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Outsourcing selective maintenance problem in failure prone multi-component systems

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RésuméIn many industrial settings, there are systems designed to perform consecutive missions interspersed with finite breaks during which only a set of component repairs can be carried out due to limited time, budget, or resources. The decision maker then has to decide which components to repair in order to guarantee a given performance level. This is known as the selective maintenance problem (SMP). This paper introduces a new variant of the SMP by specifically taking into account the maintenance outsourcing alternative. A novel integrated non-linear programming formulation where both the in-house and outsourcing maintenance alternatives are accounted for is developed and optimally solved. The effect of the outsourcing alternative on maintenance decisions is investigated through numerical experiments. The overall results obtained demonstrate the validity of the proposed approach.

Keywords: Selective maintenance, Reliability, Imperfect maintenance, Outsourcing.

1. INTRODUCTION

As organisations strive to reduce operating costs and ensure lean operations, they usually resort to alternative approaches to manage their operations, maintenance, and asset management needs. Among these alternative is the outsourcing which is a supply chain solution that enables an organisation to outsource some of its needs so that, for example, to face market demand or to keep core competition (see for example (Haoues et al., 2016) and the reference therein).

To maintain their production assets, many organisations outsource some of their maintenance activities by using a combination of in-house and contract maintenance. Contractors are generally hired for specialist maintenance where the expertise is not held in-house. Maintenance tasks to resolve basic problems are generally performed in-house.

The present paper investigates the selective maintenance optimisation of multi-component systems in the outsourcing setting. The selective maintenance is appropriate in systems designed to perform consecutive missions interspersed with finite breaks during which a limited set of component repairs or replacements can be carried out due to limited time, budget, or resources. In the airline industry for example, aircraft are subjected to overnight maintenance at night during a break interval typically varying from 2 to 8 hours (Diallo et al., 2017). To prepare the system to successfully complete its next mission, its components must be properly maintained during the scheduled intermission break. Because of the built-in redundancies, and due to the limited duration of the scheduled breaks and scarce maintenance resources, only a limited set of components have and can be maintained during the breaks. It is therefore necessary to identify an optimal subset of components to maintain to meet the predetermined reliability level required for the next mission.

The selective maintenance problem (SMP) was first introduced by Rice et al. (1998) and applied in a series-parallel system in with subsystems composed of independent an identically exponentially distributed components. In this work, the only maintenance option is the replacement at failure. Cassady et al. (2001) extended the original work of (Rice et al., 1998) for systems where lifetimes of components are Weibull distributed. Three maintenance actions are allowed : minimal repair, corrective replacement of failed components, and a preventive replacement of working components. The resulting SMP is solved using an enumeration method. To deal with the combinatorial complexity arising from large-size systems, four improved enumeration procedures are proposed in (Rajagopalan and Cassady, 2006) to reduce the computation times. An exact method based on the branch-and-bound procedure and a Tabu search based algorithm are proposed in (Lust

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et al., 2009). Khatab et al. (2007) proposed two heuristic methods, adapted from those used to solve the redundancy allocation problem in (Aggrawal, 1976; Gopal et al., 1978). The development of SMP models has since increased and dealt with extensions to include imperfect maintenance, system's components dependencies, and mission and break profiles. Imperfect maintenance in the selective maintenance setting is addressed inLiu and Huang (2010), where the age reduction coefficient approach (Malik, 1979) is used to model imperfect maintenance. An imperfect selective maintenance model was also developed by (Zhu et al., 2011) and applied to a machining line system. Panday et al. (2013) also studied the selective maintenance problem for binary systems under imperfect maintenance using the hybrid hazard rate approach introduced in (Lin et al., 2000). In (Liu and Huang, 2010) and (Panday et al., 2013), a set of maintenance levels, ranging from minimal repair to replacement, are used to improve the reliability of a system component. (Khatab and Aghezzaf, 2016) studied the selective maintenance problem when the quality of imperfect maintenance is stochastic. A non-linear and stochastic optimization problem was proposed and solved for a series-parallel system. In (Khatab et al., 2017) the SMP is addressed in systems operating missions and breaks both of stochastic durations. Schneider and Cassady (Schneider and Cassady, 2015) solve the selective maintenance problem for a fleet composed of a set of independent and identical systems. The fleet is required to execute a sequence of missions and return to a common base where maintenance actions may be performed on selected components. The selective maintenance problem has also been addressed in multi-state systems (MSS) settings. The most recent work appeared in Dao and Zuo (2017) where the authors investigated a MSS with structural relationships between its components.

All papers surveyed above do not consider the very common case where multiple repair-persons are available to carry out the maintenance actions. Only few papers attempts to some extent to overcome this restrictive assumption. (Iyoub et al., 2006) and (Maillart et al., 2009) investigated the selective maintenance problem taking into account maintenance resources allocation. For its repair, each system's component is assumed to consume an amount of a given maintenance resource. The total amount consumed from each maintenance resource must be less than or equal to the total amount of that resource allotted to perform maintenance on failed components. These approaches are however limited to systems with components of constant failure rates. In a more recent paper, Diallo et al. (2017) studied the SMP with multiple repair-persons. An integrated non-linear programming formulation was then developed and optimally solved. Numerical experiments conducted show the benefits of jointly carrying out the assignment of the tasks to repair-persons and the selection of the components to be repaired. Nonetheless, their work assume that component's replacement as the only one available maintenance decision.

In the present paper, the selective maintenance problem is investigated for systems with multiple repair-persons in outsourcing setting. The system must operate a sequence of alternating missions and scheduled breaks. Each component has a list of eligible maintenance actions ranging from minimal repair, through intermediate imperfect maintenance actions, to replacement. It is assumed that some maintenance levels are performed in-house, while the others are outsourced. To maximise the probability of the system to successfully operate the next mission, the maintenance activities are performed on the system's components during the break. Due to limited break duration, maintenance budget and repair crews, not all components are likely to be maintained. The resulting integrated optimisation model is a non-linear integer optimization model which is optimally solved and fully discussed. Numericals experiments are also provided to demonstrate the accuracy and the validity of the proposed approach.

The remainder of the paper is organized as follows. Acronyms, notation and the main working assumptions are given in Section 2. System's description and the imperfect maintenance model used are presented in Section 3. The selective maintenance model is developed in Section 4 where the new integrated selective maintenance problem is also formulated. Numerical experiments are provided in Section 5. Conclusion and future research extensions are drawn in Section 6.

2. ACRONYM, NOTATION AND MAIN WORKING ASSUMPTIONS

In this section, acronyms, the list of notation and the main working assumptions made in this paper are provided in what follows.

Acronyms

CM (PM)	Corrective (Preventive) maintenance
SMP	Selective maintenance problem
OSMOP	Outsourcing SM optimisation problem

Notation

n	Number of subsystems
i	Index of subsystems
n_i	Number of components in subsystem S_i
j	Index of component in subsystem S_i,
C_{ij}	The j^{th} component of subsystem S_i
L	The highest maintenance level available
l	Maintenance level $l \in \{0, \cdots, L\}$
t^c_{ijl}	Time of CM of level l performed on C_{ij}
t^p_{ijl}	Time of PM of level l performed on C_{ij}
c_h	Cost rate of an in-house repair-person
c_o	Cost rate of an outsourcing repair-person
A_{ij}	Age of C_{ij} at the start of a break
B_{ij}	Age of C_{ij} at the end of a break
X_{ij}	C_{ij} status at the start of a break
Y_{ij}	C_{ij} status at the end of a break
C_0	Maximum maintenance budget available
D	Limited break duration
U	Next mission duration
$h_{ij}(t)$	Failure rate of $C_{ij}(t)$
$R^c_{ij}\left(U\middle\|_{B_{ij}}\right)$	Conditional reliability of component C_{ij}
R	System reliability for the next mission

Assumptions

(1) The system consists of multiple, repairable binary components (the components and the system are either functioning or failed).

(2) During the break, system components do not age, i.e. the age of a component is operation-dependent.

(3) No maintenance activity is allowed during the mission. Maintenance activities are allowed only during the break.

(4) Multiple components can be worked on simultaneously without repair-persons colliding.

3. SYSTEM'S DESCRIPTION AND MAINTENANCE MODEL

3.1 System's description

Without loss of generality, the SMP studied in the present work considers a series-parallel system composed of n series subsystems S_i $(i = 1, ..., n)$ each of which is composed of n_i independent and possibly non-identical components C_{ij} $(j = 1, ..., n_i)$ arranged in parallel. The system is assumed to have just finished the current mission, and then turned off during the scheduled break of finite length D for possible maintenance of their components. Thereafter, the system is used to operate the next mission of duration U. At the end of the current mission, a system's component can be either in a functioning or in a failed state. Two state variables X_{ij} and Y_{ij} are used to describe the status of component C_{ij}, respectively, at the end of current mission and at the beginning of the next mission :

$$X_{ij} = \begin{cases} 1 \text{ if } C_{ij} \text{ is working at the end of the current mission} \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

$$Y_{ij} = \begin{cases} 1 \text{ if } C_{ij} \text{ is working at the beginning of the next mission} \\ 0 \text{ otherwise.} \end{cases} \tag{2}$$

Each component C_{ij} is also characterized by its age A_{ij} at the end of the current mission (i.e. at the start of te break), and its age B_{ij} at the beginning of the next mission (i.e. at the end of the break). If the component is still functioning at the end of the current mission (i.e. $X_{ij} = 1$), its corresponding probability to successfully operating the next mission is given by its conditional reliability $R\left(U|_{B_{ij}}\right)$ such that :

$$R^c\left(U|_{B_{ij}}\right) = \exp\left(\int_{B_{ij}}^{B_{ij}+U} h_{ij}(t)\mathrm{d}t\right), \tag{3}$$

Since the system's reliability block diagram is series-parallel, its reliability is :

$$R = \prod_{i=1}^{n}\left[1 - \prod_{j=1}^{n_i}\left(1 - R\left(U|_{B_{ij}}\right)\cdot Yij\right)\right] \tag{4}$$

3.2 Imperfect maintenance model

Two types of maintenance are considered either preventive or corrective maintenance, depending on the status of the component C_{ij} at the end of the current mission. Both maintenance types are modelled according to the age reduction imperfect maintenance model of (Malik, 1979). Without loss of generality, a common list $\{1, \ldots, l, \ldots, L\}$ of L maintenance levels is available for all system's components. To each maintenance level $1 \leq l \leq L$ corresponds an age reduction coefficient $\alpha_l \in [0, 1]$. If a maintenance of level l is carried out on C_{ij}, its age A_{ij} is reduced and becomes $\alpha_l A_{ij}$. Two particular values of maintenance level can then be observed. The first corresponds to the maintenance level $l = 1$ whose age reduction is equal to 1 and stands then for minimal repair. The second corresponds to the highest maintenance level L whose age reduction is 0 which is equivalent to component's renewal. It is worth noticing that minimal repair, as a maintenance level, is eligible only for failed components.

To deal with outsourcing within SMP, the maintenance list is partitioned on two subsets of maintenance levels. The first k maintenance levels $\{1, \ldots, k\}$ are the set of maintenance actions that are performed in-house, while the remaining maintenance levels $\{k + 1, \ldots, L\}$ are assumed to require more skilled repair-persons and then outsourced. To alleviate the notation, we simply write $k' = k + 1$.

4. SELECTIVE MAINTENANCE MODEL

To develop the new integrated selective maintenance optimisation in the outsourcing setting, the following decision variable z_{ijl} is defined :

$$z_{ijl} = \begin{cases} 1 \text{ if } C_{ij} \text{ is selected for maintenance and maintenance } l \text{ is performed} \\ 0 \text{ otherwise.} \end{cases} \tag{5}$$

According to the age reduction imperfect maintenance model, when a maintenance action of level l is performed on a component C_{ij}, then the effective age B_{ij} of C_{ij} is evaluated as :

$$B_{ij} = [(\alpha_{ijl} \cdot z_{ijl}) + (1 - z_{ijl})] \cdot A_{ij}. \tag{6}$$

In what follows, we compute the total time spent on components' maintenance and the total cost induced by the selected maintenance actions. To do so, let us denote by t^p_{ijl} and t^c_{ijl}, the time consumed by, respectively, a preventive and corrective maintenance level l when preformed on component C_{ij}. We also denote by c_h and c_o the cost rate (i.e. cost per unit time) induced by an in-house and outsourcing maintenance. It is reasonable to assume that outsourcing cost rate c_o is greater than the in-house cost rate c_h.

The total time spent by maintenance actions performed in-house is computed as :

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=2}^{k} t^p_{ijl}.X_{ij}.z_{ijl} + \sum_{l=1}^{k} t^c_{ijl}.(1 - X_{ij}).z_{ijl}\right). \tag{7}$$

Similarly, the total time consumed by outsourced maintenance actions is computed as :

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=k'}^{L} t^{p}_{ijl}.X_{ij}.z_{ijl} + \sum_{l=k'}^{L} t^{c}_{ijl}.(1-X_{ij}).z_{ijl}\right). \quad (8)$$

The total cost induced by both in-house and outsourced maintenance actions is computed as :

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=2}^{k} t^{p}_{ijl}.c_h.X_{ij}.z_{ijl} + \sum_{l=k'}^{L} t^{p}_{ijl}.c_o.X_{ij}.z_{ijl} + \sum_{l=1}^{k} t^{c}_{ijl}.c_h.(1-X_{ij}).z_{ijl} + \sum_{l=k'}^{L} t^{c}_{ijl}.c_o.(1-X_{ij}).z_{ijl}\right) \quad (9)$$

4.1 Problem formulation

In this section, the outsourcing selective maintenance optimization problem (OSMOP) is developed. This model deals with the case where the objective of the maintenance decision-maker consists on maximizing the system reliability to successfully achieve the next mission, taking into consideration the pre-specified maintenance budget C_0. The optimization model is drawn subsequently.

OSMOP :

$$\text{Max } R = \prod_{i=1}^{n}\left(1 - \prod_{j=1}^{n_i}\left(1 - R^{c}_{ij}\left(U|_{A_{ij}}\right)\cdot Y_{ij}\right)\right) \quad (10)$$

Subject to :

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=2}^{k} t^{p}_{ijl}.c_h.X_{ij}.z_{ijl} + \sum_{l=k'}^{L} t^{p}_{ijl}.c_o.X_{ij}.z_{ijl} + \sum_{l=1}^{k} t^{c}_{ijl}.c_h.(1-X_{ij}).z_{ijl} + \sum_{l=k'}^{L} t^{c}_{ijl}.c_o.(1-X_{ij}).z_{ijl}\right) \le C_0 \quad (11)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=2}^{k} t^{p}_{ijl}.X_{ij}.z_{ijl} + \sum_{l=1}^{k} t^{c}_{ijl}.(1-X_{ij}).z_{ijl}\right) \le D \quad (12)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(\sum_{l=k'}^{L} t^{p}_{ijl}.X_{ij}.z_{ijl} + \sum_{l=k'}^{L} t^{c}_{ijl}.(1-X_{ij}).z_{ijl}\right) \le D \quad (13)$$

$$\sum_{l=1}^{L_{ij}} z_{ijl} \le 1 \quad (14)$$

$$z_{ij1} \le (1 - X_{ij}) \quad (15)$$

$$B_{ij} = \sum_{l=1}^{L_{ij}}[\alpha_{ijl}\cdot z_{ijl} + \left(1 - z_{ijl}\right)]\cdot A_{ij} \quad (16)$$

$$Y_{ij} = X_{ij} + \sum_{l=1}^{L}(1 - X_{ij})\cdot z_{ijl} \quad (17)$$

$$i = 1,\ldots,n;\ j = 1,\ldots,n_i;\ l = 1,\ldots,L \quad (18)$$

$$Y_{ij},\quad X_{ij} \in \{0,1\},\quad z_{ijl} \in \{0,1\}. \quad (19)$$

In the above optimization model, Equations (11) is the maintenance budget constraint. Equations (12) and (13) are the maintenance break duration constraints, respectively, for the in-house and outsourcing maintenance activities. For each component C_{ij}, Equations (14) states that only one maintenance level can be selected if the component is to be maintained by an in-house or an outsourcing repair-person. The constraint (15) states that minimal repair is eligible only on a failed component. The constraint (16) updates the effective age of components at the end of the break. The constraint (17) allows to update the operating state of components.

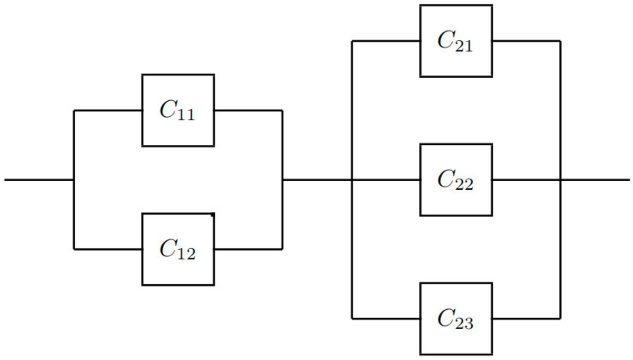

FIGURE 1. Reliability block diagram of the series-parallel system

5. NUMERICAL EXPERIMENTS

In this section, two numerical experiments are conducted. We are given a series-parallel system composed of $n = 2$ subsystems $S_i (i = 1, 2)$, its corresponding reliability block diagram is depicted in Figure (1). Subsystem S_1 is composed of $n_1 = 2$ components in parallel, while subsystem S_2 contains $n_2 = 3$ components arranged in parallel. It is assumed that the lifetimes of each component C_{ij} follows a Weibull distribution with shape parameter β_{ij} and scale parameter η_{ij}. These parameters are reported in Table (1). This table gives also the age A_{ij} and the value of the state variable X_{ij} corresponding to component C_{ij} at the end of the current mission. According to Table (1), only components C_{11} and C_{22} are failed to survive the current mission, while the other components are still functioning.

We assume that $L = 6$ maintenance levels are available each of which the corresponding age reduction coefficient is given in Table (2). Times required to perform a corrective or a preventive maintenance of level l on components are reported, respectively, in Tables (3) and (4). In the all subsequent experiments, the in-house and the outsourcing cost rates are, respectively, set to $c_h = 5$ and $c_o = 7$. The system has just finished the current mission and made available for maintenance. The next mission duration is set to $U = 70$ time units.

TABLE 1. Components lifetimes parameters, ages and status at the start of break

C_{ij}	β_{ij}	η_{ij}	A_{ij}	X_{ij}
C_{11}	3	120	110	0
C_{12}	1.5	200	175	1
C_{21}	2.7	150	175	1
C_{22}	2	150	150	0
C_{23}	2.5	170	100	1

Table 2. Age reduction coefficients values

l	1	2	3	4	5	6
α_l	1	0.8	0.6	0.3	0.2	0

Table 3. Corrective maintenance times t^c_{ijl}

l	1	2	3	4	5	6
C_{11}	0.5	1.5	2.2	2.5	2.8	3
C_{12}	0.45	1.2	2.3	2.5	2.7	2.8
C_{21}	0.6	1.1	1.3	1.8	2.3	2.5
C_{22}	0.4	0.8	1.1	1.5	1.8	2.2
C_{23}	0.5	1	1.3	1.7	2	2.5

Table 4. Preventive maintenance times t^p_{ijl}

l	2	3	4	5	6
C_{11}	1	1.9	2.3	2.7	2.8
C_{12}	0.9	1.7	2	2.2	2.5
C_{21}	0.8	1	1.2	1.4	1.9
C_{22}	0.7	0.9	1.2	1.4	2
C_{23}	0.8	1	1.1	1.7	2.2

5.1 Experiment#1

This experiments investigates the two extreme cases where the maintenance activities is completely outsourced or completely performed in-house. We then solve the selective maintenance problem for two particular values of the number k of in-house maintenance levels. In the case of complete maintenance outsourcing $k = 0$, while in the complete in-house case the value of k is set to $k = L$. In this experiment the duration of the break is set to $D = 4$, and the maintenance budget is set to $C_0 = 25$.

In complete outsourcing case, the optimal maintenance decisions suggests the maintenance plan according to which component C_{11} and C_{22} are both replaced (corrective replacement). The resulting maximal reliability to operate the next mission is evaluated to $R = 63.09\%$. This maintenance plan induces a total cost of 23.8.

In the case of complete in-house maintenance, the optimal maintenance decisions derived are such that three components are selected to be maintained. The optimal maintenance plan suggests to perform a preventive replacement on component C_{12}, a preventive maintenance of level $l = 4$ on component C_{23}, and a minimal repair on the failed component C_{22}. The resulting system's reliability is estimated to $R = 70.22\%$. This maintenance plan induces a total cost of only 20.

From the above results, it follows that the maximal achievable system's reliability when dealing with complete in-house maintenance case is better that that obtained when dealing with the case of complete maintenance outsourcing. Furthermore the optimal in-house maintenance plan is less expensive than the optimal outsourcing maintenance plan. This result comes from the fact that hiring an outsourcing repair-person is more expensive than using an in-house repair-person. Consequently, using the complete outsourcing solution limits the number of components to be maintained. However, these two extreme cases may not be realistic and one may be forced to resort to maintenance outsourcing contracts. Indeed, on one hand, not all organisations have highly-skilled maintenance operators, and, on the other hand, the maintenance budget is usually restricted. The maintenance manager should therefore strive to find an appropriate solution with mixed in-house and outsourcing maintenance activities. The following experiment investigates the effect of such combined solutions on maintenance decisions.

5.2 Experiment #2

This experiment investigates the effects of the outsourcing on the selective maintenance decisions. Using the optimisation model OSMOP, the maximum achievable system reliability will be determined for a given break duration and maintenance cost when the number k of in-house maintenance levels is taken variable. It is conjectured that the flexibility provided by the cohort with mixed in-house and outsourcing repair-persons will allow the model to reach the same or better solution than when maintenance activities are completely either in-house or outsourced.

The same system and parameters used in the previous experiments are used. The optimisation problem OSMOP is solved for different values of the allowable maintenance budget C_0. The results obtained are depicted in Figure (2).

From these results, it is clearly shown as conjectured that an appropriate combination of an in-house and an outsourcing cohort is able to achieve equivalent or better reliability than the uniform cohort (i.e. either complete in-house or complete outsourcing). For example, if we consider the case where the maintenance budget is limited to $C_0 = 25$, it follows that the maximal achievable system's reliability when dealing with a combined in-house and outsourcing solution is obtained when the first four maintenance levels are performed in-house ($k = 4$) while the highest maintenance levels $l = 5$ and $l = 6$ are outsourced. If the maintenance budget is increased to $C_0 = 30$, the maximal achievable system's reliability when dealing with a combined in-house and outsourcing solution is obtained when only the highest maintenance level $l = 6$ is outsourced. The same conclusion can also be made from the results obtained for the other values of the maintenance budget.

In general, when the maintenance budget available decreases, the maximal reliability achieved decreases. When the maintenance budget permits, the highly skilled outsourcing repair-persons can be used in an adequate combination with the in-house repair-persons. As the budget decreases, the model suggests to use the in-house persons instead of resorting to outsourcing.

6. CONCLUSION

This paper introduced a novel variant of the selective maintenance problem in multi-component system by specifically combining the in-house and the outsourcing maintenance alternatives. A novel integrated non-linear programming formulation was then proposed and optimally solved. Numerical experiments show the benefits of jointly carrying out : (1) the assignment of the maintenance tasks

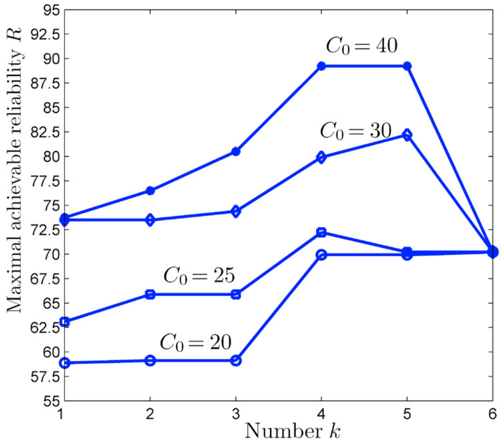

FIGURE 2. Maximal achievable reliability versus the number k : case of Experiment#2

to the in-house and outsourcing repair-persons, and (2) the selection of the components to be repaired.

Future extensions that the authors are currently working on, include the development of efficient solution methods to deal with large sized problems. Another important issue consist to investigate the selective maintenance problem where more than one objective is to be optimized. Other possible extensions may also be the generalization of the present work to consider multi-state systems and k-out-of-n systems.

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