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Integrated production quality and condition-based maintenance optimisation for a stochastically deteriorating manufacturing system

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This paper investigates the problem of optimally integrating production quality and condition-based maintenance in a stochastically deteriorating single- product, single-machine production system. Inspections are periodically performed on the system to assess its actual degradation status. The system is considered to be in 'fail mode' whenever its degradation level exceeds a predetermined threshold. The proportion of non-conforming items, those that are produced during the time interval where the degradation is beyond the specification threshold, are replaced either via overtime production or spot market purchases. To optimise preventive maintenance costs and at the same time reduce production of non-conforming items, the degradation of the system must be optimally monitored so that preventive maintenance is carried out at appropriate time intervals. In this paper, an integrated optimisation model is developed to determine the optimal inspection cycle and the degradation threshold level, beyond which preventive maintenance should be carried out, while minimising the sum of inspection and maintenance costs, in addition to the production of non-conforming items and inventory costs. An expression for the total expected cost rate over an infinite time horizon is developed and solution method for the resulting model is discussed. Numerical experiments are provided to illustrate the proposed approach.

Keywords: integrated production quality and maintenance; condition-based maintenance; degradation monitoring; reliability; optimization; stochastic processes

List of acronyms

CM	corrective maintenance
PM	preventive maintenance
CBM	condition-based maintenance
pdf	probability density function
cdf	cumulative distribution function
EPQ	economic production quantity
SPC	statistical process control

1. Introduction

1.1. Context and motivation

Planning production, preventive maintenance (PM) and quality control in an integrated manner can improve productivity and resource usage in production systems subject to stochastic deteriorations. The production system deteriorations can increase the risk of its failure and deteriorate the product quality. PM has been demonstrated to be an effective way to prevent the system deteriorations, and consequently improve simultaneously the system reliability and the product quality. The interdependence between production, system state and product quality is well-established in the literature. An ongoing field on research is the integration of production, maintenance and quality such that their interrelations are explicitly accounted for.

Thanks to the development of sensor and information technologies, we can monitor the deterioration level of systems to facilitate the prediction of failures. Therefore, the condition-based maintenance (CBM) has been widely used in maintenance engineering due to its effectiveness and efficiency. To help the manufacturer coordinate and balance production quality and CBM decisions, there is a substantial need to develop new integrated models where maintenance programs are developed

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and optimised on the basis of information collected through condition monitoring and product quality requirements. In the last decade, increasing research efforts have been made on CBM modelling and optimisation. CBM isrecognised to be more effective because it takes into account the real-time condition of the system (e.g. degradation level). In the existing literature, a large variety of mathematical models, methods as well as techniques have been developed for CBM (Pandey, Cheng, and van der Weide 2011; Liu et al. 2013; Mercier and Castro 2013; Rafiee, Feng, and Coit 2015). For a comprehensive literature review in CBM, one may refer to Jardine, Banjevic, and Lin (2006), Scarf (2007) and van Noortwijk (2009), and to the recent review by Alaswad and Xiang (2017).

In the above CBM literature, PM is done to improve system reliability and availability. However, to preserve the brand image while being highly competitive in a constantly changing marketplace, industrial companies have to constantly improve their product quality. In this context, it has been already proved that appropriate maintenance programs performed on production systems allow to improve not only the reliability of the system but also the product quality provided that the quality requirements are fully and explicitly integrated in the PM decision-making.

In this paper, the problem of optimally integrating production quality and CBM is investigated in a stochastically deteriorating production system under periodic monitoring. An optimisation model is developed as such it integrates explicitly the three aspects that are production, maintenance and quality. The objective is to determine the optimal inspection cycle and the degradation threshold level, beyond which PM should be carried out, while minimising the sum of inspection, maintenance, production and inventory costs.

1.2. Literature review

The present work is closely linked to two main research problems in stochastically deteriorating production systems. The first research problem is dealing with the integrated economic production quantity (EPQ) and PM while the second problem is focusing on the integrated of production quality in PM decisions. The following literature review is therefore conducted with respect to existing approaches developed within each of these two research problems.

Dealing with the first research problem, the EPQ model for production and inventory optimisation Harris (1913) has been extensively investigated and extended. Rosenblatt and Lee (1986) studied the effects of an imperfect production process on the optimal production cycle time. They investigated the case where shifting time (from an in-control state into an out-of-control state) is exponentially distributed and showed that the resulting optimal EPQ is shorter than the classical one derived from the conventional model. Hariga and Ben-Daya (1998) extended Rosenblatt and Lee's results to general shifting time distributions. Porteus (1986) investigated investment options to reduce quality and setup costs. The resulting optimal EPQ confirms the conclusion made in Rosenblatt and Lee (1986). Lee and Rosenblatt (1989) proposed an EPQ model integrating inspection and maintenance where restoration cost is related to the delay time of failure detection, and the shifting time is governed by an exponential distribution. The model in Lee and Rosenblatt (1989) is extended by Lin, Tseng, and Liou (1991) for a general shifting time distribution with an increasing failure rate. Wang and Sheu (2001) developed a model to compute the optimal lot size and product inspection threshold for deteriorating production system. As in Porteus (1986) and Rosenblatt and Lee (1986), their approach also concludes that it will be cost effective to produce a smaller lot size than the traditional EPQ. In Wang and Sheu (2001), the option of investing in improving process reliability was studied to demonstrate that such investment is indeed a key factor in reducing the expected total cost rate.

There are several papers dealing with time-based imperfect maintenance (Barlow and Proschan 1996; Lin, Zuo, and Yam 2000; Nakagawa 2008; Khatab, Diallo, and Sidibe 2017). Ben-Daya and Makhdoum (1998) investigated the effects of various PM policies on the joint optimisation of the EPQ and the economic design of the control chart. Ben-Daya (2002) developed an integrated model for the joint determination of EPQ, quality and PM level for a process with a general deterioration distribution and increasing failure rate. The durations of inspection intervals are chosen such that the integrated hazard rate function over each inspection interval is equal. For further details about the EPQ approaches till 2001, one may refer to Ben-Daya and Rahim (2001). Wang (2013) proposed an integrated EPQ model with rework activity in addition to imperfect preventive maintenance including minimal repair. Chen (2013) investigated an integrated EPQ model with inspection, rework and PM with an error. The correct execution of preventive maintenance reduces the system failure rate, whereas aPM error results in the system shifting to the out-of-control state with a certain probability. Chen and Lo (2006), Wang (2004), and more recently Liao (2016) studied an integrated EPQ model for an imperfect production system where items produced are sold under a warranty policy. An age-based PM has been also used in the EPQ setting to simultaneously determine the production lot size and the PM schedule for randomly deteriorating production systems producing both conforming and non-conforming items (and the references therein Chelbi, Rezg, and Radhoui 2008; Dhouib, Gharbi, and Ben Aziza 2012). Recently, Gouiaa-Mtibaa et al. (2018) studied an integrated production quality and imperfect PM in a manufacturing system subject to an increasing random failure rate and producing conforming and non-conforming items for which rework activities are allowed. Furthermore, Haoues, Dahane, and Mouss (2016), Rivera-Gomez et al. (2018)

and Zied, Sofiene, and Nidhal (2014) investigated the integrated problem of subcontracting, production and maintenance strategies in a manufacturing system subject to random failures.

For the case of multi-period multi-product capacitated lot-sizing problem in unreliable production system subjected to imperfect PM, Aghezzaf, Jamali, and Ait-Kadi (2007) proposed an integrated production and PM planning optimisation model to generate optimal integrated production and PM plans at the tactical level in failure-prone production system subjected to minimal repair at failure and PM. The model assumes that any maintenance action, minimal repair or PM, reduces the available production capacity of the system. This work has been extended under various assumption relaxations in several papers (Aghezzaf and Najid 2008; Najid, Alaoui-Selsouli, and Mohafid 2011; Nourelfath and Châtelet 2012; Zhao, Wang, and Zheng 2014; Fitouhi and Nourelfath 2014). The more recent extensions of the work in Aghezzaf, Jamali, and Ait-Kadi (2007) have appeared in Aghezzaf, Khatab, and Tam (2016), Fakher, Nourelfath, and Gendreau (2016), Nourelfath, Nahas, and Ben-Daya (2016) and Tam, Aghezzaf, and Khatab (2018).

From the above literature review, it may be seen that the maintenance policies integrated into the EPQ model are usually based on the age of the production process and on the statistical information of system lifetimes. As a result, the effect of how the production system is being used on its reliability is not accounted for. As pointed out by Singpurwalla (1995), the main drawback of lifetime distributions is that they depend only on the ageing process to evaluate whether a system is functioning or is failed. However, many real-life production systems suffer damage and deteriorate with both age and usage, such as gearboxes in a wind turbine, sawing tools, and filing or cutting machine tools. The system deterioration process is generally modelled as a time-dependent stochastic process using models such as the proportional hazard model (PHM), random deterioration rate, Markov, Wiener, Gamma or Inverse Gaussian processes; these are well known for their particular mathematical properties and clear physical interpretations (van Noortwijk 2009; Ye and Xie 2015; Zhang, Gaudoin, and Xie 2015). When the deterioration of the production system reaches a failure threshold, maintenance actions should be performed on the system. If the system degradation is appropriately modelled and measurable, maintenance actions can be carried out on the basis of the observed degradation data before the system enters an out-of-control operating mode. Maintenance programs that are developed and optimised on the basis of information collected through condition monitoring are called CBM.

In the existing literature, very few papers deal with joint CBM and production decisions. Jafari and Makis (2015) proposed a modelling approach based on a combination of the continuous-time Markov process and the PHM for the joint optimisation of EPQ and PM in a production facility subject to random deterioration and condition monitoring. The maintenance policy adopted consists of performing a corrective replacement at failure, or to perform a preventive replacement when either the failure rate reaches a pre-determined threshold or the age of the production facility exceeds a pre-specified value. The authors then developed and solved the resulting optimisation problem using the semi-Markov decision process (SMDP) technique in an infinite time horizon. Recently, Jafari and Makis (2016b) investigated the optimallot sizing and CBM policy for a partially observable production system by using a multivariate Bayesian control approach. They also extended these articles (Jafari and Makis 2015, 2016b) to deal with joint EPQ and CBM in production system composed of two economic dependent machines arranged in series (Jafari and Makis 2016a). The more expensive machine is subject to condition monitoring. Its corresponding deterioration process is modelled as acontinuous-time hidden-Markov process. The second machine is assumed to fail only with a general distributed age. Using a multivariate Bayesian control approach, an integrated EPQ and CBM optimisation model is then developed to minimise the long-run expected cost rate.

Cheng, Zhou, and Li (2017) developed an integrated optimisation model of EPQ and CBM policy in a production system composed of several economic dependent components. Each system component deteriorates randomly according to a stationary gamma process. Component conditions are revealed by inspection performed once a production lot is finished. Accordingly, a component can be correctively (preventively) replaced if its degradation exceeds the failure (preventive maintenance) threshold. The cost function is formulated in terms of production and maintenance costs. A combination of Monte Carlo simulation technique and genetic algorithm is used to obtain the optimal joint optimal values of production lot size and the PM threshold.

Peng and van Houtum (2016) consider a manufacturing system whose degradation is governed by a time-dependent stochastic process. Two specific deterioration processes are explicitly studied; the Gamma processes and the random coefficient model. The manufacturing system undergoes a corrective replacement when its cumulative degradation reaches a failure threshold. To improve the system reliability, a preventive replacement is carried out at a specified degradation threshold. The objective is to minimise the expected cost rate over an infinite time horizon. Production and maintenance costs considered are setup, inventory holding, lost sales, preventive and corrective replacements costs. The authors developed their integrated CBM and EPQ model using renewal theory and discussed the optimality conditions.

To deal with the integration of quality requirements in PM decisions, most papers in the literature jointly consider PM and statistical process control (SPC) models. Tagaras (1988) considered a Markovian deteriorating process that undergoes adjustments to restore it to the normal state whenever a process shift is detected. Furthermore, PM is carried out periodically

to prevent the deterioration of the process. Thus, an integrated model is developed for the joint cost analysis and optimisation of the maintenance operations and SPC. In another paper, Panagiotidou and Tagaras (2010) investigated the joint SPC and CBM in production systems experiencing two operational states: high and low quality. The process is monitored via a control chart to detect shifts to the out of control state (i.e. the lower-quality state). Since then, many other integrated PM and SPC models appeared in the literature, see for example Wang (2012), Xiang (2013), Yin et al. (2015) and the reference therein. As pointed out in Lu, Zhou, and Li (2016), the main drawbacks of the integrated PM and SPC approaches are related, first, to the root causes of the quality changes that are unknown and therefore cannot be included as input information in the modelling process. Second, the joint PM and SPC approaches lack the quantitative relationships between machine condition and product quality such that the product quality defects caused by machine deterioration cannot be measured.

To overcome the above-mentioned drawbacks of the joint PM and SPC approaches, Chen and Jin (2006) considered a quality-oriented-maintenance problem for tooling components in a discrete manufacturing process. In their approach, product quality deteriorates with the degradation of the tooling components. To improve the product quality, preventive replacements are carried out on tooling components. A quality-oriented-maintenance optimisation model is proposed while considering the joint and interactive impacts of multiple adjustable process variables on product quality. The optimisation model is formulated as a response model and determines the optimal replacement times for tooling components by minimising the total production cost composed of product quality loss due to process drifts, productivity loss due to catastrophic failures, in addition to maintenance costs. The work by Chen and Jin (2006) has been extended in Ji-wen et al. (2010) by considering catastrophic failures due to random shocks and the obsolescence cost in a multi-station machining system. Three maintenance policies, namely, age replacement, block replacement, and block replacement with minimal repair, are then investigated and analysed.

Lu, Zhou, and Li (2016) proposed a joint model of PM and quality improvement in a randomly deteriorating single-machine manufacturing system. During production, the machine deteriorations increase the machine failure rate and degrade product quality. Using the PHM (PHM), an integrated reliability model is formulated to consider the impacts of the adjustable process variables on machine reliability. In line with Chen and Jin (2006), a response model is provided to quantitatively describe the impact of process variables on product quality. To jointly improve machine reliability and product quality, corrective maintenance (CM) by means of minimal repair is carried out at failure, while imperfect PM is performed whenever the failure rate of the system reaches a given threshold. Anoptimisation model is proposed to determine the optimal failure rate threshold minimising the total production cost composed of product quality loss, CM and PM costs.

1.3. Paper contribution and outline

This paper investigates the problem of optimally integrating production quality and CBM planning in a stochastically deteriorating production system. The production system is subjected to stochastic deterioration and fails whenever the accumulated degradation exceeds a critical specification threshold, which can be specified according to either economic or safety standards. Failure of the system is revealed only through periodic inspections. The system is preventively maintained when the degradation level reaches or exceeds a given PM threshold. Both inspection period and PM threshold are considered as decision variables of the PM policy. The production system is expected to produce items with specified quality requirements. Such quality requirements lead to explicitly taking into account the extra actions and costs incurred by the production of non-conforming items. We develop an integrated production and maintenance optimisation model that minimises the total expected cost rate over an infinite time horizon. The objective is to determine optimal parameters of the PM strategy tominimise the sum of inspection and maintenance costs, production and inventory holding costs in addition to the cost incurred due to the production of non-conforming items. Renewal theory is used to compute the expected cost rate function involving production, inventory, maintenance and quality inspection costs.

Apart from the fact that the present paper deals with the integrated problem of product quality and CBM, the model investigated in this paper differs from that of Peng and van Houtum (2016) in two important ways. First, Peng and van Houtum (2016) assume that all items produced are defect free, meaning that the quality aspect of items produced is not accounted for. In this paper, since system degradation may impact the quality of the production, the quality aspect of items is explicitly modelled and accounted for. Second, Peng and van Houtum (2016) assume that the production system is continuously monitored, while this paper assumes discrete-time monitoring. The inspection period is therefore a supplementary decision variable, that must also be optimally determined. The present work also differs from Lu, Zhou, and Li (2016), Ji-wen et al. (2010) and Chen and Jin (2006). Here, instead of the PHM model used in Chen and Jin (2006), the production system deterioration is modelled by a stochastic process. The production system in Lu, Zhou, and Li (2016), Ji-wen et al. (2010) and Chen and Jin (2006) is also assumed to be continuously monitored, while this paper considers periodic monitoring with the inspection period as a decision variable. In Lu, Zhou, and Li (2016), the maintenance model merely relies on the failure rate threshold as the sole decision variable. In our work, two decision variables are considered: the PM

degradation threshold and the inspection period. The production rate and the customer demand are not considered in Lu, Zhou, and Li (2016), Ji-wen et al. (2010) and Chen and Jin (2006). In the present work, the production rate and the customer demand are explicitly included in the proposed integrated production quality and CBM optimisation model. As such, the constraint to meet the customer demand is included in the proposed optimisation problem. Furthermore, the present work includes the holding cost while Lu, Zhou, and Li (2016), Ji-wen et al. (2010) and Chen and Jin (2006) did not consider such a cost.

This paper is a significant extension of a recently published conference proceedings paper (Khatab et al. 2017). The conference paper deals with only one type of non-conforming item while the present paper considers two types of non-conforming products. This consideration is motivated from the fact that as the degradation increases, so does the production of items with non-conforming quality. Therefore, the first type of non-conforming item is related to the PM zone where the production system degradation level is below the critical degradation threshold. The second type of non-conforming items corresponds to items produced while the system degradation level is above the critical degradation threshold, i.e. during the sojourn time in the out-of-control state. Accordingly, to compute the cost incurred by loss of quality, a new proposition and its corresponding proof are developed. Furthermore, the present version provides more insightful experiments and sensitivity analyses in addition to an enriched literature review.

The remainder of the paper is organised as follows. Section 1 introduces the notation used and the main assumptions. This is followed by the system description and problem definition. In Section 2, cost components of the optimisation model are developed, and in Section 3, an optimisation heuristic is presented. In Section 4, numerical experiments are conducted to illustrate and to demonstrate the validity of the proposed approach. Conclusions and perspectives are presented in Section 5.

2. System description and problem definition

The following notation and assumptions are used to develop the integrated optimisation of production and CBM approach.

2.1. Notation

C_c	expected CM cost
C_p	expected PM cost
C_i	expected inspection cost
C_{nc}	expected production cost per non-conforming item
C_h	expected holding cost per unit time
$X(t)$	system's deterioration at time t
η, γ	scale and shape parameters of the Gamma deterioration process
X_f	failure or specification threshold
X_p	PM threshold
T_f	first passage time of the failure threshold
T_p	first passage time of the PM threshold
g, G	pdf and cdf of T_f, respectively
h, H	pdf and cdf of T_p, respectively
τ	inspection period
T_M	maintenance cycle (production run cycle)
$\mathcal{T}$	inventory cycle
IMC	total inspection and maintenance cost
HC	total holding cost
NCC	total cost induced by the production of non-conforming items
d	demand rate (products per unit time)
ρ	production rate (products per unit time)
E_k^p	event stating that the maintenance cycle ends with a PM at the kth inspection
E_k^c	event stating that the maintenance cycle ends with a CM at the kth inspection
α_p	proportion of non-conforming units produced when the production system degradation lies between X_p and X_f
α_f	proportion of non-conforming units produced when the production system degradation is beyond the failure threshold X_f
Q	the expected production quantity in a production cycle

2.2. Assumptions

- The system deteriorates as a monotone increasing stochastic degradation process. The damages at each time step are cumulative.
- The system fails when its degradation reaches a critical predetermined threshold.
- The system is assumed to sojourn either in *out-of-control* or in *in-control* states.
- The degradation of the system induces degradation of production quality.
- In the in-control state, the system is producing items of acceptable quality, while in the out-of-control, the system is still producing items but a portion of which is non-conforming.
- The proportions α_f and α_p are such that $\alpha_f \geq \alpha_p$, i.e. the production of non-conforming units increases with system degradation.
- The demand rate is known and constant.
- Shortages are not allowed and the fixed production cost is negligible.
- In the in-control state, inspections are carried out periodically. The inspection period is a decision variable.
- Failures are revealed only through error-free inspections.
- PM is performed when the degradation is beyond a threshold, which is a decision variable.
- After either a PM or CM, the system becomes '*as good as new*'. The duration of maintenance actions is negligible compared to the total cycle length and small enough to end before or exactly at the total depletion of the built-up inventory.
- The required amount of inspection and maintenance resources are always available.

2.3. System description

We consider a manufacturing system producing a single product at a constant rate ρ to meet a constant and continuous customer demand rate d ($d < \rho$). The system is assumed to deteriorate according to a monotonically increasing degradation stochastic process and the damages at each time step are cumulative, i.e. the system worsens over time due to ageing and accumulated wear and damage. The system fails when the degradation reaches a predetermined failure threshold X_f. The degradation is defined by a measurable scalar time-dependent random variable $X(t)$ which can take either a linear or nonlinear form. The continuous degradation of the production system is assumed to evolve according to a stationary Gamma process. The Gamma process is recognised to be suitable to characterise monotonically accumulating gradual damage over time (Abdel-Hameed 1975; van Noortwijk 2009). It has therefore been extensively used in CBM optimisation problems (Grall et al. 2002; van Noortwijk et al. 2005, 2007) and some more recent papers (Cheng, Pandey, and van der Weide 2012; Do et al. 2015). An excellent survey dealing with the application of the Gamma process in maintenance modelling and optimisation can be found in the seminal paper by van Noortwijk (2009). For more mathematical details, the reader may refer to Singpurwalla (1997) and van der Weide (1997). However, the approach in this paper is general enough that other kinds of degradation processes can be considered.

The Gamma degradation process of the production system is a time-dependent stochastic process $\{X(t) : t \geq 0\}$ with the following characteristics:

(1) $X(0) = 0$ with probability one,
(2) $X(t)$ has independent increments,
(3) for all $0 \leq s < t$, the random variable $\Delta X(s,t) = X(t) - X(s)$ follows a Gamma distribution whose pdf $f(s,t,x)$ and cdf $F(s,t,x)$ are defined for all $x \geq 0$ as

$$f(s,t,x) = \frac{x^{((t-s)\gamma-1)}}{\Gamma\left((t-s)\gamma\right)\eta^{(t-s)\gamma}} \exp\left(\frac{-x}{\eta}\right) \quad \text{and} \tag{1}$$

$$F(s,t,x) = \Pr\left(\Delta X(s,t)\right) < x) \tag{2}$$

$$= \int_0^x f(s,t,y)\,\mathrm{d}y,$$

$$= \frac{\Gamma\left((t-s)\gamma, \left(\frac{x}{\eta}\right)\right)}{\Gamma\left((t-s)\gamma\right)}.$$

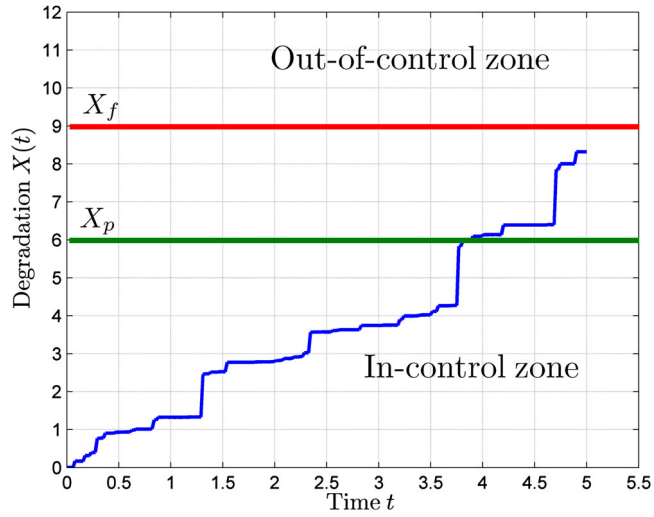

Figure 1. Production system's degradation path and its operating zones.

In the above equations, $(t-s)\gamma$ is the shape parameter and η refers to the scale parameter. The function $\Gamma(x) = \int_0^\infty u^{x-1}\,\mathrm{e}^{-u}\,\mathrm{d}u$ and $\Gamma(\alpha, x) = \int_0^x u^{\alpha-1}\exp(-u)\,\mathrm{d}u$ stands for the lower incomplete Gamma function defined for $\alpha > 0$ and $x \geq 0$. from time 0 up to time t, the expected degradation $\mathbb{E}[X(t)] = \eta\gamma t$, and its variance $\mathbb{V}ar[X(t)] = \eta^2\gamma t$. To simplify the notation, in the case where the time origin $s=0$, we simply write pdf and cdf as functions of the two remaining parameters t and x. For example, if $s=0$, pdf $f(s,t,x)$ and cdf $F(s,t,x)$ are simply denoted as $f(t,x)$ and $F(t,x)$ instead $f(s,t,x)$ and $F(s,t,x)$, respectively.

An example of a possible degradation path of the production systems is shown in Figure 1 where the degradation is governed by a Gamma process with a shape function parameter $\gamma = 2.5$ and the scale parameter is set to $\eta = 0.5$. The figure shows also the system's failure threshold X_f separating the two possible zones where the system is either in-control or out-of-control.

The lifetimes of the system are represented by the random variable $T_f = \inf\{t : X(t) \geq X_f\}$ given as the first passage time when the degradation exceeds the failure threshold X_f. Its corresponding cdf is $G(t) = \Pr(T_f \leq t)$ computed as

$$\begin{aligned} G(t) &= \Pr(X(t) > X_f) \\ &= 1 - F(t, X_f) \qquad (3) \\ &= \bar{F}(t, X_f), \qquad (4) \end{aligned}$$

where $\bar{F}(\tau, X_p - x) = 1 - F(\tau, X_p - x)$. The pdf $g(t) = \partial G(t)/\partial t$ corresponding to the system lifetime T_f is given as

$$g(t) = \frac{\gamma}{\Gamma(\gamma t)} \int_{X_f/\eta}^{\infty} (\ln(u) - \Psi(\gamma t)) u^{\gamma t - 1} \exp(-u)\,\mathrm{d}u, \qquad (5)$$

where $\Psi(u)$ is the digamma function defined as the logarithmic derivative of the gamma function:

$$\Psi(u) = \frac{\mathrm{d}\ln(\Gamma(u))}{\mathrm{d}u}.$$

We also consider the random variable $T_p = \inf\{t : X(t) \geq X_p\}$ corresponding to the first passage time when the degradation hits the threshold X_p. Its corresponding cdf and pdf are, respectively, given as $H(t) = \Pr(T_p \leq t)$ and $h(t)$, and their respective expressions are, by analogy to $G(t)$ and $g(t)$, as follows:

$$H(t) = \bar{F}(t, X_p), \qquad (6)$$

$$h(t) = \frac{\gamma}{\Gamma(\gamma t)} \int_{X_p/\eta}^{\infty} (\ln(u) - \Psi(\gamma t)) u^{\gamma t - 1} \exp(-u)\,\mathrm{d}u. \qquad (7)$$

The production process starts as an *as good as new* system and degrades while producing items, i.e. the degradation is operation-dependent. When the system's degradation is below the failure threshold X_f, the system is said to be in an

in-control state, otherwise it sojourns in an *out-of-control* state. When no maintenance is carried out, the sojourn time in the in-control state is measured via the random variable T_f whose cdf is given by Equation (4). It is worth noticing here that even if the system's degradation exceeds the failure threshold, the system is still able to operate but produces low quality items. At the beginning of the production cycle, the system is assumed to produce items of good quality. However, item quality decreases as the production system's degradation increases. When the degradation of the system exceeds the degradation threshold X_p, a first proportion α_p of low-quality items are produced between the two degradation thresholds X_p and X_f, and a second proportion α_f of unacceptable items is produced when the degradation is beyond the failure thresholdX_f.

To assess the production system's degradation and, at the same time, to improve product quality, the production system is subjected to an inspection and maintenance strategy described as follows. The system is periodically inspected at time instants $k\tau$ $(k = 1, 2, \ldots)$, where τ is the inspection interval, a decision variable. Failures of the system are revealed only through inspections (i.e. failures are not self-announcing). After an inspection, the magnitude of the system's degradation is measured at cost C_i. If the degradation value exceeds the failure threshold X_f, a CM is carried out at cost C_c. If the degradation level is larger than the given degradation level X_p (see Figure 1), then a PM is performed at cost C_p. The degradation level X_p is a decision variable whose value must almost be equal to the failure threshold X_f. It is worth noticing that the choice of the PM threshold X_p will greatly impact the performance of the production system. If X_p is chosen to be close to the failure threshold X_f, the probability of the system to shift into the out-of-control state becomes large. Conversely, lower the value of X_p, lower is the failure probability and larger is the residual life of the production system. It is assumed that after performing either a CM or a PM, the system is restored to the in-control state in an *as good as new* condition. However, a CM is more costly than a PM. If the degradation level measured is found to be less than X_p, no maintenance action is performed and the system's condition remains as is.

So far, the classical assumptions made within the EPQ model are accounted for. In particular, the demand rate is constant, continuous and demand must be satisfied. The model studied here assumes an infinite time horizon and is an integrated expected costrate-based model, where the decision-maker simultaneously determines the inspection interval τ and the PM threshold X_p; the production is continuous, as long as the system's deterioration, evolving according to a time-dependent stochastic process, lies under the failure threshold X_f. Once the preventive or failure threshold is exceeded, the system is stopped for either preventive or corrective maintenance and is restarted after maintenance. One may observe that the decision variables considered in the proposed model allow the trade-off between production quality, inspection and maintenance costs.

3. Integrated production quality and CBM model

The objective of the optimisation model is to determine the optimal values of the two decision variables, i.e. the inspection interval τ and the PM threshold X_p, that minimise the expected total cost per unit of time $C(\tau, X_p)$ over an infinite time horizon. We have a regenerative process starting and ending at the instants of completed depletion of the inventory which follows either a preventive or a corrective replacement (see Figure 2). It follows from the theory of renewal reward processes that the long-run expected total cost per unit of time $C(\tau, X_p)$ is the average total cost $\mathbb{E}[\mathcal{C}]$ during an inventory cycle divided by the average length $\mathbb{E}[\mathcal{T}]$ of that inventory cycle:

$$C(\tau, X_p) = \frac{\mathbb{E}[\mathcal{C}]}{\mathbb{E}[\mathcal{T}]}. \tag{8}$$

The average total cost $\mathbb{E}(\mathcal{C})$ in a cycle is the sum of the expected inventory holding cost $\mathbb{E}[HC]$, the expected inspection and maintenance cost $\mathbb{E}[IMC]$, and the expected cost of replacing produced non-conforming items $\mathbb{E}[NCC]$. The expected inventory cycle length $\mathbb{E}[\mathcal{T}]$ is the sum of the expected maintenance (production) cycle and the expected time required for storage depletion (Figure 2). To compute these costs, we first evaluate the expected inventory cycle length $\mathbb{E}[\mathcal{T}]$.

3.1. The expected inventory cycle length

According to Figure 2, the expected inventory cycle length $\mathbb{E}(\mathcal{T})$ is computed as

$$\mathbb{E}[\mathcal{T}] = \frac{\rho}{d}\mathbb{E}[T_M], \tag{9}$$

where ρ is the production rate, $\mathbb{E}[T_M]$ is the expected maintenance (production run) cycle which ends either by a preventive or corrective replacement. The following lemma computes the value of $\mathbb{E}[T_M]$.

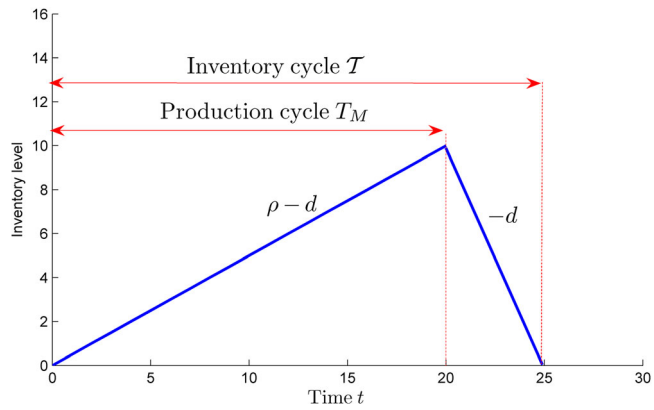

Figure 2. Inventory cycle of the EPQ model with inventory build-up and depletion period.

LEMMA 1 *The expected production cycle* $\mathbb{E}[T_M]$ *is computed as*

$$\mathbb{E}[T_M] = \sum_{k=1}^{\infty} k\tau \int_0^{X_p} f((k-1)\tau, x)\bar{F}(\tau, X_p - x)\,\mathrm{d}x, \tag{10}$$

where $\bar{F}(\tau, X_p - x) = 1 - F(\tau, X_p - x)$.

Proof The proof follows from the fact that the probability of a maintenance action being performed after the kth inspection is the probability that the accumulated system's degradation in the interval $[0, (k-1)\tau]$ is lower than the threshold X_p and the degradation $X(k\tau)$ measured at time $k\tau$ is greater than or equal to the PM threshold X_p. This probability is computed as

$$\begin{aligned}
&\Pr(X((k-1)\tau) < X_p;\ X(k\tau) > X_p)\\
&\quad = \Pr(\Delta X(0,(k-1)\tau) < X_p;\ \Delta X((k-1)\tau, k\tau) \geq X_p - X((k-1)\tau))\\
&\quad = \int_0^{X_p} f((k-1)\tau, x)\bar{F}(\tau, X_p - x)\,\mathrm{d}x.
\end{aligned}$$

■

3.2. The expected inventory holding cost

From Figure 2 and Lemma 1, the expected inventory holding cost $\mathbb{E}[HC]$ is computed as

$$\begin{aligned}
\mathbb{E}[HC] &= C_h \int_0^{\mathbb{E}[T_M]} (\rho - d)u\,\mathrm{d}u + C_h \int_{\mathbb{E}[T_M]}^{(\rho/d)\mathbb{E}[T_M]} (-du + \rho\mathbb{E}[T_M])\,\mathrm{d}u\\
&= C_h \frac{\rho(\rho - d)(\mathbb{E}[T_M])^2}{2d}.
\end{aligned} \tag{11}$$

3.3. The expected inspection and maintenance cost

According to the maintenance policy adopted, the production system is periodically inspected. Whenever either a PM or a CM is performed, the production system becomes as good as new. A PM incurs a cost of C_p while a CM incurs a cost of C_c. To compute the total expect cost incurred by inspection and maintenance, we first determine the probabilities of a cycle being ended by either a PM or a CM after an inspection. To do so, we consider two stochastic events E_k^p and E_k^c. The event E_k^p occurs when a PM is to be performed after the kth inspection, while the event E_k^c occurs when the CM is to be carried out following the kth inspection. The probabilities $\Pr(E_k^p)$ and $\Pr(E_k^c)$ are given by the following lemma.

LEMMA 2 *The probabilities* $\Pr(E_k^p)$ *and* $\Pr(E_k^c)$ *are computed as*

$$\Pr(E_k^p) = \int_0^{X_p} f((k-1)\tau, x)\left(\bar{F}(\tau, X_p - x) - \bar{F}(\tau, X_f - x)\right)\mathrm{d}x, \tag{12}$$

$$\Pr(E_k^c) = \int_0^{X_p} f((k-1)\tau, x)\bar{F}(\tau, X_f - x)\,\mathrm{d}x. \tag{13}$$

Proof The occurrence probability of E_k^p is

$$\Pr(E_k^p) = \Pr(X((k-1)\tau) < X_p; X_p \leq X(k\tau) < X_f),$$

which can be written as

$$\Pr(E_k^p) = \Pr(\Delta X(0,(k-1)\tau) < X_p; \quad X_p - X((k-1)\tau) \leq \Delta X((k-1)\tau, k\tau) < X_f - X((k-1)\tau)),$$

which in turn can be written as

$$\begin{aligned}\Pr(E_k^p) &= \int_0^{X_p} f((k-1)\tau, x)\left(\int_{X_p - x}^{X_f - x} f(\tau, y)\,\mathrm{d}y\right)\mathrm{d}x \\ &= \int_0^{X_p} f((k-1)\tau, x)\left(\bar{F}(\tau, X_p - x) - \bar{F}(\tau, X_f - x)\right)\mathrm{d}x.\end{aligned}$$

Similarly, the occurrence probability of the event E_k^c is obtained as

$$\begin{aligned}\Pr(E_k^c) &= \Pr(X((k-1)\tau) < X_p; X(k\tau) > X_f) \\ &= \int_0^{X_p} f((k-1)\tau, x)\bar{F}(\tau, X_f - x)\,\mathrm{d}x.\end{aligned}$$

■

The following proposition gives us the expected total inspection and maintenance cost in a cycle.

PROPOSITION 3 *The expected inspection and maintenance cost* $\mathbb{E}[IMC]$ *of the maintenance cycle* T_M *is computed as*

$$\begin{aligned}\mathbb{E}[IMC] = &\sum_{k=1}^{\infty} kC_i \int_0^{X_p} f((k-1)\tau, x)\bar{F}(\tau, X_p - x)\,\mathrm{d}x \\ &+ \sum_{k=1}^{\infty} \int_0^{X_p} f((k-1)\tau, x)\left(C_p\bar{F}(\tau, X_p - x) + (C_c - C_p)\bar{F}(\tau, X_f - x)\right)\mathrm{d}x.\end{aligned} \tag{14}$$

Proof This follows directly from the result of Lemma 2. If the maintenance cycle is equal to $k\tau$, it follows that from Equations (12) and (13) that the resulting expected inspection and maintenance cost is $(C_p + kC_i)\Pr(E_k^p) + (C_c + kC_i)\Pr(E_k^c)$. Therefore, the expected total inspection and maintenance cost can be written as

$$\mathbb{E}[IMC] = \sum_{k=1}^{\infty}(C_p + kC_i)\Pr(E_k^p) + (C_c + kC_i)\Pr(E_k^c).$$

After substituting $\Pr(E_k^p)$ and $\Pr(E_k^c)$ by their respective expressions, and performing basic algebraic operations, the result of the proposition is easily obtained. ■

3.4. The expected cost of producing non-conforming items

We assumed that a percentage α_p of non-conforming items are produced during the period where the production system sojourns in its in-control state though the degradation level is higher than the PM threshold X_p. In addition, while being in the out-of-control state, i.e. during the time period where the system's degradation is greater than the failure threshold X_f, a percentage $\alpha_f \geq \alpha_p$ of non-conforming items are produced. The expected total cost $\mathbb{E}[NCC]$ incurred by non-conforming items is given by the following proposition.

PROPOSITION 4 *The expected total cost* $\mathbb{E}[NCC]$ *corresponding to non-conforming items is*

$$\mathbb{E}[NCC] = \sum_{k=1}^{\infty} C_{nc}^{p}\alpha_p\rho\xi_k(k\tau)\int_0^{X_p} f((k-1)\tau,x)\left(\bar{F}(\tau,X_p-x)-\bar{F}(\tau,X_f-x)\right)\mathrm{d}x$$
$$+\sum_{k=1}^{\infty}\left(\int_0^{X_p} f((k-1)\tau,x)\bar{F}(\tau,X_f-x)\,\mathrm{d}x\right)\Bigg[C_{nc}^{f}\alpha_f\rho\zeta_k(k\tau)$$
$$+\,C_{nc}^{p}\alpha_p\rho\int_{(k-1)\tau}^{k\tau}\left(\bar{F}(t,X_p)-\bar{F}((k-1)\tau,X_p)\right)\left(\bar{F}(k\tau,X_f)-\bar{F}(t,X_f)\right)\mathrm{d}t\Bigg],$$

where $\xi_k(t)$ *and* $\zeta_k(t)$ *are defined as*

$$\xi_k(t) = \int_{(k-1)\tau}^{t}\left(\bar{F}(u,X_p)-\bar{F}((k-1)\tau,X_p)\right)\mathrm{d}u,$$
$$\zeta_k(t) = \int_{(k-1)\tau}^{t}\left(\bar{F}(u,X_f)-\bar{F}((k-1)\tau,X_f)\right)\mathrm{d}u.$$

Proof The proof is obtained by computing the conditional expectation on events E_k^c and E_k^c which represent the case where the production run cycle is ended by a CM or a PM, respectively. The probabilities of these two events are provided by Lemma 2. We have

$$\mathbb{E}[NCC] = \sum_{k=1}^{\infty}\mathbb{E}[NCC|_{E_k^p}]\Pr(E_k^p) + \mathbb{E}[NCC_{E_k^c}]\Pr(E_k^c).$$

In the case where the production run ended by a PM, $\mathbb{E}[NCC|_{E_k^p}]$ is

$$\mathbb{E}[NCC|_{E_k^p}] = C_{nc}^{p}\alpha_p\rho\int_{(k-1)\tau}^{k\tau}(k\tau-u)h(u)\,\mathrm{d}u,$$

where the integral part is

$$\int_{(k-1)\tau}^{k\tau}(k\tau-u)h(u)\,\mathrm{d}u = k\tau\int_{(k-1)\tau}^{k\tau}h(u)\,\mathrm{d}u - \int_{(k-1)\tau}^{k\tau}uh(u)\,\mathrm{d}u$$
$$= \int_{(k-1)\tau}^{k\tau}(H(u)-H((k-1)\tau))\,\mathrm{d}u. \tag{15}$$

Since $H(t) = \bar{F}(t,X_p)$, it follows that

$$\mathbb{E}[NCC|_{E_k^p}] = C_{nc}^{p}\alpha_p\rho\int_{(k-1)\tau}^{k\tau}(H(u)-H((k-1)\tau))\,\mathrm{d}u$$
$$= C_{nc}^{p}\alpha_p\rho\xi_k(k\tau). \tag{16}$$

In the case where the production run ended by a CM, $\mathbb{E}[NCC|_{E_k^c}]$ is the sum of the cost induced by the proportion α_p of non-conforming units produced during the interval $[T_p, T_f]$in addition to the cost induced by the proportion α_f of non-conforming units produced during the interval $[T_f, k\tau]$. Therefore,

$$\mathbb{E}[NCC|_{E_k^c}] = \rho\int_{(k-1)\tau}^{k\tau}\left[\left(C_{nc}^{p}\alpha_p\int_{(k-1)\tau}^{t}(t-u)h(u)\,\mathrm{d}u\right) + C_{nc}^{f}\alpha_f(k\tau-t)\right]g(t)\,\mathrm{d}t. \tag{17}$$

In a similar way as in Equation (15), the integral part $\int_{(k-1)\tau}^{t}(t-u)h(u)\,\mathrm{d}u$ of the above equation is equivalently written as

$$\int_{(k-1)\tau}^{t}(t-u)h(u)\,\mathrm{d}u = t\int_{(k-1)\tau}^{t}h(u)\,\mathrm{d}u - \int_{(k-1)\tau}^{t}uh(u)\,\mathrm{d}u.$$
$$= \xi_k(t)$$

Substituting the above result in the expression for $\mathbb{E}[NCC|_{E_k^c}]$, it follows that

$$\mathbb{E}[NCC|_{E_k^c}] = \rho C_{nc}^p \alpha_p \int_{(k-1)\tau}^{k\tau} \xi_k(t) g(t)\,\mathrm{d}t + C_{nc}^f \rho \alpha_f \int_{(k-1)\tau}^{k\tau} (k\tau - t) g(t)\,\mathrm{d}t. \tag{18}$$

Analogous to Equation (15) together with the fact that $G(t) = \bar{F}(t, X_f)$, the integral part $\int_{(k-1)\tau}^{k\tau} (k\tau - t) g(t)\,\mathrm{d}t$ may be written as

$$\begin{aligned}\int_{(k-1)\tau}^{k\tau} (k\tau - t) g(t)\,\mathrm{d}t &= k\tau \int_{(k-1)\tau}^{k\tau} g(t)\,\mathrm{d}t - \int_{(k-1)\tau}^{k\tau} t g(t)\,\mathrm{d}t \\ &= \int_{(k-1)\tau}^{k\tau} (G(t) - G((k-1)\tau))\,\mathrm{d}t \\ &= \zeta_k(k\tau). \end{aligned} \tag{19}$$

To find the integral $\int_{(k-1)\tau}^{k\tau} \xi_k(t) g(t)\,\mathrm{d}t$ in Equation (18), we note that from the definition of $\xi_k(t)$:

$$\begin{aligned} &\xi_k((k-1)\tau) = 0 \quad \text{and} \\ &\frac{\mathrm{d}\xi_k(t)}{\mathrm{d}t} = H(t) - H(k-1)\tau. \end{aligned}$$

Finding the integral by parts, one obtains

$$\begin{aligned}\int_{(k-1)\tau}^{k\tau} \xi_k(t) g(t)\,\mathrm{d}t &= \xi(k\tau) G(k\tau) - \int_{(k-1)\tau}^{k\tau} G(t)(H(t) - H((k-1)\tau))\,\mathrm{d}t \\ &= \int_{(k-1)\tau}^{k\tau} (H(t) - H((k-1)\tau))(G(k\tau) - G(t))\,\mathrm{d}t \\ &= \int_{(k-1)\tau}^{k\tau} (\bar{F}(t, X_p) - \bar{F}((k-1)\tau, X_p))(\bar{F}(k\tau, X_f) - \bar{F}(t, X_f))\,\mathrm{d}t. \end{aligned} \tag{20}$$

From the results of Equations (16), (19) and (20), the proof of Proposition 4 follows. ■

From the above results, the optimisation problem is to find the optimal joint values of the inspection time period τ and the PM threshold X_p which minimise the total expected cost rate:

$$\mathcal{J}(\tau, X_p) = \frac{\mathbb{E}[HC] + \mathbb{E}[IMC] + \mathbb{E}[NCC]}{\mathbb{E}[\mathcal{T}]}, \tag{21}$$

subject to the constraints:

$$\left(\frac{\rho}{d} - 1\right) \mathbb{E}[T_M] \geq M_D \quad \text{and} \quad X_p \leq X_f, \tag{22}$$

where M_D is the duration of maintenance, be it preventive or corrective, and is assumed to be constant. Recall here that $\mathbb{E}[HC]$, $\mathbb{E}[IMC]$, $\mathbb{E}[NCC]$ and $\mathbb{E}[\mathcal{T}]$ are, respectively, the expected holding cost, the expected inspection and maintenance costs, the expected cost of producing non-conforming items, and the expected inventory cycle length. The optimal solution determines the joint optimal inspection period τ^* and the PM degradation threshold X_p^*, which allows the computation of the resulting total expected cost rate $C(\tau^*, X_p^*)$.

4. Optimisation procedure

Finding an analytical optimal solution to minimise the objective function (21) subject to the constraints (22) is difficult. Thus, we propose a heuristic procedure for the problem based on an incremental search approach (Algorithm 4).

The following section discusses a test case to illustrate our proposed approach. The above algorithm is implemented and the obtained mathematical model is solved for various scenarios in order to derive decision-making policies for the organisations involved with the upgrade and operation of the refreshed systems. Cost structures of acquisition and upgrade are also suitably defined and commented.

Algorithm 1 Incremental Search Heuristic Pseudo-code for the minimisation of Equation (21)

1: Set $\Delta\tau$ and ΔX_p, the step sizes used to loop through values of τ and X_p.
2: Set $X_p = \Delta X_p$ and $\tau = \Delta\tau$.
3: Set $\mathcal{J}(\tau, X_p) = \infty$
4: **while** $X_p < X_f$ **do**
5: **while** $\tau < \tau_{max}$ **do**
6: Compute $(\frac{\rho}{d} - 1)\mathbb{E}[T_M]$
7: If $(\frac{\rho}{d} - 1)\mathbb{E}[T_M] \geq M_D$ Compute the value of the objective function $\mathcal{J}(\tau, X_p)$
8: Set $\tau = \tau + \Delta\tau$
9: **end while**
10: Compute the minimum value $\mathcal{J}^*(\tau, X_p)$ of $\mathcal{J}(\tau, X_p)$.
11: Store the current values of $\mathcal{J}^*(\tau, X_p)$, τ and X_p
12: Set $X_p = X_p + \Delta X_p$
13: **end while**
14: Select the lowest value of $\mathcal{J}^*(\tau, X_p)$ and its corresponding decision variables τ and X_p from all the values stored in Step 11.
15: To refine the results, use the retrieved values of τ and X_p, decrease the step sizes of $\Delta\tau$ and ΔX_p and go to Step 2.
16: Set $\tau^* = \tau$, $X_p^* = X_p$, and $\mathcal{J}^*(\tau, X_p) = \mathcal{J}(\tau^*, X_p{}^*)$

Table 1. Results obtained in Experiment #1: case of no PM.

τ^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f)$
0.44	45.01	32.96	351.94	1.05	4.19	102.84

5. Numerical examples

In this section, numerical examples are investigated to demonstrate the accuracy and the validity of the proposed approach. Data used within these experiments are randomly chosen only for illustration purposes. We start with a production system whose random degradation is governed by a stationary gamma stochastic process $\{X(t) : t \geq 0\}$ with the scale parameter set to $\eta = 15$, and shape parameter set to $\gamma = 1.8$. The production system fails whenever its degradation reaches the failure threshold $X_f = 15$. The demand and production rates are set to $d = 50$ and $\rho = 200$ respectively. The holding cost rate per production unit is set to $C_h = 0.1$. Because of the negligible maintenance time assumption, $M_D = 0$. The rest of the data are indicated within each experiment.

5.1. Experiment #1: impact of the PM threshold X_p

To show how important the value of the preventive threshold X_p is, let us first assume that no PM decision is made (i.e. $X_p = X_f$). The objective of the manufacturer is then reduced to finding the optimal value of the inspection period τ. In this experiment, expected costs corresponding to PM and CM and inspection are set respectively, to $C_p = 120$, $C_c = 350$, $C_i = 0.5$. The proportions α_f and α_f of non-conforming items produced during, respectively, the out-of-control state and in-control state are set to $\alpha_f = 40\%$ and $\alpha_p = 10\%$. The unitary cost C_{nc}^f and C_{nc}^p of non-conforming items produced during, respectively, the out-of-control state and in-control state are set to $C_{nc}^f = 10$ and$C_{nc}^p = 1$.

The optimisation problem of Equation (21) subject to constraint (22) is solved. The results obtained are shown in Table 1. From this table, the optimal inspection policy suggests inspections each $\tau^* = 0.44$ units of time. The resulting cost due to the non-conforming items production is $\mathbb{E}[NCC] = 45.01$. The expected production run cycle is $\mathbb{E}[T_M] = 1.05$ resulting then on an expected production quantity of $Q = \rho\mathbb{E}[T_M] = 208.63$. This inspection and maintenance policy results in an expected total cost rate of $\mathcal{J}(\tau^*, X_f) = 102.84$.

Now let us examine the case where the PM threshold X_p is an input parameter set by the decision-maker rather than being a decision variable. If the PM threshold is set to $X_p = 2$, the results obtained are reported in Table 2. Inspection are performed periodically each $\tau^* = 0.55$ time units. This inspection and maintenance plan maintenance results in an expected cost of non-conforming items produced of $\mathbb{E}[NCC] = 35.35$. The resulting expected production run cycle is $\mathbb{E}[T_M] = 0.59$ which implies an expected production quantity $Q = 117.56$. The expected total cost obtained in this case is $\mathcal{J}(\tau^*, X_p) = 109.68$.

Table 2. Results obtained in Experiment #1: case where $X_p = 1$.

τ^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f)$
0.55	35.35	10.37	210.16	0.59	2.35	109.68

Table 3. Results obtained in Experiment #2: case of $\tau = 0.1$ and $\tau = 0.8$.

τ	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau, X_p^*)$	Q
0.05	3.00	0.06	2.76	161.30	0.30	1.21	136.98	60.63
0.8	10.83	128.47	36.28	310.9	1.10	4.40	108.58	219.95

Table 4. Results obtained in Experiment #3: case of joint optimal solution.

τ^*	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f^*)$	Q
0.22	9.13	6.37	14.81	238.72	0.70	2.81	93.18	140.54

From Tables 1 and 2, when no PM is performed, the optimal inspection and maintenance plan allows the production to operate for a long time with an increased number of non-conforming items. The expected cost $\mathbb{E}[NCC]$ of non-conforming items produced represents approximately 44% of the expected total cost. However, this cost represents 32% of the expected total cost when the maintenance decision-maker fixes the PM level to $X_p = 1$. Furthermore, a high value of the PM threshold implies a high risk of failures which in turn increases the inspection and maintenance costs in addition to non-quality cost. In the reverse case, i.e. when the PM level is set to a low value, the number of inspections is decreased and the production run cycle is shortened leading to a lower production quantity. Therefore, it is important to find a joint inspection period together with the appropriate PM level to ensure a trade-off between production and holding cost, on one hand, and inspection and maintenance costs, on the other hand. Such a joint optimal solution is discussed in the next experiment.

5.2. *Experiment #2: impact of the inspection period τ*

This experiment considers that the inspection period is no longer a decision variable but fixed at $\tau = 0.05$ and $\tau = 0.8$. For each value of τ, and the same data used in the first experiment, the optimisation problem of Equation (21) and constraint (22) is then solved for the only decision variable, namely the PM threshold X_p. Table 3 presents the results obtained. The optimal PM plans are obtained for $X_p^* = 3.00$ and $X_p^* = 10.83$, respectively, for $\tau = 0.05$ and $\tau = 0.8$. From Table 3, one may observe that similar conclusions as those made in Experiment #1 are still valid. Besides, by comparing the results of Table 3 with the one reported in Tables 1 and 2, we note that the solutions obtained when the PM is fixed are more costly than the one obtained in the case where the inspection period is fixed.

5.3. *Experiment #3: case of joint optimal inspection and maintenance decisions*

This experiment uses the data of Experiment #1, except that the PM threshold X_p is now considered a decision variable rather than an input parameter. The optimisation problem is now solved for the joint decision variables τ and X_p. The overall results obtained are shown in Table 4. The optimal solution obtained suggests periodic inspections each $\tau^* = 0.22$ time units, and to perform a PM whenever the degradation threshold reaches the value $X_p^* = 9.13$. This inspection and maintenance plan lies between those obtained in Experiment #1. The resulting expected total cost rate is $\mathcal{J}(\tau^*, X_p^*) = 93.18$ which is lower than those obtained in the first experiment above. In fact, the joint optimal solution ensures a balance between the costs of production, holding and non-quality in addition to inspection and maintenance costs.

5.4. *Experiment #4: impact of non-conforming production costs*

This experiment aims to show how the costs of non-conforming items impacts production, inspection and maintenance decisions. The data used are that of Experiment #1 except that the cost C_{nc}^f of non-conforming items produced in the out-of-control state is varied. The results obtained are reported in Table 5. From this table, one may observe that when the cost C_{nc}^f increases, the inspection period length reduces implying that more inspection should be performed. It is also seen that more frequent inspections decrease the production run cycle, and consequently the EPQ reduces.

Table 5. Results obtained in Experiment #4: impact of the non-conforming production costs.

C_{nc}^f	τ^*	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f^*)$	Q
1	2.97	15.00	220.28	265.68	352.97	2.98	11.90	70.64	595.18
3	1.05	15.00	80.59	57.93	351.05	1.39	5.56	88.44	277.92
5	0.34	8.80	8.68	17.23	249.66	0.76	3.03	91.58	151.55
10	0.22	9.13	6.37	14.81	238.72	0.70	2.81	93.18	140.54
15	0.15	8.54	3.79	12.15	222.39	0.64	2.55	94.41	127.28
20	0.14	8.86	4.46	12.39	224.93	0.64	2.57	94.84	128.52
30	0.13	9.09	5.80	12.47	226.49	0.64	2.58	95.69	128.94
50	0.12	9.16	8.16	12.29	226.09	0.64	2.56	97.07	128.02
70	0.10	8.78	7.36	11.13	218.91	0.61	2.44	98.28	121.80

Table 6. Results obtained in Experiment #5: impact of maintenance costs.

C_p	τ^*	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f^*)$	Q
10	0.10	2.05	0.47	3.07	66.46	0.32	1.28	56.24	64.02
30	0.12	3.80	0.90	5.27	104.44	0.42	1.68	67.17	83.82
60	0.14	6.24	1.71	8.66	156.59	0.54	2.15	78.60	107.48
90	0.19	7.03	3.72	10.84	192.78	0.60	2.40	87.05	120.25
120	0.22	9.13	6.37	14.81	238.72	0.70	2.81	93.18	140.54
150	0.24	10.65	8.89	17.95	273.57	0.77	3.09	97.75	154.69
180	0.29	11.75	14.80	21.36	301.91	0.84	3.38	100.75	168.77
210	0.36	13.42	26.79	26.83	331.59	0.95	3.78	102.36	189.14
240	0.44	15.00	45.01	32.96	351.18	1.05	4.19	102.84	209.63
270	0.44	15.00	45.01	32.96	351.18	1.05	4.19	102.84	209.63
300	0.44	15.00	45.01	32.96	351.18	1.05	4.19	102.84	209.63

Table 7. Results obtained in Experiment #6: impact of the inspection cost.

C_i	τ^*	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f^*)$	Q
0.50	0.22	9.13	6.37	14.81	238.72	0.70	2.81	93.18	140.54
2.00	0.25	9.00	8.39	15.33	244.90	0.71	2.86	94.64	142.97
4.00	0.29	8.95	11.71	16.23	253.26	0.74	2.94	96.26	147.10
6.00	0.34	8.72	16.54	17.09	259.05	0.75	3.02	97.62	150.93
10.00	0.40	8.08	23.01	17.50	262.75	0.76	3.06	99.91	152.76
15.00	0.45	7.58	29.37	17.93	266.95	0.77	3.09	102.28	154.60

5.5. *Experiment #5: impact of maintenance costs*

This experiment investigates the impact of the maintenance costs on the decision variables. This experiment also uses the data of Experiment #1 except that the PM cost C_p is varied. The optimisation problem is solved and Table 6 shows the solutions obtained. This table shows that an increase in the PM cost reduces the number of inspections and, simultaneously, increases the PM threshold. Increasing the PM cost also increases both the expected total inspection and maintenance cost and the production run cycle which in turn increase the EPQ. However, the non-quality cost resulting from the non-conforming items production also increases. When the PM cost becomes significant, the solution obtained converges to that of the case where only the failure threshold is accounted for.

5.6. *Experiment #6: impact of inspection cost*

This experiment investigates the impact of the holding on the manufacturer decisions. This experiment also uses the data of Experiment #1 except that the inspection cost C_i is made variable. The optimisation model is solved for the joint decision variables τ and X_p values for different PM cost C_p. Table 7 shows the solutions. With an increase in the inspection cost, both the number of inspections and the PM threshold decrease. Increasing the inspection cost also increases both the expected total inspection and maintenance cost and the production run cycle which in turn increase the economic production quantity. However, the quality cost resulting from the non-conforming item production increases.

Table 8. Results obtained in Experiment #7: impact of the holding cost.

C_h	τ^*	X_p^*	$\mathbb{E}[NCC]$	$\mathbb{E}[HC]$	$\mathbb{E}[IMC]$	$\mathbb{E}[T_M]$	$\mathbb{E}[\mathcal{T}]$	$\mathcal{J}(\tau^*, X_f^*)$	Q
0.1	0.22	9.13	6.37	14.81	238.72	0.70	2.81	93.18	140.54
0.20	0.21	8.40	5.35	26.78	228.61	0.67	2.67	98.33	133.61
0.30	0.18	7.95	3.61	35.97	219.57	0.63	2.53	103.27	126.44
0.50	0.16	7.89	2.76	57.17	216.29	0.62	2.47	112.67	123.47
0.70	0.14	7.51	1.96	72.86	209.61	0.59	2.36	121.57	117.80
0.90	0.13	6.61	1.50	80.41	199.29	0.55	2.18	129.74	109.14

5.7. Experiment #7: impact of the holding cost

This experiment investigates the impact of the holding cost. This experiment also uses the data of Experiment #1 except that the holding cost C_h is varied. The optimisation model is solved and Table 8 shows the solutions obtained. This table shows that an increase in the holding cost reduces the PM threshold and, simultaneously, increases the number of inspections. Increasing the holding cost also decreases both the expected total inspection and maintenance cost and the production run cycle which in turn decreases the economic production quantity. The quality cost resulting from the non-conforming items production also decreases.

6. Conclusion

This paper proposes an optimisation model for integrated production quality and CBM planning. The production system is stochastically deteriorating and inspections are periodically performed to assess the actual degradation status of the system. It is assumed that the non-conforming items, those that are produced during the time interval where the degradation exceeds the specification threshold level, are replaced at a higher cost either via overtime production or spot market purchases. The resulting integrated optimisation model is solved using an incremental heuristic method and tested on some numerical cases.

Two major assumptions were made in this model: (1) the fixed cost is assumed to be negligible; (2) the holding cost is based on the average maintenance cycle. We are currently investigating the cases where these two assumptions are relaxed. Also, the current model is limited to a single product single machine. It would be worthwhile to investigate more complex cases to deal with multi-product and multi-unit production systems. An important issue that could be addressed in future research is the consideration of imperfect maintenance actions and their impact on CBM and production decisions.

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No potential conflict of interest was reported by the authors.

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