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INTELLIGENT MAINTENANCE OF COMPLEX SYSTEMS: A FRACTIONAL ORDER APPROACH

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Abstract: Compete in the global market requires high quality of products with short time of manufacture, so it needs to minimize the time that the machinery is stopped, as well as a rapid quality control of manufactured products. These process are achieved by maintenance strategies that are strongly based on the subjective knowledge of an expert. In this work we use the proficiency of the fractional order calculus to approximate complex behavior with a few parameter, providing a new tool for quickly health evaluation.

Keywords: Intelligent Maintenance, Fractional Order Calculus, Identification of Order.

1. INTRODUCTION

A failure is something does not allow a machine to keep working, because it is impossible to turn it back or safety. Typically are caused by the use of a material primed for the task, apply a force in a different direction that it was designed, cyclic loading, fatigue, wear, etc. When one of these occurs and the machine continues working tends to worsen and cause other problems. Maintenance is as good as the knowledge of the cause of failure, it is determined to thereby maintain the parts to exchange, have the tools to replace it, experts and workers, everything in place and time needed to repair the equipment before a critical failure occurs and fixing it the shortest time. Therefore, it is not enough to know that the machine is failing, but also the source of failure [1]. In order to solve this problem in literature has been proposed several strategies of maintenance trying to reduce costs of production over time.

In mechatronics, the machines typically increase cost of production and complexity, as a single product contains multiple integrated elements of several technologies (as electrical, mechanical, optical, etc.), adding components to the signals used in diagnosis. This fact complicates the task of maintenance as currently skilled workers diagnose systems based on experience, so the more complex the system, more difficult to isolate the problem and increase the economic cost of the expert, for example between 1975 and 1991 in the United States the maintenance cost increase in a 10-15 % a year [2].

In order to avoid this problem, some researchers try to automate the maintenance task, therefore some works propose the use of artificial intelligence techniques over signals usually analyzed by experts [3–5], but they are few applied because they are so complex consequently the academic level of the workers must increase, as well as the high investment on equipment without total trust in computer decisions [6].

Bearing this ideas in mind, the article is organized as follows: In section 2. introduce generally the strategy of intelligent maintenance, the section 3. present some basis of the fractional order calculus and its applications in identification of dynamical systems. In the section 4. the result of apply fractional order identification to a complex system and finally in section 5. the main conclusions are presented.

2. INTELLIGENT MAINTENANCE

In the global market, customers have suppliers of several qualities around the world. Therefore to remain competitive the factories need to produce goods of high quality and in a short time, so that satisfy the international demand of clients and customers recently acquired [7]. Consequently the production chain is more vulnerable to various disturbances, the possibility of failure and the time needed to repair it. A perfect balance only can be achieved when the factory is in operation in several shifts a day and the machines are fully functional. Therefore it requires to apply a maintenance strategy that allows to approach the ideal situation described above [8]. In the chain of production the typical problems that stop the production are [8, 9]:

- Fault present in an automated systems. It would be anything from a bad cable to internal parts.
- Failure to transmission line.
- Fault in the quality of manufactured parts. After detecting the problem has look for equipment failing and diagnose it.

- Due to fatigue of parts by repetitive motion.
- Environmental causes.
- Because of false positives in the prognosis of critical machinery, or due to a catastrophic failure to leave a machine in operation due to a false negative.

Each system in the chain presents problems due to deterioration of parts because of use or stochastic failures, such as dropped a tool, machine dovetailed bad, etc. In order to minimize costs, stop the production in order to do procedures to prevent, correct and predict failures, which together is called maintenance [10]. In literature, the strategies of maintenance are classified in [4, 10, 11]:

Correction: Over time this strategy is most expensive. The maintenance action takes place when the system symptoms are evident.

Timely: When the system has minor failures as to keep the machine in operation. When a failure is severe, takes advantage of the maintenance stop to replace all defective parts.

Preventive It is based on the information delivered by the manufacturer and experience of staff of the plant. There are planned periodic maintenance actions, in which possibly the machine is stopped. Moreover the failure may occur before the time of maintenance for unusual wear of parts or because of a random failure.

Predictive: The system must be constantly monitored and the signals analyzed at the time. When the operator observes that the machine presents a possible situation of failure in the near future, will be held the maintenance action.

If the maintenance action needs to hold the device, has three possible effects: (1) the frequency of maintenance is adequate and the machine has no additional stops, only to random failures, (2) the frequency is low and the machine failure before the scheduled maintenance action, therefore presents an additional stop for an undetected or random failure, or (3) the frequency is so high that it increases the maintenance costs unnecessarily. In the 1990s another idea starts to be used in industry, catching symptoms of equipment constantly, and when the it is abnormal it is analyzed even without stopping the production, performing the maintenance action only when it is needed [9, 12].

Make decisions on maintenance based on condition (CBM) requires a tough one to predict failure and the severity of it in the future. It has three goals: (1) Design a strategies for the maintenance of sophisticated equipment in complex operating environments, (2) reduce cost of storage of spare parts and finally (3) reduce catastrophic failures and eliminate unscheduled stops[6].

A model proposed to CBM is the layered model OSA/CBM, this consists of [13–15]:

1. Layer of sensors. This is the physical layer comprise all the sensors in the machine. It's instrumentation that can deliver signals relevant to diagnosis.
2. Layer of signal processing: The signals are typically filtered and transformed to a mathematical space where the data are easier to interpret such frequency.
3. Layer of condition's monitor : Basically compares the data obtained with the system in optimal performance and a estimated index. In the case that they are very different, it generates an alarm.
4. Layer of health assessment: Receives the indexes generated in the previous layer and diagnoses indicated the seriousness of the failure, taking into account the history of the system.
5. Layer of prognosis: Taking into account the information collected on the other layers, attempts to establish the state of the components in the future, including lifetime . There are three commonly used strategies:
    - Based on rules: Uses heuristic tools, blurred logic, machine intelligence and statistics to generate decision trees, it is strongly linked to the data collected and the manufacturer's recommendations.
    - Based on case study: Compares the signal with others in the past in the presence of failures, as well as the problem of establishing a possible solution, taking into account that corrective action has been taken previously and that had consequences. Unfortunately have a consistent database is a difficult task to achieve.

- Based on the model: generates an initial model with the machinery in operating condition and then is compared with a system identified at current time. If the margin of error between the models and larger than a fixed band, the system presents a failure and possibly informs its location. However, the complexity of the model limits the type of failure fully identifiable besides being difficult to draw in a complex machine. Alias the model contains many parameters, the identification process is slow to use it in real time.

6. Layer of decision support: With the information on the covers 4 and 5 performs recommendations indicating the corrective actions to be taken as the part becomes unusable .
7. Presentation: It shows information of all layers to the experienced operator in order to take corrective action.

As shown before, the OSA/CBM model requires a set of relevant signals to produce an index ease to interpret by a human or a software. Currently the most health evaluation are done by the subjective knowledge of the expert, as in case of interpret complex graphs like Fourier's spectrum, cepstrum, etc. [16]. Nowadays some mathematical techniques would simplify the task of obtain significant indexes, our approach consist in approximate complex interaction between several systems with a fractional order one. Basis of fractional order calculus will be studied in the next section.

3. Fractional Order Calculus

Fractional order calculus (FOC) was little used in engineering because of its complexity, the apparent sufficiency of the integer order calculus (IOC) and the lack of a simple physical or geometrical interpretation [17, 18]. However, this models more accurately the behavior of some systems in nature relating to different areas of engineering, and is used as a promising tool in bioengineering [19, 20], viscoelasticity [21, 22], electronics [23, 24], robotics [25–27], control theory [28, 29] and signal processing [30, 31] among others.

3.1. Basis and applications

In recent years these concepts have attracted the attention of engineers because of through them can models the behavior of many physical nonlinear systems in a compact way taking into account non-local features as "infinite memory" [32–34]. Examples are the phenomenon of heat diffusion [35], electrical impedance of fruits and vegetables [36], modeling the love triangles between human [37], the behavior of water in the pores of the cliffs, where the radio damping is constant regardless of the mass of water in motion [38], etc. On the other hand, directing the behavior of a process with fractional-order controllers is an advantage, since the system response is not restricted to the addition of exponential functions, therefore there is a wide range of behaviors reached where the integer response is a particular case [39].

The concept of fractional order calculus is as old as the integer order one, this can be proved across a letter from Leibniz to L'Hopital in 1695 [40]. This is a generalization of the calculation of integer order in real or complex [41]. Formally can be defined as:

$$D^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0, \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \tag{1}$$

With $\alpha \in \Re$.

One possible cause because it is little used in engineering is that the FOC has multiple definitions [34, 42], hindering their geometric interpretation, and that the IOC seemed to be sufficient to model nature. However many phenomena are better described by fractional order formulations, since it takes into account past behavior and have the ability to express with few coefficients dynamic systems considered of high-order [43, 44].

Another tool of interest in engineering is the Laplace transforms, which is still valid to simplify operations such as convolution and can be used to solve differential equations of fractional order. FOC in the Laplace transform is defined as [45]:

$$\mathcal{L}\{{}_0D_t^{\alpha}\} = s^{\alpha}F(s) - \sum_{j=0}^{n-1} s^j \left[{}_0D^{\alpha-j-1}f(0)\right] \tag{2}$$

with $n-1 < q < n, \quad n \in \mathbb{Z}$. Thus, the transform takes into account all the initial conditions from the first to the n-th derivative -1. Using this result is clear that any dynamic system of an arbitrary order could described by transfer functions of the form [46]:

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta[m-1} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \qquad (3)$$

With $\alpha, \beta \in \Re$, $\alpha_n > \alpha < n-1 > \cdots > \alpha_0$ and $\beta_m > \beta < m-1 > \cdots > \beta_0$

Many real systems can be identified from the theory of fractional systems [32, 47], whereas the transfer function is fractional order, so the response time is not approached through several exponential functions [48]. In addition the order is a variable degree of freedom that enables to adjust accurately to the system and describes it in a compact form [49]. Djouambi [50] used this fact to identify a fractal system, bringing data to the equation:

$$F(s) = \frac{K}{s^\alpha + a}, \qquad \alpha \in \Re \qquad (4)$$

Adjusted the template to find the parameters $\{K, a,\ alpha\}$ that minimize the mean error when compared with the actual data.

3.2. Application in Maintenance of Systems

Real life systems are governed by differential equations frequently interacting with other systems, complicating the mathematical description. The behavior of the system change when a failure occurs, therefore the differential equation change. Take as an example the system proposed in the Fig. 1, here 4 subsystems interact, and is difficult to have a intuitive idea of the behavior of each part. FOC can represent the system with few parameters, supposing that the system is a fractional order equivalent as shown if Fig. 2.

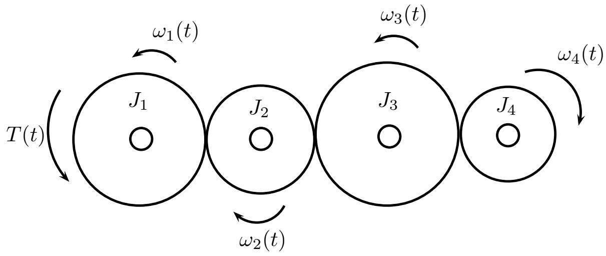

Figure 1: Case of Study. A twelve parameter linear system.

Figure 2: Fractional order equivalent system. This approximation is valid a wide band of frequencies.

4. RESULTS

In order to valuate the efficiency of the fractional order calculus to describe complex systems and how the fractional order approximation is sensitive to failures in the plant, we propose the model shown in Fig 1 for testing.

In this case the system is known and the signal provided by $\omega_2(t)$ is noise free. A general space state of this linear plant is presented in the equation 5 and table 1. The parameters used in simulation are shown in table 2.

Table 1: Space state definition.

State	Variable	State	Variable
X_1	$\omega_4(t)$	X_5	$\omega_2(t)$
X_2	$\frac{d\omega_4(t)}{dt}$	X_6	$\frac{d\omega_2(t)}{dt}$
X_3	ω_3	X_7	$\omega_1(t)$
X_4	$\frac{d\omega_3(t)}{dt}$	X_8	$\frac{d\omega_1(t)}{dt}$

Table 2: Parameters of the model

Parameter	Value	Parameter	Value
K_1	100	B_1	200
K_2	300	B_2	150
K_3	250	B_3	320
K_4	50	B_4	90
J_1	3	J_3	5
J_2	4	J_4	4

Just for test, the system is treated as unknown as a currently happens in the factory. It was exited with a sine force of amplitude 1, varying the frequency of oscillation between $1Hz$ to 1000 kHz and supposing a single sensor of displacement monitoring $\omega_2(t)$ function. With those information was

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \\ \dot{X}_6 \\ \dot{X}_7 \\ \dot{X}_8 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_4}{J_4} & -\frac{B_4}{J_4} & \frac{K_4}{J_4} & \frac{B_4}{J_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{K_4}{J_3} & \frac{B_4}{J_3} & -\frac{K_4+K_3}{J_3} & -\frac{B_4+B_3}{J_3} & \frac{K_3}{J_3} & \frac{B_3}{J_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{K_3}{J_2} & \frac{B_3}{J_2} & -\frac{K_3+K_2}{J_2} & -\frac{B_3+B_2}{J_2} & \frac{K_2}{J_2} & \frac{B_2}{J_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{K_2}{J_1} & \frac{B_2}{J_1} & -\frac{K_2+K_1}{J_1} & -\frac{B_2+B_1}{J_1} \end{pmatrix} + \begin{pmatrix} 0 \\ -T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

constructed a Bode's plot of the system in normal operation presented in the Fig. 3(a) and adding failures to the parameters K_1, K_3, B_2, B_4 shown in figures 3(b), 3(c), 3(d) and 3(e) respectively and approximated by:

$$G(s) = T(s+a)^{\alpha}, \qquad \alpha \in \Re \quad (6)$$

Fitting the parameters via a non-linear least square algorithm [51].

Note that just three parameter where needed to describe a twelve parameter system. In the table 3, the error between the integer and the fractional system is presented.

5. Conclusions

Nowadays the availability of a method to measure the quality of a machine or a product in short time minimizing production line stops is a paramount problem in the industry. In this paper were introduced some aspects of intelligent maintenance and how the fractional order calculus (FOC) would identify and evaluate some failures using just few parameters.

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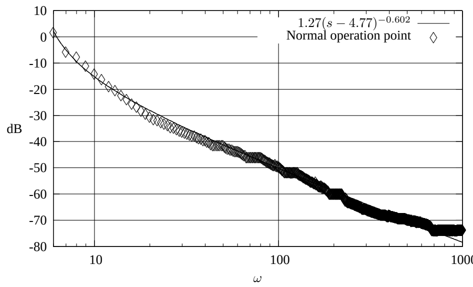
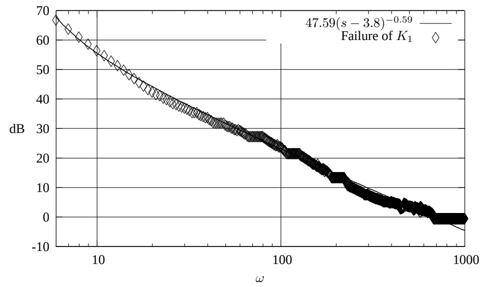
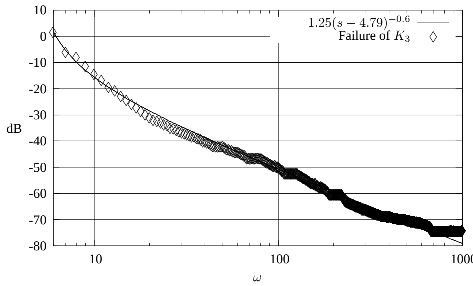


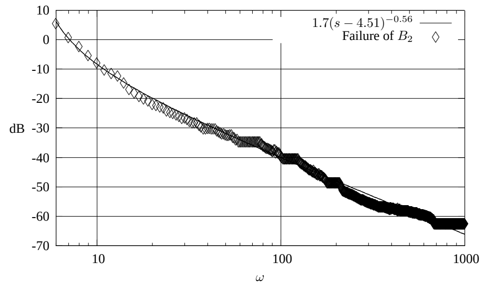
(a) Approximation of the system in "normal condition" via a fractional order system



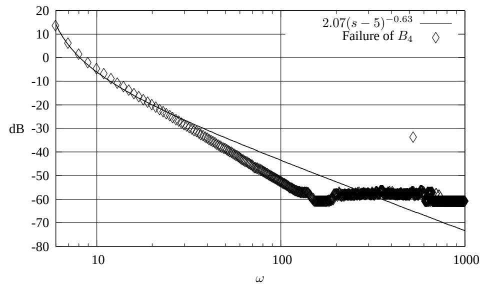
(b) Approximation of the system when the spring K_1 is boken via a fractional order system



(c) Approximation of the system when the spring K_3 is boken via a fractioinal order system



(d) Approximation of the system when the damper B_2 is boken via a fractioinal order system



(e) Approximation of the system when the damper B_4 is boken via a fractioinal order system

Figure 3: Approximation of a system with different failures via $G(s) = T(s+a)^{\alpha}|_{\{T,a,\alpha\}\in\Re}$. Note that it is a good approximation for almost all systems in the frequency band of $[1Hz - 1KHz]$.

Table 3: Equivalent fractional order system

System	Fractional order system	Mean Square Error
Normal operation	$\frac{1.27}{(s-7.77)^{0.6}}$	$5 \times 10^{-3} \pm 2.5 \times 10^{-5}$
K_1 Broken	$\frac{47.59}{(s-3.8)^{0.59}}$	$1.5 \times 10^{-1} \pm 2.5 \times 10^{-2}$
K_3 Broken	$\frac{1.25}{(s-4.79)^{0.6}}$	$5 \times 10^{-3} \pm 2.5 \times 10^{-5}$
B_2 Broken	$\frac{1.7}{(s-4.52)^{0.56}}$	$6.8 \times 10^{-3} \pm 4.7 \times 10^{-5}$
B_4 Broken	$\frac{2.07}{(s-5)^{0.63}}$	$0.5 \pm 2 \times 10^{-7}$

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