)$$

$$F_{y}^{rR} \approx C^{r} \alpha^{rR}(t)$$
(4.6)

⁵angle between the actual speed direction and the longitudinal direction of the wheels

where the slip angles of each wheel are:

$$\alpha^{fL} = \delta^{L} - \tan\left(\frac{v_{x}\tan(\beta) + \dot{\psi}L_{f}}{v_{x} - \dot{\psi}\frac{W}{2}}\right)$$

$$\alpha^{fR} = \delta^{R} - \tan\left(\frac{v_{x}\tan(\beta) + \dot{\psi}L_{f}}{v_{x} + \dot{\psi}\frac{W}{2}}\right)$$

$$\alpha^{rL} = -\tan\left(\frac{v_{x}\tan(\beta) - \dot{\psi}L_{f}}{v_{x} - \dot{\psi}\frac{W}{2}}\right)$$

$$\alpha^{rR} = -\tan\left(\frac{v_{x}\tan(\beta) - \dot{\psi}L_{f}}{v_{x} + \dot{\psi}\frac{W}{2}}\right)$$
(4.7)

c. Single-track model for lateral vehicle dynamics

Autonomous vehicles are considered fast dynamic systems, because they require short sampling times to be controlled accurately and safely. This demands fast converging times in the MPC algorithm, smaller than the sampling rate of the system. For that reason, complex models are carefully simplified, trying to maintain the most essential components of the dynamics, while omitting less significant ones. The single-track model is a simplification of the previous double-track model presented in the last section. In this model, each pair of left and right wheels are lumped together as one. Then, the four wheels are reduced to two; for that reason, the single-track model is also known as the "bicycle model". It can proved that under typical driving conditions, the behavior of the double-track and single-track model are very similar; however, the computational time in the simplified version is considerably shorter. Consequently, a great number of MPC algorithms utilize the single-track model [10, 13, 22, 61, 63, 64, 71, 73, 74]. The graphical representation is shown in the figure (4.9).

The differential equations in (4.8) were derived from the four wheels model, by setting the width of the car W = 0, along with left "L" and right "R" parameters as equal. Since the steering dynamics were considered, the order of the system increased to seven, as in the four wheels model. As in the Double-track model, the states $\dot{\psi}$, ψ , β , x, yrepresent the yaw's rate, yaw angle, body slip angle, and the Cartesian coordinates of the vehicle, respectively. The control inputs are speed v_x , and servomotor input δ_{in} .



 ${\bf Figure} ~~ {\bf 4.9-Single-track} ~~ {\rm dynamic} ~~ {\rm model}$

$$\ddot{\psi} = \frac{2}{I_z} \left(L_f(F_y^f \cos(\delta)) - L_r(F_y^r) \right)$$
(4.8a)

$$\dot{\psi} = \dot{\psi} \tag{4.8b}$$

$$\dot{\beta} = \frac{2}{mv_x} F_y^f \cos(\beta - \delta) + F_y^r \cos(\beta) - \dot{\psi}$$
(4.8c)

$$\dot{x} = v_x \cos(\psi) - v_x \tan(\beta) \sin(\psi) \tag{4.8d}$$

$$\dot{y} = v_x \sin(\psi) + v_x \tan(\beta) \cos(\psi) \tag{4.8e}$$

where $\delta = f(\delta_{in})$, δ_{in} is the servomotor input signal.

where

$$F_y^f \approx C^f \alpha^f(t)$$

$$F_y^r \approx C^r \alpha^r(t)$$

$$\alpha^f = \delta - \tan\left(\frac{v_x \tan(\beta) + \dot{\psi}L_f}{v_x}\right)$$

$$\alpha^r = -\tan\left(\frac{v_x \tan(\beta) - \dot{\psi}L_f}{v_x}\right)$$
(4.9)

4.4 Parameters for the vehicle models

In order to be complete, the vehicle models require the correct physical parameters, so that equations can match, as close as possible, the dynamics of the real system. Except for the cornering stiffness, all the parameters were obtained from measurements of the 1:5 scale robotic car. The cornering stiffnesses C_f , and C_r were estimated based on the parameters of a real-sized vehicle from [21]. The constants were scaled down proportionally to the mass of robotic car. The table (4.1) summarizes the parameters used in all the models presented in this chapter.

 Table 4.1 – Physical parameters for the mathematical models

Parameter	\mathbf{Symbol}	Value				
Total mass	m	$15.6\mathrm{kg}$				
Rotational Inertia around CG in Z axis	I_z	$0.4734\mathrm{kg}\mathrm{m}^2$				
Distance between CG and frontal wheels	L_{f}	$0.271\mathrm{m}$				
Distance between CG and rear wheels	L_r	$0.255\mathrm{m}$				
Lateral distance between wheels midpoints	$W_{}$	$0.325\mathrm{m}$				
Wheel diameter	D	$0.115\mathrm{m}$				
Frontal cornering stiffness	C_{f}	$250\mathrm{N/rad}$				
Rear cornering stiffness	C_r	$234\mathrm{N/rad}$				

4.5 Mechanical and operational bounds

Every type of machine presents physical constraints that has to be respected in order to avoid damages. Also, operational bounds may impose additional restrictions for safety, environmental regulations, etc. The bounds presented in the table (4.2) were based on the information from the chapter 2. The steering δ bounds were defined based on servomotor mechanical limits; plus, they are applicable for both, the kinematic and the dynamic models. The remaining bounds are only applicable to the dynamic models. Although the robotic can reach speeds up to 20 m/s, the maximum speed was restricted for safety reasons. The lower bound for the speed 1.5 m/s is required, because this setpoint allows the motor to achieve the minimum sufficiently torque to move the car. Finally, the bounds for the body slip angle β , and vehicle rotational speed $\dot{\psi}$ were computed based on the previous mentioned bounds, by simulating some borderline conditions.

Table 4.2 – Mechanical and operational bounds

Variables	Symbol	Range
$Steering^{\dagger}$	δ	[-0.37, 0.37] rad
Speed	v_x	[1.5, 5] m/s
Body slip angle	β	[-0.09, 0.09] rad
Vehicle rotational speed	$\dot{\psi}$	[-0.7, 0.7] rad/s

[†] Applicable for both the Kinematic and Dynamic models.



Chapter 5

NMPC controller design for autonomous driving

As has been described in previous chapters, car dynamics are inherently non-linear, because of non-holonomic kinematic constraints, highly coupled control states [17], and nonlinear behavior [38]. An autonomous driving algorithm relies on one or more controllers in order to achieve different tasks. In the present work, a Nonlinear model predictive control (NMPC) algorithm was implemented as the main controller for trajectory tracking. NMPC can manage not only non-linear models, but also non-linear constraints, depending of the trajectory complexity. Furthermore, the objective function may not only involve the primarily task of trajectory tracking, but also some other secondary tasks like minimization of travel time, and obstacles avoidance [3,33], which also involve non-linear functions.

In this chapter, three different NMPC controllers were formulated based on the vehicle models presented in chapter 4, which has also been summarized in this chapter. All NMPC controllers were transformed to NLPs applying direct methods, with the multiple shooting technique and the Runge-Kutta 4 (RK4) integrator. The RK4 integrator was chosen, since it provides a good balance between performance and execution speed.

The design of the NMPC controllers started by defining the trajectory tracking cost function. Two different cost functions and their respective boundaries were formulated, to address the kinematic and dynamic models. Also, two obstacle avoidance approaches are presented and compared. All the NMPC controllers were implemented using IpOpt with the MUMPS linear solver. The objective functions and the constraints were symbolically formulated and discretized using the Casadi framework. Furthermore, Casadi was also used to compute the gradient of the objective function, and Jacobian of the constraints, using an state-of-the-art automatic differentiation (AD) algorithm. All these functions were later exported as C++ functions using a code generation engine, also included in Casadi.

Finally, the program structure for the simulations and real-time vehicle implementation are presented. It is also explained the similarities between both programs, and the adaptations that needed to be made to the vehicle implementation software, to acknowledge the capabilities of the 1:5 scale vehicle setup. Each of the sections also include a block diagram, that explain the flow of data around the programs.

5.1 Kinematic model based NMPC controller

In figure (5.1), the general block diagram of the NMPC controller based on the kinematic model is presented. The inputs for the NMPC controller are the initial states \mathbf{w}_0 and the path reference **ref**. The outputs are the optimal states \mathbf{w}^* , and servomotor control signal δ_{in}^* .

$$\mathbf{w_0} = [x \ y \ \psi]^T \longrightarrow \text{NMPC controller} \\ \mathbf{ref} = [x_{ref} \ y_{ref}]^T \longrightarrow \text{Kinematic model} \qquad \mathbf{w^*} = [x^* y^* \psi^*]^T$$

Figure 5.1 – NMPC controller based on the Kinematic model. Block diagram for Inputs-Outputs.

a. Cost function

When trajectory tracking task is executed with an NMPC controller, the cost function is defined as the relative distance between the vehicle and road path. This cost function must be minimized online within a predefined sampling rate. For the Kinematic model, the cost function is defined below.

$$J_{track}(\mathbf{w}, \mathbf{u}, \mathbf{ref}) = P_x(x_{p,N} - x_{ref,N})^2 + P_y(y_{p,N} - y_{ref,N})^2 + \sum_{k=1}^{N-1} \left(q_x(x_{p,k} - x_{ref,k})^2 + q_y(y_{p,k} - y_{ref,k})^2 + r_\delta \delta_k^2 \right)$$
(5.1)

The vectors \mathbf{w} , \mathbf{u} , and \mathbf{ref} are the states, control inputs, and position reference, respectively. The position in each discretization interval is represented by $x_{p,k}$ and $y_{p,k}$. A terminal penalty for the last predicted position $x_{p,N}$ and $y_{p,N}$ was also included, in order to increase the system stability. Finally, the control input δ_k was likewise introduced in order to reduce the jerking or oscillations in the actuators. Notice that, in the Kinematic model, the speed is not a control variable, but only a parameter.

b. Variables bounds

In addition, physical and operational bounds are defined to ensure smooth and safe performance of the vehicle. One of the great advantages of MPC is that these bounds can be easily declared into the problem formulation. The inequalities below represent the states and controls bounds, based on the information from table (4.2). Any other states or control inputs not defined are assumed to be unrestricted.

$$-2\pi \operatorname{rad} \leq \psi \leq 2\pi \operatorname{rad}, \qquad (5.2)$$

$$-0.37 \operatorname{rad} \leq \delta \leq 0.37 \operatorname{rad}.$$
(5.3)

5.2 Dynamic models based NMPC controllers

For the two dynamic models (double-track, and single-track) based NMPC, the block diagram in figure (5.2) shows their inputs and outputs. Since both of them share the same states and control inputs, only one block is presented. As in the Kinematic based controller, the initial states \mathbf{w}_0 and the path reference **ref** are the algorithm inputs. The outputs are the optimal variables \mathbf{w}^* , and the control signals δ_{in}^* , and v_x^* .

a. Cost function

The cost function is also quite similar to the kinematic one. The main difference is the addition of the actual speed v_k , and a reference cruise speed v_{ref} . Depending on how the weights are set in the cost function, higher priority could be given to reaching the cruise control speed, or tracking the path reference. The cost function is defined as follows:

$$J_{track}(\mathbf{w}, \mathbf{u}, \mathbf{ref}) = P_x(x_{p,N} - x_{ref,N})^2 + P_y(y_{p,N} - y_{ref,N})^2 + \sum_{k=1}^{N-1} \left(q_x(x_{p,k} - x_{ref,k})^2 + q_y(y_{p,k} - y_{ref,k})^2 + r_v(v_k - v_{ref})^2 + r_\delta \delta_k^2 \right)$$
(5.4)

Figure 5.2 – NMPC controller based on the Dynamic model (Double-track or Single-track). Block diagram for Inputs-Outputs.

b. Variables bounds

The states and control inputs bounds were determined based on the information in the table (4.2), stability criteria, and safety reasons.

 $\begin{array}{rll} -0.35 \; {\rm rad} \; \leq \; \beta \; \leq \; 0.35 \; {\rm rad} \, , \\ -2\pi \; {\rm rad} \; \leq \; \psi \; \leq \; 2\pi \; {\rm rad} \, , \\ -\frac{\pi}{2} \; {\rm rad/s} \; \leq \; \dot{\psi} \; \leq \; \frac{\pi}{2} \; {\rm rad/s} \, , \\ -0.37 \; {\rm rad} \; \leq \; \delta \; \leq \; 0.37 \; {\rm rad} \, , \\ 1.5 \; {\rm m/s} \; \leq v_x \; \leq \; 5 \; {\rm m/s} \, . \end{array}$

5.3 Obstacle avoidance

As mentioned in [2], obstacles can be modeled as closed geometric shapes (ellipses) or as a potential field that penalizes the distance between vehicle and obstacles. These two approaches are briefly explained in the next lines.

a. Obstacle as a closed region

In this approach, a geometric path constrain is imposed in the trajectory tracking problem. The obstacle is modeled as a closed, convex curve. Normally, the curve chosen is the ellipse, which is very versatile since it can be shaped as almost any obstacle, by adjusting the length of the major and minor axis. Also, thanks to its smooth and convex properties, this kind of shape allows the generation of non-holonomic trajectories around the obstacles. The path constraint can be formulated as the following inequality.

$$\frac{(x_{pos} - x_{obs})^2}{a^2} + \frac{(y_{pos} - y_{obs})^2}{b^2} \ge 1,$$
(5.5)

Within the NMPC framework, the inequality is applied for one obstacle as multiple non-linear constraints for each discrete position point (x_k, y_k) k = 0, 1, ..., N. For each new obstacle considered, new "N" inequalities will be added to the constraints. An example for one obstacle is shown below.

$$\frac{(x_{pos,k} - x_{obs})^2}{a^2} + \frac{(y_{pos,k} - y_{obs})^2}{b^2} \ge 1, \qquad k = 0, 1, \dots, N,$$
(5.6)

b. Obstacle as repulsive force

Potential field functions are another approach used for obstacle avoidance algorithms. An internal point of the obstacle (commonly, the geometric center) is used as the origin of potential fields. The closer the vehicle is to the obstacle, the higher the repulsive force value generated. The potential field equation for discrete points (x_k, y_k) is shown below.

$$P_{obs,k} = \frac{K_{obs}}{(x_{pos,k} - x_{obs})^2 + (y_{pos,k} - y_{obs})^2 + \epsilon}$$
(5.7)

Within the NMPC framework, this function is added to the cost function, and penalize the distance to the obstacle. If more obstacles are to be considered, more summations has to be added into the cost function. Here, an example for only one obstacle is considered.

$$J_{avoidance} = J_{track} + \sum_{k=0}^{N} P_{obs,k}$$
(5.8)

The main disadvantage of this approach is that the obstacle geometry can not be explicitly defined. The safety region around the obstacle will depend on the obstacle gain K_{obs} , and the other gains in the cost function.

As mentioned in the chapter 2, the robotic car is equipped with a LIDAR sensor to detect obstacles. Also, the information from sensors allows the identification of obstacles shapes. For that reason, it was chosen that the obstacles will be modeled as closed shapes, as mentioned in the previous definition.

5.4 NMPC program structures

In this section, the interaction between the NMPC controller and robotic vehicle (system) is explained. Two NMPC program versions are presented; one for the simulations, and another one for real-time implementation.

a. Simulations version

The task of this program version was to test the performance of the NMPC controller before the real-time implementation on the vehicle testbed. The program is divided in four main parts (see fig. (5.3)). First, the variables and bounds for the NMPC controller are initialized. Second, the sensors simulator computes the path reference (x_{ref}, y_{ref}) for the controller. At the third part, the optimal control inputs are calculated by the NMPC controller, in which core lies the IpOpt algorithm. Finally, the optimal control inputs δ_{in}^* , v_x^* are given to the vehicle simulator, which outputs the states for the next time frame w(k + 1). The cycle is repeated until the user stops the program, or the final conditions are met.







Figure 5.3 – Simulation program block diagram

b. Real-time experimental version

This program, represented in the block diagram in (5.4), is based on the simulation version, but with few modifications. The first difference is that, the sensors and vehicle simulators are replaced by their physical counterparts.

The second main difference relies in the way the path reference is computed. After the initialization, the camera captures a new image of the road, which is sent to the image processing algorithm. The image processing algorithm provide the NMPC controller with the path's reference curve (x_{ref}, y_{ref}) . Due to the lack of an odometry unit, this curve is computed with the vehicle COG as its origin, different to the simulation, in which the world frame-of-reference is used. For that reason, the initial position of the vehicle needs to be reset in every iteration in order to have the same frame-of-reference in the camera and vehicle.

In the main algorithm, running in the on-board computer block, the path reference and the initial states are introduced to the NMPC controller, which computes the optimal control inputs. These optimal control signals δ_{in}^* , v_x^* are sent through a serial port (RS-232) to the vehicle microcontroller, which is in charge of the low-level operations, like the correct positioning of the actuators at provided setpoints.

Finally, the main program running in the computer wait until the predefined sampling rate is met, to allow the vehicle to update its position w(k + 1). The complete cycle is repeated until the user stops the main program from the GUI.



Figure 5.4 – Real-time experimental software block diagram

Chapter 6

Simulations of autonomous driving algorithms

In this chapter, the previously formulated NMPC controllers are simulated and evaluated. A total of three testing track scenarios were used in the simulation. The first track was appropriately modeled after a real running track, where the 1:5 scale robotic car was later tested. The second track was based on double lane change maneuver as mentioned in [5]. The third one was a path with obstacles, based on the test ISO 3888-2. The objective function's weights for each controller were individually tuned to achieve reliable and smooth performance. Also, three prediction horizon lengths N for each model were tested, with a sampling rate of $\Delta t = 100 \ ms$.

The results from all the simulations were compared and analyzed, in order to study the influence of the prediction horizon length, dynamic response for each vehicle model, computational time, and phase difference due to the delay in the actuators. These results were later used in order to choose the most suitable controllers to be implemented in the robotic car.

6.1 Vehicle testing tracks

In the current work, three different tracks were used in order to test the performance of each NMPC controller.

a. Circular testing track

The first one involves a circular path, modeled after a section of a real running track (see fig. 7.1a), where the robotic car was later tested. This testing track was chosen to allow the validation of the vehicle models, as well as the stability of of the NMPC controllers designed. The equation (6.1) defines the track trajectory, and its plot can be seen in figure (6.1).



Figure 6.1 – Circular track curve

b. Double lane change testing track

The second test track simulates a double lane change maneuver and is based on the equation found in [5]. The equation of the trajectory curve is shown in (6.2), and its plot in figure (6.2). This second track allowed to test how the NMPC controller would react in a fast, typical driving maneuver.

$$y_{ref} = \frac{dy_1}{2} (1 + \tanh(z_1)) - \frac{dy_2}{2} (1 + \tanh(z_2)) + y_{offset}$$

where
$$z_1 = \frac{F_{shape}}{dx_1} (x_{ref} - x_{s1}) - \frac{F_{shape}}{2}$$

$$z_2 = \frac{F_{shape}}{dx_2} (x_{ref} - x_{s2}) - \frac{F_{shape}}{2}$$

(6.2)

 $F_{shape} = 0.7, \ dx_1 = 3, \ dx_2 = 3, \ dy_1 = -2.5, \ dy_2 = -2.5, \ x_{s1} = 10, \ x_{s2} = 20, \ y_{offset} = 3.5.$



Figure 6.2 – Double lane change maneuver curve. Based on [5]

6 Simulations of autonomous driving algorithms

c. Obstacles avoidance testing track

The third and final track was based on the *evasive maneuver test*, commonly known as the "Moose test" or "Elk test". This test has been standardized in ISO 3888-2 [75]. Our path test track was designed as a straight path with two obstacles between the start and finish point. These obstacles were modeled like two identical ellipses with major, and minor axises of 0.8, and 0.4 m, respectively; they were placed at (5, -0.2) and (10, 0.2). The graphical representation is shown below. This third track tested the ability of the NMPC algorithm to avoid obstacles on the road, while also tracking a predefined path.



6.2 IpOpt configuration

Real-time operation can only be guaranteed if the NMPC problem is solved within the sampling rate of the system Δt . For that reason, the right configuration is crucial to ensure a good performance and accuracy of the control. The implementation in this works used IpOpt ver. 3.8.1 with MUMPS linear solver, distributed as a dynamic library compatible with Windows. Prior to the simulations, the following options were set up in IpOpt, and were kept along all the simulations shown in this work.

```
app->Options()->SetNumericValue("tol", 1e-3);
app->Options()->SetNumericValue("acceptable_tol", 1e-3);
app->Options()->SetIntegerValue("acceptable_iter", 0);
app->Options()->SetIntegerValue("print_level",0);
app->Options()->SetStringValue("warm_start_init_point","yes");
app->Options()->SetStringValue("mu_strategy", "adaptive");
app->Options()->SetStringValue("hessian_approximation", `\
"limited-memory");
```

The tolerance values were reduced from the default values of 10e-5, but they were maintained above the precision of the actuators of the robotic car, to ensure a good performance. Also, warm-start options were used in each iteration to reduce the computational time and increase the accuracy of the results, as in [44]. The cost function and its gradient, the model constraints (after the discretization with Multiple shooting), and the Jacobian of the constraints were all formulated symbolically using the CasaDi framework; these were later exported into C++ code, using the code-generation engine, also included with CasaDi.

The Hessian was approximated using the L-BFGS algorithm included in IpOpt. In the preliminary tests, the use of the approximated Hessian with the L-BFGS algorithm outperformed the exact Hessian provided with Casadi. For that reason, the L-BFGS approximation was chosen, as it had shown reliable performance in the simulations. Finally, since no visual output was required in the real-time implementation, the *print_level* option is set to the "0", meaning no output was printed in the command line.



6.3 Path tracking simulations

In this section, the results of the simulations done with the previous vehicle models are presented. All the controllers were simulated with three prediction horizons lengths N, to study its influence in the performance. Furthermore, each controller was simulated using two different test tracks, detailed in a previous section. All the simulation were done in a PC with an Intel Core i5 3320M @2.60 GHz, and 8 GB of RAM, running Windows® 7 OS. A precompiled version of IpOpt ver. 3.8.1 with MUMPS linear solver compatible with Windows was used. The simulation were compiled and executed on Microsoft Visual Studio® 2010.

a. Kinematic model results

The simulation results are presented in figure (6.4). The top of figure show the trajectories performed by the vehicle for different prediction horizons N in two test tracks and the reference trajectory (dashed red line). At the bottom, the optimal steering angles are shown.



Figure 6.4 – Simulation results for path tracking control with the Kinematic model in two different trajectories. $v_x = 2.22 \ m/s \ [8 \ km/h]$. Prediction horizons length $N = 5, \ 10, \ 15$

The weights used for each prediction horizon are summarized in table (6.1).

Ν	\mathbf{r}_{δ}	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P}_{\mathbf{y}}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
5	12	15	15	1	1
10	5	1	1	1	1
15	5	1	1	1	1

Table 6.1 – Weights for the Kinematic model in path tracking simulation

In the circumference track scenario, no significant difference was observed along the different prediction horizons N. However, the controller with N = 5 shows some oscillation at the beginning, which stabilized in the first three meters.

In the double lane change scenario, the three prediction horizon showed good performance, but the controller with N = 5 presented some oscillations at the exit of the last curve.

By observing the weights in table (6.1), it can be noticed that the controller with the shortest prediction horizon required significantly higher weights values. This indicates that the length N = 5 presents stability issues, due to the short predicted horizon used. Higher prediction horizon presented more stable performance, as evidenced by the figure (6.4), and the weights in (6.1). No significant improvement was observed between the controller with N = 10 and N = 15, which may indicate that higher prediction horizon lengths will not produce further improvements.

6 Simulations of autonomous driving algorithms

b. Double-track model results

The results of the simulation are shown in figure (6.5), with a structure similar to the previous section.



 $\label{eq:Figure 6.5-Simulation results for the reference tracking control with the four wheels model in two different trajectories. Different prediction horizons N were tested.$

The circumference track scenario presented no problem to any of the prediction horizon lengths tested. However, in the double lane change maneuver scenario, some problems were observed with the controller with N = 5, again. This confirms that a prediction horizon shortest than N = 5 may be unsuitable for implementation in real-time systems, due to stability related issues.
The tuned weights used for each simulation with different prediction horizon lengths N are presented in the table (6.2).

\mathbf{N}	\mathbf{r}_{δ}	$\mathbf{r_v}$	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P_y}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
5	10	0.4	5	5	1	1
10	6	0.3	2	2	1	1
15	6	0.3	1	1	1	1

Table 6.2 – Weights for the Double-track model in path tracking simulation

As in the Kinematic model case, the gains for the shortest prediction horizon N tend to be higher, in order to compensate the stability of the controller.

The controller with N = 10, and N = 15 showed both similar, and good performance in both scenarios.



6 Simulations of autonomous driving algorithms

c. Single-track model results

Finally, the results for the single-track model based NMPC are presented here. As in the Double-track model, the controller with the prediction horizon N = 5 shows stability problems. It can be concluded that, in order to have an stable NMPC with the current vehicle models and sampling rate $\Delta t = 100 \text{ ms}$, a prediction horizon must be equal o higher than 10 discrete units $N \geq 10$.



Figure 6.6 – Simulation results for the reference tracking control with the bicycle model in two different trajectories. Different prediction horizons N were tested.

Ν	$ \mathbf{r}_{\delta} $	$\mathbf{r_v}$	$\mathbf{P_x}$	$\mathbf{P}_{\mathbf{y}}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
5	15	0.4	4	4	1	1
10	10	0.3	2	2	1	1
15	10	0.3	2	2	1	1

Table 6.3 – Weights for the Single-track model in path tracking simulation

d. Performance comparison between vehicle models

Until now, each NMPC controller has been tested individually, with different prediction horizon lengths. In this part, the optimal control solutions of the three proposed models are compared, as well as the computational time required to complete each iteration.

The bar graphs in (6.7) summarize the computational time for each controller with the different prediction horizon lengths tested. The times presented are the average values of circumference and double lane change tracks. As expected, higher prediction horizons N, and complexer models required more computational time to converge to the optimal solution. Also, the computational time for the double-track model significantly increases with each increment of the prediction horizon. For the prediction horizon N = 15, the computational time for the double-track is 2.8x the time of the single-track model, and 8.62x the time for the Kinematic model.



 $\label{eq:Figure 6.7} \textbf{Figure 6.7} - \textbf{Average computational time per iteration for the simulations of the car models with different prediction horizon lengths N$

6 Simulations of autonomous driving algorithms

Next, the trajectory of the three vehicle models is compared in figure (6.8), using a prediction horizon length of N = 15, to avoid variations due to instability issues. As it can be seen, the three controllers can follow both test tracks very precisely, and they almost overlap each other.

Even though all the vehicle models produce almost the same trajectory, the steering response was different. In the double lane change maneuver (right), the response of the Kinematic model was very symmetric and smooth, since no inertial forces in the curves are considered. In contrast, the dynamic models compute different steering responses to acknowledge the lateral slip produced in the wheels, as observed between the 15th and 25th seconds.

Finally, both dynamic models were also compared. As it can be recalled from chapter 4, the Single-track model was deduced by making some simplifications in the Double-track model. It can be noticed that, the results of both models were quite similar, which supports the use of the Single-track over the Double-track model, due to shorter computational times. Taking in consideration the simulation results in this section, only the Single-track model and the Kinematic model will be used in the remaining of this work.





Figure 6.8 – Trajectory and control inputs comparison between NMPC controllers with different vehicle models. Prediction horizon length N = 15. Sampling rate $\Delta t = 100 ms$.

6 Simulations of autonomous driving algorithms

e. Phase difference of the steering angle and the input signal due to delay

In the chapter 4, the steering dynamics were modeled as a second order system. Here, the phase difference between the input control signal δ_{in} and the steering angle δ is presented, in order to emphasizes the importance of considering the steering dynamics into the vehicle modeling.

Figure (6.9) shows the phase difference of both signals, during the double lane change maneuver, previously presented. This phase difference increases when the vehicle speeds increases. Although this results only present simulated values, it allows to predict that the omission of the steering's dynamics in the NMPC controller could cause instability issues. For that reason, this discussion will be continued in the implementation chapter, in which the real impact of the delay on real-time operation could be better realized.



Figure 6.9 – Servomotor phase difference between input signal and actual position. Simulated with the kinematic model (N = 10) at $v_x = 5m/s$

6.4 Obstacle avoidance simulations

a. Kinematic model results

Next, the obstacle avoidance simulations are presented. Three prediction horizons length were simulated in a straight path with two obstacles to study its influence in the avoidance performance.



Figure 6.10 – Simulation results for obstacle avoidance with the Kinematic model. $v_x = 2.22 \ m/s \ [8 \ km/h]$. Different prediction horizons N were tested.

As it can be seen in figure (6.10), the controller with prediction horizon length N = 5had a very late reaction to avoid the obstacle, and deviated significantly from the proposed path. Due to the short prediction horizon length, the controller was unable to anticipate with enough time the obstacle. The controller with N = 10 presented a better performance at evading the obstacles, over the previous one; however, the delayed reaction at the second obstacle suggest that a higher prediction horizon length is needed. Lastly, the controller with N = 15 was able to evade both obstacles smoothly, without deviating excessively from the proposed path.

6 Simulations of autonomous driving algorithms

The tuned weights for the cost function of each simulation are shown below. As it can be seen, all the controllers used the same weights. Different values for each controller showed no improvement in the performance.

Table 6.4 – Weigh	nts for the Kinema	tic model in	obstacle	avoidance	simulation
-------------------	--------------------	--------------	----------	-----------	------------

N	\mathbf{r}_{δ}	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P}_{\mathbf{y}}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$	
5	2	2	2	1	1	
10	2	2	2	1	1	
15	2	2	2	1	1	
10	2	2	2	Ē		

b. Single-track model results

The results of the obstacles avoidance simulation for the single-track model are shown in the figure (6.11). Similarly to the Kinematic model simulation, the controller with N = 5 presented a poor performance in the obstacle avoidance test. The controller with N = 10 executed a better performance, but its reaction presented some delay, as in the Kinematic model case with the same N. Finally, the controller with N = 15 achieved an smooth, and overall good performance; also, it was able to reach and maintain the reference speed $v_{ref} = 2.22 \ m/s$, due to the higher prediction horizon.



Figure 6.11 – Simulation results for obstacle avoidance with the Bicycle model. $v_{ref} = 2.22 \ m/s \ [8 \ km/h]$. Different prediction horizons N were tested.

Master thesis Toshiharu Tabuchi

N	\mathbf{r}_{δ}	$\mathbf{r_v}$	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P}_{\mathbf{y}}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
5	2	0.3	2	2	1	1
10	2	0.3	2	2	1	1
15	2	0.3	2	2	1	1

 ${\bf Table} ~~ {\bf 6.5} - {\rm Weights} ~{\rm for} ~{\rm the} ~{\rm Single-track} ~{\rm model} ~{\rm obstacle} ~{\rm avoidance} ~{\rm simulation}$



Chapter 7

Experimental evaluation of autonomous driving algorithms

In the previous chapter, multiple simulations and analysis were done using different vehicle models. From those results, two NMPC controllers were selected and implemented in the 1:5 robotic vehicle. The controllers were based on the Kinematic and the Single-track model. The tests had two main objectives. The first one was to test the trajectory tracking performance of the NMPC controllers, using the selected vehicle models. The second main objective was to analyze the steering dynamics influence in the controllers stability and performance.

The Kinematic and Single-track NMPC controllers were tested using a prediction horizon N = 10, with a sampling rate $\Delta t = 100 \ ms$. The value of N was selected based on the simulations results, range of the sensors, and spatial resolution required for a good performance of the tracking. These criteria are expanded later in this chapter.

First, an overview of the command center software is presented. This software manage the image processing, and NMPC controller algorithms. Then, it is explained how the path reference was computed. In this work, the reference points for the NMPC algorithm are defined as a function of the speed and sampling rate, which means that at higher speeds, the look-ahead distance increases.

Second, the NMPC autonomous driving algorithms experimental results are presented. The tests were made in two different test paths, similar to the ones used in chapter 5. The first track simulates a double lane change maneuver and it was recreated in one of the TU Ilmenau's research buildings (figure (7.1b)); this track tested the controller

7 Experimental evaluation of autonomous driving algorithms

ability to manage sudden changes in the path curvature. Also, since the test track was indoors, it allowed to obtain more consistent data, at any time, independently of the weather conditions; however, the maximum speed was limited $(3.33 \ m/s \ [12 \ km/h])$ due to space restrictions. The second path was a running track, which is part of the TU Ilmenau sport's facilities. A satellite image is shown in the figure (7.1a), with coordinates 50°41'05.6"N 10°56'12.8"E. This track permitted testing the controllers consistency and stability, since the path had a constant curvature. Also, because the track was outdoors and the surface presented a high friction coefficient, it enabled higher speeds (5.56 $m/s \ [20 \ km/h]$) to be tested. In all the tests, the vehicle started from approximately the same initial position as in the simulations. This facilitated the comparison between experimental, and simulated results.

This chapter is concluded with the steering's dynamics influence evaluation in the vehicle's control stability and performance. This tests were made in parallel with the previous presented cases; then, the same scenarios and vehicle models are used. Each NMPC controller was tested in two different configurations: with the steering's dynamics considered, and without them. Even though the steering system is a fundamental component in the control of the vehicle, only few researchers address its dynamic response. The results prove that including the steering's dynamics in the vehicle model are fundamental for a good controller performance.



(a) Circular track



(b) Double lane change track



7.1 Command center software

The command center software running in the on-board PC manages all the highlevel functions for the autonomous driving system. The program was developed in Embarcadero C++ Builder XE8. Since the Embarcadero C++ compiler was not able to compile the IpOpt libraries, the NMPC controller was integrated as a dynamic library (dll), which was compiled independently in Microsoft Visual Studio 2010[®]. The sensor data acquisition and processing, as well as the NMPC controller algorithm runs over this software. A graphical user interface (GUI) (see fig. (7.2)) facilitates the monitoring of the vehicle sensors, image processing, batteries status, and operation of the autonomous driving algorithm. The GUI also provides access to some parameters of the NMPC algorithm. The user can change the weights of the cost function, sampling rate and reference speed without modifying the software code.



Figure 7.2 – User interface for vehicle control

Master thesis Toshiharu Tabuchi

7.2 Path planning

In robotics, a path is defined as a continuous curve function that maps some path parameters in the robot's configuration space [76]. The problem of path planning can be summarized as a geometric path generation, without an associated time law, from an initial to a final point, while avoiding obstacles and respecting given constraints [77]. Common approaches of path planning for non-holonomic vehicles use clothoid or polynomial curves, due to their smooth curvatures. In this thesis, a car-like robot will be tested in a track with printed lanes over, that simulates a real-life scenarios in a typical street. It will be assumed that the lanes follow smooth, non-holonomic compatible curves, and that the controller can reach practical stabilization during the path tracking as mentioned in [78]. Information of the lanes will be captured by a monocular CMOS camera. An image processing algorithm based on the work in [79], finds the lane's center line and extract its geometric features, which the car uses as reference for the trajectory tracking control (see fig. (7.3)). This task is repeated at each sampling time.

a. Reference points and look-ahead distance

The path's information reference is an important component of the objective function, since it provides the feedback to estimate the relative position of the vehicle with respect to the tracked curve. In this part, it is explained how the information obtained from the CMOS camera is processed, and what limits were defined in order to maintain a reasonable error within the measurements.

a.1. Path curve identification applying image processing

First, the CMOS camera capture an image of the road ahead, as in figure (7.3). Then, an image processing algorithm extract the geometric features of the lane lines, computes the center line, and identifies the parameters of the curve. In this work, a second degree polynomial with origin in the vehicle's COG was used.

$$x_{path} = ay^2 + bx + c \tag{7.1}$$



Figure 7.3 – Camera's raw stream(top) and geometric features extraction(bottom)

A segment of the curve (7.1) is given to the NMPC algorithm as the path reference. This segment is discretized into N points (see fig. (7.4)), in order to match the length of the prediction horizon. The length step dy_k is defined as a function of the current longitudinal speed of the vehicle v, and the sampling rate Δt . Furthermore, as seen in figure (7.4), there is an region of the curve below camera's field of view (FOV), which can not be captured. This area of the path is extrapolated backwards using the equation (7.1). It may also be possible to extrapolate the curve forward, beyond the FOV of the camera. However, in the tests, the forward extrapolation presented a very high deviation from the real values. For that reason, that section of the curve is not used in this work.

7 Experimental evaluation of autonomous driving algorithms



Figure 7.4 – Reference points obtained from the camera

a.2. Parameters and limits for the path curve parametrization

As stated in the previous section, the step length dy_k is a function of the speed and the sampling rate. In order to guarantee, to a certain degree, a good tracking performance, the length step dy_k should be maintained between a certain range. In this work, a methodology to estimate this range is formulated. This range is defined by inequalities (7.2) and (7.3).

In inequality (7.2), a maximum length step is defined, to maintain a good tracking performance of the curve. It was found that a length step below half the length of the car produces a good tracking performance. Inequality (7.2) also indicates that, in order to use higher speeds, the time step Δt , and therefore the sampling rate, should be reduced proportionally to maintain a good discretization of the curve.

$$dy_k = v\Delta t \leq \frac{L_{car}}{2}, \quad k = 0, 1, ...N$$
 (7.2)

Another limitation for the curve trajectory computation is the *look-ahead distance*, defined in inequality (7.3) as the total length of the curve that the algorithm uses

to compute the path's reference. The maximum look-ahead distance, that can be trustfully used, depends of the visual range of the camera used, defined by D_{max} . As mentioned earlier, the area beyond the FOV of the camera is not used. Also, a minimum look-ahead distance D_{min} is defined, since if it is too short, the NMPC algorithm could have problems tracking the curve, due to the lack of information. In this work, a $D_{min} = 1.5L_{car}$ is used. The equation in (7.3) also reveals that, if the prediction horizon length N is incremented, the maximum range of the sensors also need to be incremented, or the maximum speed of the vehicle reduced.

Look-ahead distance
$$= Nv_x \Delta t$$

 $D_{min} \leq Nv_x \Delta t \leq D_{max}$
(7.3)

With inequalities (7.2) and (7.3), it is possible to calculate the maximum safe speed to be used, according to the actual setup of the vehicle. Then, with $L_{car} = 0.9 m$, $D_{max} = 5 m$, N = 10, $\Delta t = 0.1 s$, it can be easily found that the maximum recommended speed for the setup is $v_{max} = 4.5 m/s$.

It is important to stress that the methodology previously defined by here must be taken as a **recommendation**. The calculated values showed good results in the tests, but this may vary depending of the vehicle or sensors setup.

7.3 Experimental results in the double lane change maneuver test track

In this section, the experimental results performed in the double lane change maneuver test track are presented (see fig. 7.1b).

a. Kinematic model experimental results

Figure (7.5) and the video links below show the optimal steering angle δ , as well as the timing results per sampling rate. As expected, the simulated and experimental results differ. The main reason could be attributed to some geometric differences between the testing track built (see fig. 7.1b) and the equation in which it is based.

There is also mismatches between the system and vehicle model. At the speed the vehicle was tested (v = 3.33 m/s), the wheels showed some signs of slippery, an inertial effect that the kinematic model does not take in consideration. This mismatch is more evident between the 4th and 6th second, in which the car made an overshoot in the steering, due the sudden change of curvature in the path. Also, the path reference from the camera and image processing algorithm may present some inaccuracies.

The computation time are presented in the bottom graph. Although the times were higher than the reported in simulations, since they kept below the sampling rate of the system, no problems were encountered during the execution of the algorithm. This issue could be attributed to different initial conditions in each iteration for the IpOpt solver, compared to the simulated version. As mentioned in the chapter 5, in the vehicle implementation, the reference frame coordinate system is reset in every iteration, due to the lack of an odometry unit. This difference translates into more iterations to achieve the optimal solution. Also, it is be possible that software-related bugs may had introduced during the porting of the code into a dynamic library, which is latter loaded by the command center software.



Figure 7.5 – Experimental results for path tracking control with the Kinematic model (N=10) . $v_x = 3.33 \ m/s \ [12 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$.

Also, the two following hyperlinks provide access to the test's videos.

CD version:

• Video: Test with Kinematic model based controller (with steering's dynamics)

Online version:

• Video: Test with Kinematic model based controller (with steering's dynamics)

In table (7.1), the tuned weights used in the test are showed. These weights may differ from the ones used in the simulation, due to system-model mismatches.

Table 7.1 – Weights for the kinematic model in the double lane change track test

N	\mathbf{r}_{δ}	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P_y}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
10	25	5	5	1	1

b. Single-track model experimental results

Next, the results from the NMPC controller based on the Single-track model are presented in figure (7.6) and the video links below. The optimal values for the steering δ and speed v are presented in the first and second graph. As in the Kinematic model case, some mismatches were expected, due to the previous reasons explained. However, due to the additional consideration of the lateral slip in the Single-track model, the vehicle managed better the curvature change, between the 4th and 6th second. Also, the reference speed given is reached in the at time t = 2 s and maintained during the whole course.

The bottom graph shows the computation time per iteration of the controller. Although few iterations exceeded the sampling rate of the system, they did not represented a major concern during the algorithm execution. This timings were higher than the reported in the simulations and may be related to different initial conditions in every iteration, due to the reference frame reset.



Figure 7.6 – Experimental results for path tracking control with the Dynamic model (N=10) . $v_{ref} = 3.33 \ m/s \ [12 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$

t [s]

Also, the two following hyperlinks provide access to the test's videos.

CD version:

• Video: Test with Single-track model based controller (with steering's dynamics)

Online version:

• Video: Test with Single-track model based controller model (with steering's dynamics) In table (7.2), the tuned weights used in the test are showed. As in the kinematic case, these weights may differ from the ones in used the simulation, due to system-model mismatches.

Table 7.2 – Weights for the Single-track model in the double lane track test

\mathbf{N}	\mathbf{r}_{δ}	$\mathbf{r_v}$	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P}_{\mathbf{y}}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
10	15	0.3	2	2	1	1

7.4 Experimental results in the circular test track

This section reports the results obtained from the circular test track experiments (see fig. 7.1a). Since the running track's surface is made of an special material with a high friction coefficient, it was possible to test higher speeds with the vehicle.

a. Kinematic model experimental results

Figure (7.7) and the video links below show the NMPC controller results based on the Kinematic model. In the first graph, it is observed that the steering angle oscillates around the simulated value. However, it should be noted that the amplitude of these oscillations were small $|\delta| \leq 1.5^{\circ}$ and did not increased over time, which indicates that the system was stable. In tests, the vehicle drove smoothly and the oscillations were not perceptible in the movement.

Compared to the tests in the double lane change maneuver, the computational times increased slightly. Although few iterations exceeded the system's sampling rate, the overall performance was not affected. As mentioned earlier, this problem may be attributed to different initial conditions in the iterations, due to the reference frame reset.



Figure 7.7 – Experimental results with the Kinematic model (N=10) in the circumference test . $v_x = 5.55 \ m/s \ [20 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$

Also, the following hyperlinks provide access to the test's videos.

CD version

- Video 1: Test with Kinematic model based controller (with steering's dynamics)
- Video 2: Test with Kinematic model based controller (with steering's dynamics)

Online version

- Video 1: Test with Kinematic model based controller (with steering's dynamics)
- Video 2: Test with Kinematic model based controller (with steering's dynamics)

In table (7.3), the tuned weights used in the test are showed. These weights may differ from the ones used in the simulation, due to system-model mismatches.

N	\mathbf{r}_{δ}	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P_y}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
10	23	2	2	1	1

Table 7.3 – Weights for the kinematic model in the circular track test

b. Single-track model experimental results

Figure (7.8) and the video links below present the results obtained using the Single-track model. Similar to the previous case, the steering angle oscillates around the simulated value. The reference speed is reached around t = 2 s and maintained until the end of the test.

The computation time graphs revealed that the algorithm exceeded the system's sampling rate in many occasions. Unfortunately, at the 7th second, one iteration exceeded the sampling rate and the system was not able to recover from the unstable state. However, during the period in which the computation time was kept below the sampling rate, the algorithm worked as expected. The time could be reduced by compiling the code using *optimization flags*. In this works, these options were not used for the controllers, since each compilation can take up several hours. Multiple tests have to be done in order to debug the code; then, using the *optimization flags* was not a viable option at the moment. Future works could test the compilation with those options.



Figure 7.8 – Experimental results for path tracking control with the Single-track model (N=10) . $v_{ref} = 5.55 \ m/s \ [20 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$

Also, the following hyperlinks provide access to the test's videos.

CD version

- Video 1: Test with Single-track model based controller (with steering's dynamics)
- Video 2: Test with Single-track model based controller (with steering's dynamics)

Online version

• Video 1: Test with Single-track model based controller (with steering's dynamics)

7.5 Influence of the steering's dynamics in the real-time performance of the vehicle

• Video 2: Test with Single-track model based controller (with steering's dynamics)

In table (7.4), the tuned weights used in the test are showed. These weights may differ from the ones in used the simulation, due to system-model mismatches.

 ${\bf Table \ 7.4-Weights \ for \ the \ Single-track \ model \ in \ the \ circular \ test \ track}$

$\mathbf{N} \mid$	\mathbf{r}_{δ}	$\mathbf{r_v}$	$\mathbf{P}_{\mathbf{x}}$	$\mathbf{P_y}$	$\mathbf{q}_{\mathbf{x}}$	$\mathbf{q}_{\mathbf{y}}$
10	10	0.3	3	3	1	1

7.5 Influence of the steering's dynamics in the real-time performance of the vehicle

In this section, an evaluation of the steering dynamics' influence in the NMPC controller stability is presented. At low speeds ($v \leq 2 m/s$), the controllers in which the steering system's inertia was not considered, were able to perform at an acceptable level. However, when the speed increased, they became unable to follow the path.

Figure (7.9) and the video links below present the kinematic model NMPC tests' results on the double lane change maneuver track, in three different configurations. The blue line defines the optimal value for the steering angle using the NMPC controller, in which the steering's dynamics were considered. This controller was able to finish the course successfully. The green line represents the kinematic model without the steerings dynamics. It can be seen that the reaction of system is slower, since the angle starts changing later than in the previous case. This caused the car to run outside the path. The previous model was also tested with a sampling rate above the transient state of the servomotor (see fig. (4.3)). The results of this configuration are plotted with orange and shows that the car was also unable to follow the path.



Figure 7.9 – Steering's dynamics influence in the performance of the NMPC controllers. Vehicle model: Kinematic (N=10) . $v_x = 3.33 \ m/s \ [12 \ km/h]$. Sampling rate $\Delta t = 100 \ ms$

Also, the following hyperlinks provide access to the test's videos.

CD version:

- Video 1: Test with Kinematic model based controller (without steering's dynamics and sampling rate $\Delta t = 100 ms$)
- Video 2: Test with Kinematic model based controller (without steering's dynamics and sampling rate $\Delta t = 300 \ ms$)
- Video 3: Test with Kinematic model based controller (with steering's dynamics)

Online version:

- Video 1: Test with Kinematic model based controller (without steering's dynamics and sampling rate $\Delta t = 100 ms$)
- Video 2: Test with Kinematic model based controller (without steering's dynamics and sampling rate $\Delta t = 300 \ ms$)
- Video 3: Test with Kinematic model based controller (with steering's dynamics)

Figures (7.10) and (7.11) show the tests' results made in the circular test track. As can be seen in figure (7.10), the kinematic model with the steering model, represented by the blue line, even tough presented an oscillatory response, the amplitude was small and did not increase overtime. In contrast, the kinematic model without the steering's dynamics, represented by the green line, the steering angle's amplitude increases overtime and the vehicle became unstable. A similar situation is observed in figure (7.11).

7.5 Influence of the steering's dynamics in the real-time performance of the vehicle

In summary, the NMPC controllers in which the steering dynamics were ignored showed an unstable and a noticeable oscillatory movement, independent of the vehicle model. The NMPC controllers with the steering model were more stable and their oscillations were not appreciable. This suggest that the steering's dynamics are very critical to the autonomous driving control, and may be even more crucial than other inertial effects.



Figure 7.10 – Steering's dynamics influence in the performance of the NMPC controllers. Vehicle model: Kinematic (N=10) . $v_x = 5.55 \ m/s \ [20 \ km/h]$.



Figure 7.11 – Steering's dynamics influence in the performance of the NMPC controllers. Vehicle model: Single-track model (N=10).



Chapter 8

Conclusions and future work

8.1 Conclusions

The work presented in this thesis has focused on the design and implementation of an autonomous driving algorithm for trajectory tracking. The algorithm was formulated using the model predictive control (MPC) framework, which is a modern, state-of-the-art control strategy. In MPC, a sequence of optimal control inputs are predicted within a short time horizon using the non-linear dynamics, and soft or hard restriction of the system. Since experimentation on real-sized vehicles involve high costs, the non-linear MPC algorithms were tested in a 1:5 scale robotic car, which allows more realistic results compared to virtual simulations.

Three mathematical model, specifically tailored for the robotic car, were formulated. Also, the steering system's dynamics were identified and included in the vehicle models to increase the precision and stability. The steering system model was identified by fitting a second order, black-box model using experimental data. For the general vehicle mathematical representation, one kinematic and two dynamic models were formulated. A Kinematic model was deduced from geometrical properties, without considering inertial effects and forces in the wheels. This model was accurate at reduced speeds, in which inertial effects like drifting and slip were not noticeable. For higher speeds, the inertial effects and forces were introduced to generate two dynamic models: a Double-track model, and a Single-track model, which is a simplification from the previous one. The MPC in continuous time was transformed into an NLP, in order to be solved with state-of-the-art NLP optimizations solvers. In this work, the interior-point based IpOpt solver was used, due to its good documentations, efficiency, and the ability to run on Windows®. The transformation to NLP was done via discretization with the Multiple shooting scheme. Within the shooting intervals, a Runge-Kutta 4 (RK4) integrator was used. IpOpt requires that the user provide not only the cost and the constraints functions, but also the gradient of the cost function, and the Jacobian of the constraints. The cost function and the constraints were symbolically formulated within the Casadi framework. Applying an state-of-art Automatic differentiation (AD) algorithm, these two elements were derived to obtain the gradient and Jacobian of the problem. Casadi also allowed to export all these functions as C++ code, which were later integrated with IpOpt.

Before the implementation in the robotic vehicle, the NMPC controllers were simulated, using to three different scenarios; the first and second track were used to test the trajectory tracking performance, while an additional third one tested the ability of the algorithm to avoid obstacles. All the simulations were executed in the same computer, with identical setups for the IpOpt solver and the Microsoft Visual Studio 2010 C++ compiler, with a sampling rate $\Delta t = 100 \text{ ms}$. The results from the trajectory tracking suggests that a minimum prediction horizon of N = 10 is required in order to achieve stability and good performance of the controller. As expected, longer prediction horizons and complexer models required more computation time.

In addition, the results from the obstacles avoidance test revealed that an even higher prediction horizon (N = 15) may be needed in order to achieve an smooth response during evasion maneuvers. Another discovery from the simulations was that Doubletrack and Single-track models presented a very similar responses for the trajectory tracking tests. For that reason, the Single-track model was chosen over the Double-track model for the implementation, due to the lower computational resources it requires.

The experimental test bed used was a self-contained, 1:5 scale robotic vehicle, equipped with an on-board computer and multiple sensors. The on-board PC runs a command center software, developed in Embarcadero C++ Builder®. Since the Embarcadero C++ compiler was not able to compile the IpOpt libraries, the NMPC controller was integrated as a dynamic library (dll), which was compiled independently in Microsoft Visual Studio 2010®. The command center software coordinates the acquisition of the data, executes the autonomous driving algorithm, and monitors the status of the sensors and batteries. For the path tracking, a CMOS camera coupled with an image processing algorithm was used.

Additionally, a methodology was proposed, in order to discretize the reference curve and to estimate the maximum recommended speed for the NMPC algorithm, based on the vehicle and sensors setup. This methodology suggests that in order to reach higher speeds, further information of the environment is required, and also smaller sampling rates to keep the discretization of the path below an acceptable range.

For the experimental tests, the Kinematic and Single-track based NMPC algorithms were implemented in the vehicle, using a prediction horizon of N = 10, and a discretization time of $\Delta t = 100 \text{ ms}$. Two test tracks were used for the trajectory tracking test. The first track recreated a double lane change maneuver and was built inside one of TU Ilmenau's buildings. The speed was limited to 3.33 m/s [12 km/h] due to the safety reasons. Both tested controllers were able to complete the track. However, the bicycle model presented a better performance, due to the additional information of the inertial effects.

The second test was done in a circular running track, which is part of the TU-Ilmenau university's sports facilities. Since the track was outdoors, it was possible to test the vehicle up to speeds of 5.55 m/s [20 km/h]. From the results, it can be concluded that at low speeds, both the Kinematic and Single-track model would perform similarly. At higher speeds, the Single-track model showed less oscillations and better performance. speeds, independently of the vehicle model used. Even though a few previous works considered basic steering models, this is the first study to our knowledge to investigate the influence of the steering's delay effects on the stability of controllers for autonomous driving applications.

Additionally, the influence of the steering's dynamics in the control was also evaluated. In the tests, the controllers without the steering's model were stable only at low speeds $v \leq 2.22m/s$ [8km/h]. The controllers with the steering's dynamics maintained stability with speeds up to v = 3.33m/s [12km/h] in the Double lane change maneuver test, and v = 5.56m/s [20km/h] in the Circular track.

In summary, the results from this work provide compelling evidence that a non-linear MPC controller could be implemented as the core of future autonomous driving algorithms, since it computes the optimal control inputs, taking in consideration the restrictions inherent to the system. However, some limitations are worth noting. Although in most of the tests, the vehicle was able to complete the test track, the computation time per iteration is still too high, which could present more complications if higher speeds are used. Future work should therefore focus on reducing the computation time, and also limiting the execution time per iteration to ensure safe, real-time operation.

8.2 Future work

One of the major drawbacks of MPC is the high computational resources it demands. In most of the tests in this work, the computation times were kept below the sampling rate of the system, but some iterations exceeded it. This suggests that, with some improvements, the NMPC algorithms presented in this work could reduce its timings below the sampling rate of the system. The easiest way to achieve this could be by using a faster, more powerful CPU, at the expense of more power consumption. However, in a real-sized vehicle, the energy requirements of this new CPU would not represent a major concern, which means that it is a very reasonable solution. Computational time could also be reduced by activating the *optimization flags* in the used compiler. In this work, these options were not used, since each compilation could take up several hours, which would have limited the number of tests made. A third alternative would be to change the linear solver within IpOpt (MUMPS was used in this work), to a more efficient one; or using a different optimization solver, more suitable for real-time operation like ACADO [80]. A fourth option could be applying parallel computation. There is already some works, like in [2], which presents promising results for reducing the computation time using parallel optimized algorithms for MPC. All these ideas mentioned below are not exclusive, and could be combined to obtain the best possible outcome.

Another minor problem in the tests was the lack of a precise odometry information to obtain the position of the vehicle in the world's frame of reference; this could allow a better comparison between the simulation and experimental results. Also, this unit may help reduce the the computational time, since better initial conditions can be loaded in every iteration of the NMPC controller. A future task could be the design and implementation of an odometry unit using a Kalman filter, to precisely estimate the vehicle position.

In the other hand, the reference information is an important component for the NMPC algorithm. A higher range of this reference, and the inclusion of more details may improve the tracking performance. This could be achieved by using sensor fusion

algorithms which combine data from the camera, LIDAR, GPS and maps to estimate the vehicle location within a map, and the details of its surroundings.



Finally, a core component of an MPC algorithm is the system's model. Improving the vehicle model, by using system identification of the parameters could increase the response accuracy. It was demonstrated in this work, that the steering's dynamics inclusion in the vehicle model dramatically increased the NMPC controller stability. The inclusion of other actuators models could improve even more the controller capabilities. Another task could focus on the tire model improvement. In the actual controller, a linear tire model was used. However, at higher speeds, this model could reach its operation limit and no longer be valid. Then, it may be necessary to use more complex models like the Magic tire formula from Pacejka or the Fiala tire model. Furthermore, the vehicle models presented in this work focused exclusively in the lateral dynamics, which represent the most fundamental part in the vehicle movement control. However, for more complex driving situations, like variable changes of acceleration and braking, the longitudinal control should also be considered.



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Master thesis Toshiharu Tabuchi

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