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A Survey of Recent Advances in Fractional Order Control for Time Delay Systems

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ABSTRACT Several papers reviewing fractional order calculus in control applications have been published recently. These papers focus on general tuning procedures, especially for the fractional order proportional integral derivative controller. However, not all these tuning procedures are applicable to all kinds of processes, such as the delicate time delay systems. This motivates the need for synthesizing fractional order control applications, problems, and advances completely dedicated to time delay processes. The purpose of this paper is to provide a state of the art that can be easily used as a basis to familiarize oneself with fractional order tuning strategies targeted for time delayed processes. Solely, the most recent advances, dating from the last decade, are included in this review.

INDEX TERMS Fractional calculus, time delay process, fractional order control.

I. INTRODUCTION

Time delays are quite frequently encountered in industrial applications, such as heat exchanges, distillation units, mining processes, steel manufacturing and so on. But they are not limited to industrial applications. Time delay processes span from biological to mechanical systems, including also economical or electrical fields. The physical phenomenon that generates time delays is the need to transport information, energy or different masses. Time lags accumulate also between interconnected systems or arise when sensors need measure and acquire signals and when microcontrollers (or other devices) compute the control signal and actuate upon the process.

From the frequency domain point of view, the presence of delay introduces an additional lag in the process phase. This results in lower phase and gain margins and ultimately complicates the closed loop control of these processes. The ideal situation is to design a controller that completely eliminates the effect of time delays. Many control strategies have been developed throughout the years to cope with time delay characteristics [1]–[4], but none of them proved to be an ideal solution. As the domain of fractional order controller gained more popularity, the control focus also reached the field of time delays processes. The desire is to combine the better performance of fractional calculus to the time delay control problem by extending fractional order design methods to the time delay field. Research output from the delay free processes suggest that using fractional order controllers can help improve robustness and closed loop response of time delay processes as well [4]–[7]. As it will be further detailed in this paper, some existing results have already accomplished better performance when using fractional order calculus in dealing with time delays compared to the traditional approach of using integer order controllers [8]–[12]. The advantages mentioned so far in existing literature refer to improved closed-loop results, considering modeling errors, due to the possibility of a more robust tuning of fractional order controllers [8]–[12]. The choice of the fractional order PID controller over classical, integer order control, is justified throughout literature in studies that focus on comparing the two control strategies tuned in similar manners for time delayed processes [8]–[10], [13]–[19]. For example, an analysis of the effect of the fractional order derivative action over

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an unstable, first order, plant is realized in [31], where the closed loop stability is investigated for fractional derivative orders in the range (0, 2). The paper clearly proves that for the same derivative gain of the controller, a fractional order differentiator with the derivative gain smaller than 1 provides a more stable system than an integer order controller. A comparison of six different tuning rules for fractional order PI and PID controllers is presented in [32] based on different approaches.

Among the years, many well-known scientists contributed to the mathematical development of non-integer order differentiation. Euler and Lagrange were the first to introduce theoretical contributions in 18^{th} century, followed by Riemann, Liouville and Holmgren, with systematic studies at the beginning of the 19^{th} century. Liouville developed function expansion in series of exponentials and definition of n^{th} order derivative by operating term-by-term, while Riemann introduced the definite integral applicable to power series with non-integer exponents. Later, Grunwald and Krug unified the results of Liouville and Riemann, with the first application of fractional calculus dating from 1823. Heaviside developed symbolic methods for solving linear differential equations of constant coefficients, while Weil and Hardy defined the differ-integral operator properties and Riesz extended the result to multivariable functions, etc. [20]. In the last decades, the number of applications for fractional-order calculus has been growing exponentially, mainly in the fields of control engineering, signal processing and system theory. The main advances were made by Bode's ideal loop transfer function, followed by Manabe's results on frequency and transient response of the non-integer integral and its application in control. The first occurrence of fractional order controller may be attributed to Oustaloup, who introduced and demonstrated the superiority of the Commande Robuste dOrdre Non Entier (CRONE) controller. The generalization of the integer order proportional-integrative-derivative (PID) controller to fractional order has been proposed by Podlubny [21]. The fractional order basic control actions, proportional, integral and derivative, add more flexibility to the set of performance specifications the closed loop system is able to fulfill. This is mainly due to the extra tuning parameters of the fractional order PID (FOPID): the fractional order of integration and the fractional order of differentiation. Even though the FOPID represents the most common fractional order control algorithm, other types of fractional order controllers have been designed, as it will be indicated later in this paper.

Review papers focusing on the use of fractional calculus in control engineering have been published recently such as [22]–[25] and provide an insight into fractional order control of different types. Analytic, numerical and rule-based tuning methods for fractional order PID controllers only has also been published [26]. Some of these methods can also be used to control time delay systems.

The main contribution of the present paper is the focus on time delayed processes and available fractional order methods for their control. As such, all fractional order controller algorithms, not just the FOPID, are included in the review. The paper provides the reader with a brief summary of the control strategies as well as relevant literature. The purpose of this paper is to provide a state of the art that can be easily used as a basis to familiarize oneself with fractional order tuning strategies targeted for time delayed processes. The most recent advances, dating solely from the last decade, are evidenced focusing mainly on the controller tuning approach, without providing any numerical examples, which can be found in the quoted literature. In this review paper, we consider processes with significant time delays and as such, tuning methods based on neglecting the time delay are not presented here.

The paper is structured as follows. The second section, after the Introduction, details the FOPID tuning methods, starting with the most commonly used design technique based on a frequency domain approach (sub-section A). Then, tuning methods for FOPIDs based on minimizing some time domain cost functions, such as Integral of Square Error (ISE), Integral of Time Absolute Error (ITAE), Integral of Absolute Error (IAE), etc. are presented in sub-section B. A third sub-section includes the tuning of FOPIDs using an extension of the popular M_s Constrained Integral Optimization (MIGO) method, called here the F-MIGO (Fractional-MIGO). Sub-section D gathers the Pontryagin and Hermite-Biehler theorems used in the tuning of FOPID controllers. Other tuning methods are included in sub-section E. All these tuning methods require a process model. Sub-section F details the auto-tuning methods for fractional order controllers for time delay processes. A process model is not required here. The common factor of all these tuning methods is that the classical closed loop system is used. However, other control schemes can be used in the control of time delay processes, such as the Internal Model Control (IMC) strategy or the Smith Predictor (SP). The next two sections of the paper include particularities in the design of fractional order controllers in the framework of an IMC (Section III) or a SP (Section IV) control structure. Section V details the existing control algorithms that combine fractional calculus and advanced control methods. Section VI is a collection of existing papers that provide the experimental validation of various types of fractional order controllers on time delay processes. Finally, the last section presents the concluding remarks and future challenges.

II. TUNING METHODS FOR FRACTIONAL ORDER CONTROLLERS (FOC) FOR TIME DELAY SYSTEMS

Classical tuning algorithms are used in the design of fractional order controllers, such as the Ziegler-Nichols rules [27], Hermite-Biehler and Pontryagin theorems [28]–[31] linear programming formulation [32], F-MIGO optimization [16], [33]. Other tuning methods based on optimizing a certain performance index were developed [8], [13], [34], [35]. The PID block diagram from Fig. 1 is based on the classic feedback control structure with the mention that the controller is of fractional order.

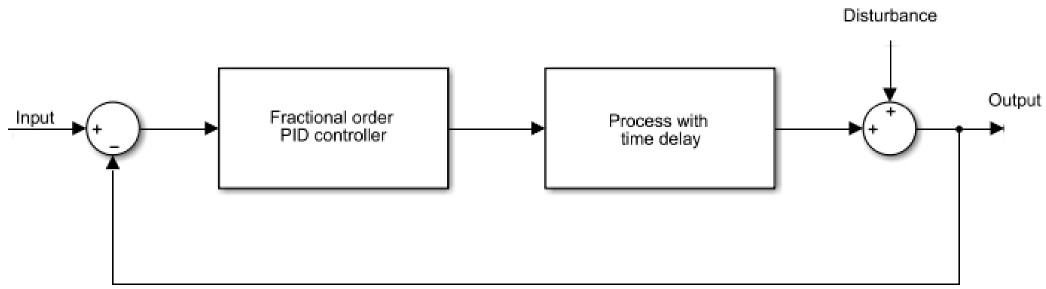

FIGURE 1. Fractional order $PI^{\lambda}D^{\mu}$ control structure.

A. FREQUENCY DOMAIN TUNING FOR FOPIDS

As a generalization of the classical PID (Proportional Integral Derivative) control approach to arbitrary orders of integration and differentiation, the fractional order PID, frequently encountered under the notation of $PI^{\lambda}D^{\mu}$, outbounds the limitations induced by integral order differentiation. The transfer function of the FOPID controller can be written as

$$H_{FOPID} = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \tag{1}$$

where k_p, k_i and k_d are the proportional, integral and derivative gains, while λ and μ are the fractional orders of integration and differentiation. Theoretically, λ and μ can take any real, positive value. However, for the controller to have physical meaning, the interval of the fractional orders of integration and differentiation is usually limited to (0, 2) [21].

One of the most popular methods for tuning the fractional order PID controller is determining the parameters of the controller by solving a system of nonlinear equations expressing specifications related to phase margin, gain crossover frequency, sensitivity functions and robustness to gain changes in a limited interval. The particularities of the system to be controlled dictates the requirements the controller needs to fulfill, determining the choice of tuning specifications. Several works such as [14], [21], and [36]–[38] approach the tuning of the fractional order PID controller through frequency domain specifications, a method with high applicability to any time delay process, regardless of the time delay amount. The constraints related to the imposed phase margin, denoted by ϕ_m, and gain crossover frequency ω_{cg} influence the damping ratio of the closed loop system, the settling time, as well as the robustness. The mathematical representations of the phase margin and gain crossover frequency are given by

$$\angle(C(j\omega_{cg})G(j\omega_{cg})) = -\pi + \phi_m \tag{2}$$

$$|C(j\omega_{cg})G(j\omega_{cg})| = 1 \tag{3}$$

The closed loop system's robustness to gain variations forces a flat phase for the open loop. Also known in literature as the time response iso-damping property, this specification ensures a constant overshoot within a certain frequency range. The range of gain variations for which the system exhibits robustness is limited by the frequency characteristics of the controlled plant and the obtained controller parameters. A flat phase translates into a constant phase, with the derivative always zero, creating the mathematical interpretation of the robustness characteristic.

$$\frac{d(\angle(C(j\omega_{cg})G(j\omega_{cg})))}{d\omega}|_{\omega=\omega_{cg}} \tag{4}$$

Robustness can also be expressed by means of sensitivity and complementary sensitivity since the closed loop system can fail to perform robustly to other process uncertainties, except for possible gain variations. The sensitivity function $S(j\omega)$ is closely related to good performance regarding disturbance rejection for a desired frequency range and can be expressed by means of

$$\begin{aligned} &|S(j\omega) = \frac{1}{C(j\omega_{cg})G(j\omega_{cg})}| \leq A\,dB \\ &\forall\omega \leq \omega_s rad \Rightarrow |S(j\omega)|_{dB} \leq A\,dB \end{aligned} \tag{5}$$

where A is the desired value for the sensitivity function frequencies [39], [40].

An uncertain plant can exhibit high-frequency noise. Specifications related to the complementary sensitivity function denoted by $T(j\omega)$ can result in the rejection of this high frequency noise:

$$\begin{aligned} &|T(j\omega) = \frac{C(j\omega_{cg})G(j\omega_{cg})}{1 + C(j\omega_{cg})G(j\omega_{cg})}|_{dB} \leq B\,dB \\ &\forall\omega \leq \omega_s rad \Rightarrow |T(j\omega)|_{dB} \leq B\,dB \end{aligned} \tag{6}$$

with B being the desired high frequency noise attenuation around the frequencies surrounding the gain crossover frequency [39], [40].

Tuning of the FOPID in (1) implies usually solving a system of nonlinear equations composed of all or part of the design constraints in (2, 3, 4, 5, 6). The resulting controller ensures a robust closed loop system in terms of gain uncertainties, with the ability of rejecting high frequency noise and disturbances, while fulfilling time domain specifications, such as overshoot and settling time. The method can be applied to plants with or without time delay.

For particular cases of the $PI^{\lambda}D^{\mu}$, such as the fractional PI or the PD controller, any three design specifications can be chosen to be solved as a system of nonlinear equations. The gain crossover frequency and phase margin specifications coupled with the differential equation for robustness to gain variations are the most common three specifications chosen to compute fractional order controllers of type PI or PD. Such an example is provided by [41] to tune a fractional order PD for a first order plus time delay plant. The work presented in [21] uses a nonlinear system formed of these commonly used specifications to tune a fractional order PI for a second order plus time delay plant. A fractional order PI controller is tuned in [9] by solving the system of nonlinear equations composed by the gain crossover frequency, phase margin and robustness constraints in order to control the level of a spherical tank. Numerical simulations compare the closed loop performance obtained with the fractional order controller to the performance achieved by integer order control tuned in a similar manner, highlighting the superiority of the fractional order approach.

Controller tuning through optimization by minimizing a frequency domain performance criterion with a constraint on the maximum sensitivity function is also presented in [34]. The optimized criterion measures the ability of the closed loop system to reject low frequency disturbance inputs.

Solving the nonlinear system composed of the five frequency domain equations for different cases of non-integer order plants with time delay (NOPDT) is studied in [42]. Different process models are identified for the same plant around different working points and several $PI^{\lambda}D^{\mu}$ controllers are tuned, all of them fulfilling the imposed specifications.

Some of the most common methods of solving the system of nonlinear equations is by using different optimization routines [21].

Another feasible approach is the graphical system solving procedure such as the one detailed in [11] in order to stabilize a first order plus time delay process with a fractional order PID controller. The controller presented is computed by stabilizing the upper bound of the derivative gain and computing the other parameters with respect to this using a graphical approach. The obtained set of stabilizing controllers provide good stability margin when compared to the conventional PID controller.

A thorough analysis regarding frequency domain stabilization of first order plus time delay systems is performed in [14] where both fractional and integer order PID controllers are tuned to simultaneously fulfill the three frequency domain design specifications previously presented: robustness, gain crossover frequency and phase margin. The paper presents a set of guidelines regarding the choice of feasible specifications for plant stabilization purposes. The achievable frequency domain boundaries are presented graphically and all possible combinations of controller parameters are verified inside the feasible regions. The paper highlights by means of numerical simulations the better performance of the fractional order controllers over the classical PIDs in terms of overall closed loop performance.

B. TUNING FOPIDS BASED ON TIME DOMAIN COST FUNCTIONS AND OPTIMIZATION ROUTINES

Variations of using the previously described frequency domain specifications can be found by combining the frequency constraints with other tuning techniques. Such an example is presented in [43] where the authors tune a PI controller based on Bode's ideal transfer function for the open loop system. Iterative optimization routines are used to minimize a quadratic cost function based on the sum of squares of the control signal input and the sum of the integral squared error between the closed loop system with the PI controller and the time response desired process response. The optimization features two additional constraints regarding the sensitivity and the complementary sensitivity equations from (5) and (6). The proposed method is highly versatile, being validated on both integer and fractional order models as well as processes with considerable time delays. A set of tuning rules to determine both integer and fractional order controllers for first order plus time delay plants are highlighted in [13]. The procedure minimizes the integral absolute error (IAE)

$$IAE = \int_0^{\infty} |e(t)|dt = \int_o^{\infty} |r(t) - y(t)dt \tag{7}$$

with a constraint applied to the maximum sensitivity function from (5). Set-point tracking and achieved performance IAE indexes, as well as disturbance rejection are considered to asses the proposed method. The conclusion of this study, based solely on simulated data, states that the fractional order derivative action improves the overall system performance, while the fractional order integral doesn't bring significant improvements when compared to integer order PID controllers tuned in the same manner.

In [8], both fractional and integer order PID controllers are computed using genetic algorithms that minimize a given cost function. The process considered is the oxygen generation of the heart-lung machine, approximated to a third order plus time delay system. Performance criteria such as IAE from (7), ISE

$$ISE = \int_0^T e^2(t)dt \tag{8}$$

and ITAE

$$ITAE = \int_0^T t|e(t)|dt \tag{9}$$

are minimized in order to obtain the controllers. The genetic algorithm searches for a solution starting from a random population and the parameters of the controller are computed. Simulations prove the superiority of fractional order control compared to integer order PID.

Minimization of the ISE performance combined with frequency domain specifications has been achieved in [35]. The paper targets processes modeled as first order plus time delay with a normalized time delay between 0.1 and 3.5 seconds. Differential Evolution (DE) algorithms are used to tune parallel 2-Degrees of Freedom PID for load frequency control of interconnected power systems in [44] and [45]. The design of the controllers is also viewed as an optimization problem where the DE searches for the optimal solution by minimizing the Integral of Time multiplied by ITSE and ISE. ITAE is also used to further increase the performance as well as time domain specifications such as settling time and overshoot, weighted by coefficients. The technique is compared to the Craziness based Particle Swarm Optimization (CPSO) and the proposed method proves its superiority. System uncertainties are introduced by varying the process parameters, testing and validating the robustness of the optimized controller.

C. F-MIGO METHODS

Fractional M_s Constrained Integral Optimization, defined as F-MIGO through the specialized literature is the fractional extension of the MIGO algorithm designed by Astrom et. al. as an improvement to the simple and tuning-friendly Ziegler-Nichols rules. The procedure consists of optimizing the load disturbance rejection by constraining the maximum sensitivity function. Comparisons between the F-MIGO and other fractional order design strategies is realized in [19] by analyzing and comparing the closed loop system's performance in terms of time domain and frequency domain specifications. The method is based on the assumption that the transfer function of the process is known, linear, with a finite number of poles and exhibits a single singularity at infinity. Using equations (5) and (6) one can define the sensitivity

$$M_s = \max_{0<\omega<\infty} |S(j\omega)| \tag{10}$$

and complementary sensitivity margins as

$$M_p = \max_{0<\omega<\infty} |T(j\omega)| \tag{11}$$

The integer order MIGO approaches showed that choosing the sensitivity margin, M_s, as a design parameter provides significant changes in the time response of the closed loop system. Since there is a trade-off between the sensitivity and complementary sensitivity, it is also important to keep the value of M_p at a minimum. The problem is overcome by choosing a circle enclosing M_s and M_p as the design parameter such that the F-MIGO optimization problem is stated as follows

$$\begin{aligned} f(k, k_i, \omega, \alpha) &= |1 + C(j\omega)G(j\omega)|^2 \\ f(k, k_i, \omega, \alpha) &\geq R^2 \end{aligned} \tag{12}$$

where R is the radius of the circle enclosing both M_s and M_p. The design procedure is the optimization of k_i, the integral gain, with respect to the sensitivity constraint from (12).

A practical method for tuning fractional order PI controllers for first order plus time delay process models using F-MIGO are presented by [15] and [33]. The tuning rules consist of applying the relative time delay of a first order plus time delay model in order to determine the optimal fractional order of integration and proportional and integral gains. The presented tuning rules from (10, 11, 12) are validated through numerical simulations in [15] and [33], proving that the proposed F-MIGO tuning rules are applicable to systems of any complexity, not only first order processes with time delay.

The superiority of fractional order control over classical, integer order control techniques for processes exhibiting time delay uncertainties is raised in [16]. A fractional order PI controller is computed using the F-MIGO method, modified to simultaneously maximize the ITAE performance index from (5) and the jitter margin defined as

$$\begin{aligned} O_2(x) &= \frac{1}{\delta_{max}} \\ \delta_{max} &= \min_{\omega} \epsilon[0, \infty]| \frac{1 + G(j\omega)C(j\omega)}{j\omega G(j\omega)C(j\omega)} \end{aligned} \tag{13}$$

The method from [16] is applied to a set of a hundred first order plus time delay processes with varying time constants and time delay values. The study shows that for delay dominant systems, one must sacrifice the ITAE performance index in favor of the jitter margin. The paper summarizes the F-MIGO optimization based method by approximation of the optimum gain parameters and the fractional order of the PI controller.

D. PONTRYAGIN AND HERMITE-BIEHLER THEOREMS

Pontryagin and Hermite-Biehler theorems are used to determine the fractional controllers for delayed plants in [28], [30], [31], and [46]. As stated in the quoted literature, the Hermite-Biehler theorem is described by the following equation

$$\delta^*(j\omega) = \delta_r^*(\omega) + j\delta_i^*(\omega) \tag{14}$$

where δ_r^* and δ_i^* are the real and imaginary parts of the complex function δ^*. The function is guaranteed to be stable if (1)

1) $\delta_r^*(\omega)$ and $\delta_i^*(\omega)$ have only simple real roots that are interlaced.
2) There exist some $\omega = \bar{\omega}$ in R such that $\delta_i^{'*}(\omega)\delta_r^*(\omega) - \delta_r^{'*}(\omega)\delta_i^*(\omega > 0)$ where $\delta_i^*(\omega)$ and $\delta_r^*(\omega)$ are the derivatives of the real and imaginary parts.

The purpose is to ensure that all the roots of δ_r^* and δ_i^* are real, which can be done using Pontryagin theorem that can be written as

$$-2l\pi + \eta \leq \omega \leq 2l\pi + \eta \quad (l = 1, 2, 3, \ldots) \tag{15}$$

where η is a constant such that the highest degrees in δ_r^* and δ_i^* are kept for $\omega = \eta$ and the real and imaginary parts must have precisely $4lN + M$ roots with N and M the order of the integer order part numerator and denominator polynomials. Extending this into the fractional order domain, Pontryagin theorem states that δ_r^* and δ_i^* must have $4l([N]+1)+[M]+1$ roots where [.] gives the integer part.

The study from [30] uses the Hermite-Biehler combined with the Pontryagin theorems to develop a fractional order PI controller for a first order time delayed plant. The paper presents thoroughly the influence of the proportional gain, k_p, and the integral gain, k_i on the closed loop system's stability. The same tuning procedure is found in [28] which uses the two theorems to design a fractional order PI controller for a process characterized by a second order plus time delay model. Performance indexes such as IAE and ISE are used to asses the tracking performance of the closed loop system as well as the influence of time delay and destabilizing factors.

Stability analysis based on an extension of the Hermite-Biehler theorem applied to quasipolynomials is preformed in [31] and [46]. In [46], the mathematical approach analyzes the characteristic equations of fractional order time delayed processes. Also, fractional order PI controllers that stabilize the process are obtained using the aforementioned theorem. Numerical simulations regarding step response of the closed loop in the the stability region confirm the proposed method. A similar approach is focused on stabilizing an uncertain first order plus time delay process in [31].

E. OTHER TUNING METHODS FOR FOPIDS

The five parameters of a fractional order PID designed to control a fractional order process with time delay are determined based on a regression model in [47]. The general regression equation is

$$z_n(x)^n = z_0 + z_1x^1 + z_2x^2 + z_3x^3 + \ldots + z_{n-1}x^{n-1} + z_nx^n \tag{16}$$

where $z_n(x_n)$ is the required system data, while z_0, z_1z_n are system data points at different conditions. The regression values for every point, with ω_n being the weight of the exponential function,

$$z_n(x)^n = \frac{1}{1 + exp\,\omega_n(E(s) * K * e^{-\theta s}(\frac{K_ps+K_i+K_ds^2}{(Ts^2+s)}))^n} \tag{17}$$

combined with the standard fractional system gives

$$\begin{aligned} z_n(x)^n &= z^0 + \frac{1}{1 + exp\,\omega_1(E(s) * K * e^{-\theta s}(\frac{K_ps+K_i+K_ds^2}{(Ts^2+s)}))^1} \\ &\quad + \ldots + \frac{1}{1 + exp\,\omega_{n-1}(E(s) * K * e^{-\theta s}(\frac{K_ps+K_i+K_ds^2}{(Ts^2+s)}))^n} \\ &\quad + \frac{1}{1 + exp\,\omega_{n-1}(E(s) * K * e^{-\theta s}(\frac{K_ps+K_i+K_ds^2}{(Ts^2+s)}))^n} \end{aligned} \tag{18}$$

The regression function is the popular prediction and forecasting technique which is computed using an iterative algorithm based on the parameters of the time delay process, the regression model being strongly related to the weights of the exponential functions. Numerical simulations comparing the proposed method with the well known Ziegler-Nichols or Wang tuning procedures prove the validity of the presented work.

The D-decomposition technique defined by Real Root Boundary (RRB), Complex Root Boundary (CRB) and Infinite Root Boundary (IRB) is used in [10], [48], and [49]. Defining the characteristic fractional order equation of a time delayed system as

$$P(s) = p_ks^{q_k} + \ldots + p_1s^{q_1} + p_0 \tag{19}$$

where p_i are coefficients and q_i represent fractional orders. Considering P as a parameter space, one may split the stability and instability regions through a thrice defined boundary

- Real Root Boundary (RRB): the imaginary axis is crossed by a real root $s = 0$, determining the RRB by substituting $s = 0$ in the fractional order characteristic equation.
- Complex Root Boundary (CRB): the imaginary axis is crossed by a real root at $s = j\omega$, making the system unstable.
- Infinite Root Boundary (IRB): the imaginary axis is crossed by a real root $s = j\infty$.

Controlling unstable first order time delay systems is given by [48]. The design of the fractional order PID controllers revolves around the system's time delay. Firstly, the ranges of the stabilizing controller parameters are determined using the D-decomposition technique to graphically visualize the stability regions. Furthermore, the parameters of the controllers are incremented by small quantum and the closed loop system stability is analyzed. Finally, a set of fractional order PID controllers are obtained that stabilize the given time delay process. A simple stabilization approach for integrating time delay systems through fractional PD controllers is presented in [10]. The stability region of the derivative action is determined. A set of stabilizing fractional order PD controllers are obtained for arbitrary integrating time delay processes. Numerical simulations prove that fractional order PD controllers provide larger stability regions than the more limited, integer order PD control approach. A generalized stabilization method of fractional order time delay processes by using fractional order PID controllers based on determining the stability ranges of the fractional parameters is detailed by [49]. The method is also applicable to guarantee imposed gain and phase margins as well as stability.

The work presented by [32] presents the problem of determining the parameters of fractional order PID for time delay processes as a linear programming exercise. The Euler-Lagrange equations are derived as a fractional boundary problem taking into consideration the time delay using calculus of variations and the Lagrange multiplier. The Grunwald-Letnikov approximation is used to reduce the fractional boundary problem to a linear programming task. The study provides numerical simulations and comparisons with other similar methods and proves the veracity of the proposed algorithm for the initial problem, as well as for optimal control problems of fractional nature.

F. AUTOTUNING OF FOPID CONTROLLERS

Whilst the other subsections are abundant with recent works revolving around fractional order tuning for time delay processes using a mathematical model of the system to be controlled, autotuning methods for FOPIDs are relatively scarce. One recent study is presented in [27] where a fractional order PI controller is determined. Even if the method is of an experimental nature, the authors use only numerical simulations to prove the veracity of the proposed approach. Also, the method is focused around plants exhibiting large amounts of time delay. A relay test is used to find the frequency of the process to be controlled and an integer order PI control law is computed using the previously determined process frequency and the classical Ziegler-Nichols tuning rules. Starting from the integer order controller, an extension into the fractional dimension is done by varying the fractional order of integration and analyzing the time domain closed loop system response.

In [50], the autotuning procedure is based on the relay test. The FOPID tuning procedure is divided into two parts: first a design of a FOPI controller is achieved, followed by the design of a FOPD controller with a filter. In both designs, the iso-damping property, a gain crossover frequency, and phase margin are employed as design specifications. The method is exemplified for a first order system with time delay. Two types of fractional order PI controllers are designed in [51] using an autotuning method also based on the relay test. The fractional order controllers are designed such as to ensure that the phase Bode plot is flat at a given frequency called the tangential frequency. Several relay feedback tests are used in an iterative procedure to identify the plant gain and phase at the tangential frequency. Only simulation results are presented for simple processes, but the method can be used for time delay systems as well.. An iterative procedure combined with the relay test is also used for designing fractional order lead-lag compensators [21]. The same relay test has been proposed in [52], where fractional order $PI^{\lambda}D^{\mu}$ controllers are designed for second order plus time delay plants. Firstly, the process dynamics are modeled by using an offline relay-based method, where a maximum of four unknown parameters are determined in the simplest possible way. This is a clear limitation of the method. Then, a relay with hysteresis is used to obtain the describing function. The gain and phase margins, as well as iso- damping property are used to tune the five parameters of the controller. The proposed method is validated through simulation studies in a class of process models, and also verified experimentally on a coupled tank system. Other autotuning methods based on the relay test have been proposed, such as for the design of a FOPI controller for real-time steam temperature control [53].

In [54], the same three performance specifications are used for the autotuning of FOPI/FOPD controllers. Although the method is exemplified only for delay free systems, the procedure can be easily appplied to processes with time delay, as well. A simple sine test on the process is used to determine its magnitude, phase and phase slope at the gain crossover frequency. Then, either a graphical approach or an optimization routine can be used to solve the resulting system of nonlinear equations. A similar approach, deriving from the same single sine test in [54] is also used in the autotuning method presented in [55], called the FO-KC autotuner. Here also, the same three performance specifications are used to tune the parameters of either a FOPI or a FOPD controller. The advantage of the FO-KC autotuning principle is that the nonlinear system of equations that needs to be solved to estimate the FOPI/FOPD parameters is no longer required. Instead, the design is based on using the phase margin requirement to define a forbidden region in the Nyquist plane that the loop frequency response should avoid. Moreover, to ensure the iso-damping property, the optimal FOPI/FOPD controller is determined such that the difference between the slope of the loop frequency response and the slope of the forbidden region border is minimum, at the gain crossover frequency. The method can be applied to time delay processes as well. In [56], the proposed autotuning procedure is limited to systems with delay and order greater than one. Firstly, the identification of the process at the desired crossover frequency is performed. Next, the parameters of the fractional order PID controller are determined using the same set of performance specifications: crossover frequency, phase margin and the iso-damping property.

III. FRACTIONAL ORDER CONTROLLERS IN AN IMC CLOSED LOOP SCHEME

The Internal Model Control (IMC) approach is among the most popular control schemes and algorithms for tuning controllers for time delay processes. The advantages of IMC have been also exploited in the tuning of fractional order controllers. Several examples can be cited here, such as [12] and [57]–[59]. Quite frequently, the IMC is chosen as a design methodology to yield a fractional order controller to be used in a Smith Predictor control structure. The standard IMC block diagram that stands as a basis to the design procedure is shown in Fig. 2.

In [12], one of the algorithms presented attempts to design fractional order IMC controllers as simple IMC controllers with a fractional order filter. The tuning is based on selecting the parameters of the fractional order filter such that better closed loop and robust performance are obtained, as compared to a classical IMC controller. Several case studies are presented, for processes described by first-order and second-order transfer functions with time delays. Comparisons with integer order IMC controllers are presented to validate the results and to demonstrate that the proposed fractional-order IMC controller ensures an increased robustness to modeling uncertainties.

The Internal Model Control approach (IMC) is explored in [59] by combining it with a fractional PID filter designed for a second order plus time delay process. The resulting tuning procedure leads to a cascading structure composed by PID controllers and fractional order filters. Lag dominant processes as well as processes with considerable time delays were chosen to validate the proposed method through numerical simulations. The validity is confirmed through sensitivity function analysis and robustness to process uncertainties both for reference tracking and disturbance rejection performance.

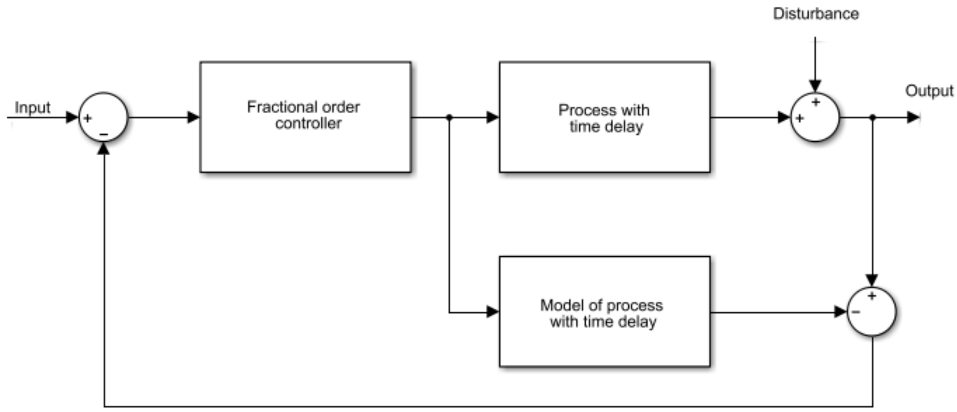

FIGURE 2. Internal model control (IMC) block diagram.

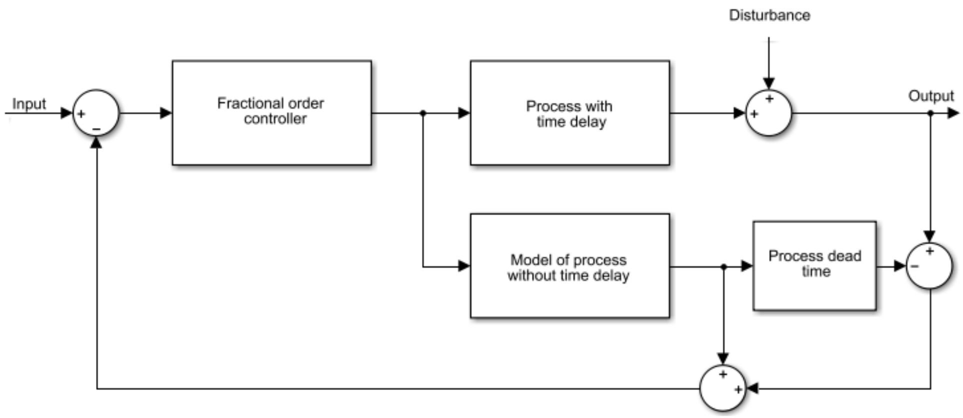

FIGURE 3. Smith predictor control structure.

IV. SMITH-PREDICTORS AND FOC

A couple of researchers have proposed to solve the problem of controlling time delay systems based on combining various types of FO controllers and classical or modified Smith Predictor structures (Fig. 3).

In [60], two Smith Predictor based controllers are used in the comparison. The paper uses a mismatched model to analyze the performance of the control strategy. The controller chosen is a fractional order CRONE controller, which is well known for increased robustness and performance tradeoff. The conclusion of the research is that the use of an improved Smith predictor is not necessary to obtain good performance, even though it can be used as a means of simplifying the design of the (robust) controller. In [61], a new fractional-order PI controller embedded in a Smith predictor is proposed. The design is based on fractional calculus and Bode's ideal transfer function. In this case, the model considered in the design is of a first-order-plus-dead-time, with the analytical tuning rules derived by using the frequency domain. Later, the tuning rules are easily applied to various dynamics, including both the integer-order and fractional-order dynamic processes. The approach benefits from a couple of advantages, such as a simple design scheme, a straightforward method, which can be easily implemented in the process industry. Numerous numerical examples are included to show the superior closed-loop performance, for both servo and regulatory problems, in comparison with other reported controllers in terms of the minimum integral absolute error with a constraint on the maximum sensitivity value.

Most of the times, the design of the fractional order controllers is done using frequency domain approach, due to a simpler representation of fractional order systems in this domain. However, time domain approaches have been also reported, in [5] and [62]. In the case of [5], a new approach to the design of fractional order PI controllers in Smith Predictor structures for varying time delay systems is proposed. The design is focused on ensuring robustness of the closed loop system against time delay uncertainties and it is based on time domain specifications, rather than the more widely used frequency approach. Apart from this, the proposed tuning method relies on additional robustness to plant uncertainties, achieved by maximizing open-loop gain margin, through an iterative procedure. The simulation example provided demonstrates the efficiency of the proposed method, in comparison to classical integer order PI controller.

In [62], a simple and efficient analytical method to design a fractional order controller for time delay integer order systems is proposed, considering a Smith predictor structure. The design procedure is based on time-domain specifications, such as the percentage of overshoot and settling time. The tuning formulas are then derived based on ideal closed-loop transfer function. The major advantage of this design method consists in only two tuning parameters, which can be obtained using an explicit set of tuning formulas. Two simulation examples are used, including a robustness test considering plant uncertainties, to demonstrate the performance of the proposed controller compared to those provided by several well-known design techniques. A time domain approach is also considered in [63] and [64]. In [63], a new methodology to design fractional order integral controllers combined with Smith predictors is proposed with the target of ensuring the robustness to high frequency model changes, especially changes in the time delay. The case study considered here is the water distribution in a main irrigation canal pool. The fractional order integral controllers perform better and are more robust than the integer order PI or PID controllers. Additionally, they also show less sensitivity to high frequency measurement noise and disturbances. Comparisons with more complex control techniques such as predictive control and robust H_∞ controllers, demonstrate that the proposed fractional order integral controllers have better or similar performances than these. In [64], the idea in [63] is reiterated. Time domain specifications are used in the design of a fractional order PI controller combined with a Smith Predictor, which is later compared to an integer order PI controller in a Smith Predictor structure. The extra tuning parameter of the fractional order controller is used to maximize the robustness to variations in plant parameters. Simulated results show the robustness improvements achieved with this controller compared with a conventional PI controller.

In [57], an original model based analytic method is developed. The method attempts to design a fractional order controller combined with a Smith predictor and a modified Smith predictor that yield control systems, which are robust to changes in the process parameters. The proposed method is not limited to fractional order systems and works well for integer order systems as well. The tuning procedure is based on Bode's ideal transfer function and the IMC (internal model control) principle. In this case also, only simulation results are presented, which demonstrate the successful performance of the proposed method for controlling integer as well as fractional order linear stable systems with long time delay.

In [65], the differential evolution algorithm is used to determine the five parameters of the FOPID controller. The tuning is based on user-specified peak overshoot and settling time and has been formulated as a single objective optimization problem. The control structure used is the Smith Predictor. Simulation results are provided that show that the FOPID controller performs better than its integer order version.

Another FOPID controller is incorporated in the Smith Predictor control structure generalized for plants exhibiting time delay in [66]. The fractional order controller is determined by solving the set of the five non-linear equations related to gain crossover frequency, phase margin, robustness, sensitivity and complementary sensitivity. The proposed control structure is validated on numerical simulations for plants with different amounts of time delay.

For first order uncertain systems, an interval-based stabilization method using stability conditions of the non-commensurate elementary fractional order transfer function of the second kind is developed in [67]. The design of all stabilizing controllers is based upon some analytic expressions, including those derived for a robust performance, such as the iso-damping property. The fractional order controller obtained is combined with the Smith Predictor to control a first-order system with time delay. Only numerical examples are included here, also.

The Smith Predictor is also the chosen control structure in [68], where a FOPI controller is designed for a second order time delay system. The tuning is based on phase margin and gain crossover frequency specifications, combined with a condition to maximize the loop gain margin. A hardware-in-the-loop approach is considered to validate the results, with the digital FOPI controller combined with the Smith Predictor implemented in LabVIEW.

The long time delay compensating Smith Predictor based control scheme is also proposed in [69]. The FOMCON toolbox with MATLAB/Simulink is used to determine a fractional order PID controller for a coupled tank system. The design is based on an iterative optimization technique. Comparisons with a PID controller based on the Smith Predictor, designed in a same way, are also included.

In [70], several fractional order control algorithms are investigated and compared, when integrated within a Smith Predictor control structure. Simulations are presented to evaluate the performance of the proposed fractional order control algorithms, on a heat diffusion system, selected as the case study.

Two tuning algorithms for fractional-order internal model control (IMC) controllers for time delay processes are presented in [12]. One of these tuning algorithms is based on Smith Predictor structure, where the equivalency between IMC and Smith predictor control structures is used to tune a fractional-order IMC controller as the primary controller of the Smith predictor structure. The design of the fractional order IMC controller is done with the purpose of enhancing the closed-loop performance and robustness of classical integer order IMC controllers. Several numerical examples, as well as an experimental validation are provided for a multivariable system.

A multivariable case study is presented in [58], as well, where the design method of fractional order Smith Predictor controller was proposed. The idea is also based on the equivalency between the Smith Predictor and IMC structures. A decoupler is used to achieve non-square system decoupling, while the particle swarm optimization technique is used to reduce the complexity of the decoupled system from a high order to a first order plus time delay model. Finally, based on the IMC principle, a fractional order controller was designed, and the controller parameters were tuned by using the dominant pole placement method. The simulation results show that the proposed method is simple, and can provide a better dynamic performance and robustness.

In [71], a genetic algorithm is used to determine a fractional order model for a closed loop system with a Smith Predictor structure. Then, a fractional $PI^{\lambda}D^{\mu}$controller is proposed to improve the controlled system performances. The tuning of this controller is done based on an optimization routine that minimizes the position error taking into account the sensitivity and the complementary sensitivity conditions. Several simulation examples are considered to show that the proposed Smith predictor enhances the closed loop control system performance.

V. ADVANCED CONTROL STRATEGIES AND FOC

This section focuses on design strategies rarely encountered throughout the specialized literature that combine advanced control strategies with the benefits of fractional calculus.

The studies in [72]–[75] are among some of the most recent ones on fractional order Sliding Mode Controllers (SMC) for delayed systems.

Consider the master system

$$x^{(n)} = f(x, t) \tag{20}$$

and the slave system given by

$$y^{(n)} = g(y, t) + b(y, t)u \tag{21}$$

satisfying

$$\begin{aligned} &0 < b_{min} \leq b \leq b_{max}, \quad 0 < a_{min} \leq \dot{b} \leq a_{max}, \\ &\hat{b} = (b_{min}b_{max})^{1/2}, \quad \beta^{-1} \leq \hat{b}b^{-1} \leq \beta, \\ &\beta = (b_{max}/b_{min})^{1/2}|\Delta f| < f_1, \quad |\Delta \dot{f}| < f_2, \\ &|\Delta g| < g_1, \quad |\Delta \dot{g}| < g_2 \end{aligned} \tag{22}$$

Also, the tracking error signal is e = y âĂ" x. The existence problem of the SMC is defined as

$$\begin{aligned} S &= s_1(t, \dot{e}, D^{\lambda}\dot{e}, \ldots, D^{n+\lambda-1}\dot{e}, u) \\ &= k_0\dot{e} + k_1D^{\lambda}\dot{e} + k_2D^{\lambda+1}\dot{e} + \ldots + k_nD^{n+\lambda-1}\dot{e} \end{aligned} \tag{23}$$

and the tracking is denoted by

$$\begin{aligned} \dot{S} &= s_2(t, \ddot{e}, \frac{d}{dt}D^{\lambda}\dot{e}, \ldots, D^{n+\lambda-1}\dot{e}, \dot{u}) \\ &= k_0\ddot{e} + k_1\frac{d}{dt}D^{\lambda}\dot{e} + k_2\frac{d}{dt}D^{\lambda+1}\dot{e} + \ldots + k_n\frac{d}{dt}D^{n+\lambda-1}\dot{e} \end{aligned} \tag{24}$$

The problem statement of the SMC controller is that $S\dot{S} < 0$ such that the motion of the sliding mode is asymptotically stable.

In [72], the new approach provides stability of the delayed process completely independent of the time delay. Also, two theorems are provided for the sliding phase and the finite time. For the sliding phase, the theorem guarantees the asymptotic stability independent of the time delay, while the finite time ensures the occurrence of the reaching phase. In [73], the fractional order sliding mode control strategy is based on the support vector machine (SVM). The design procedure is successfully applicable for fractional order systems both in case of state and control delays. The paper also compares the performance of the fractional order sliding mode control to the integer order version, proving the superiority of the fractional order scheme.

Tang *et al.* [4] propose a novel fractional sliding mode strategy with a dynamic essence in order to control several delay based chaotic systems in a master slave configuration. The control strategy is simulated on a multivariable delay chaotic robot by taking into consideration the chattering problem of the sliding mode control. Another innovative sliding mode strategy is detailed in [74] with applications upon a nonlinear robotic exoskeleton. Parameter uncertainties and external disturbances lead to a time delay estimation based model-free fractional order nonsingular fast terminal sliding mode control (MFF-TSM), The fractional order controller is designed to for tracking performance, fast speed of convergence, and chatter-free control inputs lacking singularities. Asymptotical stability is also investigated through the Lyapunov theorem. Cascade control structures for delayed systems are exemplified in [76]. Both master and slave controllers are of fractional order. For the slave controller, a fractional order PD is chosen for the time delayed process, while for the master control, the fractional order SMC law is employed. Numerical simulations validated the proposed approach in terms of time domain performance and stability of the closed loop system. Bode's ideal transfer function and internal model control principles are used in [57] to analytically develop a fractional order controller combined with a modified Smith predictor targeting robustness to process' uncertainties. The simulations prove the veracity of the method for systems with long time delays. Another control combination is done in [77] where a hierarchical structure has an event-based supervisor and a lower level fractional order PI controller applied to a wind turbine. The purpose of the supervisor is to analyze and determine the states of the process, while the fractional order controller's main purpose is to ensure maximum power generation with peak performance and reliability.

Wavelet Kernel Neural Networks (WKNN) are employed by [78] to tune fractional order PID controllers for processes with time delays. The wavelet and kernel functions are combined to check the availability of the neural network. The WKNN approach is compared to classical neural network strategies and prove the rapid convergence of the wavelet to the desired solution. Furthermore, the WKNN approach is combined with a fuzzy logic rule to successfully control a pressurized water reactor.

Over the network control with variable time delays is studied by [79]–[82]. Fractional gain scheduling strategies are designed in [79] to compensate the effects of time-varying network delays. An experimental platform is controlled over the internet with the purpose of eliminating the time delay caused by the network communication. A fractional order PI controller is combined with the scheduling strategy that proves to introduce optimal delay-adaptive gains. In [82], an online estimation method for the uncertain network delay is used as an input parameter to tune a fuzzy adaptive PID controller using optimization techniques such as Particle Swarm Optimization and Genetic Algorithms. The method is validated through simulation and proves to be superior to traditional PID approaches for network applications.

Fractional optimal control problems with time delay are also a topic of interest during recent years. One of these optimal methods is based on Bernstein polynomials for dynamical systems with constant delay where the fractional differentiation is defined in the Caputo sense [83], [84]. A novel wavelet derived from a class of orthonormal polynomials that leads to optimality, named Chelyshkov orthonormal wavelet is presented in [85]. Shifted Legendre orthonormal polynomials for fractional integral defined by Riemann-Liouville and fractional Caputo derivative are employed to minimize the performance index in [86]. Finally, [87] presents conformable fractional derivatives to define fractional differentiation and integration leading to a linear programming problem.

VI. EXPERIMENTAL VALIDATIONS

Most of the previously presented works validate the presented concepts and control strategies by means of numerical simulations. Scarcely any available literature provides real life experimental data of applied fractional calculus to time delayed plants. Apart from strains brought by the physical nature of the controlled processes, the difficulties in experimental implementation are also caused by the fact that when implementing fractional order control, one is faced with the challenge of properly determining a discrete-time integer order approximation of the control action. Fractional order PID type controllers for time delay systems and their practical implementation are presented by [17]–[19] and [41]. Comparing the presented methods as well as disseminating the practical results is impossible due to the dissimilarity of the tackled processes. Hence, the emphasis of this section revolves around the physical nature of real time delay plants and on the hardships encountered in practically implementing the the fractional order control action.

A Vertical Take-Off and Landing platform formed by a rotating cantilever beam around a fixed point as a process exhibiting a time delay of 0.8 seconds is presented in [41]. The controller is developed in order to control the pitch of a Vertical Take-Off and Landing (VTOL) platform, characterized by a second order plus time delay transfer function. The study shows the design of a fractional order PI controller by imposing frequency domain specifications such as gain crossover frequency, phase margin and robustness to gain changes. The approximated discrete form of the FOPI controller is obtained using a novel discretization algorithm that computes directly a 7 order discrete-time controller [88]. The complex control action is implemented on the experimental

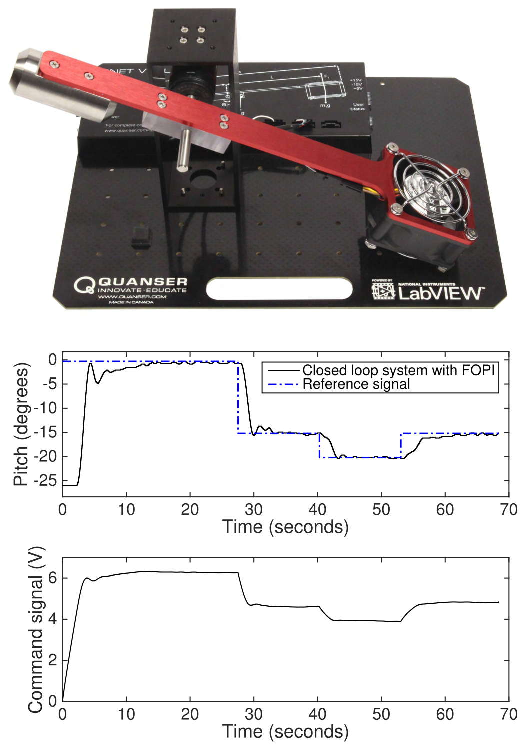

FIGURE 4. VTOL setup and FOPI experimental results [41].

setup through LabVIEW and a real-time NI Elvis board. Several experiments are realized by testing reference tracking around different operating points and disturbance rejection. Also, robustness is challenged by physically altering the parameters of the process (such as the weight of the beam). All the performed tests validated the chosen approach in tuning the FOPI controller for a time delay process. The experimental setup along with closed loop reference tracking results are presented in Fig. 4.

In [19], the F-MIGO method is used to tune the fractional PI controller for a first order time delayed system, which is further implemented on an experimental platform consisting of a DSPACE card and a first order plus time delay process. The controller's robustness is validated experimentally in terms of reference tracking and disturbance rejection as seen in Fig. 5.

Another similar practical implementation of fractional order PI controllers is realized in [17] on a Heat Flow Experiment modeled by both fractional and first order plus time delay dynamics. Two fractional order PI controllers are determined for the integer and fractional order processes, which are successfully validated on the laboratory unit. Malek *et al.* [17] and Hmed *et al.* [19] provide no information regarding the digital implementation of the fractional order controller.

A fractional order PID is tuned in [18] for a first order plus time delay plant using frequency domain specifications. The controller is implemented on the National Instruments sb-RIO9631, programmed through LabVIEW, tocontrol a

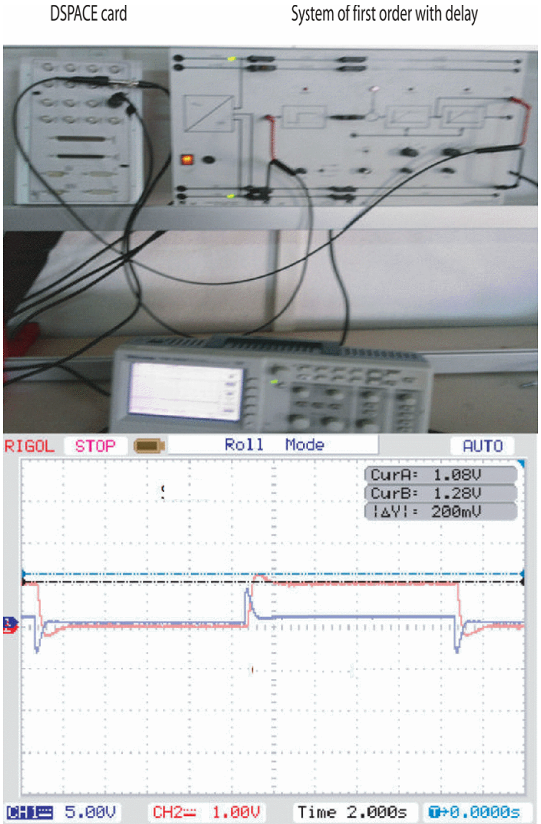

FIGURE 5. DSPACE and first order plus time delay process and closed loop step response of the F-MIGO controller [19].

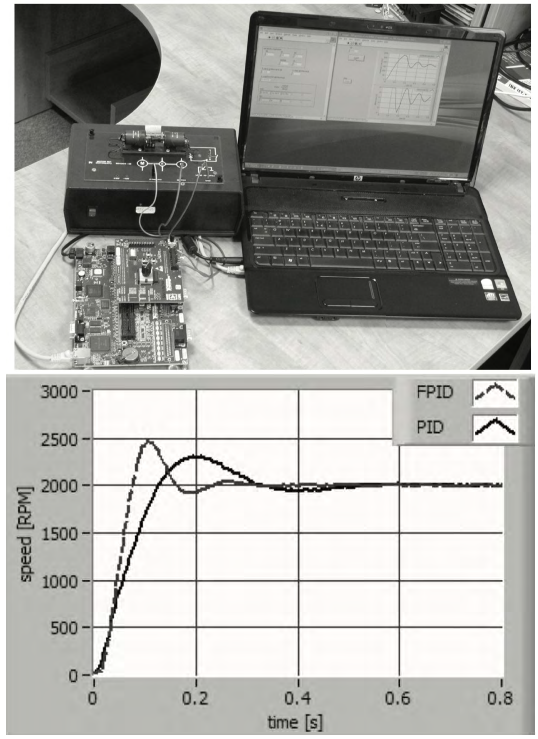

FIGURE 6. Experimental DC motor setup and closed loop results with FOPID [18].

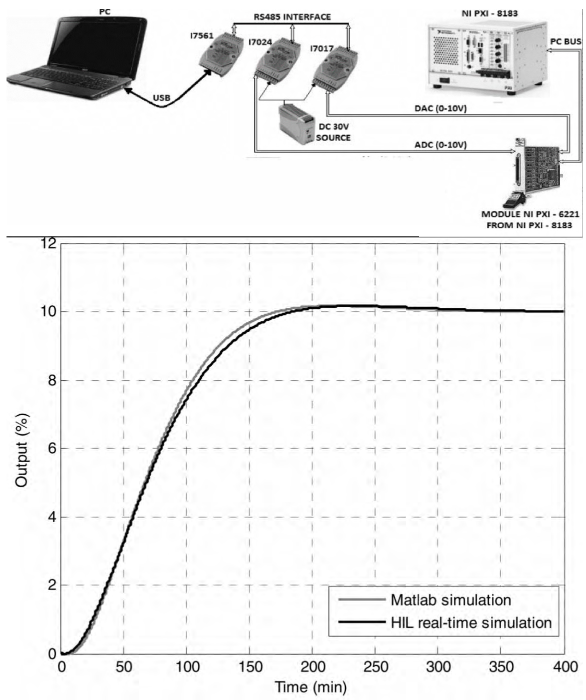

FIGURE 7. Hardware in the loop and closed loop FOPI results [68].

DC motor. Several experimental tests (Fig. 6) are provided that compare the capabilities of the fractional order controller and integer order PID, proving once more the superiority of the fractional order PID. The digital realization of the controller is realized by approximating the fractional order PID to a division of integer order polynomials using the Continued Fraction Expansion (CFE) formula that is further discretized to a 4^{th} order discrete transfer function.

Also, several recent works were found that combine FOPI control to the Smith-Predictor structure [68], [89]. Evaluating closed loop performance of systems without the actual need of implementing them on the process itself is realized in [68] using a hardware in the loop real-time simulator. The paper focuses around a hardware in the loop setting suitable for testing fractional order PI controllers inside a Smith-Predictor structure for a second order plus time delay process. The setup and one experimental test are presented in Fig. 7. Nominal and uncertain operating conditions are experimentally tested for the proposed control strategy. The same approach is tackled by [89] for thermal processes, modeled by first order plus time delay dynamics.

In [12], experimental results are also provided, for the design of a multivariable fractional order IMC controller in a Smith predictor structure for a quadruple-tank system. Also, fractional order IMC controller is implemented and experimentally tested on a a thermal process in [89].

Other papers offering experimental validations of fractional order controllers can be found in [50], [52], and [56].

VII. CONCLUSIONS

The review study familiarizes the reader with recent advances in fractional order control for time delayed plants. Due to recent advances in fractional order control, the paper focuses on publications later than 2010. The study is categorized into sections based on the closed loop control structure used (feedback control, IMC or the SP control scheme). The study also delimits the several existing tuning methods for FOPID controllers and the fractional order extensions to advanced control strategies.

Fractional order Proportional Integrator Derivative, Internal Model Control, Smith Predictor, Sliding Mode Control are the main structures detailed, providing the user a brief summary of the design method and its requirements while also specifying relevant literature where the design methods were applied on time delayed processes. Analyzing the amount of relevant literature present in every category, it is clear that the fractional order PID in the classical negative feedback structure is the most popular control approach. Another important remark is the purely analytic aspect of all the cited works.

Among this review there have been about 85 studies cited targeting fractional order control design procedures. However, less than 7% of the papers have experimental examples in real life implementation of fractional order control, the others providing only numerical examples or simulations at best.

Despite the extensive reference list presented here, including citations dating from 2010 and earlier, fractional calculus and fractional order systems, in general, are not a panacea. The results gathered in this review paper show that fractional order control has a great potential, although many issues are still left unexplored. All the cited works that tackle the comparison between fractional and integer order control prove that the fractional order approach provides better overall closed loop system performance. To conclude, this review is a clear statement that fractional calculus is a powerful tool in the control engineering field for time delayed plants.

As indicated in Section VI, there is currently a reduced number of practical applications of fractional order controllers for time delay systems. Future challenges of fractional order calculus in control applications in general, and specifically in dealing with time delay systems, revolve around the practical implementation of these systems to make them more appealing to the industrial engineer, especially since most industrial applications are characterized by time delays.

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