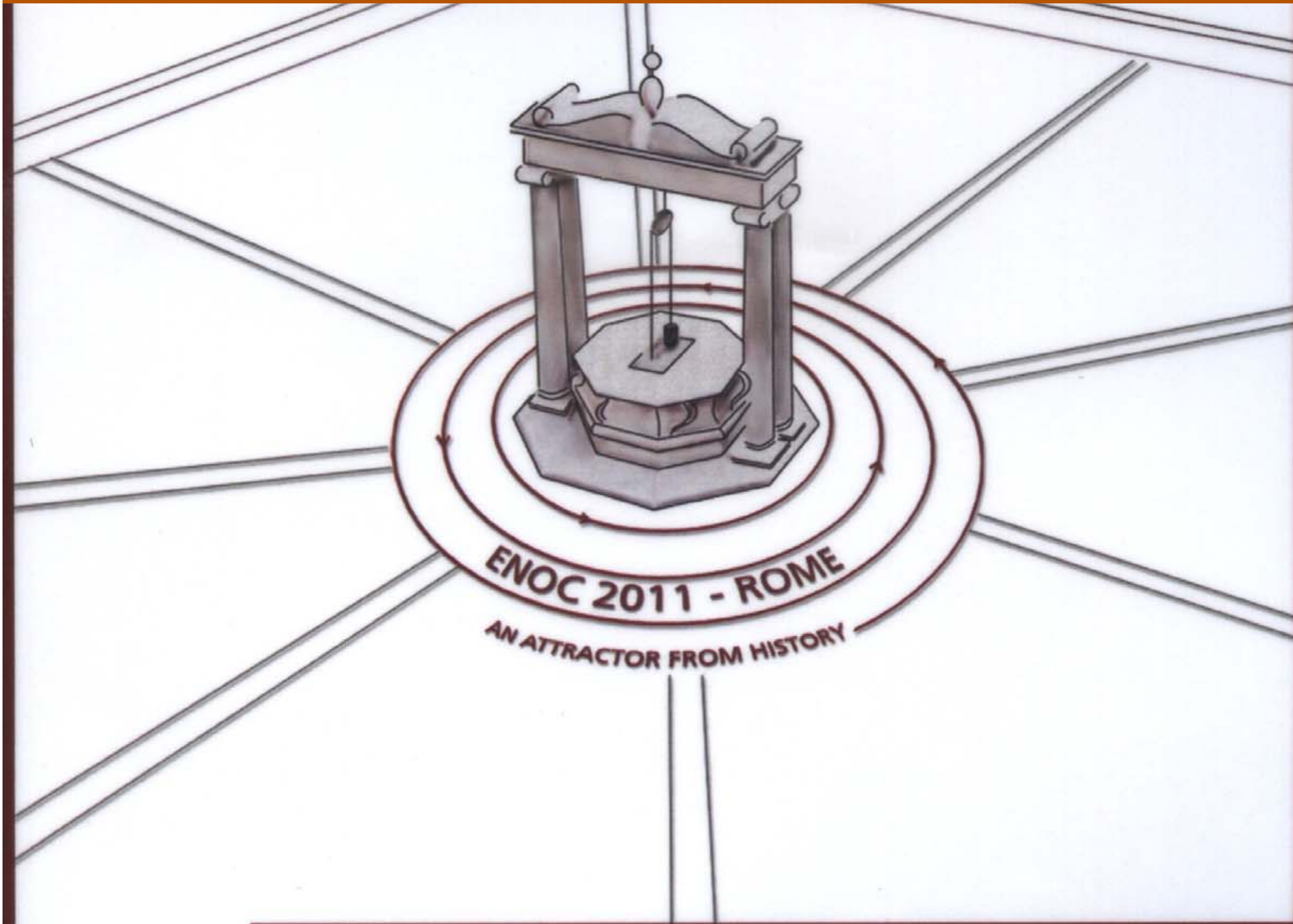

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Identifying Economic Periods and Crisis with Power Law

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Summary. Power law (PL) and Fractional Calculus are two faces of phenomena with long memory behaviour. This paper applies PL approximations to analyze different periods of the business cycle. With such purpose eight important stock market indexes, namely the CAC, DAX, Dow Jones, FTSE, HSI, IBEX, MIBTEL and SSMI are studied over time. It is observed that the PL curvefitting constitutes a good tool for separating the signal dynamics from the noise while revealing the main system characteristics.

The analysis under the lens of the PL shows that the indexes have characteristics similar to those of fractional noise and it permits to identify patterns in the stock markets specific to each economic expansion period.

Introduction

Modern economies have important swings in their economic activity. While in some periods most industries are growing and unemployment is low, in other periods most are operating well under capacity and unemployment is high. The reasons behind such behaviour are complex and normally involve inflation, monetary policy and business sentiment. Periods of economic prosperity are called expansions and periods of economic decline are called recessions. The combination of these movements is called the business cycle [1]. In the United States the business cycle is formally followed by the National Bureau of Economic Research (NBER). The NBER's Business Cycle Dating Committee maintains a chronology of the U.S. business cycle, identifying the dates of peaks and troughs that frame economic expansion or recession [2]. Between trough and peak, the economy is in an expansion. A recession begins just after the economy reaches a peak of activity and ends as the economy reaches its trough. That is why investors normally monitor the economy, looking for inflection signs. As the inflection tends to be gradual and to take many months, the stock markets are normally faster to react than the economy and are, therefore, considered by many as a leading indicator of the business cycle. Each period of the economic cycle tends to be characterized by specific believes and rules, making the business cycle a good strategic framework for investment. However, during each period these believes and rules can change and may lead to sudden corrections in the market. Extreme examples are stock market crashes, being the most notorious the Wall Street Crash of 1929. Nevertheless, while crashes are normally associated with bear markets, they do not necessarily go together. For instance the important crash of 1987 did not lead to a bear market. From that day on the beliefs, the rules and practices associated with program trading (considered by many the cause of the crash) changed dramatically but the stock markets did not continue to decline.

Economic recessions are the primary factor that drives fluctuations in the volatility of stock returns [3]. It is not surprising that changes in economic activity have strong consequences on stock markets-stock values are based on corporate earnings which are greatly determined by the business cycle. Business cycle forecasting is a popular effort in stock markets not because it is successful, but because the potential gains are so large. In fact, such prevision is a very difficult task and most of the times it is not correct, as illustrated by a famous Paul Samuelson [4] quote "Wall Street indices predicted nine out of the last five recessions". Therefore, although the stock markets normally identify coming recessions, there tend to be many false alarms.

This paper is expected to contribute to the improvement of business cycle forecasting practices by developing a method of analysis of the trend.

Data and Methodology

Our data consist of the n daily close values of $S = 8$ stock markets, six European, one American and one Asian markets,listed in Figure 1 (left), from January 2, 2000, up to December 31, 2009, to be denoted as $x_i(t), 1 \leq t \leq n$, $i = 1, \cdots, S$.

i	Stock market index	Abbrev.
1	Cotation Assiste Continu	cac
2	Deutscher Aktien Index	dax
3	Dow Jones Industrial	dji
4	Footsie	ftse
5	Stock market Hong Kong	hsi
6	Iberia Index	ibex
7	Italian Bourse	mibtel
8	Swiss Market Index	ssmi

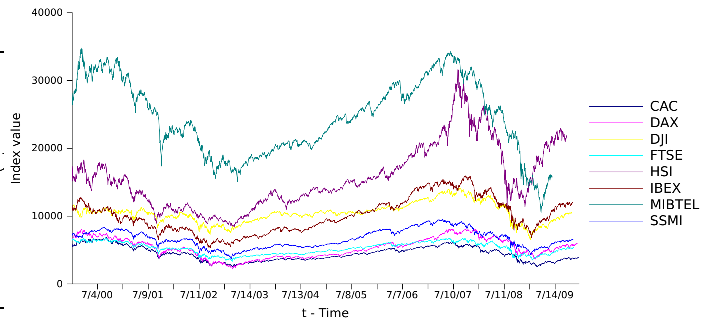

Figure 1: Eight stock markets (left). Time series for the eight indexes from January 2000, up to December 2009 (right)

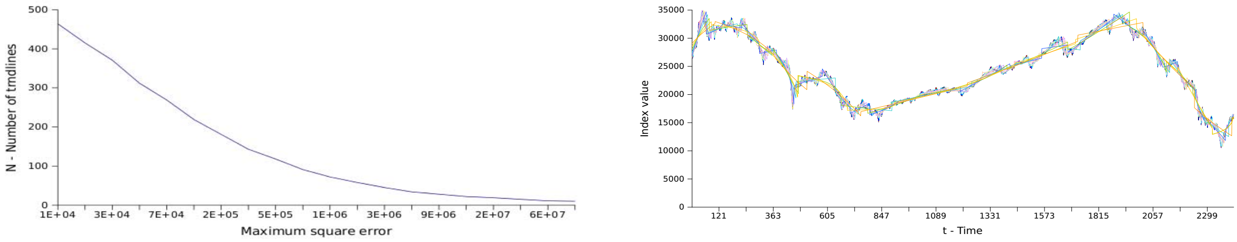

Figure 2: Number of trendlines v maximum square error (left). The temporal evolution and PL trendline for the Mibtel index from Jan 2000 to Dec 2009 (right).

The data are obtained from data provided by Yahoo Finance web site [5] and they measure indexes in local currencies. Figure 1 (right) depicts the time evolution, of daily, closing price of the twenty five stock markets versus year with the well-know noisy and "chaotic-like" characteristics.
In order to examine the behavior of the signal, for each of these partitions, a Power Law (PL) trendline is calculated according the following equation, where t is time:

$$x_k(t) \approx a_k|t + c_k|^{b_k},\ a_k \in \mathbb{R}^+,\ b_k, c_k \in \mathbb{R}, k = 1, \cdots, S,\ \ 1 \le t \le n \tag{1}$$

In this approximation the parameter a_k describes the "magnitude", b_k is related with "velocity of change" and c_k to the "center" of the trend. Therefore from the point of view of financial analysis the parameter b_k is clearly the most relevant. We must mention that PL and fractional dynamics seem to be manifestations of the same type of phenomena, that is to say, of dynamical systems with long memory behaviour, while the relation between the two faces is not yet clearly understood of the mathematical considerations underlying Fractional Calculus.
Based on a visual analysis of the pattern of the indexes chart, we see that we can subdivide each of them into several different partitions. According with the pattern of the indexes chart we decided to consider a variable number of the trendlines according with a maximum of the N square error.
Figure 2 (left) shows the number of de trendlines, for the Mibtel index, *versus* the value of the maximum square error ε. Obviously, the larger the N the smaller the ε. For each one of the partitions we superimpose the corresponding values of the PL trendline mappings over the original data. Figure 2 (right) depicts the partitions and the trendlines approximation for the Mibtel index.
Figures 3 shows the chart of the parameters $\{a,\ b,\ c\}$, respectively, of the Mibtel index in the perspective of the PL.

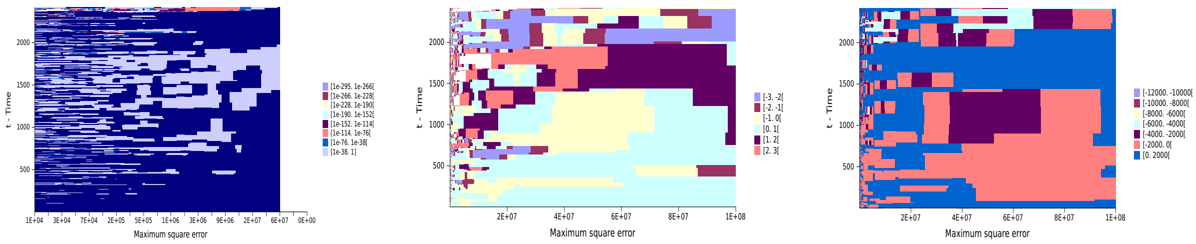

Figure 3: Locus of the parameters $\{a, b, c\}$ (left, midle, right) respectively vs (ε,t), based on equation (1), for the Mibtel, from Jan 2000 to Dec 2009.

The charts reveal that for small/large values of ε we get an high/low number of PL approximation. Obviously the smaller the number of windows N the larger the generalization and scope of the conclusions, but the higher the error. Therefore, some value in the "middle" represents a compromise between the two extreme situations.

Conclusions

Economy cycles are the cumulative result of a plethora of different phenomena. Therefore, financial indexes reveal a complex behaviour and their dynamical analysis poses problems not usual in other types of systems. In this paper it was studied a PL trendline as a manifestation of the long memory property of systems with fractional dynamics.
The PL trendline proved to constitute a tool capable of retaining the dynamical properties of the economic cycles while providing a global perspective of its evolution.

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