Contributions of the epistemological and didactic analysis: question-answer maps in engineering and in teacher education

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DOCTORAL THESIS

Title	Contributions of the epistemological and didactic analysis: question-answer maps in engineering and in teacher education
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ABSTRACT

During the past decades there has been an important movement claiming for a change in school institutions and more precisely in the organization of teaching and learning processes, a movement characterised by general pedagogical principles such as “active learning”, “inquiry-based” and “student centred”. However, this transition from a transmission-based paradigm to a new pedagogical paradigm lacks a systematic didactic approach enabling researchers and teachers to design, manage and analyse the new study processes. It also lacks an epistemological foundation to reformulate and make sense of the new mobilised knowledge.

The aim of this research work is to study to what extent the framework of the Anthropological Theory of the Didactic (ATD) provides the epistemological and didactic tools that are required to support the new paradigm. To do so, we study the implementation of Study and Research Paths (SRPs) and the role played by Question-Answer maps (Q-A maps) in engineering education and in lecturer education. More specifically, we study the conditions and constraints enabling and hindering the use of Q-A maps and how they help make the raison d’être of the knowledge at stake explicit. We also study the role of the Didactic Engineering (DE) methodology as a tool for lecturers to systematically design inquiry study processes. In order to address these issues, we have carried out three empirical studies: (1) an SRP implemented in a Strength of Materials course, (2) an SRP in an Elasticity course and (3) a course for secondary mathematics teachers. The results show that Q-A maps have been adopted by both lecturers and students. Specifically, lecturers have used them together with researchers in the design of SRPs and their analysis. Q-A maps have also been used by students in the two implemented SRPs to describe the paths followed during the inquiry process as well as to communicate their progress and to assign and share tasks. In addition, Q-A maps enabled the collaboration between researchers in didactics and lecturers. In our study cases, their use cannot be separated from the use of DE, which became the main task design tool for lecturers. Finally, this research shows that Q-A maps are a productive epistemological tool in study processes where the knowledge to be taught is not a pre-established body of knowledge but an initial open question. Furthermore, our research shows how Q-A maps and the DE methodology play a crucial role as communication tools when the design,

implementation and analysis of SRPs is not exclusively done by researchers in the ATD but by mixed teams of lecturers and researchers

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PUBLICATIONS

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Florensa, I., Bosch, M., Gascón, J., & Winsløw, C. (in press.). Study and Research Paths: A New tool for Design and Management of Project Based Learning in Engineering. *International Journal of Engineering Education*. **(See section 4.2)**

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Florensa, I., Bosch, M., & Gascón, J. (2015). The epistemological dimension in didactics: Two problematic issues. *In CERME 9 - Ninth Congress of the ˇ European Society for Research in Mathematics Education* (pp. 2635–2641). Praha. **(See Introduction)**

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Florensa, I., Bosch, M., Gascón, J., & Ruiz-Munzon, N. (2017). Teaching didactics to lecturers: a challenging field. *In 10th Conference of European Research in Mathematics Education - CERME10*. Dublin. **(See section 5.2)**

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Book chapters

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The specific contribution of the doctoral student in each of the three-peer reviewed international journals articles was as follows:

(1) The first author of the article published in the European Journal of Engineering Education is the teacher who implemented the REI with engineering students. The author participated in the design and implementation as a support researcher and observer. In the article, he was responsible for the theoretical framework, the research methodology and the discussion of results, leaving the description of the experimentation to the first author.

(2) This article published in the International Journal of Engineering Education focuses on an SRP designed and experienced by the author himself. The article has been drafted mainly for him during his stay at the University of Copenhagen with Professor Carl Winsløw. His contribution was mainly support in the structuring and writing of the final version of the article.

(3) The third article accepted at the Educaçao Matemática Pesquisa has been written mainly by the author based on the experience already mentioned in the training of engineering professors.

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1 INTRODUCTION

1.1 A change of paradigm in education

During the past decades, there has been an important movement claiming for a change in school institutions and more precisely in the organization of teaching and learning processes. The origin of this change can be traced back in the first half of the 20th century with the works of Dewey, Pólya and Piaget among others (Dewey, 1938; Piaget, 1974; Pólya, 1945). The main characteristics of this new pedagogical paradigm are usually described by general principles such as active learning, student-centred processes, open and contextualized activities and independent learning among others (Felder & Brent, 1996; Prince, 2004; Prince & Felder, 2006).

This paradigm shift has spread globally and has been promoted by governments and international organizations. Interesting examples of these initiatives are the development of new curricular documents such as the New Generation Science Standards in the United States of America or the creation of the Centre for Educational Research and Innovation by the OECD. In addition, numerous international research projects have also been developed, such as the PRIMAS Project for compulsory education (2010-2013) or the Working Group on the Modernisation of Higher Education (2016-2018), both founded by the European Union.

In order to implement and promote these changes, many teaching formats have been developed and analysed in educational research during these past decades. Problem-Based Learning (PBL), Project-Based Learning (PjBL) or Active Learning (AL) are examples of these. From now on, we will refer to all these kind of teaching formats using the term Inquiry Based Learning (IBL). Even if the first research experiences with IBL come from the decade of 1970, their implementation in school institutions—including universities— remains still a challenge for both researchers and teachers. IBL study processes are usually defined and characterised by general principles such as the fact that the starting point is an open problem, question or project, that knowledge is introduced as a tool to address these problems, questions or projects that teachers act as facilitators more than instructors, etc. A large variety of teaching formats are proposed, but the focus is put on how to design, manage and assess them, more than on which are the modifications needed to adapt or even newly elaborate the knowledge that is to be taught and learnt.

This transition from one pedagogical paradigm to a different one was well theorized and described by Chevallard (2015) on the occasion of the Felix Klein medal award ceremony speech. Chevallard describes the old paradigm as the "visiting works paradigm" or "monumentalism". This paradigm is characterised by regarding knowledge as split into isolated pieces to be transmitted by teachers to students. The selection of these pieces of knowledge is not based on their (current or past) *raisons d'être* but on their intrinsic importance or historical value. Chevallard (2015) states that: "students are reduced to almost mere spectators, even when educators passionately urge them to 'enjoy' the pure spectacle of [the proposed] works." An important segmentation of these pieces of knowledge in well-established (and closed) domains and fields is also a characteristic of this paradigm. In contrast, Chevallard (2015) proposes to consider the counterparadigm of "questioning the world". In this new pedagogical paradigm what is valued is not "what people know" but "what they can *learn*—and *how* they can do so" (Chevallard, 2015, p. 177). Knowledge to be conquered—in contrast to already known knowledge—is central and proposed to be reached by study processes led by open and relevant questions.

A main assumption of our research is that one of the open problems hindering the implementation of IBL study processes is related to the change of the very nature of knowledge involved in these formats, even if this change usually remains implicit. The way knowledge has traditionally been conceived in school institutions is static, hierarchically organised structured and crystallized. In contrast, the implementation of IBL teaching formats makes knowledge become dynamic, provisional and collective. This is a crucial change: the way to describe and manage knowledge must change both at the research and school levels. However, IBL study processes do not always include or propose an explicit enough epistemological change. This is an important open problem in the research of IBL study processes: how to model or describe the changes in knowledge to be taught required by these new type of teaching proposals.

A second open issue in education research can be located more at a design level, even if it cannot be separated from the conception and theoretical foundations of IBL teaching formats. It corresponds to the way the general principles defining the new paradigm are concretised in a specific and systematic methodology to design, manage and analyse study processes and its associated knowledge. An example of this need is the lack of specific tools to select a relevant, and productive "open-problem" or "initial question" for IBL study processes, a problem that have been raised by many researchers (Lima et al., 2012; Moreira, Mesquita, & Hattum-Janssen, 2011; Servant, 2016). Servant (2016) also states that the way a generating question (or project) initiating an IBL study process is chosen depends heavily on the school institution where the process takes place and that there is not often a clear and explicit methodology to do it.

In summary, IBL teaching formats can be considered as a way to promote the change from the paradigm of visiting works to the paradigm of questioning the world, as far as they locate inquiries, projects and open problems at the core of teaching and learning processes. However, their implementation does not always take into account the prevalence of the paradigm of visiting works, which happens to hinder IBL activities, for instance when inquiries, projects and problems are proposed as a way to motivate or introduce some previously established knowledge works. There is a lack of a systematic methodology, not only to design and manage this kind of teaching processes, but especially to deal with the important modifications that affect the knowledge to be taught, a knowledge that has usually been transposed according to the paradigm of visiting works.

1.2 The Anthropological Theory of the Didactic and the change of paradigm

The shift of paradigms from visiting works to questioning the world is not specific of the teaching and learning of a given domain. Engineering, medicine, science and mathematics education have experienced the transition and the previously described problems. In the case of mathematics education, different theoretical frameworks have been developed to help researchers to design, implement and analyse study processes and, in particular, IBL activities (Artigue & Blomhøj, 2013).

One of these theories is the Anthropological Theory of the Didactic (ATD) that appeared in the framework of the "epistemological approach in didactics" during the decade of 1970 (Sriraman & English, 2010). The central and distinctive point of the *epistemological approach* is the status given to the knowledge to be taught and learnt, and to the knowledge actually taught and learnt. Many educational approaches consider the knowledge to be taught as a given, and focus on the best conditions or practices to teach and learn it: the knowledge is not problematic, the relationships of the students to it are (Chevallard, 1999). In contraposition, the epistemological approach in didactics—and ATD in particular—locates the *epistemological problem* at the core of the didactic analysis. A double assumption is meant by this. First, that phenomena affecting teaching and learning processes, at school as well as in other social institutions, are strongly dependent on the content to be taught, actually taught and learnt, and also on how this content is considered by the participants of teaching and learning processes. Second, that the study of these phenomena is also strongly dependent on the way knowledge is considered and modelled by researchers in didactics. This foundational postulate of the epistemological approach contrasts with many IBL approaches based on general pedagogical principles that do not propose an explicit way to model, modify or analyse the knowledge involved in study processes.

Under this perspective, ATD intends to develop a whole *science of the didactic* theorizing how knowledge is created, studied, modified and transmitted in societies (Chevallard, 2007). ATD, as any other framework in educational research, pursues a set of more or less explicit *research and education ends*. The *educational ends* in ATD can be summarized in the "questioning the world" paradigm, which relies heavily on these four concepts: *inquiry* and *being herbartian, procognitive* and *exoteric* (Chevallard, 2015). Regarding the *research ends* pursued by the ATD they could be summarizedfollowing Bosch and Gascón (2014) as the study of didactic phenomena arising when doing, teaching, learning, diffusing, creating, and transposing mathematics or any other kind of knowledge in institutional settings, as well as the study of the conditions and constraints affecting these activities.

Consequently, not only pedagogical phenomena (dealing with problems of general education) are studied by ATD, but specially phenomena regarding the way knowledge is conceived, modified and transmitted between social institutions (Chevallard, 2007). We will talk about the *didactic dimension* of a phenomenon when the concrete mathematical activities organised by the teacher and carried out by the students, as well as any other fact affecting the delimitation, construction, management, evolution and assessment of these activities are taken into account. In a way, didactics of mathematics was created by Guy Brousseau in the 1970 as a new research field that integrates both the pedagogical and the epistemological dimensions of teaching and learning phenomena. This integration appears in the first formulations of the theory of didactic situations (TDS) under the expression that didactics deals with the teaching and learning phenomena that are *specific to mathematics* (Brousseau, 1997).

According to this conception, research and study of *didactic phenomena* will need a bigger empirical basis including not only individual cognitive processes but also data about the institutions where knowledge is selected, modified and transmitted. At this respect, the institutional perspective on teaching and learning processes is one of the main contributions of ATD. In the next sections we summarise the main sub-theories and notions of the ATD used in this dissertation.

1.2.1 Didactics and pedagogy: a crucial integration

We consider that there are two main reasons for the epistemological-didactic integration that will later on be at the basis of the problematic issues this dissertation wishes to deal with. The first one is the dependence between the dominant epistemologies of mathematics (or of any other field of knowledge) in an educational institution and the way teaching is organised in this institution. In other words, we assume that the way a specific field and its specific bodies of knowledge are considered in a given institution, usually as implicit assumptions, affects the conditions established for its learning. In this sense, we can say, rephrasing Brousseau (1997), that teaching organisations are supported by spontaneous epistemologies appearing to the members of the school institution as the unquestionable and transparent way to conceive the content to be taught.

The second main reason, also put forward by Brousseau, is related to the implementation of research results wishing to improve teaching and learning. Whatever general strategies or conditions we may find at the pedagogical level, teachers will always have to specify them in terms of what ties them to the students: the knowledge-based learning activities. Of course, it is possible to delimit general pedagogical phenomena affecting any content to be taught and propose general pedagogical actions in order to improve teaching and learning processes. However, these actions will always need to be concretized and converted into didactic phenomena and strategies, that is, into specific ways of organizing mathematical contents and designing mathematical activities for the students.

Let us say that the epistemological problem addresses the issue of how to conceive, interpret, delimit, talk about, etc. the knowledge at stake in teaching and learning processes. Once the necessity to integrate the "epistemological problem" into the "teaching and learning problem" is assumed, there are different levels to take the epistemic dimension of teaching and learning processes into account. In some cases, the focus can be put on a given piece of knowledge ("proportionality", "limits of functions", "linear equations"); in other cases on a whole area, field or domain ("algebra", "calculus", "statistics"), thus considering specific models relying on a more or less explicit conception of what mathematics is and how it can be described. Therefore, the consideration of the didactic problem has to include, in one way or another, a specific answer to the epistemological problem. To properly interpret the deep interrelation between the epistemological and didactical problems, we will now present a historical development of the object of study and their respective empirical basis.

1.2.2 Modelling knowledge: praxeologies

The prominent role of knowledge in the epistemological approach, and in ATD in particular, forces researchers to avoid blindly accepting the way knowledge is conceived in a specific institution. This forces researchers to explicitly model knowledge, even if knowledge is a broad notion in ATD and includes not only concepts, notions or theorems, but also their production, use and dissemination. ATD proposes to model knowledge in terms of *praxeologies*. Praxeologies are "living" entities evolving and changing according to the institutions where they are. They are defined by a set of four elements $[T / \tau / \theta / \Theta]$, according to the ATD principle that any activity combines a "practice part" (or "know-how") known as the *praxis* block, and a "knowledge part" (or "know-that") known as the *logos* block. The *praxis* block involves a specific *kind of tasks* (T) and a set of associated *techniques* (τ) enabling to develop the tasks. The *logos* block includes the technology (θ) and a theory (Θ) justifying and interpreting (in a more or less formal and explicit form) the *praxis*, its *raison d'être* and its results (Chevallard, 1999). An interesting characteristic of praxeologies is that "know-that" and "know-how" cannot be detached, like the two sides of the same coin. This fact makes an important difference with other approaches in education where knowledge-that and its corresponding knowledge-how are considered more or less independently.

Praxeologies enable researchers to systematically describe knowledge existing in a school setting and to describe the different elements of the praxeology and its degree of explicitness. In addition, this way to characterise knowledge allow researchers to detach from the traditional way of conceiving knowledge in school institution, a way usually based on finished and closed elements (such as notions, concepts, properties, procedures and definitions in mathematics).Consequently, they appear to be a productive tool to characterise the *dominant epistemology* in a school institution related to a given piece or field of knowledge. Praxeologies can be used to describe the type of tasks that are actually associated to this piece of knowledge, the way these tasks are carried out, how they are described and justified, how they are related to other pieces of knowledge and also its *raison d'être*, that is, the main questions this piece of knowledge allegedly helps to addressing. This use of praxeologies—also called the *praxeological analysis*—cannot be detached from the elaboration of alternative (to *dominant epistemology*) conceptions of knowledge by researchers trying to overcome undesired *didactic phenomena* caused by the *dominant epistemology*. Illustrative examples of the use of praxeologies to model *dominant epistemologies* and to propose alternative knowledge conceptions exist in different domains of mathematics education: proportionality (García, 2005), numeral systems (Sierra, 2007) and elementary algebra (Ruiz-Munzón, 2010), among others.

The notion of praxeology is also used to model teaching and learning processes. In this case we talk about *didactic praxeologies*. Unlike mathematical or scientific praxeologies, and due to the youth of didactics as a scientific field, the logos block of didactic praxeologies is not well structured nor systematized. Most of its components remain implicit and are more or less sustained by the prevailing pedagogical paradigm. Therefore, it is difficult to fully describe the universe of types of tasks teachers and students carry out in their participation in teaching and learning processes. The descriptions depend on the kind of institution considered, with specific terminologies and delimitations. We can talk of an underdeveloped *logos* linked to an unquestioned and traditionally-based praxis. One of the main ends of research in didactics is to contribute to the development and dissemination of didactic praxeologies.

1.2.3 Institutional relativity of knowledge

ATD considers that institutions (and school institutions in particular) play a crucial role in how teaching and learning processes exist (or fail to exist) and that most of these phenomena cannot be explained only by a cognitive approach. Two main sub-theories have been developed within the ATD to describe the institutional relativity of knowledge and the institutional constraints and conditions affecting it: the Theory of Didactic Transposition (see Figure 1) (Bosch & Gascón, 2006; Chevallard, 1985) and the levels of didactic co-determinacy (see Figure 2) (Chevallard, 2002b).

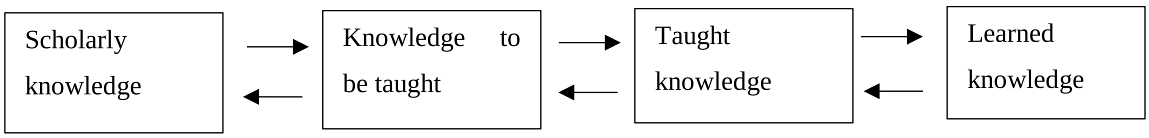

Figure 1 Process of didactic transposition

Civilization ⟷ *Society* ⟷ *School* ⟷ *Pedagogy* ⟷
Discipline ⟷ *Domain* ⟷ *Sector* ⟷ *Theme* ⟷ *Question*

Figure 2 Scale of levels of didactic co-determinacy

The Theory of Didactic Transposition models how a piece of knowledge (for example *real functions of one real variable*) suffers important modifications from the moment of its genesis, the moment of its selection to be taught, until the moment when it is actually taught. This theory highlights that the knowledge that is taught at school institutions is externally created by scholars and modified by external actors such as governments and policy makers. The study of this elaboration process is important to understand what products will be taken as knowledge to be taught and to what extent this knowledge is relative to the considered school institution. In addition, the product of this selection and elaboration process will generate a set of limitations affecting the type and form of study processes.

This sub-theory enables researchers to detach from the ingenuous conception that a specific label (*real functions of one real variable*) defines a unique piece of knowledge and consequently a set of specific tasks, techniques and theoretical justifications that are the same all over the world. Didactic transposition has been widely used in different research works framed within the ATD to take distance from the way knowledge is defined by the *dominant epistemology* (García, Bosch, Ruiz-Higueras, & Gascón, 2006). Another interesting example is the research on how knowledge is selected, modified and structured to exist in informal learning settings such as museums (Achiam, 2014).

An important notion in ATD is the scale of levels of *didactic co-determinacy*. This scale states that some *didactic phenomena* existing in the classroom when a study process is implemented cannot be modified by *only* changing conditions at the classroom level. The conditions and constraints affecting the development of study processes—such as the lack of epistemological tools, the available teaching materials among others—often depend on other levels and cannot be modified by the teacher. This enable researchers not to take certain conditions for granted, such as the discipline segmentation or the need to implement one hour-sessions. Consequently, the levels of didactic co-determinacy help researchers to explicitly detach themselves from the teacher's position and formulate research questions involving also other levels that usually remain transparent to teachers (Barquero, Bosch, & Gascón, 2013; Bosch & Gascón, 2006).

The levels of didactic co-determinacy can be used to determine the kind of conditions that may affect—and are affected by—the introduction of changes in teaching and learning processes. For instance, teachers' work usually remains at the lower levels of the question and theme, because the mandate they receive is to teach a given set of works organised in disciplines, domains and sectors. They can at most decide on the order of teaching the sectors and domains, but not on the kind of domain that has to be taught. On the contrary, in an inquiry process, one cannot know in advance what tools will be needed to address the questions raised, not even the disciplines or domains these tools belong to. This implies an important modification of the scale structuring the conditions at the different levels (Chevallard, 2011; Bosch, 2018).

1.2.4 Reference epistemological models

The methodology used in ATD to develop and design new study processes starts with an explicit characterisation in terms of praxeologies of the knowledge involved and the *prevailing epistemology* regarding this knowledge, in order to identify some *didactic phenomena* affecting the study process. The characterisation of this institutional conception is done by researchers through the analysis of empirical material such as textbooks, curricula, observation of class episodes among others. Diverse research works have described the *prevailing epistemology* in different school institutions regarding elementary algebra (Ruiz-Munzón, 2010), mathematical modelling (Barquero, 2009), statistics (Wozniak, 2005) or numeral systems (Sierra, 2007), among others.

An important assumption in ATD is that any attempt to modify a *didactic phenomenon* has to include an explicit change in the way knowledge is conceived in the institution of study. In other words, the design and subsequent implementation of a new study process in a school institution must be based on a researchers' previously developed conception of knowledge. The new conceptions of the knowledge to be taught can be materialised on the so-called *reference epistemological models* (REM). One should consider a REM as a scientific hypothesis developed by researchers intending to modify a *didactic phenomenon*. In the past two decades many research works have developed REM in different fields of mathematics: numeral systems (Sierra Delgado, 2007), elementary algebra (Ruiz-Munzón, 2010), modelling (Barquero, 2009), proportionality (F. J. García, 2005) and differential calculus (Lucas, 2015) among others. The material form of a REM is diverse: different tools have been used ranging from a set of praxeologies of increasing complexity (Ruiz-Munzón, 2010; Sierra Delgado, 2007) to an arborescence of questions and answers initiated by a generating question (Barquero, 2009; Lucas, 2015).

In the cited works, REMs have been presented in different ways. For example, Sierra (2007) presents a REM as a sequence of praxeologies of increasing complexity, the limitations of one praxeology leading to the emergence of a more complete praxeology (Figure 3). Ruíz-Munzón (2010) uses a similar structure when presenting a REM in the field of elementary algebra: three levels of algebrisation are defined and, again, the limitations of the lower levels lead to the following levels (Figure 4). In contrast to this representation, Barquero (2009) and Lucas (2015) present the REM as an arborescence of questions and answers initiated by a generating question (Q_0), taking the notion of Herbartian schema as standing point.

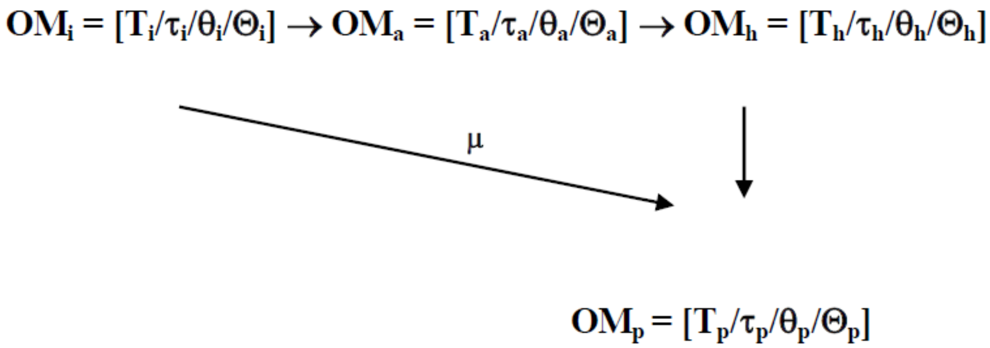

Figure 3 Reference Epistemological Model for numerical systems. Starting with additive systems followed by additive-multiplicative systems and finishing with positional systems (Sierra, 2007)

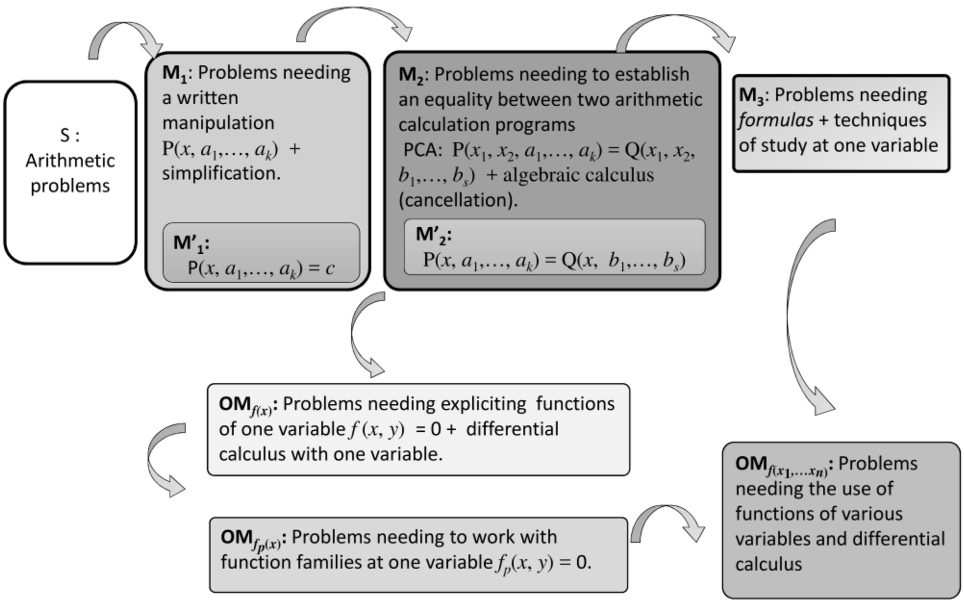

Figure 4 Reference Epistemological Model proposed in Ruiz-Munzon (2010) about elementary algebra

The REM acts as a necessary emancipating tool (Gascón, 2014) enabling the researcher to detach from the school and the scholar institutions and to propose explicit alternative models for the knowledge to be taught. In consequence, the REM plays a crucial role in the analysis of didactic transpositive processes, the study of didactic phenomena and the design of new processes of study.

1.2.5 *The Herbartian schema: modelling study processes*

Chevallard (2008) developed the notion of Herbatian schema (see Figure 5) as a representation facilitating researchers to describe and analyse different aspects of study processes (Bosch & Winsløw, 2016).

$$[S(X ; Y ; Q_0) \circlearrowright \{Q_1, Q_2, \ldots, Q_m; A^{\Diamond}_{m+1}, A^{\Diamond}_{m+2}, \ldots, A^{\Diamond}_{n}; W_{n+1}, W_{n+2}, \ldots, W_p\}] \hookrightarrow A^{\heartsuit}$$

Figure 5 Herbartian schema

The first part of the schema represents the didactic system $S(X; Y; Q_0)$ formed by a set of students (X) a set of guides of study (Y) that together face the task to generate an answer to an open question Q_0. The second part of the schema describe the process of elaboration of an answer ($A^{\heartsuit}$) of the community of study to the generating question Q_0 This part is composed of two elements interacting through a dialectic: the questions ($Q_1, Q_2, \ldots, Q_m$) and answers and works ($A^{\Diamond}_{m+1}, A^{\Diamond}_{m+2}, \ldots, A^{\Diamond}_{n}$ and $W_{n+1}, W_{n+2}, \ldots, W_p$). The hallmarked answers and works are preexisting developed knowledge in different institutions that the community of study will access in different *media* (Bosch & Winsløw, 2016). This information obtained is then studied, deconstructed and adapted to the (sub)question addressed and incorporated to the *milieu.* The potential of the Herbartian schema is not only its capacity to systematically model inquiry study process by easily incorporating the *question-answer* and *media-milieu* dialectics but also its adequacy to compare any study process ranging from traditional lectures to more innovative formats. For example, in a more transmissive—traditional setting in where lectures are central, the Herbartian schema reveals that only one answer will be available to the community of study and that this answer will coincide with the one presented by the teacher. In addition, the question leading to the knowledge will remain in the shadow. In contrast, in an open study process initiated by a question the community of study will search for available answers, will modify and incorporate them. The difference between teaching formats is made explicit in terms of diversity of elements of the Herbartian schema mobilised during the process.

1.2.6 *Study and research paths*

Study and Research Paths (SRPs) are a teaching format proposed by the ATD to foster the transition from the paradigm of visiting works to the paradigm of questioning the world. SRPs are initiated by an open question posed to a community of study (a set of students X and a set of guides of the study, Y) that will generate moments of *study* of available information in the media, and moments of *research* and development of new solutions to generate an answer to the initial question. The implementation of an SRP under the ATD perspective is often twofold. On the one hand, SRPs can be considered as a tool to reach the education ends of the ATD. On the other hand, and from a research perspective, SRPs can be implemented to empirically validate how a REM overcomes (and to what extend) a specific undesired didactic phenomenon. This double character of SRPs is crucial: they are research tools enabling researchers to generate answers to their research questions, and also teaching tools to implement new study processes. In order to understand the genesis of an SRP it is crucial to describe its associated methodology. This is done in the next section.

1.2.7 Didactic Engineering as an SRP design methodology

The design, implementation and analysis of SRP follow a specific *research methodology* that can be considered at the same time as a *task design methodology* (Barquero & Bosch, 2015). This double character (research methodology / task design methodology) places SRPs at the crossroad between research in scholar settings and action in school institutions. The existence of an explicit methodology makes an important difference with IBL teaching formats. IBL study processes base their design and implementation on general principles that are not always easy to materialise in a specific school institution. In addition, IBL teaching formats remain and propose changes at pedagogy level while other upper and lower levels of the scale of co-determinacy remain unchanged.

Didactic Engineering (DE) (Michèle Artigue, 2014; Barquero & Bosch, 2015) has its origins in the decade of 1980 as a methodology allowing researchers to generate and modify specific didactic phenomena. DE is structured in four-phases: (1) preliminary analyses; (2) design and *a priori* analysis; (3) the *in vivo* analysis and (4*) a posteriori* analysis and validation.

The preliminary analysis includes the analysis at three levels: epistemological, economical and (Gascón, 2011). The epistemological analysis includes a specific analysis of the dominant epistemology in the considered institution and a characterisation of the associated didactic phenomena. In addition, an alternative conception of the knowledge to be taught (REM) a an hypothesis trying to overcome the detected didactic phenomena has to be explicitly described. The ecological analysis takes into account the conditions and constraints existing in the school institution that will facilitate or hinder the implementation of the SRP. The economic analysis studies, among other things, the way a teaching process can be managed in a specific institution.

The design and *a priori* analysis (phase 2) include the selection of a generating question of the SRP that should at the same time be capable to engage the community of study to a self-sustained process and to incarnate parts of the REM developed in the previous phase. This phase also includes the evaluation of the potential study paths that the initial question is supposed to be able to generate. In this phase researchers also decide which kind of data of the SRP will be collected: students and teachers' productions, observations, final interviews and questionnaires, etc. Researchers will use this data as an empirical basis to analyse to what extend the REM and the implemented SRP overcome the tackled *didactic phenomenon*. Finally, they will decide, together with the teachers, on the practical aspects such as time planning, assessment and deliverables.

The *in vivo* analysis (phase 3) includes the implementation and management of the SRP and the data collection according to what was planned in the second phase. Analysing what is done and still to be done during an SRP is obviously important for the teachers and the researchers. But not only. It is also an important aspect of the SRP management to share with the students a description and validation of the different steps of the inquiry process.

The final phase includes the *a posteriori* analysis of the SRP implementation and the study of the gathered data to evaluate to what extent the hypothesis set in the preliminary analysis was valid and to what extent. In this phase, it is also important to take into account the conditions and constraints that have enabled or hindered the implementation of the SRP, as well as the role played by the different institutions involved in the experience.

1.2.8 Question-answer maps as an epistemological tool

The design, implementation and analysis of SRPs by researchers and, more generally, the need to represent knowledge in new ways raise the question of how to provide teachers and researchers with new specific epistemological tools. In other words, DE methodology requires to develop an important epistemological analysis in each phase. Winsløw, Matheron and Mercier (2013) present the *question-answer maps* (Q-A maps) as a research tool to model "mathematical knowledge from a didactical perspective". Q-A maps are rooted tree representations of the inquiry followed in an SRP. They start from the generating question and include all the partial answers and the derived questions appearing during the whole process (see Figure 6). Describing inquiry processes using sequences of questions and answers is not new. It has its foundations in the Socratic method. In the philosophy of science, dialogical logics and the Interrogative Model of Inquiry (IMI) are interesting examples of modelling scientific inquiry using alternating questions and answers (Hakkarainen & Sintonen, 2002; Hintikka, 1982).

The use of Q-A maps by researchers in the ATD has spread in the past decade with two main uses. On the one hand, they have been used as the materialisation of a REM (Barquero, 2009; Lucas, 2015). On the other hand, Q-A maps have been used as tools to describe and model study processes by, for instance, Barquero (2007) when implementing an SRP about population growth, and by Hansen and Winsløw (2010) during the implementation of a bidisciplinary SRP in mathematics and history. In fact, Winsløw, Matheron and Mercier (2013) present and propose to systematize the use of Q-A maps as a tool enabling researchers to describe, design and analyse process of study, because these highlight "how knowledge is constructed in the process" (Ibid, p. 281).

In summary, Q-A maps have been progressively incorporated by researchers as an epistemological tool to generate new material representations of REM and to describe the evolution of SRPs.

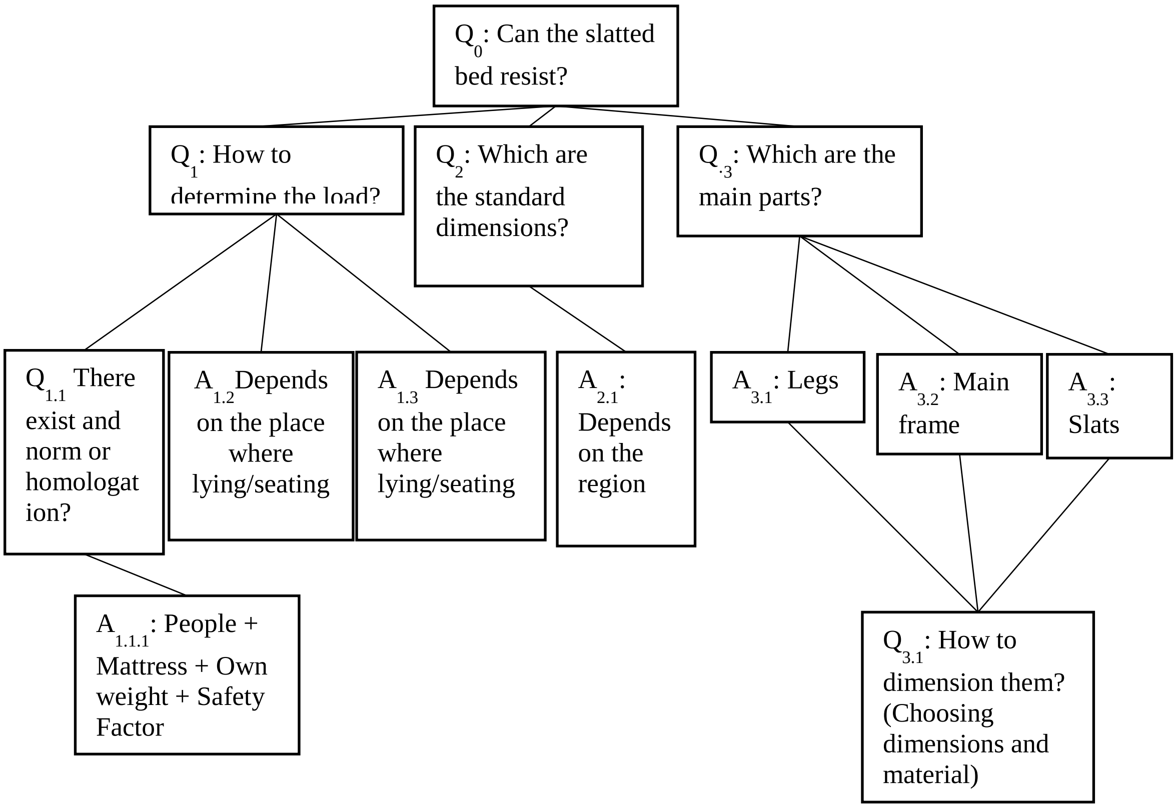

Figure 6 Example of a Q-A map used during the implementation of an SRP in engineering education

In contrast to this use at research level, Winsløw, Matheron and Mercier (2013) hypothesise that "such a representation is sufficiently close to teachers' concerns, and also captures such essential parts of a didactic design, that one could use it as a tool for collaboration and communication with and among teachers, regarding concrete teaching designs" (p. 281). Therefore, this alternative use of Q-A maps is not placed at the researchers' level but at school level. In these settings, Q-A maps could be generated by teachers and students to design, manage and deal with knowledge when implementing SRPs. This is a central issue in this dissertation.

1.3 Open issues in research

The previous sections briefly outlined how the design, implementation and analysis of SRPs call for the activation of specific didactic—and epistemological—techniques and create new types of didactic tasks. The implementation of SRPs generates some problematic issues at the crossroads of epistemology and didactics. As presented before, ATD considers that the design of study processes cannot detach the pedagogical level from the knowledge at stake. We consider that this integration generates two movements that appear in the design, management and evaluation of teaching and learning processes and can be briefly described as follows (Florensa, Bosch & Gascón, 2015):

(1) Starting from the analysis of teaching and learning processes, and considering a large enough empirical basis to include the processes of didactic transposition, all this empirical work provides tools to design specific REMs for the main mathematical contents or areas that are designed as knowledge to be taught. We can define this movement as *using didactic facts and phenomena to produce epistemological models.*

(2) Conversely, we can start from the principles and criteria that have guided the construction of a REM for a specific area of school mathematical activity and, in particular, the contrast between the *raison d'être* assigned by the REM to this area and its official (explicit or tacit) role in school settings. All these provide some epistemological and didactic tools to design, manage and evaluate teaching and learning processes based on SRPs sustained by that REM. This movement can be defined as *using the epistemological model as the core of didactic tools*. This double movement raises different open issues which are at the starting point of this dissertation.

1.3.1 New didactic needs

We have seen how previously elaborated REM on mathematical contents or problematic questions together with DE (obviously complemented with other methodological design tools) provide criteria for the design and implementation of teaching and learning processes that are closer to the paradigm of questioning the world. In principle, they aim at organising activities that should allow the students to carry out new tasks and techniques in a more autonomous, functional and justified way, starting from open questions that are related to a (possible) *raison d'être* of the contents to be taught. The "mathemagic" games in the case of elementary algebra (Ruiz-Munzón, 2010) or the different inquiry processes described in Winsløw, Matheron, & Mercier (2013) are good examples of this enrichment. Obviously, these new didactic organisations should be made available to the study community and their viability in different school institutions should be tested.

It is important to emphasize that all approaches and theories in didactics are also based on general models of didactic activities, what we have defined as the *education ends*. These general models are a particular way to interpret the school activity and to conceptualize the study process of mathematics or any other field (teaching, learning, diffusion and application). Even though these models are not always clearly spelled out, they remain an essential feature of theoretical approaches, as they strongly affect the type of research problems this approach can formulate. Two crucial questions arise:

- How to transform the REM into possible didactic organisations that could live in current school institutions? How to make this process available to the school institutions, especially to the profession of teachers?
- How to take into account the interrelation between the REM and the didactic phenomena appearing in the implementation of these new didactic organisations?

1.3.2 New epistemological needs

The empirical analysis of the study processes taking place in various institutions (for example, but not exclusively, in schools) clearly shows that didactic praxeologies are closely related to the epistemological tools available in the institution to describe and manage the praxeologies to be taught. For example, in the institutions where the dominant model is so-called Euclidean (Gascón, 2014), teaching and learning processes are conceived and described in terms of didactic activities around "definitions", "concepts", "theorems", "proofs" and "applications". In this case, these didactic activities tend to be hierarchically structured according to the logical construction of mathematical concepts (real numbers before limits, limits before derivatives, etc.).

If, instead of analysing traditional teaching processes, we look at those based on didactic research, the situation is very similar: how didactic processes and the dynamics of mathematical praxeologies are designed, described and managed also depends on the tools provided by the epistemological model which upholds, more or less explicitly, the didactic approach considered. The further this research-based epistemological model is from the dominant epistemological model at schools and scholarly institutions, the more difficult it becomes for teachers to carry out innovative teaching proposals designed within this frame.

In all these cases, the most remarkable feature is the shortage and inadequacy of tools available in the teaching institution to describe, manage, and evaluate the dynamics of mathematical activity. This lack of tools could in the first place be attributed to the scarcity of the components of the spontaneous epistemological models.

- Which new notions or tools are needed to describe and manage the dynamics of the didactic activity that will take place in study processes?
- How to describe these tools depending on the role addressed (didactic researcher, teacher and students) and how to make them available in the teaching institution and to the participants of the didactic process?

An interesting example of this phenomenon is described by Barquero (2009) showing up problems when teachers and students deal with notions such as mathematical model, validation, testing the model, etc. because these notions are usually absent from the school institutions.

1.3.3 The evolution of didactic-epistemological models

In order to establish an alternative and rich enough REM of a specific mathematical domain or questioning, it is necessary to take into account the didactic phenomena taking place in teaching institutions. This leads to an enrichment of the prevailing epistemological model during the first design of the REM. However, it is important not to take this REM as a final proposal but to keep its development running during the implementation and evaluation of the teaching proposals based on this REM. The consequent evolution of the REM is a clear example of the dynamic and provisional nature of the epistemological models elaborated by didactics, evolving from its initial proposals through the analysis of empirical facts.

From a mathematical perspective, these continuous evolutions of the REM can be seen as the incorporation of new notions and organisations into the field of knowledge at stake. This phenomenon can be related to the transformation of some paramathematical notions into mathematical concepts—as happened with concepts such as “set”, “function”, “continuity”, “graphs”, etc.—, a transformation which takes place as long as researchers deal with new problems. For instance, in the case of elementary algebra, the notion of “computation programme” is a new and crucial element of the proposed REM. In the experiences described by Ruiz-Munzón (2010), this notion initially played a very ambiguous role in the management of the teaching and learning processes, given the fact that it did not belong to the official mathematics to be taught and the teacher did not feel at ease with it. A similar phenomenon happened when implementing SRP on population dynamics with notions such as "quantities", "model", "system", "mixed and separated generations", etc.

- What degree of explicitness hould be adopted with the new epistemological models necessary to design, implement and evaluate new teaching and learning processes depending on the participants of the study communities addressed (students, teachers, mathematicians, etc.)?.

1.3.4 Q-A maps and DE: possible tools to generate an answer

The problematic questions raised by our research revolve around the didactic and epistemological needs both at research and school levels. In fact, we consider that the problematic issues that we present are crucial to transfer results of research works in didactics to school institutions.

We can summarize the problematic questions as:

- How to empower school institutions to design study processes—such as SRPs— based on previously established REMs?
- Which new epistemological tools and models are needed to describe and manage the dynamics of these didactic processes in school institutions?
- Which degree of explicitness should be adopted with the new epistemological tools and models to design, implement and evaluate new teaching and learning processes in school institutions?

We hypothesize that the transposition of research tools such as Q-A maps and the DE methodology to school institutions could be helpful to generate an answer to these questions. In fact, we consider that Q-A maps and DE can play an important role as tools enabling teachers to describe and manage the dynamics of the inquiry activity. As said before, both have been widely used as *research* tools, but their use as *teaching and learning* tools remains an open problem. We consider that this movement of research tools to school institutions can be understood as a (meta)didactic transposition movement (Arzarello et al., 2014).

This dissertation presents the design, implementation and analysis of four empirical studies in order to generate answers to these questions and to understand what phenomena appear when these tools are implemented. These four experiences are:

- A course for secondary teacher education
- The design, experimentation and analysis of an SRP in general elasticity,
- A lecturer's course on didactics, and
- The design, experimentation and analysis on an SRP in strength of materials developed with one of the participants of the lecturers' course.

As we will see in Chapter 3, in all these experiences, the role attributed to Q-A maps and DE is central both in the teacher education courses and in the experienced SRPs.

2 RESEARCH QUESTIONS

The main research question of this dissertation can be formulated as:

To what extent the use of Q-A maps and DE as epistemological tools in school institutions—including universities—are useful to enable teachers and students implement SRPs based on REMs, and describe, manage and analyse the dynamics of the inquiry process? In other words, what role can Q-A maps and DE play as part of the logos block of didactic praxeologies—the didactic *technology*—mobilised by teachers and students when implementing, describing and justifying didactic processes?

This general questioning will be fulfilled by developing answers to the following research questions:

- RQ 1: Regarding the Q-A maps in engineering education:
  - RQ 1.1: How can Q-A maps be explicitly used by lecturers and students when carrying out SRPs? What are their potentialities to describe and manage knowledge, during the design, implementation and analysis of SRPs? Which are the conditions and constraints enabling and hindering their use?
  - RQ 1.2: To what extent the use of Q-A maps helps make the *raison d'être* of the knowledge at stake explicit in contrast to the traditional paradigm of visiting works?
- RQ 2: Under what conditions can Q-A maps be used in teacher and lecturer courses as tools to develop epistemological questionings?
- RQ 3: What can be the role of the DE methodology as a tool for lecturers in engineering education to systematically design inquiry study processes?

3 THESIS STRUCTURE

3.1 Published papers in the dissertation

This dissertation is based on the following papers published or accepted to be published in international peer-reviewed journals:

Chapter 4:

Bartolomé, E., Florensa, I., Bosch, M., & Gascón, J. (2018). A 'study and research path' enriching the learning of mechanical engineering. European Journal of Engineering Education, 1–17. https://doi.org/10.1080/03043797.2018.1490699 **(See section 4.1)**

Florensa, I., Bosch, M., Gascón, J., & Winsløw, C. (2018). Study and Research Paths: A New tool for Design and Management of Project Based Learning in Engineering. *International Journal of Engineering Education, 34*(6), 1-15. **(See section 4.2)**

Chapter 5:

Florensa, I., Bosch, M., & Gascón, J. (in press). Análisis a posteriori de un REI-FP como herramienta de formación del profesorado. *Educaçao Matemática Pesquisa*. **(See section 5.1)**

3.2 Dissertation structure

The introduction in chapter 1 is based on the text of an oral communication to the 9th Congress of European Research in Mathematics Education (CERME9, Prague) where the general aim of the thesis is presented. This text is complemented by an oral communication presented at the 6th International Conference on the Anthropological Theory of the Didactic (CITAD6, Grenoble) including the specific issue of the form and function of REMs in school institutions.

In the CERME9 paper (chapter 1), we present some problematic relations between epistemology and didactics. The problems identified in this paper are the initial questions of this PhD research work. They led to the design and implementation of different experimental teaching proposals, which generate the empirical data used in the elaboration of the partial answers. Let us remind the main problematic issues this paper states. The first one is related to the relationships between REMs and SRPs and the need of new epistemological and didactic tools for researchers and teachers. The second question refers to the need of specific tools and notions to describe and manage the knowledge involved in the new inquiry processes. Finally, a question regarding the evolution of the didactic-epistemological models is stated: what degree of explicitness should be adopted with epistemological models when dealing with teachers and students?

In the CITAD6 paper we expand the problematic of the role of the tools needed in school institutions to design, manage and analyse study processes. Specifically, we elaborate on the role of REMs when used by teachers and students. As we have presented in the introduction, a problematic question when implementing new study processes (SRPs in particular) is the important change experienced by knowledge. The paper studies the possibility to use a transposed version of the REM in school institutions to describe and institutionalise knowledge involved in SRPs. Specifically, we conduct a review on the different material forms REMs have taken in previous ATD research works and how these research tools can be adapted and used in school institutions.

In chapter 4, we present the design, implementation and analysis of two SRPs in engineering education. These SRPs have been implemented in the same institution (Escola Universitària Salesiana de Sarrià – Universitat Autònoma de Barcelona) in two different subjects. Section 4.1 includes a paper published in the *European Journal of Engineering Education* in June 2018. This implementation took place during the 2016-17 academic year in a Strength of Materials course, a second-year subject of a Mechanical Engineering degree. A singular characteristic of the implemented SRP is that the SRP is designed for the whole semester and that seminars and specific lecturers will be implemented depending on the development of the study process. Regarding the research aspects, the SRP was designed in order to generate empirical material regarding research questions RQ1 and RQ2. We analysed the viability of implementing an SRP in engineering education with a team of researchers and a lecturer with no experience in didactic research. Two main questions were analysed: firstly, the use of the Didactic Engineering methodology when used by lecturers and researchers to systematically design, manage and evaluate a study process. And, secondly, we analysed the role played by epistemological tools such as Q-A maps and media-milieu dialectics when used not only by researchers but also lecturers and students. In years 2017-18 and 2018-19 new implementations of the SRP have been carried out at EUSS-UAB.

In section 4.2 we present a paper published in the *International Journal of Engineering Education* in October 2018. This SRP was implemented during academic year 2015-16 in an Elasticity course of the third year of a Mechanical Engineering Degree. This SRP was designed by researchers and implemented into groups of students: one taught by a researcher and the other one by a non-researcher lecturer. Contrasting to the SRP implemented in “Strength of Materials” this SRP combined the traditional organization of the course with the implementation of the new study process. The first 9 weeks of the semester kept the classical organization with 2-hours of lecture and 2-hours of problem solving. The last 5 weeks were entirely devoted to the SRP implementation. The use of Didactic Engineering methodology took a central role in the research process and the Q-A maps were a crucial tool during all the phases. These facts allowed us to generate empirical material to elaborate on an answer the RQ1 and RQ2 questions. This SRP has also been implemented with small modification in years 2016-17, 2017-18 and 2018-19.

In chapter 5 we present the work addressing objective RQ3, regarding teacher education and the possibility to incorporate Q-A maps as an epistemological tool. Section 5.1 includes an accepted paper accepted to the *Educaçao Matematica Pesquisa* journal describing the implementation of an online course addressed to secondary education teachers in the frame of a Master’s Degree in Mathematics Education. In this course, teachers that had already experienced and SRP-TE use Q-A maps to analyse their mathematical work as well as the work developed by their students. The course lasted for four weeks and four different activities were proposed to students including the use of Q-A maps as tools to analyse and design study processes.

Section 5.2 includes a paper presented to the 10th Congress of European Research in Mathematics Education (CERME 10, Dublin). This paper presents the design, implementation and analysis of a course addressed to lecturers in an Engineering School in Barcelona. The specific objectives of the course are twofold. On the one hand, the experience pretends to enrich the empirical available material to generate an answer to research questions RQ3 of this thesis. On the other hand, we consider that this course is a starting point of ATD-based research on lecturer education. In fact, this field is almost non-existent even if lecturers’ activity always include teaching as an important activity.

In chapter 6, the main results of the presented works are discussed, and a provisional answer is stated to the research questions of this dissertation. Chapter 7 includes this final answer in terms of conclusions regarding the initial objectives RQ1, RQ2 and RQ3.

Finally, appendix 1 includes other documents that we consider relevant to this dissertation. In section A1.2 we present a book chapter published by the Institut for Naturfagenes Didaktik, Københavns Universitet. This chapter describes how the initial problematic presented in chapter 4 is linked to the implementations of SRPs and teacher education courses described in chapters 5 and 6 respectively. Section A1.2 includes a paper presented at the 13th International Conference on Mathematics Education (ICME 13, Hamburg). This paper presents the initial design of the course addressed to lecturers presented in chapter 5.2. We consider it relevant because it was the only contribution to ICME 13 regarding lecturers' professional development. Finally, section A1.3 includes a paper presented to the Second Conference of the International Network for Didactic Research in University Mathematics (INDRUM2018, Agder; Norway). In this paper, we present a wider perspective of lecturer education using the scale of levels of co-determination and studying the questions lecturers raise regarding their teaching activity.

4 DESIGN, EXPERIMENTATION AND ANALYSIS OF STUDY PROCESSES IN ENGINEERING EDUCATION

4.1 A ‘study and research path’ enriching the learning of mechanical engineering

This section is published in:

Bartolomé, E., Florensa, I., Bosch, M., & Gascón, J. (2018). A ‘study and research path’ enriching the learning of mechanical engineering. *European Journal of Engineering Education*, 1–17. https://doi.org/10.1080/03043797.2018.1490699

4.2 Study and Research Paths: A New tool for Design and Management of Project Based Learning in Engineering

This section is published in:

Florensa, I., Bosch, M., Gascón, J., & Winsløw, C. (2018). Study and Research Paths: A New tool for Design and Management of Project Based Learning in Engineering. *International Journal of Engineering Education 34*(6) 1848-1862

5 QUESTION ANSWER MAPS IN TEACHER EDUCATION

5.1 A posteriori analysis of an SRP-TE as a teachers training tool

This chapter is published in:

Florensa, I., Bosch, M., & Gascón, J. (in press). Análisis a posteriori de un REI-FP como herramienta de formación del profesorado. *Educaçao Matemática Pesquisa*.

5.2 Teaching didactics to lecturers: a challenging field

5.2.1 Introduction

Traditionally, lecturers' development courses have not been considered relevant by research in teacher education. This is a normal phenomenon considering universities' criteria when hiring lecturers and evaluating those already lecturing: mainly research activities and merits are considered. In contrast, lecturers' didactic or pedagogical education is usually ignored or, at most, considered as a positive complement. The absence of regular lecturers' teaching training is a worldwide phenomenon with few – and not always successful – exceptions. In the United Kingdom, the Higher Education Academy (HEA), the UK Professional Standards Framework (UKPSF) and its accreditation process made a first attempt to incorporate lecturer training as a requirement to teach in UK universities (Department for Education and Skills, 2003). Nevertheless, this program that was thought to be central in lecturers' professional development has finished as a volunteer training and accreditation schema for both individuals and institutions involved in teaching at higher education (The Higher Education Academy, 2011)(The Higher Education Academy, 2011).

We consider that, as long as their activity has a clear twofold character based on research and teaching, in addition to the traditional training in research (Master's Degree and PhD program), lecturers also need an explicit pedagogical and didactic education. In fact, universities are among the sole existing teaching institutions where teachers are not required an explicit training course on teaching and learning processes. We consider that this crucial difference should not be accepted as a given: the conditions of existence of a university teacher education course have to be studied, especially with the possibility to base it on contents emerging from research in didactics.

In order to have a first set of empirical data to evaluate the conditions of existence of such a course for lecturers at university level we designed a course for 14 lecturers of an Engineering School in Barcelona (www.euss.es). Lecturers participating in the course teach Analysis (3), Strength of Materials (4), Physics (2), Electronical Technology (2) and Informatics (2). We took as starting point the frame of "study and research paths for teacher education" (SRP-TE) based on recent investigations in the Anthropological Theory of the Didactic (ATD) for pre-service and in-service secondary teachers. The lecturers' course was experienced in February 2016. We present the design principles and results of this first edition, as well as the subsequent re-design for new editions, to overcome the experienced difficulties and take advantage of its potential strengths.

5.2.2 University teacher education: a field to be explored in ATD

Courses for pre- and in-service lecturer professional development are an unexplored field in research. There exists very little literature regarding this subject and the few experiences reported involve only general pedagogical contents not taking into account the very nature of the knowledge involved in the teaching and learning processes. It is important to highlight that no paper on this field was presented at the last CERME9 (neither at TWG 14, University Mathematics Education; or at TWGs 18, 19 and 20, Teachers' Knowledge, Practices and Education), or at groups regarding teacher training or university teaching at the last ICME 13, except for a preliminary version of this paper (Florensa, Bosch, & Gascón, 2016b). The structure of ICME13 Topic Study Groups about teacher education is especially revealing at this respect: there were four groups on teacher education, two (in and pre-service) centred on the elementary level and two on the secondary level, but none on the tertiary level. At the recent conferences on Mathematics Education in North America, only Ellis presented research on teacher assistants training (Ellis, 2014, 2015).

Regarding the presence of papers in journals about lecturers' education we have found very little production: only two papers (Guasch, Alvarez, & Espasa, 2010; Postareff, Lindblom-Ylänne, & Nevgi, 2008) and the Handbook on Teaching and Learning in Higher Education (Fry, Ketteridge, & Marshall, 2009). We have developed a research from the initial year of publication to the end of 2015 in these journals: Educational Studies in Mathematics, Higher Education, Journal of Mathematics Teacher Education, Mathematical Thinking and Learning, Journal of Teacher Education, Recherches en Didactique des Mathématiques, REDIMAT, RELIME.

As said before, we consider that research in didactics can be taken as the basis for courses on lecturer education regarding teaching and learning processes. We assume as starting hypothesis that results emerging form secondary teacher education can be used at this level. The results presented in this paper will be used to partially confirm this assumption. The Solid Findings in Mathematics Education on Teacher Knowledge (Education

Committee of the EMS, 2012) state explicitly that "content knowledge" (CK) is necessary but not sufficient for teaching. The report of the Education Committee highlights as crucial notions to be developed in teacher education the "pedagogical content knowledge" (PCK) (Shulman, 1987) and the different dimensions of the "mathematical knowledge for teaching" (MKT) (Ball et al., 2008). Both approaches clearly go further than the traditional conception of teaching as transmission of knowledge and consequently ask for changes in teacher education concerning the way mathematical knowledge should be approached.

We use the Anthropological Theory of the Didactic (ATD) as a main framework for the design, experience and analysis of the course. The last investigations on teacher education in ATD show that the use of notions such as PCK and MKT do not ensure researchers/educators to include a questioning of the nature, selection and organization of the contents to be taught (Ruiz-Olarría, 2015). Under the ATD approach, the role of teacher education is not limited to enrich teachers' pedagogical performance, but also to provide them with tools to contest the so-called dominant epistemology and emancipate from it when designing study processes (Gascón, 2014).

This questioning and reorganization of the knowledge to be taught is not spontaneous for teachers (nor for lecturers) because they tend to assume the institutional dominant epistemology as their own. The way proposed by ATD research to locate it at the core of teacher educational processes has very much evolved in this last decade. It started with a first experience in secondary teacher education based on the "questions of the week" (Cirade, 2006) and nowadays takes the form of an inquiry-based device called "study and research path for teacher education" (SRP-TE), which starts from a problematic question appearing in the field of the teacher profession and leads to the search, development and analysis of alternative teaching proposals (Barquero et al., 2015, 2016). The main idea of the SRP-TE is to generate a practical and theoretical questioning of the school activities linked to the teacher professional initial question. It is structured in five modules:

- M0: Formulation and first exploration of the generating question Q0 of the SRP-TE, for instance one of the kind: "How to teach (a specific content)?" which is to be partially answered at the end of the process.

- M1: Living a “study and research path” (SRP) as a student. The main goal is to make teachers encounter an unfamiliar inquiry-based activity related to Q0 that could exist in a normal classroom of the considered educational level.
- M2: Adaptation of the lived SRP to be experienced in a real school situation. During this adaptation, many of the institutional restrictions teachers should face are expected to show up. They can thus be afterwards analyzed from an epistemological, didactic and ecological perspective (what can “live” and under what conditions in a given educational setting).
- M3: Experimentation, management and carrying out of in vivo and a posteriori analyses of the adapted teaching proposal.
- M4: Joint elaboration of a critical analysis of traditional teaching practices and the possibilities (and limitations) of introducing new proposals, as well as generation of a partial answer to Q0.

During the development of SRPs-TE for secondary school teachers, an epistemological tool has been adapted and developed to facilitate the analysis of the SRP and the questioning of school contents: what we call “question-answer maps”. Following other authors, we consider these maps, which are used as a key tool in ATD research, as a powerful instrument for teacher education:

We hypothesize that such a representation is sufficiently close to teachers’ concerns, and also captures such essential parts of a didactic design, that one could use it as a tool for collaboration and communication with and among teachers, regarding concrete teaching designs (Winsløw et al., 2013, p. 281).

Some preliminary and promising experiences exist in using these maps in teacher training courses to describe the dynamic and collective aspects of mathematical activity (Barquero et al., 2016; Florensa, Bosch, & Gascón, 2016a; Jessen, 2014). The work with the maps seem to be useful for teachers in order to describe knowledge in inquiry activities and to act as a counterpoint of the official curricular organization of contents.

5.2.3 Research questions

The work presented in this paper is considered as an exploratory design (Singh, 2007) to obtain and analyse a first set of data from the first implemented course and to redesign it to be applied in another institution. The specific research questions that will be studied are:

- RQ1. The role played by question-answer maps in teacher education: Do they help lecturers describe, analyze and design inquiry and modelling processes and the involved knowledge?
- RQ2. Does the course empower lecturers to identify the dynamic and collective nature of the lived SRP in contrast to the static, individual and compartmentalized dominant conception of knowledge?

5.2.4 Course description

The engineering school where the course was implemented keeps a four-hour time slope with no teaching for all lecturers all Wednesdays: they use this time for professional development, meetings, pedagogical courses or activities. In fact, it is a Salesian university with a special concern about teaching and learning processes, as well as students' personal evolution. The course was structured in six two-hour sessions during three weeks, and the central question to be partially answered was: "Could modelling be the main motivation of my subject? Which conditions enable and which constraints hinder this modelling activity?"

Because of the time restriction, the five-module structure of the SRP-TE had to be adapted. The six sessions appeared to us (designers and course leaders) as a short course. However, they finally seemed to be enough for the work planned. Of course the true work is to be carried out afterwards, when lecturers decide to introduce some new proposals in their subject based in the work initiated at the course. During this application phase teachers implementing SRPs asked for help to the researchers-educators, thus extending the real duration. We planned the course as follows:

- 1st session: Explicitly state the professional question Q0 and shortly present the ATD framework including the notions of praxeology, Herbartian schema and media-milieu dialectics, topogenesis, mesogenesis and chronogenesis (Barquero & Bosch, 2013). These notions were well understood and some of them were mobilised during the 4th session.
- 2nd and 3rd sessions: A SRP was proposed to be carried out in groups of up to three lecturers. "Taking into account the incidence index of the last 9

months of a dengue outbreak: could you forecast the incidence index for the next 3 months (already known)?" (Figure 7)

- 4th session: Lecturers generated a question-answer map of the lived SRP including aspects such as media-milieu dialectics. One of the generated maps can be seen in Figure 8.
- 5th session: Lecturers are invited to create new small groups with the colleagues teaching the same subject. They are asked to design a SRP by choosing a generating question in their field trying to overcome some observed didactic facts such as the absence of *raison d'être*, the disconnections of topics or the poverty of the experimental work, among others.
- 6th session: Sharing some possible teaching proposals and conclusions of the course.

Year	Incidence index
0	31
1	179
2	438
3	454
4	587
5	1176
6	1543
7	1859
8	2373
9	696
10	0,1
11	0,05
12	0

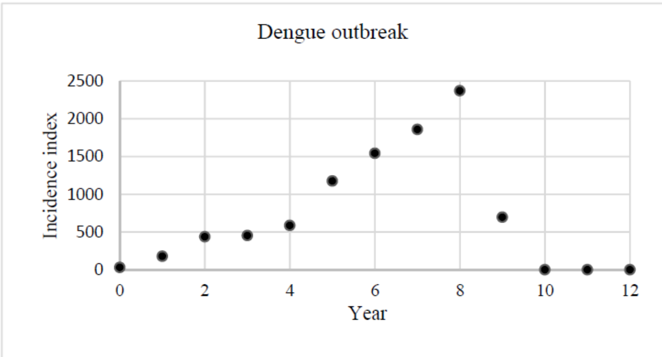

Figure 7 Data used for the lived SRP

In the introduction to the 5th session, lecturers were invited to identify didactic facts that they would like to overcome through the new didactic proposal. The goal was not to implement the inquiry by itself, but to identify how the dominant epistemology in the institution is related to these problematic phenomena and roughly propose new possible epistemological and didactic organizations to face them. The question-answer maps were the tool provided to lecturers to carry out this work. During the implementation of the course, some of the contents that we initially considered as difficult had an easier reception than expected (especially the notion of media-milieu dialectics) and, on the contrary, some basic notions were difficult to share with the participants, for instance the description of contents in terms of questions instead of topics.

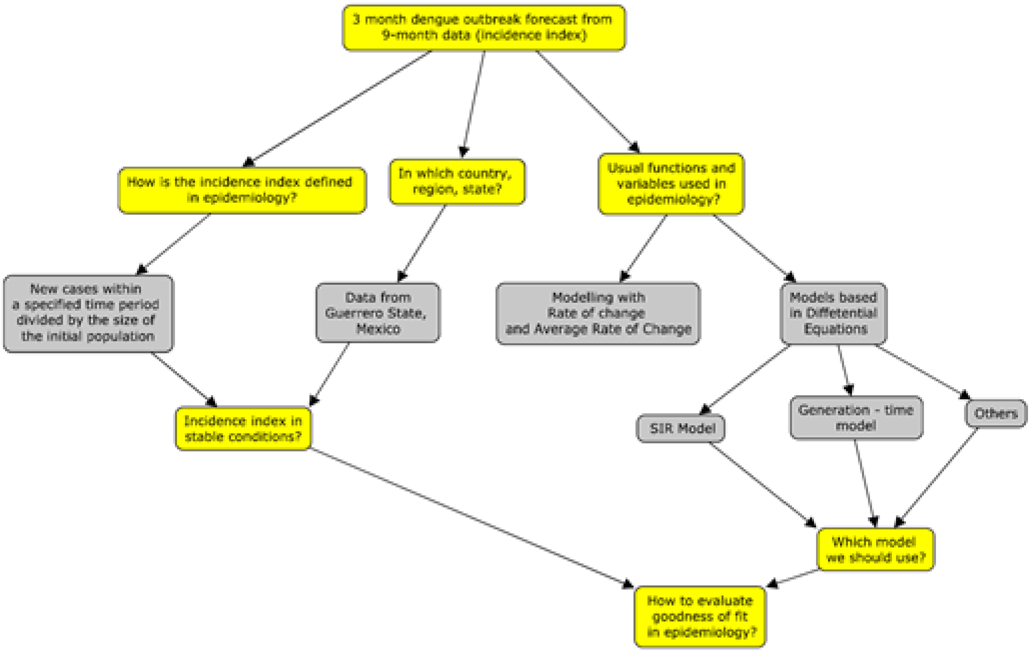

Figure 8 Question – answer map of one of the groups

In order to obtain data to evaluate the course, all the questions-answer maps of all groups, both from the analysis of the modelling lived activity and from the a priori design of the SRP, were collected. We have also obtained data from a final survey filled in by all lecturers attending the course. The survey was structured in three main blocks. The first block addressed general aspects of the course such as duration, balance between individual and team work, time structure, etc. The second block asked about content-related aspects of the course like the work developed with question-answer maps and with the media-milieu dialectics. Finally, the survey asked the lecturers about the possible consequences of the course on their teaching activities: changes in the conception of knowledge, dynamics and collective aspects of activities, and availability of new designing and evaluating tools.

5.2.5 Results and discussion

The question-answer maps regarding the dengue outbreak SRP shew up how the inquiry was capable to connect fields usually disconnected in the traditional curricular organization of contents. For example, the map of Figure 8 reveals that functions, differential equations, regression, average rate of change and epidemiological notions are deeply interrelated. An interesting fact emerged when analyzing different maps from different working groups: depending on their lecturing field, they approached the problem quite differently. For instance, Mathematics lecturers' work was centred on finding a mathematical model fitting the data, whereas Chemistry lecturers' work evolved around the epidemiological data, the notion of "incidence index" and searching scholar literature regarding other similar outbreaks. The different teaching fields of lecturers permitted to share different visions of the knowledge at stake in the proposed SRP. The use of the maps was a key factor to describe this connection of fields usually lacking in school institutions.

The second part of the survey about the content of the course reveals that the work developed by lecturers with the question-answer maps and the media-milieu dialectics was difficult for them (more than 70% of the teachers found it hard or very hard) but at the same time they identified this work as "easily applicable to design and manage new teaching and learning processes" (more than 70% of the lecturers found contents and tools of the course easy to use and to implement). Regarding the consequences of the course on the lecturers' teaching practices, the survey showed that it helped (more than 90% totally agreed) to change their previous conception of knowledge towards a dynamical-collective conception in terms of modelling activities.

The third source of evidence are the maps generated by the lecturers as a priori analysis for an SRP to be experienced in their subjects. In total, six maps where generated by lecturers, all of them with a generating question and making explicit the didactic facts intended to be overcome. Two of these a priori SRP designs where experienced during the spring semester, starting just after the lecturers' course. These two emerging SRP have been experienced and managed only by lecturers that followed the course and did not have any other didactic experience or training. This fact is especially interesting because with the analysis of these experiences a first set of data can be collected regarding the conditions of existence of SRPs at the university level led by lecturers with almost no direct connection with research in didactics. This first experience in lecturer education seem to preliminary validate Winsløw et al. (2013) hypothesis about the use of question-answer maps in teacher education and confirm Barquero et al. (2016) results. Lecturers have worked with the maps and have used them to both model a lived study process and a priori analyse their own designed SRP. Moreover, the maps have been used to compare the knowledge mobilised during a specific SRP and the school knowledge. The Q-A duplets appearing in the map were used as the elements to contrast with curricular requirements.

The course also appears as a good tool to empower lecturers to question and put under vigilance the dominant epistemology at the university. It produced a discussion (and thus enabled a reflection) on what knowledge has to be taught at the university and how the modelling activity with its dynamics and collective aspects could be considered. Regarding the conditions of existence of a lecturer course based on the ATD, it seems that the described conditions make it viable and that some lecturers have taken it as an opportunity to redesign their teaching and learning activities. However, an important aspect to take into account is the fact that one of the leaders of the course is also a lecturer in the considered Engineer School, what certainly affected the good predisposition of the attendees due to his personal leadership in the institution. This particular condition has to be considered in new editions of the course and the question of its reproducibility remains open.

6 DISCUSSION OF RESULTS

6.1 Implementing SRPs in engineering education: the role of Q-A maps and didactic engineering (DE)

We have evaluated the use of Q-A maps and DE methodology through their implementation as tools used by teachers and students in two courses in a Mechanical Engineering Degree at the Escola Universitaria Salesiana de Sarrià.

The first SRP (see section 5.1 for further details) was implemented in a Strength of Materials course. The SRP was designed in order to cover all the sessions of the course (16 weeks, 4 hours per week). The design of the SRP was done by the author of this dissertation together with the lecturer in charge to implement the study process. The team researcher and lecturer decided to follow the DE methodology, which unexpectedly appeared to play an important role. First, its four-phase structure allowed the team of researcher and lecturer be systematic and share a roadmap. Second, the DE *preliminary* and *a priori* analyses forced the team to be explicit with respect to the problematic didactic phenomena considered. In fact, this mixed team and the shared methodology made the lecturer aware of the *researcher's problem*: to modify the prevailing epistemology that causes undesired phenomena (extreme *algorithmisation* of proposed techniques, *atomisation* of different chapters and total detachment from real engineering activity, etc.) in order to overcome them. In addition, the design of the new study process was at the same time an answer to the *lecturer's problem* (how to teach Strength of Materials in a satisfactory way). The ATD *educational ends* played a crucial role in these phases: importance of the initial question, openness of the process, students' responsibilities in managing the inquiry, etc. In a more or less explicit way, they were taken as a reference for the new study process. The selection of an initial question was not only based on general principles, but an explicit epistemological work was conducted to justify it.

Regarding the use of Q-A maps, these were used in the *a priori, in vivo* and *a posteriori* analyses by all the participants: the researcher, the lecturer, *and the students*. First, they were used during the *a priori* analysis to study different alternatives and select the initial question. In this situation, Q-A maps were used as a partial representation of the REM:

they modelled how knowledge could be generated and evolve during the new inquiry process.

A singular aspect of this SRP has to be highlighted: during the *a priori* analysis, the lecturer and the researcher decided to explicitly train the students to create and use Q-A maps in order to help them describe the inquiry process (see **¡Error! No se encuentra el origen de la referencia.**). The use of Q-A maps also involved the students during the *in vivo* analysis: they were used as the main tool to describe the evolution of the inquiry process (see **¡Error! No se encuentra el origen de la referencia.** for the whole map and **¡Error! No se encuentra el origen de la referencia.** to **¡Error! No se encuentra el origen de la referencia.** for the Q-A maps elaborated by students). The maps not only described how knowledge was evolving but also played a crucial role in the institutionalisation: in each session, the incorporation of a question or an answer to the map was the main tool to made decisions about the next steps to follow. In addition, the maps allowed the community of study to assign tasks regarding specific sub-questions.

The second SRP was implemented in the same engineering school and degree but in a different subject: General Elasticity, a third-year course. This SRP was implemented in the second part of the semester while the first part kept the traditional structure (lectures and problem sessions). The design and analysis of the SRP were carried out by a team of ATD researchers, while the implementation was done in two groups: one led by the author of this dissertation and another managed by a non-researcher lecturer.

The design, implementation and analysis of the SRP also followed the DE methodology. The preliminary analysis revealed an interesting epistemological phenomenon regarding the nature of the elasticity model: the model itself was the *raison d'être* of the subject while the problems the model enabled to solve remained in the shadow. This didactic phenomenon was due to the fact that analytically solvable problems were far from real problems. However, the emergence of numerical methods did not affect how and why the subject was taught. During this phase, another phenomenon was detected such as the isolation of lab sessions proposed in the previous organization. As in the previously described experience, the ATD *educational ends* were used as a reference to analyse the didactic and epistemological phenomena arising in the previous organization.

The new SRP designed during the *a priori* phase and its initial question were analysed using the Q-A maps. The mixed team (researcher and lecturer) developed the Q-A of **¡Error! No se encuentra el origen de la referencia.**. During the *in vivo* analysis, Q-A maps were also used as a tool to communicate between students and lecturers as well as a tool between the two lecturers to describe how knowledge was evolving. Also, the use of Q-A maps in the *a priori* analysis helped to evaluate to what extent the phenomena identified during the preliminary analysis would be overcome.

Making the non-researcher lecturer a participant of the DE methodology and, in particular a Q-A map user, helped to make her aware of the changes the new SRP would cause, not only at the pedagogical level but also at the epistemological one. She stated this fact in these terms: "Implementing the SRP and the first phases have changed how I teach the first part of the course, including lectures and exercise sessions. Although they are very similar to the previous year's sessions, I have changed the way I teach. Enabling students answer the SRP generating question has become the *raison d'être* of the taught knowledge. Now I feel that my teaching task has a rationale, the Navier, Cauchy and Lamé model has changed somehow, and it changed my idea of what I teach. And this fact makes sense when training engineers to face real problems…" In addition, the final survey revealed than more than 65% of the students changed their mind about what Elasticity was.

Regarding the role played by Q-A maps during the implementation and the *a posteriori* analysis of the SRP, we would like to highlight an interesting fact: the use of Q-A maps by students was difficult during the first sessions. For example, even if students were requested to present a map on their weekly reports, many groups did not include it. The statement of one of the students is revealing: "I failed the first weekly report: I did not understand what teachers meant about questions and answers… I was only worried about designing a gear for the bike and in just one week we did not have any concluding result…". This fact contrasts with the SRP implemented in Strength of Materials, where the initial activity helped students detach from the idea that maps should only include final and well-confirmed knowledge. In contrast, at the end of the SRP, the use of Q-A maps was considered useful by most of the students. According to students' statements, they acted as a productive management tool: "Q-A maps became very useful when managing the work load: we assigned questions to everyone" or as a communication tool: "Q-A maps helped us to inform the teacher where we were in the project…."

Even if there are important differences between both experiences, they reveal that Q-A maps were integrated in the design and implementation phase of SRPs in engineering education. They were used by researchers and lecturers to describe a new study process and to partially evaluate to what extent the new proposal overcomes the identified didactic phenomena. Regarding the use of Q-A maps by students, it seems interesting to incorporate an explicit training of students. The fact that the maps highlight aspects usually absent from school culture like hesitation, hypothesis development and estimation, among others, causes an important change in the *didactical contract* that can be facilitated by this training. This proposal can be related to the phenomenon of metacognitive shift (Brousseau 1997), which appears when the general methods or tools used to solve problems become the core of the teaching process. However, learning how to use the new tools does not seem unavoidable. The risk of shifting from engineering content to Q-A maps use should then be kept under the researchers' and lecturers' epistemological vigilance.

In addition, in both cases, the DE methodology helped the researcher and the lecturers collaborate in the design and implementation of the study processes. DE served as a common framework for lecturers to develop—together with researchers—an indispensable epistemological work in the design, implementation and analysis of SRPs within the ATD framework. In addition, the incorporation of the DE methodology empowered lecturers to see knowledge to be taught as something relative that can and has to be questioned in order to overcome problematic aspects in teaching and learning processes.

6.2 Q-A maps in teacher and lecturer education

Q-A maps were also implemented in a teacher and a lecturer professional development course. The Q-A maps were used in these courses by participants in two situations. First, they were used as tools for teachers and lecturers to describe the development of a study process experienced by them in the "student position". Secondly, they used Q-A maps as tools to *a priori* study and analyse the potentiality of a generating question. In both uses, the maps were applied to compare the new SRP study processes (experienced or potential) with the previously existing ones.

The first course is an online course addressed to secondary teachers. Q-A maps were used in two parts of the course. In the first part, the teachers had to create a Q-A map after having experienced an SRP initiated by a question about sales forecasting. All groups of teachers developed Q-A maps that reveal interesting facts. First, most of the maps included questions and answers that usually remain out of what is considered "mathematical knowledge". For example, some of the teachers' teams included contextual questions such as the need to consider GDP of the country as a variable to predict sales or the difference between short and long term. Second, some of the groups abandoned the description of their real inquiry path and proposed a map that reproduced the classical curricular structure of teaching functions at school. This is a revealing point: even if teachers used the maps, they cannot really detach from the *prevailing epistemology* in school institutions.

In the second part of the course, the use of the maps was to *a priori* evaluate the potential of a generating question in the domains of "Functions" and "Statistical regression", and to compare the new study process and the potential tasks with the existing school ones. Regarding its use, we can say that the maps revealed to be very useful when used as counterpoint of school mathematics. In fact, the way mathematical activity is described highlights many of the constraints of the school institution. For instance, the fact that maps include a dynamic knowledge organisation, showing abandoned or non-fruitful paths and usually overcoming the traditional limits of domains and disciplines.

This first experience with Q-A maps in teacher education generates interesting results. Teachers easily adopted their use both to describe previously experienced processes and to *a priori* evaluate new study processes. A second aspect of teachers' use of Q-A maps is to contrast the genesis and evolution of knowledge with the way knowledge is described in more traditional settings and in official curriculum documents. We could say that they were an important tool to empower teachers to be aware of the institutional relativity of knowledge.

The second experience of using of Q-A maps in teacher education courses took place in a lecturers professional development course at the Escola Universitaria Salesiana de Sarrià. Lecturer professional development in education has not traditionally been considered as a relevant field in teacher education even if lecturing is a central activity in lecturers' profession (together with research). This fact makes this experience an exploratory research study aiming not only to evaluate the use of Q-A maps but also the conditions of a course about didactics addressed to lecturers.

In this course, Q-A maps were also implemented by lecturers as a tool to describe an experienced SRP (regarding the modelling of the number of cases of a Dengue outbreak) and to *a priori* evaluate generating questions of SRPs. Q-A maps analysing the experienced SRP made explicit that the activity was able to connect traditionally isolated contents in curricula. In more general terms, the maps acted as a tool enabling lecturers compare the knowledge generated during the SRP with the traditional curriculum. When Q-A maps were used to *a priori* analyse a generating question for an SRP, most of the lecturers find this activity difficult. However, at the same time they considered the maps as very useful to compare how and to what extent the generating question modifies the current organization of knowledge.

Additionally, the work with Q-A maps raised the debate about how—and in what direction—the implementation of SRPs would modify knowledge at the university. This led to interesting debates between participants questioning aspects such as "where does knowledge to be taught comes from" that are usually blindly accepted or the freedom of lecturers to select and redesign study processes.

7 CONCLUSIONS

The main research question of this dissertation has been formulated as:

¿To what extent the use of Q-A maps and DE as epistemological tools in school institutions—including universities—are useful to enable teachers and students implement SRPs based on REMs, and describe, manage and analyse the dynamics of the inquiry process? In other words, what role can Q-A maps and DE play as part of the logos block of didactic praxeologies—the didactic *technology*—mobilised by teachers and students when implementing, describing and justifying didactic processes?

This general questioning has been deployed into the following research questions:

- RQ 1: Regarding the Q-A maps:
    - RQ 1.1: How can Q-A maps be explicitly used by lecturers and students when carrying out SRPs? What are their potentialities to describe and manage knowledge, during the design, implementation and analysis of SRPs? Which are the conditions and constraints enabling and hindering their use?

The results obtained in two case studies of SRP implemented in courses of Strength of Materials and Elasticity in an engineering school of Barcelona reveal that Q-A maps have been used by both lecturers and students. Specifically, lecturers have used them together with researchers in the *a priori* design of the courses as an epistemological tool to evaluate changes that the SRP will cause on the previous knowledge organization and conception. Lecturers have adopted the maps easily and these have been crucial to make explicit how the new instruction proposal overcomes some important didactic phenomena (such as thematic confinement, algorithmization or labs isolation).

Q-A maps have been used by students in the two implemented SRPs to describe the paths followed during the inquiry process as well as to communicate their progress and to assign and share tasks. Regarding the use of Q-A maps by students, we consider that an explicit training on their use seems to overcome constraints related to the prevailing *didactic contract* and facilitate students to make traditionally hidden aspects explicit, such as hesitation, estimations and unfruitful work.

In summary, Q-A maps have shown up great potential as communication tools during the implementation of SRPs. In both experienced SRPs, the maps became the tool used to communicate the progress of the study process between students and between students and lecturers. In addition, the maps also appear as a managing tool: students and lecturers used them to assign specific tasks to students.

Results show that a specific training addressed to the students involved in the SRP led to a faster incorporation of the use of Q-A maps helping to overcome some difficulties associated to their use. These difficulties appear because maps highlight aspects of study processes that traditionally remain at the shadow such as unfruitful tasks or hesitation. We consider relevant that the fact of involving lecturers in mixed research-lecturers teams using the maps in the *a priori* phase facilitates its use during the implementation.

- RQ 1.2: To what extent the use of Q-A maps helps make the *raison d'être* of the knowledge at stake explicit in contrast to the traditional paradigm of visiting works?

In the design analysis, Q-A maps locate at the core of study processes the potential questions that can be addressed during the inquiry process. Elaborating a Q-A map of a whole subject such as Strength of Materials or Elasticity, makes it necessary to make these questions explicit, which soon leads to an epistemological study about the *raison d'être* of the main contents and organisation of the courses. This has not traditionally been the lecturers' concern. The collaborative work with researchers in didactics and lecturers and Q-A maps as an essential tool seems to have made it accessible. In our study cases, this result cannot be separate from the use of DE, as we will see below.

- RQ 2: Under what conditions can Q-A maps be used in teacher and lecturer courses as tools to develop epistemological questionings?

Q-A maps have been incorporated in an online teacher education course and in a face-to-face lecturer education course. In both cases, they were used after teachers and students had carried out a SRP in the position of the student. The maps were proposed as a descriptive tool to highlight the different steps followed during the SRP, both the expected and unexpected ones. They also served as a counterpoint to analyse official curricula and promote an epistemological questioning to see that other knowledge organisations are also possible. However, due to the short length of both courses, Q-A maps were used as an isolate tool for the epistemological analysis, without any specific methodology to address the teachers and lecturers' professional questions. For instance, the DE methodology that revealed to be useful in the work with lecturers was absent in these courses.

- RQ 3: What can be the role of the DE methodology as a tool for lecturers in engineering education to systematically design inquiry study processes?

The notion of DE appears for the first time in the Theory of Didactic Situation as a way to test the epistemological models proposed in terms of situations and to generate new didactic phenomena related to these models. When the notion of DE is retaken by Barquero and Bosch (2015), it is presented as a task design methodology according to the main principles of TDS and ATD. The use of DE as a mediator instrument between lecturers' and researchers' problems appeared in the engineering SRPs in a total unexpected way. We then discover that presenting a methodology in four phases with its corresponding sub-goals was a strong guidance to lecturers, together with the use of Q-A maps and the researcher's assistance. DE and Q-A maps also appeared as a systematic way to select and validate the generating question of inquiry processes, which still remains an open question in research related to IBL.

Our doctoral research started with the aim of deepening into the relationships between epistemology and didactics and, more specifically, in the new role of reference epistemological models (REMs) in the new paradigm of questioning the world. In previous research within the ATD, REMs were always related to a given body of knowledge that was part of the curriculum and, therefore, were in some extend still linked to the paradigm of visiting works. When the starting point of a study process is not a work to be learnt but an open question to be addressed, questions themselves should become central in the way to describe or model the knowledge at stake. Our contribution to this problem is to incorporate Q-A maps as a central epistemological and didactic tool to analyse the content to be taught, to propose alternative knowledge organisations and to implement new instructional proposals. Given that Q-A maps are only a partial description of inquiry process in terms of the Herbarian schema, we consider that our contribution could be enriched with the dialectics supporting the dynamics of the inquiry, especially, but not only, the media-milieu dialectic and the individual-collective one. In a way, the Herbartian schema is an important contribution of the ATD to the general epistemological problem of modelling knowledge and inquiry processes.

A second important contribution of our research can be located at the interface between research and teaching. Up to the date, the ATD proposals of implementing SRPs as a way to move towards the paradigm of questioning the world has remained at a very experimental level, almost all new instructional proposals being designed and implemented by ATD researchers. In our case, not only the design, implementation and analysis of SRPs have been developed in the new field of engineering education, but this work has been done in collaboration with lecturers non-specialists in didactics. The combination of Q-A maps with the DE methodology appears to be a fruitful framework for the dissemination of SRPs in university institutions. This is only a first incursion into a very difficult problem that remains open in many faces: the implementation of collaborative settings and strategies to disseminate results from didactics in current school institutions and to make them operational.

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APPENDIX 1. OTHER RELEVANT PUBLICATIONS

This appendix presents three publications related to our research that have appeared in conference proceedings and research books:

A1.1 Florensa, I. (2016). Contribution of the epistemological analysis to the design , experimentation and analysis of Study and Research Paths. *In C. Winsløw & M. Achiam (Eds.), Educational design in math and science: The collective aspect* (pp. 32–39). Copenhaguen: Institut for Naturfagenes Didaktik, Københavns Universitet.

A.1.2. Florensa, I., Bosch, M., & Gascón, J. (2016). Lecturer Education: a course design. *In 13th International Congress on Mathematical Education*. Hamburg.

A.1.3. Florensa, I., Bosch, M., Cuadros, J., & Gascón, J. (2018). Helping lecturers address and formulate teaching challenges: an exploratory study. *In V. Durand-Guerrier, R. Hochmuth, S. Goodchild, & N. M. Hogstad (Eds.), INDRUM 2018: Second conference of the International Network for Didactic Research in Univeristy Mathematics* (pp. 373–382). Kristiansand, Norway: University of Agder and INDRUM.

A1.1 Contribution of the epistemological analysis to the design, experimentation and analysis of Study and Research Paths

Introduction to the research problem

The implementation of SRP at school institutions plays a crucial role in the new paradigm of "questioning the world" proposed by Chevallard (2006b). The previous works developed in the Anthropological Theory of the Didactic (ATD) revealed important difficulties caused by the ecological constraints that appear when designing and implementing Study and Research Paths (SRP) as didactic devices at both secondary and tertiary levels (Barquero, 2009; Ruiz-Munzón, 2010; Winsløw et al., 2013). In fact, the design and implementation of these devices usually shows up aspects traditionally banned from school such as collective work or the search of relevant literature in real inquiry processes.

In the ATD, the notion of "ecology" is used as a metaphor in the following sense:

The ecology deals with the scientific study of the interrelations between organisms and their environments. In fact, ecology studies the physical and biological factors that influence these relations and that are influenced by them. By a deeper analysis of the etymology of the word ecology (oikos logos) it can be observed that in Greek oikos means “place to live”, so literally “ecology" is the study of living organisms (in our case knowledges) “at their home”, in their environment (in our case it would be the specific institution). (Ruiz-Munzón, 2010, p. 21)

The epistemological component and, more specifically, the prevailing epistemological model in the considered institutions appear to be in the root of many of these institutional constraints.

In order to better explore these constraints, researchers need to assume an external position from the institutions involved in the didactic transposition process. The external position is needed to be able to question knowledge conception at the institution. The strategy is to elaborate an alternative epistemological model known as the Reference Epistemological Model (REM). This model is supposed to be in a continuous evolution: it is not a static model but an initial scientific hypothesis, which is supposed to be modified as long as it is used in every experimentation. However, it is not usually described as such in the aforementioned research works. In these works the REM is always presented in its final and finite form.

When considering the construction and role of REM in the design, implementation and analysis of SRP, new problematic questions emerge at the crossroads between epistemology and didactics. These questions are taken as the starting point of the research work:

- The first question approaches the relationship between REM and SRP: How to describe an SRP in order to implement it in a school institution? How to transform a given REM into possible didactic organisations that could live in current school institutions? This problem also includes the need to make this process available to the school institutions, especially to the profession of teachers.
- Another important and difficult question is the degree of explicitness that should be adopted with the new epistemological models and tools necessary to design, implement and evaluate new teaching and learning processes depending on the participants of the study communities addressed (students,

teachers, mathematicians, etc.). Do teachers need an explicit definition of REM? Is a question-answer map a rich enough material form for the REM?

- Finally, we also wish to study which new notions or tools are needed to describe and manage the dynamic nature of the mathematical activity that will take place in study processes. In fact, how the SRP and REM descriptions can take into account its dynamic nature? How to describe these tools depending on the role addressed (didactic researcher, teacher and student) and how to make them available in the teaching institution and to the participants of the didactic process.

These questions arise from a study of previous ATD works and the need to transpose research results to the teacher profession. A more detailed justification of the questions was presented in the 9th Congress of European Research in Mathematics Education (CERME9) (Florensa et al., 2015). Actually, these questions are taken as the first step of the research work presented here.

First research phase: a teachers' professional development course

In order to establish a preliminary answer to the research questions, the first work of the research phase was centred in designing an on-line teachers' professional development course. Prior to the design of the course implementations of SRPs at secondary and university levels (Barquero, 2009; Ruiz-Munzón, 2010; Winsløw et al., 2013) were analysed as well as teacher training works in the ATD framework. Teacher education in the ATD is based on the use of the so called SRP in teacher education (SRP-TE). This device has been described in the works of Ruiz Olarría (2015) as well as in the works of Barquero and colleagues (2015). These two works establish a formal methodology in teacher education structured in five modules. Other theoretical approaches about teachers' knowledge have also been analysed: especially in what concerns the so-called "Pedagogical Content Knowledge" (PCK) (Shulman, 1987) as well as the notion of "Mathematical Knowledge for Teaching" (MKT) (Ball et al., 2008). Other devices such as the Lesson Study or the Multidisplinary Teaching (Miyakawa & Winsløw, 2009) should be analysed in order to incorporate some aspects to further editions of the course.

Assuming these contributions, a teacher's training course on "The nature of mathematical thinking" has been designed and implemented on March 2015. Actually, the students had already lived a SRP-TE in the sense of Ruiz-Olarría in a previous course in November 2014. Because of this previous course, the designed course is centred on the epistemological analysis of the mathematical activity developed during this previous course. Both courses are part of an online Master in Mathematics Education coordinated by the CICATA (Mexico) and were coursed by the same students, all of them teaching mathematics on the secondary level at different Latin-America countries. The initial course (SRP-TE) is presented in detail in (Barquero, Bosch, & Romo, 2015). It includes an adaptation of the five modules of a SRP-TE presented in the work of Ruiz- Olarría (2015). Specifically, the SRP-TE took as generating question: "How to teach mathematical modelling and regression in secondary level?"

The aim of the course is to provide the teacher-students with ATD tools to carry out an analysis of the mathematical activity that enables them to emancipate from the school dominant viewpoint on mathematics. For that, the notion of reference praxeological model (RPM) and Herbartian schema become crucial for us as educators and designers, even though they were not explicitly defined in the course. The main mobilised tool during the course are the question-answer maps: they play an important role as a partial representation of the RPM.

Four activities were presented to the teachers during the course. In the first activity, students were asked to work in five teams and generate a tree-map of all the questions and answers that emerged when they lived an SRP as students. This initial map will be used as an analysing tool during the subsequent activities. The second activity was centred on the use of the question-answer map to analyse curriculum-related documents. During the first phase of the activity, students were asked to collect official documents (such as textbooks, curricula extracts…) related to "functions" and "regression". These domains appeared in the first course when teachers designed and experimented a sketch of the SRP with their students. The second phase of this activity was the analysis of the collected documents and its comparison with the lived and experimented SRP. This analysis was proposed in terms of praxeologies and its articulation and rationale. The third activity asked the students to extend the map from the first activity or to propose new generating questions and its associated questions-answers maps. The main goal was to include new questions and answers in order to include all curriculum requirements in the fields of functions and regressions that may had not been considered on the first version of the map. In fact, students were asked to evaluate the (new) proposed generating questions to possibly cover all notions from curriculum. The last activity intended to highlight the importance of having an explicit conception of the “nature of mathematical thinking” and more precisely of the concrete school mathematics activities and domains that is a RPM, when tackling with didactic research questions. The activity included the analysis of a textbook based on the principles of the socioepistemology (Montiel & Cantoral, 2001) and its comparison with the ATD principles used in the course.

The use of the question-answer maps appear to be a powerful tool for teachers to analyse knowledge mobilised during and inquiry process. In fact, the construction of the maps was quite natural to them to describe the followed path during the inquiry. Moreover, by using maps students are capable to describe mathematical aspects usually ignored in the school mathematics such as “real” inquiry processes (in the sense that mobilised knowledge is not pre-defined) and collective work. In fact, the collective aspect appear crucial in teachers work when designing the SRP. In fact, the analysis of the followed path includes the analysis and the work with a "lesson plan". This guide was generated by teachers during the first phase as a way to transmit the knowledge acquired to their colleagues. In fact, the geneses of this document could be deeper studied by the approach of Gueudet and Trouche (2012). In fact, we could assume that teachers participating in the course form a Community of Practice, especially when they generate a common material to transmit new didactic tools.

For researchers, maps also appear as a good tool to connect different blocks of contents. However, despite of the real experience where regression and functions were co-used, teachers showed big difficulties in accepting alternative paths compared to the official ones: at the end of the course, only one team really connected the work with families of functions and regression. We can attribute this confinement in disconnected themes to the strength of the didactic phenomenon labelled by Chevallard as “thematic confinement” (Barbé et al., 2005).

The materials generated by the teachers as well as the analysis of the final survey filled by the participants are being analysed. In fact, the analysis of their answers shows up interesting preliminary results. For example, 72 % of the teachers accepted that “describing and understanding mathematical activity as a question-answer sequence allowed them to incorporate new knowledge and pedagogical tools to their practices”. In conclusion, the empirical work during this first phase enables us to generate a partial answer to the first and second research questions presented previously. In fact, the use of these maps as a partial and material representation of the REM empower teachers to make explicit mobilised knowledge and enable them to carry out an epistemological analysis of the mathematical activity. Moreover, teachers use these maps to describe the activity in the lesson plan both in a priori and a posteriori analysis.

Second research phase: An SRP in general elasticity

Mathematical modelling

In order to continue to generate partial answers to the research questions previously stated, we have designed, experimented, and analysed an SRP on General Elasticity. The main goal of this work was to analyse the dynamics of SRP, to develop tools in order collect this kind of data and to study the role played by mathematical modelling in an engineering course. We consider engineering as an interesting field to develop an SRP because of the important presence of mathematical modelling. The designed SRP is developed in a third year General Elasticity course of a Mechanical Engineering Degree.

The integration of mathematical modelling into current educational systems has been tackled by numerous investigations but still remains a major challenge. Many examples of mathematical modelling in various domains of engineering education exist: modelling acoustic properties of materials (Hernández, Couso, & Pintó, 2015) or the works of engineering teaching in US high schools (English & Mousoulides, 2011).

Numerous theoretical approaches agree on the need to incorporate mathematical modelling in mathematics and engineering teaching in consequence. As a result, some new curricular approaches try to introduce mathematical modelling in some university degrees (Gould, Murray, & Sanfratello, 2012; Kirby & Dangelmayr, 2003). Some studies consider that mathematics in engineering play such an important role that engineering could not exist without them. Because of this strong interdepence between mathematics and engineering, the classical modelling cycle approach cannot be applied in this case (Biehler, Kortemeyer, & Schaper, 2015).

However, many institutional constraints and limitations appear when designing and implementing modelling devices in university teaching institutions (Barquero et al., 2008). The institutional ecology plays a crucial role in the study of these conditions and constraints. The Anthropological Theory of the Didactic (ATD) framework enables us to describe these conditions and constraints affecting the implementation of mathematical modelling in scholar institutions, especially at university level. The necessary conditions for mathematical modelling at the undergraduate level have been studied in the case of first-year students of a business administration degree (Barquero, Serrano, & Serrano, 2013).

Course design and a priori analysis

Beyond the mathematical role played by mathematics as a service subject and the importance of mathematical modelling, a second motivation justifies the adoption of a SRP for the General Elasticity course. Until the last academic year this course was structured in mixed theory and problem sessions, and practical sessions. The latter included six 2-hour sessions on the following topics:

- Tensile test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Charpy test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Finite Element Method (FEM) simulation of a tensile test (using SolidWorks simulation as software).
- Oral presentation about failure criteria in different family materials.

During the practical sessions in the past two academic years three didactic facts were observed. First, a thematic confinement in the sense of Barbé et al (2005) explicitly appeared. This means that all four activities were ‘lived’ as independent by the students even if the activities were closely connected. For example: FEM simulation (3rd session) simulated the real test carried out in the 1st session. The second didactic fact is related to the role played by the computer during the FEM simulation. Students introduced geometrical data, loads and meshing conditions to obtain the required results. Important difficulties appeared when they tried to understand “how the computer solved the problem” and “validating the results obtained”. The students tended to validate all the results without any validating process. Both factors can be understood as a “black box” phenomenon: computer simulation is not understood by students and thus hinders them when judging the adequacy of the results obtained. And thirdly, we detected a clear absence of rationale in the four practical sessions. Both for students and for lecturers the presence of these sessions was more due to its “classical” character in elasticity than to a well-founded and justified didactic choice.

It seems that the adoption of a SRP based on a substantial enough generating question may partially overcome these limitations. The choice of the generating question emerges from the question “Why is General Elasticity taught in engineering?” which necessarily leads to the missing rationale. Once this question is posed, it is clear that the main reason to teach the subject is to provide engineers with tools enabling them to design any part of a machine working under an elastic regime. The connection between themes comes up immediately. To begin with a specific issue, the two lecturers teaching the subject agreed to start the SRP with the generating question: “How to choose one material (with unknown mechanical properties) from a set of three and design a part for a bike given in advance (brake lever, crank, gear, and bike lock key)?”

As an a priori analysis of the SRP, we have studied what kind of knowledge is expected to emerge when the students work on the design process. As a partial representation of this mobilised knowledge a question- answer map has been used (Figure 9). This tool was already used when modelling knowledge geneses from a generating question (Jessen, 2014; Winsløw et al., 2013).

Experimentation, data collection and analysis

The SRP has been experimented in December 2015 and January 2016 with two groups of 25 students. One of the groups was taught by a teacher without any didactic training and the other group was taught by the author of the paper.

The students have work in the mechanical laboratory during eight 2-hour sessions. The laboratory is equipped with a universal tensile test machine, a Charpy test machine, computers with simulation software and two 3D-printers. Each large group of students have been divided in groups of 4 or 3 students: each small group will have one specific part to be designed.

Each group is asked to design a specific part of a bike. At the end of the eight sessions they were asked to write a final report addressed to a fictional “bike design company”. The report included:

- Specific dimensions of the part including its dimensional plans,
- Estimated loads
- Justification of the choice of the material
- Estimated strains that it will suffer while being used
- The adopted safety factor for stresses and strains
- Justification of the results regarding the computer simulation and the mathematical model used
- Final cost of the whole design process calculated by using given prices.

The requirement of explicitness of these aspects are expected to partially “enlighten” the existing “black boxes” such as computer simulation and mathematical models.

During the first session each small group of students received three samples of different metallic materials, whose mechanical properties are totally unknown to the students. Then students were asked to write a first partial report that had to be delivered after the first week. It included:

- Time planning for the whole design phase
- Initial budget
- First questions that have emerged and that are planned to be solved during the following week.

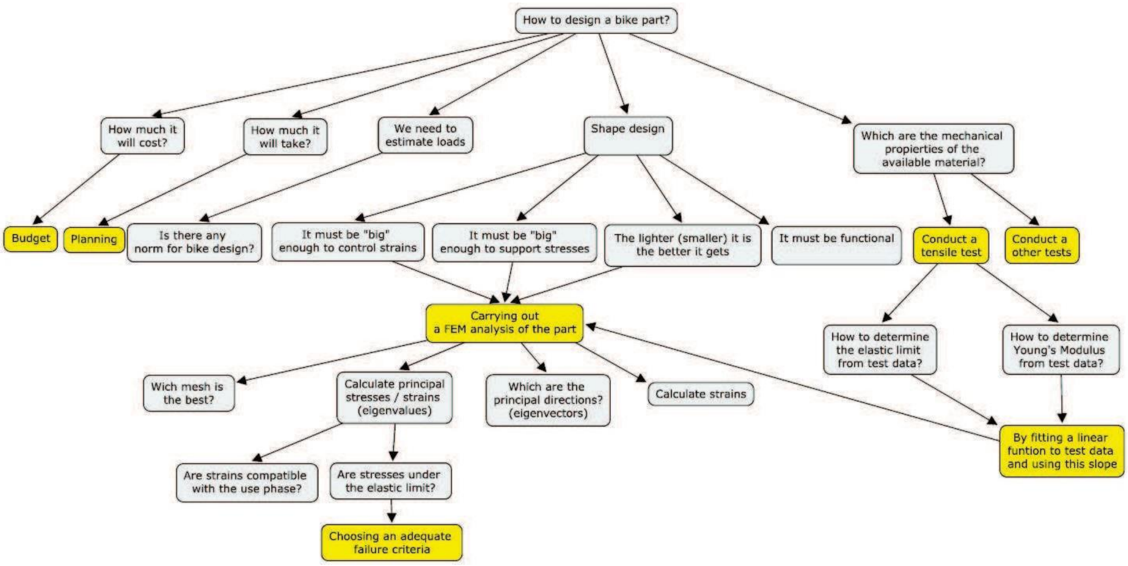

Figure 9 A priori question answer map

During the first session each small group of students received three samples of different metallic materials, whose mechanical properties are totally unknown to the students. Then students were asked to write a first partial report that had to be delivered after the first week. It included:

- Time planning for the whole design phase
- Initial budget
- First questions that have emerged and that are planned to be solved during the following week.

After this first report, a weekly report was generated by the students. The content of the weekly reports is intended to collect data from the dynamics of the activity. In order to collect this kind of data the proposed content was:

- An updated time planning
- The questions that the team planned to ask during the week
- A description of the tasks carried out even if obtaining wrong results
- The obtained and validated answers that they have obtained (and how) from the questions of the week and derived questions.
- New questions for the next week

To evaluate the SRP the students had filled in a survey at the end of the course and now seven semi structured interviews are planned: one to the teacher without didactical training, two to retaking students, two with good marks at the reports and two to students that have not passed the subject.

Preliminary results of the survey show that most of the students (80%) consider that weekly reports have helped them to follow the SRP and that the "lived" SRP has helped them to change their initial idea of the rationale of General Elasticity. The 75% of the students considered very positive their participation to the project. A deeper analysis of the results of the collected data is now carried out.

Conclusions

Both teacher training course and the engineering course allow us to establish preliminar answers to the research questions. On the one hand, working with question-answer maps help teachers to develop an epistemological analysis and to question mobilised knowledge during teaching-learning processes. On the other hand, asking students to make explicit their study process in weekly reports enabled students to make explicit aspects that are usually absent from traditional scholar settings. Both tools are easily adopted by both communities making explicit in some degree the epistemological model sustaining the lived SRP.

A1.2Lecturer education: a course design

The education of university teachers: does it exist?

Research in teacher education has not paid much attention to the development of lecturers, that is, teachers working at the university level. In most universities, leaving research merits aside, a Master's Degree or a PhD is the sole requirement to become a lecturer: no didactic or pedagogical training is required (Lomas, 2004). However, some countries have created institutions (such as the Higher Education Academy in the UK, HEA) in order to encourage lecturers develop their teaching professional development, or at least to accredit them. Other initiatives start from universities, for instance by offering pedagogical courses addressed to lecturers as an optional part of their ongoing professional development. In both cases, the courses propose general teaching techniques and strategies, without really entering in the analysis of the knowledge to be taught and learnt. For instance, the HEA will only provide what they call "discipline specific support" from 2016 on.

In what concerns research in university teacher education, at the 9th Congress of European Research in Mathematics Education (CERME9, Prague February 2015), none of the papers of the thematic working group University mathematics education (TWG14) or the groups about Teachers' Knowledge, Practices and Education (TWG18, 19 and 20) deals with the education of lecturers. Published papers in journals are also rare: works from Gibbs and Coffey (2004), Postareff et al (2008) and Guasch et al (2010) are some of the few existent examples. And the only Handbook for Teaching and Learning in Higher Education was published more than 15 years ago(Fry et al., 2009) without any reedition since then.

We state that as long as the activity of lecturers has a twofold character, research and teaching, their professional development cannot be only based in research activities but needs to be completed with an explicit training in teaching and learning processes. Consequently, if lecturer education is necessary, then didactic research should be involved in it, as it does with the education of other teachers. The differences between other educational levels are to be investigated, not assumed without further questioning.

Main principles for teacher education according to the ATD

We initially consider that teacher education (at primary and secondary levels) does not essentially vary from lecturer education. A recent review about research on teacher knowledge, carried out by the Education Committee of the European Mathematical Society (2012) as Solid Findings in Mathematics Education on Teacher Knowledge, highlights that teachers need a high level of Content Knowledge (CK) but that CK is not sufficient. Using Shulman's notion of Pedagogical Content Knowledge (PCK), the research findings reported show the need to enrich what is usually thought as "(school) mathematics" with new components related to the teaching practices and students' activities (difficulties, misconceptions, etc.). These findings seem to break down with the traditional conception of mathematical knowledge as something teachers already know and only have to transmit. However, the approach on teacher knowledge does not usually include the necessity to put under question the very nature, selection and organization of the knowledge to be taught, and to provide teachers with the tools provided by didactic research to build alternative perspectives of curricular organizations.

During these last decade, research on teacher education in the ATD (Chevallard, 2005) has evolved towards the use of what we call "study and research paths for teacher education" (SRP-TE). SRP-TE is an inquiry-based educational device based on the postulate that teacher education should start from the approaching of professional problematic questions and introduce didactic tools as far as they appear necessary to approach them. In addition, it assumes that these problematic questions are open questions also in research: educators are not supposed to provide definitive answers but to help teachers build their own ones, jointly with other teachers and educators. SRP-TEs combine a practical and theoretical questioning of the school mathematical activity and try to provide teachers with epistemological and didactic tools to analyze and reconstruct it in the class (Barquero et al, 2015). A SRP-TE is structured in five modules:

- M0: This module states explicitly the rationale of the SRP-TE and is transversal to all the process. It starts from a generating question of the kind: "How to teach XXX (a specific content)?" which is to be partially answered at the end of the process. Teachers are asked to search existing answers to this question and consider their appropriateness for implementing them in the classroom, as well as the conditions needed to do so.

- M1: Teachers are proposed to act as students of a teaching activity presented as a possible answer to M0. The main goal is to make teachers encounter an unfamiliar mathematical activity (for instance based on the study of an open question and requiring collective work, etc.) that could exist to some extent in an ordinary classroom, in spite of the usual restrictions that should be faced (pedagogical tradition, time schedule, etc.).
- M2: Once students have lived the teaching proposal as students, they are asked to adapt it and design their own version for a specific group of students. During this adaptation, many of the institutional restrictions teachers should face are expected to show up. They can thus be analyzed from an epistemological, didactic and ecological perspective.
- M3: Experiment, manage and carry out an in vivo and a posteriori analyses of the designed teaching proposal.
- M4: Use the analysis and experiences of the previous modules to elaborate a critical analysis of the traditional teaching practices and the possibilities of introducing new proposals, as well as a realistic view on their possibilities.

Course design

We are preparing a new proposal of SRP-TE for university teachers to take place in February 2015 with a group of 8-10 lecturers of an Engineering school in Barcelona (Spain), the authors of this paper acting as educators. The course is structured in 6 two-hour sessions. This time restriction forces us to adapt the five-module structure. The course takes as generating question (M0): “How can a real mathematical modelling work be incorporated in my teaching sessions? Under what conditions and for what purpose?” The first session will initially include a presentation of the course and a brainstorming activity about what modelling is and why it is important as a learning activity for future engineers. The end of the first session and the second one will include the M1 module based on the question Q_0. “How many time it takes to empty a tank full of fluid by a small hole in its bottom part? Does the shape of the tank play a role in the emptying time?” In fact, this is a classical engineering question and it has been addressed by Meyer (2010)in a TED talk about rethinking mathematical education. We consider this question particularly interesting because it allows many variations of the original question as well as an experimental validation of the results. Educators will leave lecturers a total freedom to generate an answer. These will work in teams and will be asked to collect the derived questions appearing during the inquiry process as well as the tools used to validate their answers.

For the third session, lectures will be asked to prepare a lesson plan including the problem approached plan to explain to another lecturer how to implement the activity in a real class situation (M2). They will thus need to take into account, without even noticing it, the institutional constraints in their didactic organization. This lesson plan will be shared and discussed during the fourth session. Before that, in the third session, lecturers will be asked to analyze the mathematical work carried out by generating a question-answer rooted tree starting from the generating question and including all derived questions and answers as well as the (theoretical and practical) knowledge tools mobilised.

Questions-answers tree maps have been used in research and also in teacher education courses as an epistemological tool to describe the knowledge mobilised during inquiry processes and to relate it to official curriculums (Winslow et al, 2013; Jessen, 2014; Barquero et al, 2013). The role of the question-answer map is crucial as it enables teachers to elaborate knowledge organizations in a completely different way from the usual curriculum, mostly based on the theoretical construction of concepts. The map thus becomes a useful epistemological tool to question school knowledge and a didactic guide to elaborate new teaching proposals. The maps elaborated during the third lesson will then be used in the fourth and fifth ones to comment on the lesson plans proposed. They will be used as a counterpoint of traditional teaching practices carried out by lecturers. In fact, question-answer maps highlight aspects from mathematical work that usually remain implicit or absent from university sessions, which tend to focus on the concepts construction or on the stereotype sets of problems assigned to them. This analysis is expected to generate preliminary pieces of answer to the generating question stated in M0.

During the last session, lecturers will be asked to design a revised lesson plan according to the results previously obtained. The main goal of this work is twofold. On the one hand, lecturers will have to explicitly state the need for new teaching devices to manage the inquiry process in the classroom: how to organize the students' teamwork, how to balance between the freedom of the inquiry process and the available time, how to assess the work done, etc. On the other hand, lecturers will be led to describe new forms of knowledge and overcome the classical thematic partitioning of mathematical knowledge. We do not expect these aspects to be taken into account in the initial version of the lesson plan, where lecturers will certainly assume the traditional (and transparent) university pedagogy.

The research related to this first course for university teacher education wishes to address the following questions:

Q1: Can SRP-TE experimented in primary and secondary teacher education programs be extended to lecturers? What changes should be made and what new conditions need to be created?

Q2: A central aim of SRP-TE and, more generally, a main principle for teacher education courses based on the ATD is to provide teachers with epistemological and didactic tools to avoid taking the curricular traditional organization of school mathematics for granted and be able to elaborate alternative ones, together with their conditions for implementation. In the case of teachers at university level, “school” and “scholar” knowledge are closer than in the other educational levels. Will that affect the development of the educational course? In what aspects?

During the conference, we will present the preliminary results of the implementation of the course.

A1.3 Helping lecturers address and *formulate* teaching challenges: an exploratory study

Introduction: the problems teachers face

Since 2009, our research group has designed and implemented two different courses to provide university lecturers and research assistants with educational tools enabling them to better design, implement and analyse teaching and learning processes. The first course took place at IQS – Universitat Ramon Llull in Barcelona, an institution offering degrees and master programmes in engineering and management. This first course was addressed to PhD students teaching at the institution or planning to teach soon. The PhD students' research domains and subjects taught were diverse and included econometry, finance, mathematics and engineering, among others. The course was structured into thirteen 2-hour sessions and lasted 4 academic years. In the first session, the participants were asked to raise teaching questions they would like to address in the course. The collected questions were then classified according to the level of co-determinacy they affected (Chevallard, 2002 we will come back to this notion later). The subsequent sessions were each devoted to addressing the questions that belonged to one of the levels, starting from the general ones (Civilisation, Society) and finishing with the content-specific ones (Domain, Sector, Theme, Question). At the end of the course students were asked to design a teaching project for a subject of their specialty, including a syllabus, the planning of learning goals and a detailed description of three teaching activities: a lecture, a student-centred task and an autonomous out-of-class activity.

The second course was held at EUSS-UAB in Barcelona, an engineering school offering Mechanical, Electronic, Electrical and Management engineering degrees. The participants were in-service teachers with different educational backgrounds and research fields. The course was organised in six 2-hour sessions. It was based on a study and research path for teacher education, an inquiry-based teaching format focused on the study of a professional teaching question (Florensa, Bosch, Gascón, & Ruiz-Munzon, 2017). The question addressed was "Could modelling be the main motivation of my subject?" It was approached through different phases where participants experienced an inquiry study process in the position of the students, then analysed the process experienced and finished by designing an inquiry study process for their subjects.

Both courses started by asking the participants to provide a list of questions or difficulties they would like to address with the help of the educators. In all the cases, we were surprised to find there was only a small number of questions that dealt specifically with the knowledge to be taught. Teachers mainly mentioned general issues related to assessment, class management, coordination or student characteristics (diversity, lack of motivation, the role of mathematics in their subject, etc.). They rarely included their subjects in the questions and, when they did, the problems formulated were very general.

We compared this result with an investigation research carried out by Cirade (2006) in pre-service teacher education in France within the anthropological theory of the didactic (ATD). In this research, during 3 editions of a 25-week course in 3 academic years, the participants who were doing an internship in secondary schools were asked to formulate a question every week. These "questions of the week" constituted the basis of the course, despite the fact that only a small sample of them could be addressed – all in all, more than 7,000 questions were collected. Cirade provides a systematic gathering and analysis of the teacher-students' spontaneous questions and uses them to identify the mathematical difficulties teachers encountered and their trouble in making them explicit. The kind of questions raised at the beginning of the course – which coincided with the beginning of the academic year – were initially very general, and were related to how to behave in class, how to manage the students' behaviour, what to do in a meeting with parents, etc. Then, as the teacher education course progressed and certain tools coming from the field of didactics of mathematics were introduced, teacher-students became more and more able to state questions related to the knowledge to be taught. In a sense, we can say that they stopped taking the knowledge to be taught as a given and dared to state questions about their own field of expertise. For instance, they ended up asking questions such as "How to justify the need of sketching functions given their analytical expression?", or "Why do we need to measure angles in radians in addition to degrees?", etc.

Following Cirade (Chevallard & Cirade, 2010; Cirade, 2006), we postulate that educational courses for university teachers cannot ignore the way teachers problematize their professional practice and teachers should take their concerns and difficulties as the starting point of educational processes. Besides, as researchers in mathematics education, we also agree with the importance of approaching these questions from a discipline-based level. As stated by Berthiaume (2009, p. 215):

“For some time now, educational researchers have investigated the idea that, in order to be effective, higher education teaching may have to be ‘discipline-specific’. In other words, teaching in higher education has to take into account the specific characteristics of the discipline being taught. This means that developing an understanding of teaching and learning is not sufficient to become an effective teacher in higher education. Rather, one must also develop an understanding of the teaching and learning requirements of one’s own discipline. This has been termed ‘discipline-specific pedagogical knowledge’”.

We consider essential for university teachers to be able to formulate their difficulties, not only as general issues concerning students and class management, but also including the knowledge to be taught as a key element. Even if teaching problems are initially perceived as general in their manifestation, the way to address them will necessarily involve knowledge-based activities. From the perspective of the ATD, taking the knowledge to be taught into account means more than simply including it as a variable or parameter of the problem formulation. It also means considering it as an institutional construction, questioning its current shape and searching for possible new reorganisations, taking into account – without assuming – the epistemologies and pedagogies prevailing at the university (Barquero, Bosch, et al., 2013).

The aim of our study, which is still at an exploratory stage, is to analyse the kind of questions university teachers are able to state at the beginning of an educational course – as the ones we implemented – and locate their questions at different levels of specificity/generality regarding the knowledge to be taught. We postulate that knowledge in didactics is important to provide university teachers with conceptual and methodological tools not only to improve their professional practice, but also to describe, interpret, conceive and question it in a more productive way. The first step to make progress in this direction is to start understanding how lecturers spontaneously formulate the challenges faced during their daily practice.

What problems do lecturers set forth?

We collected a total of 143 questions from the 4 courses, 35-40 per course, each of which was attended by 10-15 participants. In all of the cases, teachers attending the course were asked the following: “Write down two or three problems, difficulties or doubts that you find, or you think you may find during your teaching practice.” There was a lot of redundancy in the questions, so we eliminated repetitions even if the phrasing was different. We are presenting this selection according to the questions' generality, using the scale of levels of didactic co-determinacy. This tool was introduced by Chevallard (2002) in the didactic analysis to include aspects of the institutional organisation of teaching and learning processes that are usually taken for granted (M Artigue & Winsløw, 2010)(Artigue & Winsløw 2010; Chevallard & Sensevy, 2014). It helps distinguish the conditions and constraints affecting teaching and learning processes that are originated within the discipline, and the generic levels common to the teaching of any discipline:

Upper levels	*Lower levels*
Humanity	↓↑
↓↑	Discipline
Civilisation	↓↑
↓↑	Domain
Society	↓↑
↓↑	Sector
School	↓↑
↓↑	Theme
Pedagogy	↓↑
↓↑	Question

Figure 10 Scale of levels of didactic co-determinacy

Civilisation and Society

The upper levels of the scale refer to the conditions that are set up by our society or, when these are common to several societies, by the civilisation they belong to. We identified the following questions at this level:

- What to do in a culture in which effort and reward are no longer related?
- How does a social situation influence the effectiveness of a course or a teaching format? For example, the students' attitude seems different in times of crisis…
- To what extent should study plans be aligned with the labour market?
- What is "academic freedom" and what are its limits?
- Where do competencies come from? How are they established?
- Clashing of (generational or social?) values: sometimes it is difficult to act as a teacher, a guide or a referent when our own values seem to be obsolete (or to strongly contrast with those of our students). For example: the value of

effort, the gratification of work well done, the fact that money cannot buy everything or that not everything is on the web, the importance of culture, of thinking, that there are things that are "right" and others that are "wrong" (e.g. cheating in an exam is "wrong"), etc.

- What to do with students who act as "clients"?
- The application of Bologna is an adaptation of the learning process and an evolution or change: more participative students, more teacher-student interaction, etc. Adaptability is therefore considered a consequence of an evolution, but if we analyse it, we are giving the same classes, in the same environment, with the same student profile. Can we do anything to make the context change?
- How is the knowledge of the different subjects selected and what criteria are used?
- How far should we, as teachers, arrive in our role in and out of class? What are teachers educated for?

As we can see, all these questions refer to dimensions or difficulties related to university teaching that do not depend on the specific institution considered – many of them can be extended to any kind of teaching and to other educational levels. What is questioned is the way our societies – more or less explicitly – conceive, organise, and manage the dissemination of knowledge, and the general roles assigned to teachers as guides, leaders or knowledge disseminators.

School (here University)

The School level includes the conditions and constraints that depend on the specific teaching institution considered, in our case, the University with its own specificities:

- Is the number of students per class important in terms of effectiveness of the teaching? Is there an optimal number? Are there exceptions?
- How are decisions regarding time-schedule, session duration, etc. taken?
- To what extent are university facilities important? Are there optimal premises? How to adapt to the ones available?
- How to respond to the pressure of introducing ICT in the classroom? Is it used because of real educational reasons or is it cost saving? Is it just a trend?

- How to ensure a good coordination between teachers of the same subject? What happens when they have a different conception of the subject to teach?
- How to establish more synergies between colleagues, sharing methodologies?
- How to ensure a coherent programme? What relationships exist between subjects?
- How to integrate the different subjects to obtain a more global education?

These questions also reveal the aspects teachers think can be changed and the ones they take for granted, not even considering them questionable. For instance, in the fifth question, only the coordination with teachers of the same subject is considered, according to the traditional compartmentalization of knowledge in higher education. Together with the sixth question, they reveal the lack of a professional culture that might include coordination between teachers. However, the sixth question seems to consider that this coordination only affects “methodologies”, which again appears to be a vague and general dimension of teaching. The last two questions are content-related, but only with respect to the relationships between subjects, as these are considered to be previously determined – and, therefore, untouchable.

Pedagogy

The level of pedagogy is common to the different subjects or disciplines that can be taught. It includes all the resources, formats, and strategies teachers and students activate – many times without even noticing it – for teaching and learning processes to take place. We gathered numerous questions that can be located at this level of the scale and organised them into two main blocks: students and lecturers.

Students

- How to manage long projects, where students slack off and decrease their work intensity? How to reach a balance between establishing milestones and letting students work independently?
- What to do with the students that chat, are unfocused, use their mobile phone, do other things than what is requested of them?
- How to deal with student diversity?
- How to arouse the students’ interest in subjects that are not at the core of the degree?

- How to motivate students beyond the minimum required “pass” grade?
- How to encourage students to participate in a large group?
- Should students be monitored closely or should they work more independently?
- How to motivate students to behave in class?
- How to encourage students to be more competitive through the intrinsic values of the subject that is taught?
- Students are not previously taught how “to learn”. How will this affect our job?
- How to teach students to listen and maintain their attention?

Lecturers

- How to improve oral and body expression?
- How to organise assessment in a fair and impartial way?
- How to assess core (non-disciplinary) competencies?
- Is it better to use final examinations or continuous assessment? How to measure long-term student learning outcomes?
- How to design contents, planning and methodologies of the subject that take into account the student diversity?
- How to reach all the students and not only those who have more knowledge, excluding the ones that got lost? How to find the balance between maximising student learning and the amount of information provided?
- How to ensure an individualised methodology considering the time limitations?
- How to become the best teacher for each student?
- How to deal with the so-called “decline in student knowledge”?
- Does the decline in student knowledge correspond to something real or is it just what each generation says about the previous one?
- Does it make sense to give lectures nowadays?
- How to improve teaching resources and methodologies using ICT?
- What to do after the class? How to analyse what happened and what the teacher did? How to assess teacher performance?

As can be seen from the questions above, most of them focus on specific teaching practices, but they do not refer to the difficulties of the corresponding subjects. The questions are mainly related to what the teachers can or might do, and they are very general. Only two of them refer to specific teaching formats: projects and lectures. There is no mention of the activities organised (labs, tutorials, problem solving or case study sessions, outdoor activities, etc.) and the way to better implement them. The need to implement new kinds of activities is not mentioned. The questions mainly have to do with the teacher and the teacher's actions. For instance: assessment is always considered as a lecturer's task; "motivation" is assumed to be generated (only) by the teacher. The questions thus reveal many features of the traditional pedagogical contract, which seems to be fully assumed by the lecturers.

Discipline

As said before, we were astonished to find so few questions at this level, which corresponds to the conditions and constraints directly linked to the content taught and learned. They can be related to the general discipline the content belongs to (Mathematics, Engineering, Economics, Management, etc.) or to the different components of the discipline, according to the way it is structured or delimitated in the considered institution. The general terms used to specify these levels are: domains, sectors, themes, tasks or questions. The divisions and boundaries established in a discipline or field of knowledge are institutional constructions. They vary from one institution to another and from one historical moment to another. The collected questions at this level remain very general; none of them specifies the difficulties related to the teaching or learning of a given piece of content. The first one, for instance, is very similar to those located at the School level: it depends on whether we interpret the question as affecting the design of an entire programme or the possible actions in one discipline:

- How to better connect the different subjects of the programme?
- How to highlight the multi-disciplinary nature of the subjects?
- How to select the learning goals of the subject? What content should be included?
- What should the level of the learning goals be?
- How to relate the subject with the real world?
- How to balance learning goals between specialisation and generalization?

Why these questions? An interpretation from the atd

The assumed educational contract between lecturers and educators

The first reason that came to our mind when trying to understand why lecturers did not ask any content-related question is the kind of implicit didactic contract that was assumed by them at the beginning of the course. Given the fact that the course was about university teaching, they might have expected to learn certain generic tools to help them in their teaching practice; not something related to their specific subject. The educators were seen as specialists in teaching processes and the questions were stated at this general level. Either way, this shows a first important phenomenon: lecturers expect to receive help with general teaching practices that are only a part of their daily practice. A lot of their teaching work (elaboration of the syllabus; choice of textbooks, reference books and other kind of resources or materials; selection, design and organisation of activities, cases or problems; decisions about the kind of in-class and out-of-class activities students should carry out; renewal of the subject matter; etc.) does not seem to have been included in the objectives of the course.

A problem of legitimacy

The second reason we put forward is related to what we call a problem of legitimacy. University teachers are often also researchers or at least experts in the subject they teach. Therefore, they may be reluctant to accept the idea that their teaching difficulties might come from problems with the subject matter they are supposed to master. Their lack of expertise can only be attributable to what is external to the discipline they teach. This reinforces the previous reason about their expectations from the educational course.

The divide between pedagogy and didactics

There is another important and more general factor that may explain the lack of content-related questions. It corresponds to the dominant interpretation of teaching and learning phenomena that has been called "pedagogical generalism" (Gascón & Bosch, 2007) or the "didactic divide" between pedagogical and subject-matter knowledge (Bergsten & Grevholm, 2004). It tends to introduce a strict separation between instructional processes and the "content" of these processes, that is, using the scale of didactic co-determinacy, between the level of Discipline and the level of Pedagogy. The main point in crossing the boundary between the two levels is the way knowledge is conceived in the considered teaching process or, in other words, which aspects of the subject-matter are questioned and which ones are assumed as a given.

When a teacher – or a lecturer – is asked to teach a given piece of knowledge k, the first question she will first ask herself is "what should I do to teach k?", not "what is this k I should teach?" What the theory of the didactic transposition (Chevallard, 1985; Chevallard & Bosch, 2014) states is that instructional processes rely on the fiction that there is only one way to define k and that this is the k that is taught and learned. Questioning the knowledge to be taught, asking about its origin, selecting and applying a given instructional process rarely occur. This is why it is normal the participants of the course did not set forth questions of that kind. In the questions stated, knowledge always appears as a given, not as a variable.

The "pedagogical generalism" that is found in the teachers' questions is not an isolated fact. If we look at the teaching support some universities offer their (new) faculty, we see that only the Pedagogical level is addressed, and possibly some aspects of the School level. For instance, in the Teacher Training in Higher Education (FDES) programme proposed by the Autonomous University of Barcelona, the structure of the programme is presented as follows:

- Activity 1. Teaching in the new context of learning and teaching
- Activity 2. Practicing oral discourse
- Activity 3. How to assess university students' learning?
- Activity 4. Experiences in educational innovation
- Activity 5. Observation in the classroom
- Activity 6. Teaching planning: from study programmes to syllabus
- Activity 7. Teacher's portfolio

Similar programmes can be found at other universities. For example, some years ago, the Teaching Engagement Program of the University of Oregon posted a list of frequently asked questions (FAQs) organised according to the following headings: "Getting ready to teach; Presenting and facilitating information; Motivating students; Questions of respect; Assessment; Managing the classroom climate". None of the questions was content-related. It seemed as if, once certain answers were provided to the pedagogical issues, their specification to the subject-matter was considered evident or, at least, non-problematic.

Conclusion: a lack of tools and notions

One of the consequences of "pedagogical generalism", that can partially be seen in the questions stated by university teachers, is the lack of terms and concepts to go below the level of Pedagogy and start questioning the levels of Discipline. University teachers develop their professional activity at institutions where little is said about the way knowledge should be selected, arranged, updated, organised, "elementarised", put-into-practice, problematized, etc. in order to teach it or to help students to learn it. This is a crucial aspect in which the results obtained from research in Didactics of Mathematics, both practical and theoretical, can assume an important function.

From the experience of the courses here presented, we have seen how introducing certain elements of the ATD (the notions of praxeology, didactic contract, didactic moments, Herbartian schema, media-milieu dialectics, didactic ecology, etc.) provides lecturers with a productive enough framework to talk about and start questioning a larger part of their teaching activities (Florensa et al., 2017). The more is said about didactic processes, the more dimensions of these processes can be questioned and tentatively changed. Our hope is that the education of lecturers, as is the case with primary and secondary school teacher education, will have the power to make this state of things evolve. Our experience with the courses presented lets us be moderately optimistic in this respect.