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SELF-SIMILARITY IN WORLD ECONOMY

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Abstract. Self-similarity is a property of complex networks, commonly observed in many distinct areas of science. Many real networks are said to be Şscale-freeŤ since they show a power law behavior. In this paper, we focus in self-similarity of economical phenomena. For that purpose we analyze data and their corresponding statistical properties. We approximate data by trendlines, whose parameters are unequivocally related with each type of phenomenon.

Self-similarity and power law behavior. Self-similarity is a characteristic of complex nonlinear dynamical systems. It means that a system looks roughly the same, despite of the scale considered. A self-similar structure is infinite and is not differentiable in any point. Examples of self-similar systems are the fractals, observed in coastlines, turbulent flows, bacteria cultures and lungs. In music, structure and repetition are general features. A self-similar system, $S(x)$, obeys the scaling equation, given by:

$$S(\lambda x) = \lambda^s S(x) \tag{1}$$

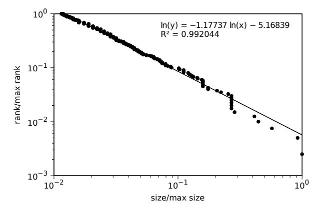
where s is the scaling factor, independent of x, and λ is a constant factor.

A power law (PL) tell us that the size of an event is inversely proportional to its frequency. Researchers usually represent a log-log plot of these quantities and verify if a straight line is obtained. The probability function of a discrete random variable following Pareto distribution is given by:

$$P(X = x) = Cx^{-\alpha} \tag{2}$$

A direct observation of equations (1) and (2), reveals that a PL satisfies the scaling condition.

Power law behavior in economy. The study of the Pareto's law validity for describing income or wealth distribution is very important. Though validity of PL behavior is restricted to a very small amount of income or wealth distribution, this amount can influence greatly the global economy. We analyzed the last fifteen years of the Forbes 400 list, from 1996 up to 2010. For each year the collected data was sorted, ranked, normalized, and represented in a log-log plot. A PL was adjusted to the data, using the least squares algorithm. Figure 1 depicts the result obtained for the year 2000. In that year the richest person was Bill Gates, with a fortune of 63 billions of dollars. At the bottom of the list were Christel Dehaan, Malcolm Glazer and Richard Haworth, with a fortune of 0.725 billions dollars. It can be observed that the distribution of wealth unveils a clear statistical regularity in a range of almost two orders of magnitude.



Rank/frequency log-log plot of the wealth of the 400 richest Americans in 2000

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