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Neutron star oscillations

Linking gravitational waves to microphysics

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Fast rotating isolated neutron stars are strong sources of gravitational waves when deformed. One possible source of such a deformation is the occurrence of unstable oscillation modes in the star. Their properties strongly depend on the equation of state, describing the behavior of matter at extremely high densities. A future detection of gravitational waves from rotating neutron stars can thus provide insights not only for relativistic astrophysics, but for nuclear physics as well.

1 Scientific background

When a star reaches the end of its lifespan, the nuclear fusion powering it comes to an end, resulting in the collapse of the stellar core. Depending on the mass of the initial star, the result of the collapse can be one of three objects: a white dwarf, a neutron star or a black hole.

Lighter stars (such as our Sun) leave a white dwarf, which is supported against further collapse by the degeneracy pressure of the electrons. A white dwarf as massive as the Sun is only about as large as Earth.

The remnant of higher mass stars can not be supported in the same manner. Instead, it collapses further, squeezing electrons and protons tight enough together to form neutrons, whose degeneracy pressure now stabilizes the newly formed neutron star (NS). These objects have densities comparable to an atomic nucleus, meaning that a typical 1.4 solar mass NS is only about 10 km in radius.

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Even heavier stars form a black hole as the remnant of their collapse. Not even letting light escape their influence, black holes generate the most extreme gravitational effects. They do not, however, possess any material qualities and their behavior is completely determined by a very small number of parameters.

1.1 Neutron stars

NSs are the densest material astrophysical objects and as such provide unique opportunities to study matter under most extreme conditions. Their existence was already proposed in the 1930s, while the first direct detection happened in the 1960s. Since then, several thousand NS have been observed, some of them with additional interesting properties such as extremely intense magnetic fields or very high rotation rates of nearly 1 kHz. While those observations all relied on electromagnetic observations, in 2017 the first detection of gravitational waves from the merger of two NSs was accomplished (B. P. Abbott, R. Abbott, T. D. Abbott, Acernese et al., 2017).

1.2 Gravitational wave astronomy

Being the subject of the Nobel prize in physics 2017, gravitational wave astronomy has recently gained a lot of attention. While classical astronomy relies on electromagnetic waves as carriers of information (be it radio, optical or gamma-rays), the first direct detection of gravitational waves (GWs) in 2015 (B. P. Abbott, R. Abbott, T. D. Abbott, Abernathy et al., 2016) has opened up a completely new window for the observation of astrophysical objects.

Unlike electromagnetic waves, gravitational waves are essentially unaffected by matter they pass through, since they do not rely on electromagnetic fields to propagate but only on the fabric of spacetime itself, as described by Einstein's general theory of relativity. Locally, they manifest as tiny changes in distance between otherwise fixed points. In order to detect GWs, those tiny changes have to be measured to very high precision which is realized by extremely sensitive interferometer setups.

The strongest sources of gravitational waves are the mergers of very massive objects such as black holes or NSs. When two such objects orbit each other, they emit GWs and thus loose orbital energy, bringing them closer together. The closer they are, the stronger the emitted GWs are, and the faster their orbits shrink. While it can take millions of years until the two objects come close enough together to touch and merge, it will happen inevitably. Just when they touch, the gravitational wave signal is strongest and gets weaker, while the objects merge and form a single new black hole or NS. Therefore, the typical signal for such a merger is characterized by a signal with a distinctive peak in amplitude and a monotonously increasing frequency. Several such signals have been observed over the past years and provided valuable information about the masses and several additional parameters of the objects involved.

1.3 Gravitational waves from neutron stars

While these mergers yield the strongest signals (and therefore the easiest to be detected), they are not the only sources of GWs. In this project we are mainly interested in the continuous signal that can be emitted by fast rotating deformed NSs. A perfectly axial-symmetric rotating NS does not emit GWs, but as soon as there is a deviation from symmetry, GWs are being generated.

Just as a water drop in zero gravity (or any other physical system for that matter), a NS has distinct frequencies it can naturally vibrate at. These vibrations can involve a plethora of hydrodynamic and general relativistic quantities and forces, as well as different geometrical patterns. A large number of these vibration patterns (or oscillation modes) lead to at least short-lived deviations from axial symmetry. Since such oscillations are typically damped by one mechanism or another, the GWs created by these asymmetries are usually very weak.

However, if certain conditions on the star's rotation rate and/or the oscillation frequency are fulfilled, it is possible for one or more oscillation modes to become *unstable* via the so-called Chandrasekhar-Friedman-Schutz (CFS) mechanism (Chandrasekhar, 1970; Schutz et al., 1975). An unstable mode can siphon energy off the star's rotation and increase its oscillation amplitude further and further. With the amplitude increasing, so does the deviation from axial symmetry and therefore also the gravitational wave amplitude.

Since we can observe NSs rotating fast enough to drive one or more modes unstable, this process cannot go on indefinitely (which would mean ripping the NS apart), but saturates at a certain (unknown) amplitude. With several models for the saturation mechanism being proposed, knowledge of the saturation amplitude is an important ingredient in properly understanding the mechanism and also gaining information on hydrodynamic properties of the NS (e. g. viscosity).

The CFS mechanism can therefore lead to rather strong gravitational wave emission from fast rotating NSs, the detection of which would allow to draw conclusions on the structure of the NS itself.

In the immediate future, however, the detection of GWs from mergers of binary NS systems is more likely, with one such event already recorded. While the broad structure of those signals primarily depends on the NS masses, its finer details still carry imprints of the matter oscillations. Given a clean enough signal, similar conclusions can be drawn.

However, all of this requires precise knowledge of different features of the oscillation modes, the most important one being the oscillation frequency. These properties strongly depend on the behavior of matter under the extreme conditions inside a neutron star, which is encoded in the equation of state.

1.4 The equation of state

The matter in the interior of NSs has very unique properties that can neither be found in any other astrophysical environment nor be recreated in a laboratory. In the core of a NS, particles are squeezed together more tightly than in an atomic nucleus. The behavior of matter under such conditions is one of the greatest unsolved problems of modern nuclear physics, the solution of which would allow us to better understand the very early history of the universe.

For our purposes, all the microphysical interactions can be encoded in the so-called equation of state (EoS), connecting macroscopic, thermodynamical quantities such as pressure, (energy) density or temperature. Many different models, computations and theories have led to a huge number of proposed EoSs. Determining which is the »correct« one is a very important step in solving this problem.

Since the EoS describes the behavior of the matter the NS consists of, it has a huge impact on its properties. Different EoSs will lead to a different mass-radius relationship for example, but also to different properties of the oscillation modes. By investigating the behavior of possibly unstable oscillation modes and the resulting gravitational wave emission under the assumption of different EoSs, we can therefore constrain the EoS with future GW observations from fast rotating NS. This will boost our understanding of the behavior of supranuclear matter and thus the early universe.

1.5 State of the art

Although oscillations of relativistic stars have been studied for more than half a century and many of the oscillation properties have been revealed, the focus was on non-rotating or slowly rotating objects. The main reasons were the numerical challenges presented in the non-linear treatment and the size of the perturbation equations in linear theory.

During the last decade, our group considerably advanced the field by studying the oscillations and instabilities of relativistic stars in the Cowling approximation, i. e. freezing the spacetime perturbations (Gaertig and Kokkotas, 2008; Gaertig and Kokkotas, 2009; Krüger et al., 2010; Kastaun et al., 2010). The Cowling approximation is known to provide very good qualitative results, which, however, are known to deviate from the exact solutions by typically 10-30 %, at least in the case of non-rotating stars. Thus, in this linear formalism our group developed asteroseismological relations which can be used to extract the parameters (mass, radius, rotation rate and EoS) of the stable and unstable fast rotating neutron stars (Gaertig and Kokkotas, 2011; Doneva, Gaertig et al., 2013; Doneva and Kokkotas, 2015). Furthermore, the criteria for the onset of CFS instability and its efficiency were also discussed in (Gaertig, Glampedakis et al., 2011; Passamonti et al., 2013), while the saturation amplitude of the instabilities was studied in (Pnigouras et al., 2015; Pnigouras et al., 2016).

This expertise led to proposing a novel scenario, according to which the post-merger remnant of colliding neutron stars, if it is not so massive as to collapse directly to a black-hole, will rotate quite fast, near its Kepler limit and will be rotationally unstable (Doneva and Kokkotas, 2015). The efficiency of the process depends on three factors, the accurate knowledge of the oscillation frequency, the growth time of the instability and the saturation amplitude. All three cases are known in Cowling approximation, but in order for the process to be detectable and astrophysicaly relevant, i. e. to be able to extract the parameters of the star, more accurate calculations are necessary.

In this direction we are concentrating our effort, hoping to be ready for the next binary neutron star mergers and to contribute to the global effort.

2 Numerical setup

The main goal of this project is to accurately investigate the properties of various oscillation modes of NSs and thus create a better understanding of the microphysics of matter under extreme conditions via future gravitational wave observations. To this end, we want to perform a number of fully non-linear relativistic three-dimensional numerical simulations of the evolution of isolated NSs. Each simulation assumes a certain NS model by choosing a mass, rotation rate and EoS. For each model, several different initial perturbations of the equilibrium model have to be considered in order to excite various oscillation modes. These simulations largely rely on the Einstein Toolkit[1] (Löffler et al., 2012), an open source software platform for relativistic astrophysics, and are mostly carried out on the bwForCluster BinAC[2].

The results of each simulation need to be analyzed further in order to extract the relevant properties of the oscillation modes. This task is carried out on the same machine by a custom Python code.

2.1 System of equations

For a working numerical model of the evolution of a NS, we need two key ingredients. The first is a formulation of the field equations of general relativity, since the Newtonian theory of gravity is not applicable to the extremely strong gravitational fields governing the evolution of a NS. The second are the equations of relativistic hydrodynamics, as we can use a perfect fluid description for the matter inside the star, with all the microphysics encoded in the EoS. The actual time integration is carried out using the method of lines. This way, we do not have to take care of the coupling between the hydrodynamic quantities and the spacetime quantities explicitly.

[1] Einstein Toolkit: Open software for relativistic astrophysics: `http://einsteintoolkit.org/`, (visited on 17. 06. 2018).

[2] bwForCluster BinAC: `https://www.binac.uni-tuebingen.de/`, (visited on 17. 06. 2018).

2.1.1 The field equations

Einstein's field equations describe the relation between the mass-energy content of the universe and its spacetime geometry:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{1}$$

Here, $G_{\mu\nu}$ denotes the Einstein tensor describing the curvature of spacetime. The stress-energy tensor $T_{\mu\nu}$, on the other hand, describes the energy content of matter and fields. Due to the strong interconnection shown by these equations, we cannot treat the evolution of spacetime and matter separately, but have to include the state of the one in the evolution equations of the other.

However, being four-dimensional tensor equations describing the entire spacetime geometry at once, the field equations are not very well suited for numerical treatment. Instead, it is possible to pose the problem as a time succession of three-dimensional space geometries, i. e. an initial value problem.

Here we are using the very successful BSSN (Baumgarte, Shapiro, Shibata, Nakamura) formulation (Shibata et al., 1995; Baumgarte et al., 1998). It achieves such a reformulation by decomposing the field equations into three parts: a set of time-evolution equations for a number of abstract spacetime quantities, a set of constraint equations that need to be fulfilled at all times and a set of gauge conditions.

The first set is used to evolve the spacetime quantities from one timestep to the next, the second set is important in the construction of initial data and can serve as an error estimate during the evolution, while the proper choice of gauge conditions is important to obtain a stable evolution.

2.1.2 Hydrodynamics

While for the evolution of the rather smooth spacetime variables a finite-differences method works very well, applying the same to the hydrodynamic quantities would introduce large errors. The reason is that fluid variables can vary rather rapidly in space and develop discontinuities (shock waves) even from smooth initial data. Treating a shock wave with a finite-difference method would quickly smear out the (physical) discontinuity and thus fail to describe the correct evolution.

Therefore, we use a finite volume method for the treatment of the hydrodynamic evolution instead. To this end, we recast the equation of relativistic hydrodynamics (the continuity equation, the energy equation and the Euler equation) into so-called flux-conservative form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}^i}{\partial x_i} = \mathbf{S}. \tag{2}$$

It is possible to find »conserved« variables $\mathbf{q}$ instead of the »primitive« hydrodynamic variables, such as pressure, density or velocity, for which the equations of relativistic hydrodynamics takes this particularly simple form. This way, we can split up the computation of the right-hand-side for the method of lines into the treatment of a source term $\mathbf{S}$ and a flux-term $\mathbf{F}^i$, which are both functions of the $\mathbf{q}$.

The main advantage of this procedure becomes apparent when we take the cell average of these equations by integrating over a single grid cell $\mathcal{V}$ of volume V

$$\frac{1}{V}\int_{\mathcal{V}} \mathrm{d}V\, \partial_t \mathbf{q} + \frac{1}{V}\int_{\mathcal{V}} \mathrm{d}V\, \partial_i \mathbf{F}^i = \frac{1}{V}\int_{\mathcal{V}} \mathrm{d}V\, \mathbf{S}. \tag{3}$$

If we now choose the discretization scheme for the hydrodynamic variables $\mathbf{q}_{(i,j,k)}$ to represent the cell average (instead of the value at the cell center) over the cell with spatial indices (i, j, k), we arrive at

$$\partial_t \mathbf{q}_{(i,j,k)} + \frac{1}{V}\int_{\partial\mathcal{V}} \mathrm{d}\mathbf{A}_i\, \mathbf{F}^i_{(i,j,k)} = \mathbf{S}_{(i,j,k)}, \tag{4}$$

using Gauss' law for the second term and thus transforming the volume integral to a surface integral. To compute this, we need to evaluate the flux terms on the six cell interfaces, even though we only know the cell-averaged $\mathbf{q}$. This is where the last crucial ingredient for a shock-capturing scheme comes into play. Instead of trying to interpolate the $\mathbf{q}$ and computing the flux terms from this (which would essentially result in the same disadvantages as using a finite difference scheme), a so-called Riemann problem is set up at each cell interface and the fluxes are being computed from the solution to this. A Riemann problem is set up as initial data with a single discontinuity between two constant states on its left and right hand side.

The solution to such a problem can be obtained (semi-) analytically and captures shock waves properly (after all, this initial data results in the formation of a single shock, a contact discontinuity and a rarefaction wave). For the expert reader, let us note that we are typically using the Marquina solver with PPM reconstruction.

A more extensive description of the implementation can be found in the description of the GRHydro component of the Einstein Toolkit (Baiotti et al., 2005). The EoS can in principle be provided either as an analytic expression, e. g. for a polytrope, or as tabulated values for more realistic descriptions.

2.2 Grid setups

We are using two different three-dimensional grid setups for our simulations. The first one is a traditional Cartesian grid with several layers of mesh refinement, as provided by the Carpet module of the Einstein Toolkit (Schnetter et al., 2004).

The second one is a spherical grid setup we developed specifically for this project. One of the limiting factors for such simulations on Cartesian grids is the occurrence of numerical viscosity effects, which tend to damp out the very oscillations under investigation after a relatively short period of time. The main contribution to this effect stems from the mismatch of the more or less spherical NS surface and the Cartesian grid.

The spherical grid has performed much better in this regard in initial tests using the Cowling approximation (i. e. keeping the spacetime background static and only evolving the hydrodynamic quantities). We hope to see similar advantages for the full implementation we are currently working on.

The main drawback of the spherical grid is the clustering of grid cells at the center and the axis, which severely limits the possible time step via the Courant–Friedrichs–Lewy (CFL) condition (Courant et al., 1928). Additionally, it scales worse to higher numbers of cores used, since it typically uses fewer grid cells than the Cartesian setup. However, the code can still produce results in a reasonable amount of time on the current hardware. Utilizing accelerators (GPUs) for this setup could possibly increase the performance in the future.

2.3 Post-processing

Using the ingredients sketched above (and many more we could not mention here), we can simulate the evolution of our chosen NS model in full general relativity. This is, however, only the first step. The results of the simulation are time series for different physical quantities at a large number of grid points. In order to extract information on the oscillation modes, this data needs to be analyzed further.

To this end, we are in the process of developing a flexible post-processing code to extract the frequencies and eigenfunctions of the individual oscillation modes present in the star, which can in turn be used to compute further properties such as the damping times.

Providing this information for a large number of NS models and different EoSs will be crucial for posing constraints on the EoS from future observations.

2.4 Computational cost

The simulations are computationally very expensive and could not be realized without a cluster as provided by the bwForCluster BinAC or the bwUniCluster of the bwHPC initiative.

While the individual simulations are tailored to the specific problem (size of the NS, required resolution etc.), we here want to provide some ballpark numbers on the computational cost involved for the Cartesian setup. Since the spherical grid setup is not yet ready for production use, we do not have comparable numbers for it.

A typical simulation run in the Cowling approximation (keeping the spacetime fixed) requires keeping track of three time levels of ten hydrodynamic quantities and several helper variables and of one time level of 24 constant spacetime variables as well as another large number of helper variables such as coordinates, quality estimates and so on. At a grid size of 100^3 cells, memory consumption is at least about 5 GB, but can be much higher in certain steps of the computation. After over 60,000 time steps, such a simulation creates 200 GB of relevant data and takes about 35 hours on 6 nodes (168 processors) of BinAC.

For the simulations in full general relativity, also the spacetime variables need three time levels. Additionally, we need a much larger computational domain to also keep track of gravitational waves. This is accomplished with so-called box-in-box mesh-refinement techniques, which allow for a much higher resolution at the center than at the outer boundaries by placing one refinement level at half the size and twice the resolution in the center of the next coarser level. We typically use 6 refinement levels with 100^3 cells each. This leads to over 35 GB of minimum memory consumption and creates more than 350 GB of data. Such a run takes about 48 hours on 10 nodes (280 processors).

The post-processing code works in two steps, the first one of which takes about 5 hours on 6 processors to prepare the second step, which takes about 45 minutes per investigated mode (typically less than 10) on a single processor. However, this post-processing code is still under development.

We have already performed production simulations of 10 models and want to do 20 more in the next 6 months, once the post-processing code is completed.

3 State of the project and current efforts

In a number of experimental simulations, different Cartesian grid resolutions, placements and treatments of the boundary as well as several gauge choices have been tested for their impact on performance and accuracy.

The thoroughly investigated BU series (the second, uniformly rotating series of Dimmelmeier et al. (2006)) of increasingly rapidly rotating polytropes at a fixed central energy density has been chosen as a first benchmark set of models for simulation. The evolution of all ten members of the BU series has successfully been simulated with the chosen set of parameters. This data is also being used to assess the performance of the post-processing code.

Additionally, first steps were taken to handle not only polytropic, but also more realistic EoSs that cannot be described by an analytic function, but are typically provided as tabulated data. This requires a different numerical treatment, both in the construction of the initial data and in the actual evolution.

The analysis code provides convenient access to the various data produced by both simulation codes. Extracting frequencies and wave forms is already possible for non-rotating stars. Our current efforts in this regard are to extend the extraction procedure to rotating stars, and to employ this data to obtain damping times for the various modes using the quadrupole formula.

The code operating on the spherical grid setup can handle fast rotating neutron stars in the Cowling approximation. Those simulations yield promising results at greatly reduced numerical viscosity. We are looking forward to the results the full general relativistic version of the code (currently under development) will produce.

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