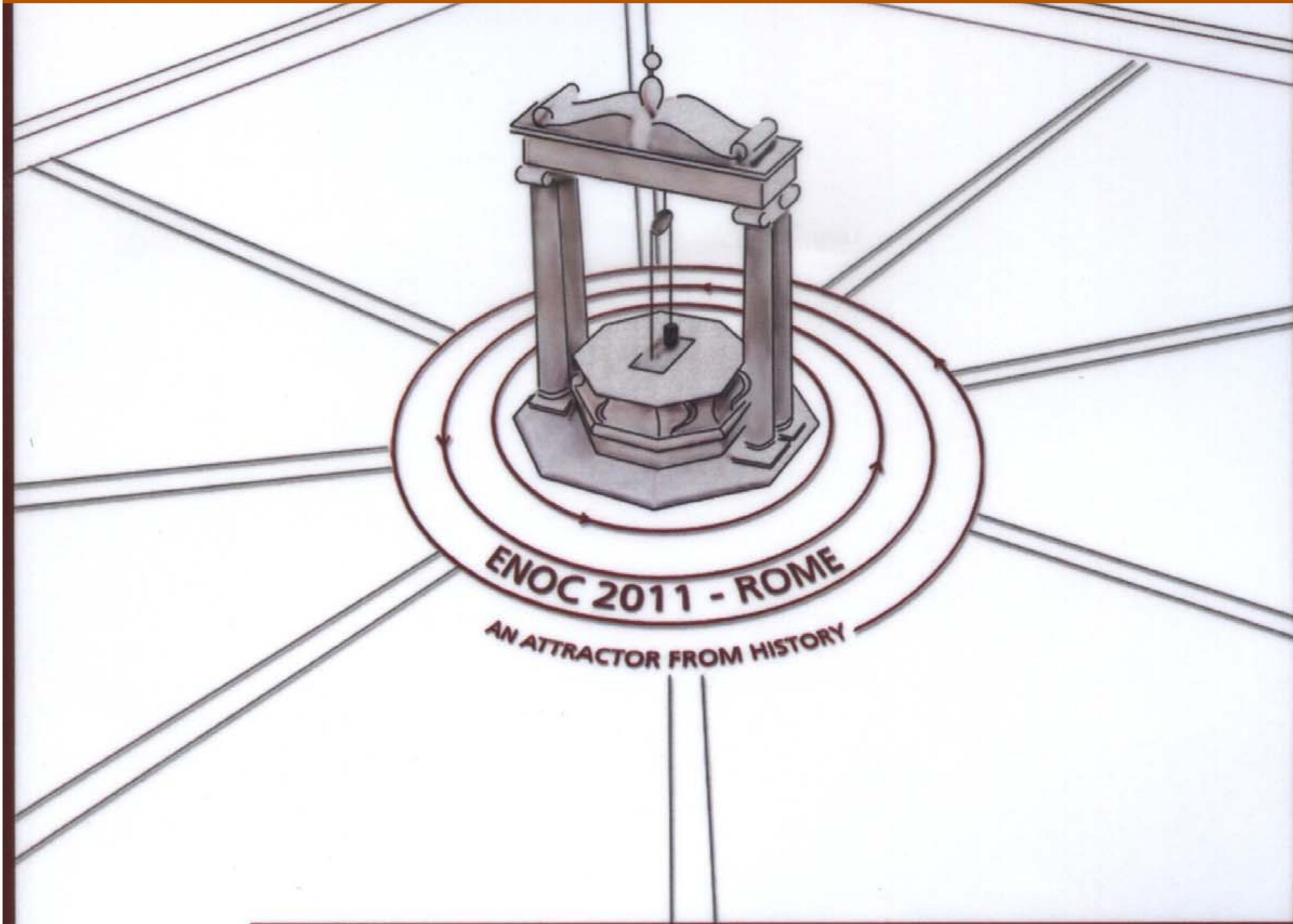

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A Computational Approach for Obtaining the Root Locus of Fractional Systems

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Summary. In this paper a new method for the calculation of the root locus of fractional systems is presented. The proposed algorithm takes advantage of present day computational resources and processes directly the characteristic equation. The results demonstrate the feasibility of the method for different types of expressions.

Introduction

During the last decades Fractional calculus (FC) verified a strong development which is demonstrated by the large volume of research published. In spite of its popularity, FC requires some efforts towards the development and adoption of standard tools often adopted in system analysis and control [7, 8, 14, 9]. This paper addresses the development of an algorithm for obtaining the Root Locus (RL) of fractional order expressions of any type, easy to implement using today's computational resources. In this line of thought the next section introduces the algorithm for the calculation of the RL of fractional order expressions and presents several examples.

Root locus of fractional systems

The RL is a classical tool for the stability analysis of integer order linear systems [1, 2, 5, 3], but its application in the fractional counterpart poses some difficulties. Recently the RL was considered for the stability analysis of fractional systems by tacking advantage of commensurable expressions that occur when truncating real valued integrodifferential orders up to a finite precision [12]. This strategy allows the use of built in routines available in several engineering packages, with a good level of integration of commands and editor, but, on the other hand, limits the precision and the type of symbolic expressions. Bearing these ideas in mind, it was decided to implement a numerical tool for construction the RL without the previous limitations. The algorithm consists in searching the complex plane s for possible solutions of the closed-loop characteristic equation. In general, in the s-plane it is adopted a two stage calculation scheme, with a large spaced grid of points for a preliminary evaluation of candidate solutions and, for each solution, a more precise calculation by inserting a local small spaced grid of points. Due to the importance of solutions in the real and imaginary axis it was decided to search directly those cases. Therefore, the structure of the algorithm is designed as follows:

1. Definition of the system characteristic equation
2. Definition of parameters:
    - 2.1 The limits of the search in the complex plane
    - 2.2 The number of points with the large spaced grid for testing possible solutions
    - 2.3 The accuracy threshold for entering the high precision root evaluation
    - 2.4 The number of points in the small spaced grid for calculating more precise solutions.
3. Calculation of solutions in the imaginary axis with two phases
    - 3.1 Test solutions with large grid
    - 3.2 If feasible solution found, then larger precision evaluation by searching within a local small spaced grid
4. Calculation of solutions in the real axis, directly with small spaced grid.
5. Calculation of solutions in the complex plane with two phases
    - 5.1 Test solutions with large spaced grid
    - 5.2 If feasible solution found, then larger precision evaluation by searching within a local small spaced grid

For the implementation of the algorithm were tested both compiler and interpreted computational packages. It was verified that, although popular in some areas, interpreted codes were much slower and, therefore, in the sequel the experiments are based on code implemented with the Lazarus compiler [6].
For the purpose of testing the algorithm are considered several fractional characteristic equations $Q(s)$ proposed in the literature, namely:

- Example 1, by F. Merrikh-Bayat et al [10] $Q(s) = s^2 - 3s^{1.5} - 2s + 2s^{0.5} + 12 + k\left(s^{0.5} - 1\right)$
- Example 2, by I. Podlubny et al [13] $Q(s) = 0.7943s^{2.5708} + 5.2385s^{0.8372} + 1.5560 + k$
- Example 3, by I. Jesus et al [4] $Q(s) = 1 + ke^{-3.0\sqrt{\frac{s}{0.042}}}$

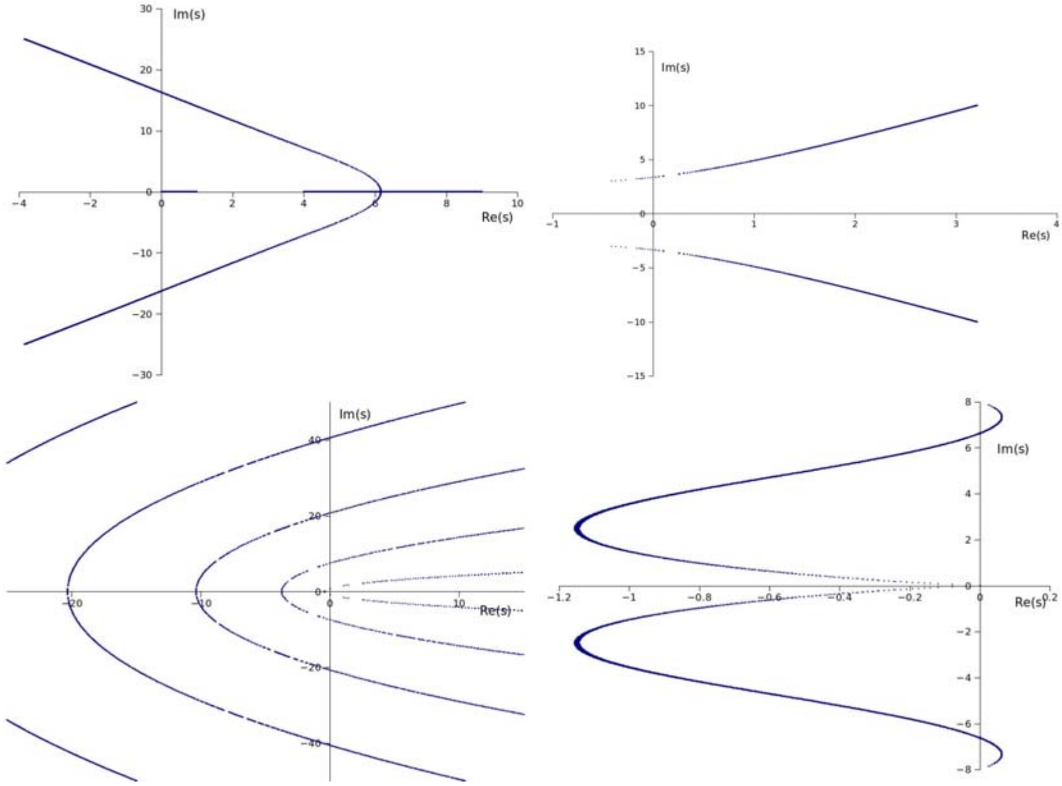

Figure 1: Root locus for $k \geq 0$ of the fractional-order characteristic equations: $Q\left(s\right) = s^2 - 3s^{1.5} - 2s + 2s^{0.5} + 12 + k\left(s^{0.5} - 1\right)$, $Q\left(s\right) = 0.7943s^{2.5708} + 5.2385s^{0.8372} + 1.5560 + k$, $Q\left(s\right) = 1 + ke^{-3.0\sqrt{\frac{s}{0.042}}}$, $Q\left(s\right) = s + k\left(s^{0.5} + 1\right)e^{-\sqrt{s}}$

- Example 4, by F. Merrikh-Bayat et al [11] $Q\left(s\right) = s + k\left(s^{0.5} + 1\right)e^{-\sqrt{s}}$

where s represents the Laplace transform variable. Figure 1 depicts the corresponding RL obtained through the proposed computation scheme. The results are consistent with those reported based on the adaptation of the standard RL, when available, or with stability conditions based on other methods, for those cases where the RL was not obtained. It was also confirmed that the quality, precision and speed can be easily tuned through the parameters defining the grid points and the precision threshold. From the operational point of view we find that we loose the automatisms provided by common engineering packages, since we have to insert the code for each characteristic equation but, on the other hand, we gain computational speed and freedom of analyzing a larger set of fractional-order expressions. In conclusion, the proposed algorithm provides a simple platform for the stability analysis and control design of closed loop linear fractional systems.

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