A DESIGN EXPERIMENT EXPLORING THE INFLUENCE OF VISUAL AND KINESTHETIC TOOLS IN LEARNING GRADE 8 LINEAR ALGEBRA IN A NAMIBIAN SECONDARY SCHOOL

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DECLARATION OF ORIGINALITY

I, Enos Kalua, student number: 16K7503, declare that this thesis, *A design experiment exploring the influence of visual and kinesthetic tools in learning Grade 8 linear algebra in a Namibian secondary school*, is my own work written in my own words and has not been submitted at any other university. Where I have used other people's ideas and words, I have acknowledged them according to the Rhodes University Education Department Referencing Guide.

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ABSTRACT

Based on a broad literature review, understanding algebra is a challenge among learners in middle schools around the world. Early researches also indicated that algebra is often taught through inherent symbols and procedures. This does not exclude Namibian learners in secondary schools whom I have worked with for over 10 years. Examination reports (2014, 2016 and 2017) serve as evidence that learners performed poorly (below 45%) in the area of algebra, with these reports indicating that teachers need to strongly emphasise the issue of solving linear equations. Therefore, this study presents a proposed design research in an attempt to help learners develop meaningful understanding of linear algebra at Grade 8 level. Eight learners whose ages ranged from 13 - 14 years from one Namibian secondary school in Oshikoto region, in northern Namibia, were the participants in this study. The learners represented different groups of learning abilities, ranging from low learning abilities to high learning abilities.

The designed programme for this intervention consisting of eight lessons was planned for three weeks and the lessons were conducted in the afternoon to avoid any interruption with normal learning hours. The study used four tools for data collection, namely, benchmark tests (pre-test and post-test), observation, focus groups and unstructured interviews. The data collected for this study was inductively analysed.

The purpose of this study was to determine whether and how the specific visual and kinaesthetic teaching tools (diagrams, expansion box and balance method) used may have contributed to learners' understanding of algebraic concepts and techniques (variables, expressions and equations). The study used diagrams (geometrical plane shapes) for separating terms, an expansion box for expanding brackets and the balance method for solving linear equations. The study revealed the use of diagrams helped the learners in understanding the separation of variable and constant terms when simplifying expressions through addition and/or subtraction. Moreover, the study also revealed that the use of an expansion box was useful for the learners in understanding expansion of brackets in expressions with more than one term. Regarding the use of the balance method, the study showed that learners were already able to solve linear equations by the transfer method, hence, the balance method was not necessary.

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DEDICATION

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CHAPTER ONE

BACKGROUND AND CONTEXT OF THE STUDY

1.1 INTRODUCTION

This chapter provides an orientation to the study of the use of visualisation and multiple representations in teaching Grade 8 linear algebra at a secondary school in the Oshikoto region in Namibia. In order to do this, it seeks to introduce the reader to the context and background of the study before presenting the rationale and value; the goals and research questions of the study are also included in this chapter. This chapter also provides the orientation of the research and methodology of the study before ending with an outline of the whole thesis.

1.2 BACKGROUND AND CONTEXT OF THE STUDY

Since secondary school, algebra has been the area of mathematics that has appealed to me most. Focusing on it in this research, therefore aligns with my personal and professional interest in this branch of mathematics. However, algebra, its definitions and teaching methodologies has a long history within Western mathematical thinking.

Algebra is a mathematics domain that substitutes letters for numbers (Russel, 2018). There are branches of algebra learning described by Russel (2018) in middle school mathematics which are: a) elementary algebra – deals with the general properties and the relations between them, b) abstract algebra – deals with abstract algebraic structures rather than the usual number system, and c) linear algebra – focuses on linear expressions and equations such as linear functions and their representations through matrices and vector spaces.

Learning of algebra links to that of arithmetic and includes the generalisation of numbers through the use of letters and symbols referred to as variables (Subramaniam, 2011). Algebra is described as a gateway to the learning of all mathematics as it provides the language in which mathematics is taught (Kieran, 1992). However, the procedures of computing arithmetic expressions do not equip learners to generalise algebraic expressions (Subramaniam, & Banerjee, 2011). Sfard (1991) suggested that to ensure success in the learning of algebra, learners need to suspend operations for a while and focus on properties which can be used to develop the meaning of algebraic statements such as, expressions and equations.

Kaput (1989) argued that learners' problems in learning algebra is influenced by the innate difficulties in working with the accurate but hidden meanings of formal algebraic symbols. He further pointed out the lack of connections between symbols and other representations that would provide clear feedback on the appropriate action. Because of this, learning of algebra needs to be made meaningful for the learners to see its relevance (Stacey & Chick, 2004). They go on to state that objects and processes of algebra need to be made meaningful for the learners to develop ideas of what algebra is beyond symbolic manipulation. The search for symbolic meaning in algebra has prompted a variety of teaching approaches in the field of mathematics (Stacey & Chick, 2004).

To succeed in manipulating algebraic symbols, there is a need to understand the structural properties of mathematical operations and relationships (Kieran, 2007). Sfard and Linchevski (1994) asserted that "the sense of meaningfulness comes with the ability of 'seeing' abstract ideas hidden behind the symbols" (p. 224). Though drawing meaning from the algebraic structure can be elusive, the source of meaning is fundamental to the learning of algebra (Kieran, 2007).

The general issues in understanding algebra apply very much to the Namibian context. Namibia as a nation has a problem of poor performance in mathematics at the junior secondary phase. Despite all the interventions made since independence in 1990, the failure rate is still very high. For example, in 2015, 70% of high school learners in the Kunene, Karas and Hardap regions failed Mathematics (Ministry of Education, 2016). At a mathematics congress in 2010, the Minister of Education announced that the pass rate in Mathematics for the last five years was unimpressive and not worth celebrating (New Era Newspaper, 2010). As mathematics is a compulsory subject in calculating final points aggregates, poor results directly affect the learners' final results. Examination reports (2014, 2016 and 2017) showed that learners performed poorly (below 45%) in algebra. The same reports indicated that teachers needed to give greater emphasis to the issue of solving linear equations. The weighting of algebra in all the combined mathematics papers is

40%, so an improvement in understanding algebra is crucial. One of the possible causes of this problem could be the poor understanding of algebraic concepts.

Secondary school mathematics in Namibia is divided into seven themes, namely: numbers, algebra, measurements, money and finance, geometry, trigonometry, statistics and probability. According to Jupri, Drijvers and Heuvel-Panhuizen (2014) algebra is recognised by many learners and educators as the most difficult, yet it has a strong influence on other areas of mathematics. One of the key algebraic concepts from junior secondary school to university is linear equations (Wati, Fitriana, & Mardiyana, 2018). Linear equations are essential for learners to master advanced topics. According to Wati et al., (2018) learners' ability to solve linear equations is low (39.32%) in Indonesian schools. The same applies to Namibia. For example, in the 2016 Junior Secondary Certificate Mathematics Examiners' report, algebra emerged as challenging to most candidates. On Question 17, Paper 2 the examiners remarked that, "this question was extremely-poorly answered. Candidates didn't know the difference between an equation and an expression" (Examination Report, 2016, p. 228). Research studies cited in Wati et al. (2018) indicated that the learners who have difficulties in solving linear equations will also have severe learning difficulties in other subjects, such as physics and engineering. For this reason, it is essential to devise better teaching strategies to help the learners in the early stages of their mathematics learning.

From my ten years personal teaching experience, I have come to observe persistently poor performance in algebra among learners across Grade 8 to 12, especially in dealing with linear algebra. My observations suggested to me that the learners did not learn algebraic manipulation of expressions and equations with understanding but rather by trying to memorise procedures. A different instructional intervention to understanding algebraic concepts among junior secondary phase learners might contribute to changing this situation. This study investigated the use of visualisation (diagrams) and multiple representation (expansion box) in a teaching programme for linear algebraic concepts and examine how this may deepen the algebraic understanding of learners.

1.3 RATIONALE AND VALUE OF THE STUDY

My motivation to conduct this study initially was rooted in the Namibian learners' poor performance in algebra mainly at the junior secondary phase (Grade 8-10). The teaching of algebra has been widely researched outside Namibia and it was found that learners taught through mere procedures do not do well in algebra. This could suggest one of the reasons that learners have a poor understanding of algebra.

Secondly, algebra has been my favourite topic in mathematics since high school and I found it fun to solve problems that required algebraic skills. As a mathematics teacher, I try to incorporate algebra in teaching other mathematics topics to demonstrate how algebra can help the learners understand those topics. Both the poor performance of learners country-wide, and my own interest in algebra, prompted me to undertake a study on how my teaching will best be of help to learners in understanding linear algebra and developing a love of algebra.

Therefore, my argument on the teaching of algebra is that if learners continue to be taught through procedures, they are being deprived of seeing the underlying structures of the algebraic concepts. Through teaching other topics in mathematics, such as, geometry and mensuration, I have observed that the use of manipulatives and visuals stimulate learners' interest in learning these areas of mathematics. These tools also appear to provide assistance in developing conceptual understanding in the learners. The kinaesthetic tools referred to in this study refer to the use of concrete objects used in the programme to enhance the difference between unlike terms in algebraic statements. The use of a balance scale to solve linear equations further aligns with the definition of kinaesthetic tools.

It is thus against this background that I suggest making use of visuals, namely diagrams and an expansion box, as described in Chapter Three, to boost learners' understanding of abstract/symbolic algebraic representation forms. The hope is that this will help learners see the underlying deep structures of the concept, and hence develop a deep and lasting understanding.

1.4 RESEARCH GOALS AND QUESTIONS OF THE STUDY

The aims of this study are to determine whether and how the specific visual and kinaesthetic teaching tools (diagrams, expansion box and balance method) used may have contributed to learners' understanding of algebraic concepts and techniques (variables, expressions and equations).

The three specific teaching tools used were:

- Diagrams (geometrical plane shapes) for separating terms;
- An expansion box for expanding algebraic brackets;
- The balance method for solving linear equations.

The development of the use of diagrams and an expansion box is described in Chapter Three and the balance method is discussed in Chapter Two.

The goals of the study stated above will be achieved through responding to the following research questions:

1. *What was the possible influence of the use of diagrams on learners' capacity to separate terms?*
2. *What was the possible influence of the use of an expansion box on learners' capacity to expand brackets?*
3. *How did the use of the balance method influence the learners' understanding in solving linear equations?*

The learners' experience of the programme will be briefly detailed in chapter 4.

1.5 RESEARCH ORIENTATION AND METHODOLOGY

This study is located in the interpretive paradigm and is therefore concerned with understanding the world we live in. Creswell (2003) claimed that interpretivist researchers discover reality through participant's views, and their own background and experiences.

This study researches a teaching intervention and takes the form of a design experiment. A design experiment "entails both 'engineering' a particular form of learning, and systematically studying those forms of learning within the context defined by means of supporting them" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). Roosevelt Hass (2001) defined the design experiment as a type of research that puts educational experiments in real world settings to see what works in practice.

When planning a design experiment there are three phases a researcher has to consider. These are described in more detail in Chapter Three, but briefly:

Phase 1: *Preparing for the experiment:* The aim of this phase of a design research experiment is to formulate a local instruction theory that can be refined and explained while conducting the intended design (Gravemeijer & Cobb, 2006).

Phase 2: *The design experiment*. After the preparation work has been done, the next phase is to conduct the design experiment. During the enactment of the instructional activities in the classroom, the researcher analyses the actual process of the learners' participation and learning (Gravemeijer & Cobb, 2006).

Phase 3: *The retrospective analysis.* This is the other aspect of design research that is conducted of the entire data set collected during the experiment. The focus of the introspective analysis depends on the theoretical intent of the design experiment (Gravemeijer & Cobb, 2006).

1.6 THESIS OUTLINE

This thesis consists of six chapters.

Chapter one provides the overview of the study. This chapter provides the background, context and the rationale of the study. The research goals and questions used to guide the study as well as a brief description of the research orientation and methodology are all included in this chapter. This chapter ends with the thesis outline.

Chapter two provides the review of the literature. This chapter provides the general views of algebra learning and reviews the learning of specific concepts in algebra. This chapter also theorises the learning of algebra by means of multiple representations and visualisation. In other words, it presents the theoretical views of using multiple representations and visualisation in mathematics and in algebra in particular.

Chapter three presents the research design and methodology of the study. In this chapter, I describe and motivate my choice of the interpretive orientation in which my study is located. The chapter provides the description of the design experiment, the participants and the research site. The methods and the tools used to collect data are also provided in this chapter together with their meaning and the motivation for choosing them for this study. These include the benchmark tests, observations, focus groups, journals and unstructured interviews. Lastly, this chapter highlights how the data was analysed and how ethics and validity were ensured for the study.

Chapter four presents the data collected for the study. In this chapter, I present the raw data as obtained from the participants in response to the research questions.

Chapter five provides the analysis and the discussion of the data. The data analysed and discussed in this chapter was drawn from the benchmark tests and the observations.

Chapter six provides the conclusion and final findings of this study, along with recommendations for future research.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter presents the literature review. Firstly, it discusses the views of algebra learning in general from other researchers. Secondly, it discusses the views of algebra on specific concepts, namely: understanding variables, separation of terms, expansion of brackets and solving linear equations. The last part of this chapter discusses theoretical views of learning through multiple representations and visualisation.

2.2 GENERAL VIEWS ON ALGEBRA LEARNING

When learning algebra, learners are challenged by common errors and misunderstandings of algebra that arise from the meaning of symbols including letters, the transition from numerical data to variables with functional rules or patterns and the recognition of structure (Kieran, 1989). Literature reveals that some of these challenges may be caused by a teaching approach that mainly focuses on the process of calculation rather than on relational and/or structural aspects (Poon & Leung, 2010). This implies that learners often adopt formal rules and procedures without conceptual understanding.

Kaput (1989) as cited in Kieran (2007) argued that the learners' problem in learning algebra is compounded by the inherent difficulties and challenges in dealing with the formal symbols, and the lack of linkages to different representations that might give feedback on the appropriate actions taken.

In general, the teaching of algebra is one dimensional, meaning that it is merely taught through procedures which turn out to be ineffective (Kieran, 2004). Noss, Healy and Hoyles (1997) argued that even though some learners are able to apply the correct methods and procedures to solve problems, they often cannot articulate general relationships in a natural language. Reinforcing the argument, Stacey (1989) explained that quite a small number of learners often respond to questions that require explanations to provide mathematical justifications. This indicates that there is a need for a method that prompts understanding in the teaching and learning of algebra. The quality of activities given to learners generally requires them merely to work out or solve problems presented through procedures rather than demanding meaning and justification (Stacey, 1989).

Capraro and Joffrion (2006) reiterated that learners often focus on computational procedures and rules rather than understanding. This has resulted in learners resorting to memorisation of rules and procedures, leading to a belief that algebra is all about rules and procedures with no conceptual understanding (Kieran, 2004). Stacey (1989) added that learners have a tendency to use algebraic procedures but that they do not consider the reason why a certain rule or procedure is needed. She proposed that the learners need a balance of the conceptual understanding and procedural skills as they develop understanding of algebraic concepts.

According to Kieran as cited in Chow (2011, p. 9), “students’ learning difficulties are centered on the meaning of letters, the change from arithmetical to algebraic conventions, and the recognition and use of structure”. Learners do not perceive algebra as an extension of arithmetic experience, instead they take it as a concept that is completely distinct from their previous numerical experience. This comes as a result of lack of exposure to the algebraic thinking during arithmetic learning, with some researchers asserting that “if students are not exposed to the ideas and connections in algebraic thinking, then their opportunities to learn these ideas are severely limited” (Chappel & Strutches, 2001). However, some researchers have different views about the connection between arithmetic and algebraic thinking.

Sfard (1995) asserted that there is a typical justification that the behaviour of algebraic expressions is expected to be similar to that of arithmetic expressions. If a learner lacks proficiency in arithmetic calculations, then he/she is likely to face challenges when learning to manipulate algebraic expressions (Drijvers, 2010; Bokhove, 2011). They further expound that learners see addition as meaning the same as multiplication, thus, ending up multiplying terms over addition. According to Larbi and Mavis (2016) it is the insufficient knowledge of arithmetic structures and understanding of operations that hinders the learners’ acquisition of algebraic concepts. This point is supported by Drijvers and Jupri (2014) who stated that the lack of proficiency in the use of rules of arithmetic calculations resulted in learners committing errors when dealing with algebraic expressions.

As an example, the various uses of the minus sign (-) in algebra is a challenge to most early algebra learners and can constitute a significant conceptual barrier to their ability to give meaning to algebraic symbols and processes (Kieran, 2007). Researchers highlight the challenges that early algebra learners have with the subject, indicating for example, that the learners (a) believe the equal sign represents an operator that gives the output of the right hand side as a result of the left hand side (b) do not recognise the distributive property and (c) do not use the mathematical symbols to represent the relationships among quantities (Carraher & Schliemann, 2007).

The structure of an algebraic representation refers to the mathematical features and relationships such as the number, type, position of quantities and variables, position of operations, equality, relationship between quantities, operations, equality/inequality, complexity etc. (Star et al., 2015, p. 16). The structure of algebraic statements refers to the relationships among objects or quantities rather than finding solutions (Khuluq, 2015). When we use algebraic problem solving strategies to solve problems, we basically convert word problems into algebraic sentences that consist of terms (constants and variables), expressions of equality/inequality. Learners need to understand and manipulate these expressions by exploiting the structure of the representations (Khuluq, 2015). This understanding leads to separation and combination of terms (checking separation by + or - or combinations such as ×, ÷, ( )), expansion, substitution and factorisation etc. These skills are applied when algebra is used to solve problems (solving equations accurately). An illustration of structure in algebraic representations is shown below:

Consider these three equations:

$$2x + 8 = 14$$

$$2(x + 1) + 8 = 14$$

$$2(3x + 4) + 8 = 14$$

Though the equations appear to differ, they have similar structures: in all three equations, 2 multiplied by a quantity, plus 8, equals 14.

Figure 2.1: **Adapted from IES (2015, p. 16)**

Paying attention to the structure helps the learners make connections, note similarities and differences in an expression and understand the characteristics of algebraic expressions and equations presented in any format (symbolic, numeric, verbal, diagrammatic or graphic) (Star et al., 2015).

The use of precise language that reflects mathematical structure, modelling, modelling precise language, use of reflective questioning to stimulate noticing of structure (similarities and differences), use of diagrams and use of multiple algebraic representations, can help the learners to understand the structure of algebraic representations (Star et al., 2015). In practice learners are expected to apply these skills to notice that 4y + 4z is different from 4y + 4 etc.

In the case of

Y = mx + c and y = 3x + 4

A cross examination of the structure should help the learners to notice the similarities and differences.

***Figure 2.2:* Adapted from IES (2015, p. 16)**

- Similarities on figure 2.2: (a) they are both equations of a straight line; (b) gradient is 3; (c) y intercept is + 4.
- Differences on figure 2.2: (a) gradient form; (b) intercept explicitly.

Expressions are part of the algebraic concepts that learners often fail to understand. Expressions can be referred to mathematical phrases that consist of numbers and letters (variables), e.g. $2x$, $x + 3$, $y + x$ (Kieran, 2004). In mathematics, an algebraic expression is built up from integer constants, variables and basic operators (add, subtract, multiply, divide) that connect them together. Kieran (2004) emphasised that the underlying objects of algebraic expressions and equations are variables. These are discussed in further detail in the next section.

2.3 UNDERSTANDING VARIABLES

A vital aspect of algebra is understanding the meaning of a variable. Linsell, Cavanagh and Tahir (2013) described variables as pronumerals which they defined as "letters representing numbers".

Learners often treat a pronumeral as representing an object 'itself' rather than representing a number of objects – this leads them to committing errors in their practice of algebra. Poor understanding of variables hinders the learners' meaningful understanding of algebraic concepts: their expressions and equations (Macgregor & Stacey, 1997). Examples of errors learners tend to commit frequently, are seen in simplifying expressions, say $25x - 3$ to $22x$ or $3xy - y$ to $2x$. Kieran (2006) termed this type of error as "deletion". She further suggested that these types of errors are prevalent because of poor understanding of variables as learners view letters as labels for concrete objects and not representing a number of objects. Kieran (1992) indicated that when working with algebraic expressions, learners attempt to "slap a veneer of names on an arithmetic base, but all the work remains in the arithmetic" (p. 4). Learners often perceive variables as abbreviations or labels rather than representing quantities, assigning values to letters based on their alphabetical positions and unable to operate with letters as different quantities rather than specific values (Capraro & Joffrion 2006).

However, variables do not always represent quantities or unknown numbers as many learners may think. In geometry, variables often represent points, for example: $\triangle$; if we use the variables P, Q and R, then when we write if PQ = QR, then triangle QPR is an isosceles. Usisikin (1988) added that many times learners tend to believe that variables are always letters which is not the case. Usiskin (1988) gave an example to justify his point: $2 + y = 5$ and $2 + \triangle = 5$. Learners consider the first equation to have a variable whereas the second equation does not, but the two equations are both desiring a solution to the equation. Logically, y and $\triangle$ are equivalent. "Trying to fit the idea of a variable into a single conception oversimplifies the idea and in turn distorts the purpose of algebra" (Usiskin, 1988, p. 10).

The other perception among the learners on variables is that they believe that variables represent unknowns (Tekin-Sitrava, 2017). Tekin-Sitrava (2017) further highlighted that the way learners understand variables is brought about by some curricula that define variables as unknowns.

2.4 SEPARATION OF TERMS IN EXPRESSIONS

Understanding the differences between algebraic terms can be difficult for learners. Tall and Thomas (1991) highlighted the issue of scientific language that complicates learners' understanding of expressions. They stated that it is common for learners to take 'ab' to mean 'a + b'. This error is said to have originated from subject areas that do not differentiate from conjoining, for example, in chemistry, adding oxygen to hydrogen produces H_2O, (Stacey & MacGregor, 1994). Tirosh, Even and Robinson (1998) indicated that learners understand open expressions as incomplete and tend to complete them. Learners need to recognise that algebraic expressions, for example, 2a + 5b, have a dual nature, representing a calculation process and also being an algebraic object on its own (Drijvers, 2010).

From a teaching perspective, Davis (1989) as cited in Tirosh et al. (1998) indicated that most teachers teach learners first to collect like terms before they introduce simplifying algebraic expressions. He added that this is normally done by presenting a list of terms with variables and learners choosing like terms based on the sameness of the letters accompanying each number. Tirosh et al. (1998) stated that this helps learners see and understand that unlike terms are kept separate and not combined because their letters are different. They further add that when learners are to deal with expressions such as 2a + 5b, they regard it, for example, as 2 apples and 5 bananas, viewing the expression as an entity in its own right.

Tall and Thomas (1991) asserted that learners often face "parsing obstacle", the inability to separate the order in which the algebraic expressions must be understood and processed. For instance, when learners deal with algebraic expressions like 5x + 4, they may read it as 5 and 4 giving 9 and consider the whole expression to be the same as 9x; given 8 - 3y, learners may read it as 8 - 3 giving 5 and consider the final result to be 5y (Tall & Thomas, 1991). This misunderstanding might be caused by a lack of perfection or complete transition from the world of working with numbers (arithmetic) to the world of working with symbols (algebra) (Bokhove, 2011). Hall (2000) argued that the order of operations in which an expression is presented leads learners to think an expression is dealt with according to that order.

Another view could be that this difficulty is a result of the teaching that focuses on calculation rather than on understanding the meaning of an expression (Jupri et al., 2014). In this regard, Kieran (1992) spoke to the understanding of algebraic expressions as a process and object. She stated that learners view expressions, for example, $x + 3$ as a mere process while in algebra it is a process and a product at the same time. In other words, learners view an expression $x + 3$ as a procedure of adding 3 to x but algebraically it is also an object. This has, however, caused a challenge in understanding algebraic expressions as objects. Matz and Davis as cited in Kieran (1992) thus referred to this challenge as the "process-product dilemma". In the literature, it was also found that learners grasp $3x + 7$ only as a process to be performed, a thought that is brought about by the interpretation of the symbols such as (+) with a view of action to be performed, thus leading to combining the terms (Tirosh et al., 1998). Adding to this is Panasuk and Beyranevand (2010, p. 5) who reiterated that "when a mathematical concept is learned, its conception as a process precedes its conception as an object".

Tirosh et al. (1998) indicated that the tendency of combining terms, for example, $2x + 4$ to get 6x or 6 comes as a result of cognitive difficulty that learners face in "accepting lack of closure" as they view the expression as incomplete and resort to completing it. They further add that algebraic expressions have a typical similar behaviour to arithmetic expressions which expect a single-termed answer.

2.5 EXPANDING BRACKETS IN EXPRESSIONS

Inappropriate expansion of brackets is rooted in the use of the distributive property of multiplication over addition (Drijvers & Jupri, 2014). A study carried out by Seng (2010) indicated that learners saw the coefficient of x as 0 instead of 1 and this misconception was caused by the invisible coefficient 1. He further goes on to give an example: for expressions $3x + x$, learners tended to give 3x as an answer, and this implied that they had taken the coefficient of x to be 0.

MacGregor and Stacey (1997) talked about the learners' inability to recognise the coefficient 1 in an expression a. They stated that the ignorance is rooted in the lack of understanding that a is a product of 1 and a ($1 \times a$) and this ignorance leads the learners to committing errors in complex algebraic expressions. Learners do not only ignore the coefficient 1 of a given variable; according to Seng (2010), learners tended to ignore the negative sign when it preceded a term. He further reasoned that learners treated (+) and (-) as mere operations that required actions rather than the signs of the number as well. Ayres (2000) affirmed that learners tended to detach the negative sign from the terms/expressions because they saw it as an operation.

The study of Kraja (2012) which talked about misconceptions with regards to the use of brackets among learners revealed that for expressions 2(x + 1), learners tended to give 2x + 1 as a result after expanding. This type of error occurs because learners do not treat the expression (x + 1) as a single quantity but rather as a process (Tall, 1992).

Kieran (1979) stated that errors committed by learners when working with brackets in algebra are hosted in the poor emphasis of brackets in their arithmetic learning. She stressed that there is little emphasis of brackets usage in arithmetic expressions and this inhibits learners' understanding of use of brackets in algebra.

Some of the misconceptions or errors committed by the learners in working with brackets in algebra are highlighted in the study of Kaur (1990, p.36). His study indicated that learners perceived brackets as meaning multiplication. Below are the examples of the misconceptions:

Q.	item	"Error" answers	% giving error answer
1(e)	$(2x - y) + y$	$2xy - y^2$	12%
1(g)	$3x - (2x + y)$	$-6x + 3xy$	8%
		$-5x - y$	4%
1(h)	$(x+y) + (x-y)$	$x^2 - xy + xy - y^2$	16%
		$x^2 - y^2$	20%
1(i)	$(3x+2y) - (x-2y)$	$2x - 6xy - 2xy - 4y$	8%
		$5xy - 2xy$	4%
		$3x^2 - 2xy - 6xy - 4y^2$	16%

Figure 2.3: **Adapted from Kaur (1990, p. 36)**

Welder (2012) associated these misconceptions with the rule of order of operations. Welder stressed that learners do not see the need for the rules in working with these types of expressions. Learners resort to dealing with these expressions based on the arrangement of the terms in the expression (Welder, 2012). Gardella (2009) claimed that the concept of order of operations is poorly developed and that learners do not get the meaning of the concept and believed that the lack of conceptual development of this concept was the major cause of learners forgetting the rule.

2.6 SOLVING LINEAR EQUATIONS

Equations in one unknown can be written in the form ax + b = c where a, b and c are constants and a $\neq$ 0.

Understanding linear equations and algebraic relationships is important in preparing learners to prosper in their understanding of advanced concepts of algebra (Capraro & Joffrion, 2006). In this regard, scholars have established that learners in middle schools need to develop representational skills and techniques for a strong understanding and fluency with algebraic equations (Silver, 2000). Equation solving involves formal procedures of performing the same on both sides of the equal sign though this is not generally what is taught to the learners in their first encounter (Kieran, 2004). In conformity with arguments made by a string of scholars (Silver, 2000; Capraro & Joffrion, 2006), Kieran (2004) further highlighted some of the methods used to solve equations taught to learners. These included:

(a) Use of number facts;
(b) Use of counting techniques;
(c) Cover-up;
(d) Undoing (working backwards);
(e) Trial and error substitution;
(f) Transposing (i.e. change side-change sign);
(g) Performing the same operation on both sides.

These methods are referred to as formal methods (Kieran, 1992). However, Kieran went on to criticise the last two methods by stating that they do not prepare the learners with conceptual understanding of equation solving, rather with mere procedures. On a different note, the perception of equality among middle school learners was found to play a critical role in their equation solving and verbal problem solving achievement (Knuth, Steph, McNeil, & Alibali, 2006). They argued that learners with a better understanding of the equal sign are able to solve equations correctly as compared to learners with less understanding of the equal sign. This argument is echoed in Ertekin

(2017) who also argued that learners should be made aware of the relational meaning of the equal sign as "the same as" apart from the operational meaning as "do something". Knuth et al. (2006) asserted that the understanding of the equal sign as relational is helpful when learners encounter learning how to solve algebraic equations that require calculations on both sides of the equal signs, for example, $4x + 2 = 2x - 6$.

Understanding linear equations is a complex process that involves many levels of abstraction (Wati et al., 2018). Wati et al. (2018) indicated that learners who struggle with the solving of linear equations have factual difficulty (poor knowledge of symbols, signs and notations), conceptual challenges (lack knowledge of abstract ideas used in an equation), operational challenges (lack subject related knowledge such as counting and algebraic workmanship; they are not accurate) and principle difficulties (lack the knowledge of axioms, theorems and relationship formula). Mathematics educators need to understand the difficulties faced by the learners so that they can plan better approaches/strategies that help the learners to succeed in solving linear equation problems.

Magruder (2012) pointed out three sub-components which middle school learners find difficult when solving linear equations which are: a) understanding of the symbols; b) the meaning of equal signs; and, c) the reliance on procedural knowledge without conceptual understanding. Learners struggle to balance the procedural knowledge and conceptual understanding, thus they find it hard to understand solving linear equations with one unknown (Saraswati, Somakim, & Putri, 2016). This comes as a result of memorisation of the formulae and procedures teachers employ when teaching solving linear equations (Jupri, 2015). Saraswati et al. (2016) highlighted that the learning and teaching of linear equations is inhibited by the lack of fundamental understanding of the concept that views linear equations in real life contexts.

Khuluq (2015) stated that learners struggle with understanding solving linear equations with one variable because they find it difficult to understand 'equation' as a structure. This failure can be realised through three conditions highlighted by Kieran (2007) which are:

1) strategic errors when solving algebraic expressions;

2) learners’ ignorance to treat variables as objects and,

3) the misunderstanding of equal signs.

This argument implies that learners need to have a strong fundamental understanding of arithmetic as this will nurture their understanding of algebra since algebra is viewed as generalised arithmetic (Linsell, 2008). But, Kieran (1992) proposed that arithmetic can be strengthened to help learners comprehend algebraic concepts by involving some “arithmetical identities” (Khuluq, 2015) with some implicit numbers in presenting equations to the learners. Commenting on the issue of understanding the ‘equal sign’, Khuluq (2015) stated that learners perceive the equal sign as a signal for getting a single answer, as that is how they understand it from an arithmetic perspective. Khuluq (2015) further indicated that this perception by the learners becomes a challenge to them when they are required to solve equations particularly with unknowns on both sides. Among other methods of solving equations, two methods, namely: the transfer (transpose) method and the balance method have been often used and extensively researched (e.g. Khuluq, 2015; Atem, Andam, Anoako, Obeng-Denteh & Wiafe, 2017) in teaching solving linear equations locally and globally.

2.6.1 Transfer method

Most research indicates that transposing and performing the same operation on both sides are the most used methods for solving equations in middle school algebra. The teaching of equation solving is dominated by the use of the transpose (transfer) method (Hall, 2002a). Transposing is a change side-change sign technique used as a way of solving algebraic equations (Kieran as cited in Hall, 2002a). Learners who only use the transpose method to solve equations are “less sure of the underlying structure of an equation than those used to using what might appear to be the less sophisticated trial and improvement method” (Hall, 2002a, p. 2).

On the same point, Kieran (1992, p. 400) echoed that,

> The emphasis (on symmetry) is absent in the procedure of transposing. There is some evidence to suggest that many students who use transposing are not operating on the equation as a mathematical object but rather blindly applying the Change Side-Change Sign rule.

Tall and Lima (2007) stated that learners’ preference of using the transfer method in solving linear equations is influenced by their understanding of simplifying expressions such as, 2x + 4y + 3x, whereby 2x can be shifted next to 3x. They stated that learners see the shifting of symbols as "an application of human embodiment, picking up things and moving them around" (p. 4). However, dealing with equations, the shifting requires extra actions (change side-change sign) which if not presented in a meaningful way may be seen as 'magical' actions by the learners and eventually leads them to committing errors (Tall & Lima, 2007). Kieran (1981) also found that the common procedural phrase of change side-change sign is meaningless to the learners and leads learners to making mistakes. To the learners, it appears easy to move symbols around the equal sign as learners merely see symbols as "physical entities" (Cortés & Pfaff, 2000) that can be passed to the other side where a learner feels appropriate, with just a change of sign.

2.6.2 Balance method

A linear equation can be referred to as a statement of equivalence of two expressions (Kieran, 1992). Balance method is another method that can be used to learn solving linear equations.

> The advantage of this model is that it has a meaning in everyday life situations, and students can make a mental image of the balance very easily. Moreover, the balance emphasises the static character of the equation; the concept of equivalence remains in the foreground as the solution procedures are carried out. The major limitation of the balance model – of all physical and visual models for that matter – is the restrictions of its applicability to equations involving negative terms and negative solutions. (Kieran, 1992)

Balance scale facilitates the understanding of the operation of removing the same term from both sides of the equal sign (Brown, Eade, & Wison, 1999). Nevertheless, Vlassis (2002) argued that learners experience major difficulties in solving equations that include negative integers and cancellation errors of terms is unavoidable when using the balance scale.

Sfard (1991) argued that learners are taught to solve equations through the use of a balance model with the view that the balance pivot is around the equal sign without any visualisation. The use of visual proofs in learning equations help learners treat algebraic expressions as objects by experimenting and presenting with a real balance scale (Khuluq, 2015). Moreover, the experience of using a balance scale in the learning of equations helps learners develop multiple representations of the equations (Suth & Moyer-Pockenham, 2007) and also stimulates learners to present their understanding through drawing and verbalisation (Khuluq, 2015).

Though the use of the balance model is deemed appropriate in helping learners understand equation solving, it carries challenges in dealing with some aspects of the equations. Khuluq (2015) accentuated that the use of balance models is incapable and confuses the learners when working with reversible operations. This argument has been refuted by other scholars mainly because it overlooks the agency of learners. Van Ameron (2002) pointed out the incapability of the balance model to present unknowns or algebraic expressions involving negative terms.

Vlassis (2002) briefly discussed two theoretical tendencies with conflicting views of the usefulness of models, namely, the oponents and the defenders. This discussion is quoted verbatim below:

> ***The opponents:*** Filloy and Rojano (1989) observe that the models (geometrical and balance) do not allow students to deal with the unknown value. Pirie and Martin (1997) believe that the balance model makes no sense to modern students, as contemporary scales are no longer based on the principle of the two sides balancing. Moreover, these authors argue that using this model gives rise to errors that are caused by the model itself (such as removing a negative integer to cancel it out).
>
> ***The defenders:*** On the other hand, Brown et al. (1999) are of the opinion that "for many students and many teachers proficiency in specific concretizations forms the backbone and principal motivation of activity within the classroom." (p. 68). The results of Radford and Grenier (1996) show that "the idea of the scales facilitates the use of the rule of elimination of like terms (rule of the al-muqabala)" (p. 264). Moreover, according to Linchevski and Williams (1996): "people attempting to solve mathematical problems often make use of several models in the process of finding the solution. Different parts of the problem may lend themselves to the use of different representations, including the combination of concrete thinking with abstract formal reasoning." (p. 352).

It is worth noting that both opponents and defenders agreed that the use of the model does not help in solving equations that involve negative integers.

Bush and Karp (2013) stressed that solving an equation through the balance model helps learners understand the relationship of equality between two expressions and also develop an understanding of inverse operations. Literature on learning equations has also underscored the importance for learners to understand the equal sign as a relational symbol as it helps them perform inverse operations (Asquith, Stephens, Knuth, & Alibali, 2007). In this vein, Vlassis (2002) affirmed that the balance model "provides a mental picture of the manipulation to be carried out and the associated concepts (the meaning of equality and expressions, the properties of equality" (p. 355).

Section 2.6 has presented researchers' views on solving linear equations, with some of the difficulties attached to understanding and therefore to teaching this aspect of mathematics that provide a context to understanding the data generated in this study. The next section turns to a consideration of specific teaching methodologies which can and have been used to teach algebra.

2.7 THEORISING THE TEACHING TOOLS

This study focuses on different ways of teaching algebra, which until now has been taught in a largely abstract, disconnected and procedural way. It seeks to understand whether different learning gains can be made if non-standard teaching tools are introduced, proposing multiple representation and visualisation as the foci.

The study made use of visual and kinesthetic tools. Therefore, this study is informed by two theories which are: multiple representation and visualisation. Multiple representation is the broader theory in which visualisation is also embedded.

2.7.1 Multiple representations

Representations can be defined as tools that can be used for solving and communicating scientific data, problems and ideas (Giere & Moffatt, 2003). "The term *representation* refers both to process and product – in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself" (NCTM, 2000, p. 67).

Mathematics is a subject that deals with ideas that are represented by different types of abstract symbols or representations, such as, words, variables, operations etc. that also enable the creation of visual representations of Mathematics information (Matheson & Hutchinson, 2014). On a basic level, "good problem solvers usually construct a representation of the problem to help them solve it" (van Garderen & Montague as cited in Matheson & Hutchinson, 2014, p. 2). Multiple representations in mathematics are used to enhance the learners' understanding and ability to make connections (Ross & Wilson, 2012).

Learners learn algebra contents through engaging themselves in problem solving, reasoning, making connections and using multiple representations (Strickland & Maccini, 2012). They further indicated that some learners find it easy to use non-algebraic representations (symbols) in solving algebraic problems and communicating their algebraic ideas to their peers and teachers through mathematical language.

In addition, it can help learners to know more than one representational technique: “A common justification for using more than one representation is that, it is more likely to capture a learner's interest and, in so doing, play an important role in promoting conditions for effective learning” (Ainsworth, 1999, p. 146). An external representation “is typically a sign or a configuration or signs, characters, or objects’ that can symbolise something other than itself” (Panasuk, 2010 p. 238).

2.7.1.1 Benefits of using multiple representations

There are different forms of representations that can be used in learning algebra, such as, graphic, tabular and diagrammatic forms of representations (Giere & Moffatt, 2003). They further pointed out the use of tables, stating that tables provide a good organizational structure of concepts that allow learners to clearly see details of algebraic statements.

Ross and Wilson (2012) indicated that the conceptual understanding is developed when learners are provided with choices of concept representations and make use of their understanding through actively engaging in a problem-based situation. The use of multiple representations facilitates learners’ learning and has the potential to deepen their understanding (Suh & Moyer, 2007). Boggan, Harper and Whitmire, (2010) echoed the same view that using different representations assists learners to effectively build mathematical understanding.

In mathematics, representations are needed as ways of enabling learners’ understanding and abilities to develop meaning of the mathematics concepts (Ross & Wilson, 2012). Ross and Wilson (2012) further stated that enactive representations such as manipulatives or visuals are used to provide a solid foundation of understanding underlying concepts such as algebra that is inherently presented in abstract symbols. Ntsohi (2013) indicated that the use of multiple representations reveals information for a particular situation as opposed to a method using a single representation. A different representation of information may bear superfluous features as well as those that are unique to each form, or each may encode different aspects of the domain (Ainsworth, 2006). Ntsohi

(2013) stated that the use of multiple representations enable learners develop a conceptual understanding of the concept at hand by seeing the structures in detail. “The cognitive linking of representations creates a whole that is more than the sum of its parts… it enables us to “see” complex ideas in a new way and apply them more effectively” (Kaput, 1989 as cited in Ntsohi, 2013, p. 60)

Ainsworth (2006) stated that using multiple representation enables learners acquire deeper understanding when learners integrate information from multiple representations to obtain insights deemed difficult through the use of a single representation. Through the use of multiple external representations, learners develop references that reveal the underlying structures of the concept or domain at hand (Bransford & Schwartz, 1999). Ainsworth (2006) stated that learning through combination of different representations helps when one representation constrains interpretation of the other representation, for example, diagrams are often used for simulations alongside complex representation such as symbols. Schoenfeld, Smith and Arcavi (1993) noted that though external representations are deemed appropriate to promote learners’ understanding, on the other hand, learners find it difficult to translate between representations.

The use of external representations (diagrams) requires learners to understand how each representation encodes information and relates to the domain it represents (Ainsworth, 2006). Roth and Bowen (2001) similarly maintained that external representations become more effective if the learners understand the relation between the representation and the domain that it represents. For instance, learners need to understand which operators to apply to a representation to retrieve the relevant domain information (Ainsworth, 2006). Ainsworth (2006) further stated that matching types of representations to the demands of learning of a particular concept can remarkably improve performance and understanding of the learners. Learners need to contextualise the representation within the domain being taught in order to understand its use and what it helps to reveal (Jong & Meij, 2006). Failure to notice regularities and discrepancies between representations can entirely inhibit learning (Ainsworth, Bibby & Wood, 2002). Representations that are not compatible to the task at hand are not helpful to the learners as they may lack support of the required cognitive process (Booth & Koedinger, 2011).

Scaife and Rogers (1996) suggested that external representations vary in their role and in the degrees to which they help learning in computational offloading and re-representation. By computational offloading they refer to what degree the representation assist learners minimize mental effort to solve the same problem. For instance, the use of diagrams can exploit the perceptual process by grouping information together and making the recognition of the underlying structure easier (Larkin & Simon, 1987). Re-representation refers to how external representations with the same abstract structure impacts the problem solving (Ainsworth, 2006).

Sitompul, Budayasa and Masriyah (2018) explains that for a learner to be mathematically literate one should have the following five basic skills: Communication, Mathematising, Representation, Reasoning and Argument. Mathematics representations may be in the form visual symbols or verbal symbols. The way a learner accepts information that is in verbal symbols or in visual symbols depends on that individual's cognitive style: is the learner a verbalizer or visualizer? (Kozhevnikov, Hegarty & Robert, 2002).

Kieran (1992) argued that if algebra is only taught as a study of expressions and equations, it can pose serious challenges to effective and meaningful learning of algebraic concepts. She indicated that the use of symbols alone barely gives meaning of the algebraic concepts to learners, therefore, a representation of a distinct nature may serve to give meaning.

2.7.1.2 Challenges in using multiple representations

The benefits of multiple representations do not always come easily, as they all have their own cognitive demands. One of the challenges highlighted by Ainsworth (2006) is the difficulty of interpreting and relating one representation to another. He argued that some mathematical representations are difficult to interpret as opposed to the concept they represent, such the use word problems in algebra may be a challenge to the learners as some of them may have language problems. Ainsworth (2006) also discussed the challenges faced by learners in algebra when using graphic representations to solve equations. He stated that learners commit errors in plotting point and then misinterpret intervals for points.

Molina-Geraldo, Carvajal, Alvarez-Meza and Castellanos-Dominguez (2015) discussed the problem of translation between representations. They indicated that learners often struggle to translate a different representation into another form of representation. Learners often face ‘difficulties in maintaining the semantic congruency of [algebraic expressions] even when they display an understanding of the initial or final representation (Molina-Geraldo et al., 2015). It is important for learners to understand the mutual dependency between the representations so that they can perform the correct translation successfully (Kieran, 2007).

2.7.2 Visualisation as a representation

Following the logic of the argument above, the teaching programme of this research has incorporated diagrammatic representations to generate understanding of the algebraic elements.

Arcavi (2003, p. 235) defined visualisation as;

> Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.

Mathematical relationships and ideas can be expressed in different representations of which visual representation is one (Panasuk, 2010). She goes on to suggest that the role of any type of representation is to communicate a mathematical idea in a way that it is easily comprehensible by the learners and conveys different meanings of a single concept.

Visual representations come in the form of a picture, chart, diagram or symbolic representations that could be external (e.g. a diagram) or internal (e.g. schematic images), and both can be used by learners to solve problems (Matheson & Hutchinson, 2014). Diagrams as part of external visualisations enable the learners to show how information is related, organised and they help work out solutions to the problem, without errors of omission (Kolloffel, Eysink, de Jong, & Wilhem, 2009).

As discussed by Strickland (2010), the connection between pictures of physical manipulatives and abstract notations help learners develop understanding when solving problems. The use of visualisation is one of implicit instruction, which involves a number of teaching behaviours that assist learners to engage actively in mathematical activities. It has a long history as an effective strategy (Strickland & Maccini, 2010). Panasuk (2010) added that when learners demonstrate that they are able to recognise relationships or concepts presented to them in different ways (for example, symbolically or diagrammatically), they have developed greater conceptual understanding of these relationships and have moved a step further from procedural skills which he termed “process perception” to structural skills such as “object perception”.

Despite the studies that indicate the potential of visualisation in fostering understanding for mathematics learning and problem solving, some learners often display difficulties and reluctance to use visual support systems (Eisenberg & Dreyfus, 1991). The connection of symbols and visual structures in mathematical activities brings its own cognitive demands. It is emphasised that the use of visual (external or internal) representations alone does not improve the performance of learners in Mathematics, and that verbal and written forms should be integrated with the visual tools (Gersten et al., 2009; Suh & Moyer as cited in Matheson & Hutchinson, 2014).

Some learners have difficulties in reading diagrams and recognising the meaning implied in them (Kaput, 1989), thus, the need to balance visualisation with conceptual modelling, which was not done due to the scope of the programme. Diagrams may be seen as helpful, but their impact depends on what they are used for (Mayer, 2005). It has also been observed that the lack of clear interpretation of diagrams on the part of learners also inhibit their level and scope of conceptualisation of the underlying structures of algebraic concepts (Booth & Koedinger, 2011).

The use of diagrams can only be beneficial to the learners in a particular concept if their use is well designed to support the cognitive process of the learners (Booth & Koedinger, 2011). According to Booth and Koedinger (2011), “a common proposed strategy for improving learning involves creating a smooth transition from what students already know, which is often more concrete, to the desired knowledge, which is often more abstract”. Koedinger, Alibali and Nathan (2008) suggested that instructions of teaching algebra may be clearer and more effective in abstract symbolic representation if it is built from the existing understanding of a concrete or visual representation. Failure to establish concrete-abstract representation connections would preclude successful problem solving (Koedinger et al., 2008).

Sengul and Uner (2010) indicated that the use of cartoons is very effective in raising the performance of more qualified individuals. Cartoons include diagrams that can be used to model concepts. These diagrams are used to demonstrate concepts to the learners to stimulate their thinking and inquire their knowledge (Uguerel & Morali, 2006). Dabell (2004) added that diagrams help learners to question their thoughts, solve their daily life problems, broaden their horizons and provide different perspectives for the events. Sengul and Uner (2010) argued that the use of cartoons or diagrams in teaching helps learners to acquire thinking skills which allows them to understand scientific concepts which increases their motivation.

English and Sharry (1996) asserted that the use of concrete or pictorial representation adds value to conceptual learning as it mirrors the structure of the concept and thus enables the learner to use the structure of the representation to build a mental model of the concept. They further highlighted that when a learner gains confidence in using pictorial or concrete representation of the concept, they can easily make the required "abstraction shift" where the generality itself becomes the object of the thought. Schwartz (1995) affirmed that visual representations expose implicit structures of the concepts they represent and that this assists learners develop or construct references when solving problems in abstract notation.

Although the use of diagrams has advantages, among learners with learning difficulties the transition from the use of diagrams to the abstract stages turns out to be more challenging, as they struggle to generalise learned materials and conceptualise abstract algebraic concepts and tasks (Hudson & Miller, 2006).

Internal visual representation is a mental exercise that involves the creation of visual imagery to represent information (Matheson & Hutchinson, 2014). The majority of mathematics teachers perceive these learners who use mental pictures, as not using diagrams to get their mathematics solutions. It is advisable to allow the learners to explain how they got their answers, and learners can be asked to draw images that they have used in their heads. Matheson and Hutchinson (2014) explained that for a learner to create mental pictures that are used to solve a problem, "they combine information from the problem with their prior knowledge of the topic" (p. 13). These cognitive process are difficult to teach or learn and the teacher has to scaffold the learning process by giving explicit instructions or by drawing diagrams.

Not all theorists agree. Chu, Rittle-Johnson and Fyfe (2017) argued that "it remains unclear whether providing diagrams in conjunction with symbolic equations can benefit problem solving as well" (p. 273). The outcome of their study revealed that diagrams increased problem solving accuracy and the use of informal methods (Chu et al., 2017). Panasuk (2010, p. 17) stated that "pictures provide an external sketchpad where students can represent and connect pieces of information and that generating pictorial representations facilitate the conceptualisation of the problem structure and form the basis for a solution".

External representations such as diagrams influence learners' understanding by changing how learners internally represent problems, which ultimately impact their way of solving problems. Chu et al. (2017) observed that "specifically, diagrams may elicit students' intuitive, informal knowledge, and strategies, which may allow students to connect this knowledge to formal, symbolic problem solving formats" (p. 275). Schnotz (2002) argued that symbolic or textual representation allows ambiguity in the interpretation of the concept presented which turns out not to be the case when the concept is represented diagrammatically. Ainsworth (2006) echoed the same view that iconic representations are useful and often clear, as detailed information can be extracted from them.

2.7.3 Visualisation in algebra

Algebra is one of the strategies that may be used to "solve problems, hence we have algebraic representations that comprise symbolic, numeric, verbal or graphic forms" (Star et al., 2015, p. 37). The use of algebra as a key problem solving strategy, however, has advantages and disadvantages, as Niemtus (2016) pointed out, "algebra with all its signs, symbols and substitutions can leave pupils bored and confused" (p. 1). Nevertheless, Star et al. (2015. p. 1) asserted that "understanding algebra is the key for success in future mathematics courses e.g. geometry and calculus".

The use of numbers and letters in algebraic statements raises difficulties for most learners (Hembree as cited in Toh, 2009). To develop an understanding of algebraic concepts among learners, Pashler et al. (2007) suggested that there is a need to integrate and connect concrete and abstract representations within our teaching to aid comprehension of the concept taught, as well as to transfer the concept to a variety of situations. Ferrucci, Kaur, Carter and Yeap (2008) indicated that diagrammatic representations increase algebraic thinking and problem solving skills via visualisation of algebraic relationships.

Witzel, Mercer and Miller as cited in Saraswati et al. (2016) commented that the learners' understanding of the abstract concepts in algebra is simplified by teaching that incorporates manipulation and pictorial representations.

Khuluq (2015) suggested that to help students "understanding of algebraic representations, algebra learning should be started on a concrete level, in which the students can produce and reflect on their own symbols against the true situation. This will make the symbols meaningful to them" (p. 29).

Ross and Willson (2012) stated that to build a solid foundation for knowledge and understanding of underlying concepts in mathematics, for example in algebra, enactive representations such as manipulatives and visuals are the most effective tools to use. Chappell and Strutchens (2001) agreed, and identified algebraic tiles as the most useful tools to facilitate the understanding of the variable concept.

Learners master algebraic concepts well when first taught through the use of manipulatives before being introduced to the use of numbers, symbols and variables to solve the task (Strickland & Maccini, 2010). Using manipulatives assists learners to easily acquire conceptual knowledge through play which eventually increases their proficiency in dealing with the concept (Picciotto & Wah, 1993). A study conducted by Witzel et al. (2003) showed that the use of concrete representation, diagrams or pictures to teach algebraic concepts was a more effective intervention than using abstract notations for helping 7^{th} graders and for learners with mathematics learning difficulties.

According to Ives (2007), graphic organisers have also been discovered to help the effective learning of algebraic skills for learners beginning to learn algebra and for those with learning difficulties. Graphic organisers are "visual displays that demonstrate relationships between concepts which guide learners' thoughts as they fill in and build upon a visual map" (Teaching and Learning with Graphic Organisers, 2018). Geary (2004) on the same note, also suggested that the use of graphics be incorporated into the teaching of algebra to assist learners to organise symbolic notations and to arrange multiple steps in complex manipulations of algebra.

The study by Fong and Lee (2009) explored the use of diagrammatic representations in improving understanding of algebra of a sample of Singapore learners in three stages – starting with word problems then moving to diagrammatic representations of these and then to their algebraic, or symbolic forms. Solving equations using diagrams helps learners to “focus on both representing and solving a problem rather than on merely solving it” (Cai, Morris, Moyer, Fong, & Schmittau, 2005).

The use of non-algebraic representations such as tiles are also regarded as visual tools that can be used to assist learners in understanding some algebraic concepts (Larbi & Mavis, 2016). The use of representations e.g. tables and tiles, assist learners separate algebraic expressions into parts (Larbi & Mavis, 2016). Russel (2011) claimed that the use of algebra tiles promote efficiency in multiplying algebraic expressions. The inability to use algebra tiles or other manipulatives limits learners’ acquisition of mathematics concepts especially algebra (Larbi & Mavis, 216). The use of tools enables learners to split terms apart when dealing with algebraic expressions and helps them with mental processes (Russel, 2011). By using [an expansion box], learners tend to develop greater connections with the ability to use distributive law (Russel, 2011).

The use of algebra tiles helps learners develop visual understanding and acknowledges the distributive property which is mainly taught through the use of mere symbols (Picciotto & Wah, 1993). Larbi and Mavis (2016, p. 56) added that the use of algebra tiles guides learners in developing an understanding of the distributive property in the form: “multiply every block on the left by every block across the top”. Moreover, they went on to state that algebra tiles enabled learners to develop sufficient knowledge and understanding of the basic operations and to also help treat them as different.

Ntsohi (2013) explored the use of a spreadsheet to learn algebra concepts and the usage of a spreadsheet appears to correspond with the use of an expansion box used in this study. His study indicated that the use of a spreadsheet enabled learners to identify terms and their roles in a given algebraic expression.

Fyfe, McNeil and Borjas (2014) argued that one of the solutions to assist learners to learn algebra and develop a conceptual understanding, is the use of concrete manipulatives such as, the balance scale to teach solving linear equations. The use of visuals activates real world understanding during the learning process and enhances learners to develop their own understanding of abstract concepts (Brown, McNeil, & Glenberg, 2009).

The use of manipulatives/visuals in solving equations is also supported by Davis (as cited in Hall, 2002, p. 270), as he wrote:

> An example may make the point more clearly. Some teachers want a student to begin solving the equation 2x + 3 = x + 8 by thinking, "I'll move the 3 across the equals sign and change its sign." These teachers hope the student will then write 2x = x + 8 - 3 but this approach is surely a case of regarding mathematics as a collection of small, meaningless rituals. Why "move the 3 across the equals sign?" And why on earth should such an act "change (the) sign of the 3? Certainly, if we want the student to think of mathematics as consisting of reasonable responses to reasonable challenges, it will be far better if we encourage the student to think, "I can subtract 3 from each side of that equation, without changing its truth set." If the student has seen pictures of balance scales or has worked with actual balances, there can be very straightforward imagery underlying the idea of subtracting the same thing from each side of an equation.

Bohan and Shawaker as cited in Chappell and Strutchens (2001) stated that it is crucial to ensure the connection between the concept presented with visuals and the knowledge they represent to build the learners' understanding. For example, when using algebra tiles, learners need to mirror the actions performed on algebra tiles in relation to the understanding required to expand or factorise algebraic expressions (Thomson, 1994). Chappell and Strutchens (2001) indicated that the incorporation of visuals in the teaching of algebra is quite complex in regard to deciding *when* to use *what.* They highlighted that some learners benefit from learning through the use of models first and then seeing how the 'paper-and-pencil' is derived from the model and some learners benefit from learning the other way.

> One risk of this approach, however, is that if students manipulate the symbols without meaning, they may not fully understand the concepts, or they may regard using the concrete models as unnecessary or child's play. When using any concrete model, teachers should help students make connections with the abstract symbolism; otherwise, the use of the model could become trivial. (Chappell & Strutchens, 2001 p. 21)

However, McNeil and Jarvin (2007) argued that the use of mere manipulatives in the teaching does not guarantee success in learning. They highlighted that despite the effectiveness of the use of manipulatives in learning mathematics concepts, the application of the understanding in the required context remains a challenge to the learners. This challenge is brought about by poor linkage between the visuals and the symbols (Gravemeijer, 2002).

2.8 CONCLUSION

This chapter presented the literature review of the study. It discussed general views of algebra learning and views on learning specific concepts of algebra. Theoretical views on the use of multiple representations and visualisation to learn mathematics and algebra in particular, were also discussed in this chapter. Considerations of all these factors lead to the research design presented in the next chapter, Chapter Three, of this thesis.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

The research goal of this study is to determine whether and how the specific visual and kinaesthetic teaching tools (diagrams, expansion box and balance method) used may have contributed to learners' understanding of algebraic concepts and techniques (variables, expressions and equations). In particular, to determine whether and how the specific teaching tools used may have contributed to learners' understanding of algebraic concepts and techniques. Briggs and Coleman (2014) defined a research design as "the schema or plan that constitutes the research study" (p. 107). This chapter presents the orientation of the research, the methodology, the development of the programme, the participants and the research site. Furthermore, this chapter discusses the methods used to collect data, i.e. benchmark tests, interviews, observations and artefacts. It also discusses the data analysis approach and tools and presents validity issues and ethical aspects considered for this study.

3.2 ORIENTATION

This study is located in the interpretive paradigm, and is therefore concerned with understanding the world we live in. Creswell (2003) claimed that interpretivist researchers discover reality through their participant's views and their own background and experiences as indicated in Chapter One. On the same note, Willis (2007) indicated that, "interpretivism usually seeks to understand a particular context, and the core belief of the interpretive paradigm is that reality is socially constructed". In line with this, research focused on the use of visuals and representations in teaching linear algebra to understand how these tools help learners develop understanding of the concept. A qualitative approach to analyse the unstructured interviews, observation and analysis of the artefacts was employed. By artefacts I mean the work learners produce through activities they were prompted to complete.

3.3 METHODOLOGY

This study researched a teaching intervention and took the form of a design experiment. A design experiment "entails both 'engineering' particular forms of learning, and systematically studying those forms of learning within the context defined by means of supporting them" (Cobb et al., 2003). Roosevelt Hass (2001) defined the design experiment as a type of research that puts educational experiments in real world settings to see what works in practice. These types of studies carry out formative research to test and refine educational problems, solutions and methods (Botha, 2000). This study is a design experiment that researched the design and implementation of a programme that intended to refine problems, solutions and methods associated with learning algebra. Literature discussed in Chapter Two indicated some of the challenges and difficulties that learners face in dealing with linear algebraic concepts. Therefore, the teaching programme designed for this study aimed to help learners develop understanding of such concepts.

There are three phases to design research a researcher has to consider when planning and designing the design experiment. To expand on the comments already made in Chapter One, these are:

Preparing for the experiment phase. The aim of this phase of a design research experiment is to formulate a local instruction theory that can be refined and explained while conducting the intended design (Gravemeijer & Cobb, 2006). Local instruction theory refers to the theory that offers explicit guidance on how to better help people learn and develop (Reigeluth, 1999) in this particular area. Cobb et al. (2003) asserted that the local instruction theory consists of conjectures about possible learning processes together with conjectures about possible means of supporting that learning process. In short, this phase consists of formulating local instruction theory, conjectures about the possible learning process, conjectures of possible means of supporting the learning process and anticipation of learners' thinking and understanding that might emerge (Gravemeijer & Cobb, 2006).

The design experiment phase. After all the preparation work has been done, the next phase is actually to conduct the design experiment. During the enactment of the instructional activities in the classroom, the researcher analyses the actual process of the learners' participation and learning (Gravemeijer & Cobb, 2006). It is on this basis that the researcher makes decisions about the validity of the conjectures that are embodied in the instructional activity. According to Simon

(1995), the researcher tries to find out to what extent the actual thinking process of the learners corresponds with the hypothesised ones during the enactment of these activities. He further accentuated that it is during this phase that the researcher understands the relationship between the learners' participation and the conjectured mental activities. The data depends on the theoretical intent of the design experiment (Gravemeijer & Cobb, 2006).

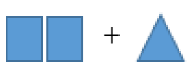
The retrospective analysis phase. This is the other aspect of design research that is conducted on the entire data set collected during the experiment. The introspective analysis depends on the theoretical intent of the design experiment (Gravemeijer & Cobb, 2006) and focuses on the process of learning in the experiment (Cobb et al., 2003). The form of the analysis will necessarily involve an iterative process of analysing the entire data (Gravemeijer & Cobb, 2006). The data included all video recordings conducted with learners to assess their mathematical learning, copies of all the learners' written work, field notes and audio recordings. To ascertain the credibility of the analysis, all phases of the analysis process have to be documented, including the refining and refuting of conjectures (Gravemeijer & Cobb, 2006).

All three phases were reflected in this study. The study was framed within the broad learning theory of visualisation and multiple representations which each played a definitive role in informing the design. There was also a structured programme with lesson plans designed to guide this experiment with a schedule of specific activities as means of supporting the learning processes. During and after the implementation of the design experiment, learners' thinking and understanding of the algebraic concepts was investigated to provide insight into their learning and the possible influence of the learning experience. The design of the current study is discussed in more detail in the next section.

3.4 DEVELOPMENT OF THE PROGRAMME

This programme was developed with the ideas obtained from earlier studies. The study by Garderen (2007) conducted in New York on the use of diagrams to teach learners with learning difficulties to solve mathematical word problems, gave me an opportunity to think of a slightly different way of using diagrams to teach linear algebraic concepts. Her study was an intervention type of study aimed at improving the performance of the learners with learning difficulties in solving word problems. She incorporated the use of diagrams to generate meaning of the word problems by encouraging learners to use diagrams to represent and solve the word problems. “Diagrams have often been cited as a powerful visualisation strategy for representing word problems, as they can be used to help unpack the structure of a problem and thus lay a foundation for its solution, simplify a complex situation, and make abstract concepts more concrete and, as a result, familiar” (Garderen, 2007, p. 540). Her use of diagrams was prompted by the considerable difficulties showed by learners to transform the language and numerical information in word problems into an algebraic representation.

Her study made me think of which diagrams I could use and *how* I could use them to help learners develop an understanding of linear algebra concepts. During my time of teaching mathematics, I have noted that early algebra learners persistently have difficulties in separating terms and I thought of using diagrams (geometrical plane shapes) to represent terms in the expressions to assist them in recognising the differences/similarities in those terms. For example, 2x + y would be represented as:



The study conducted by Chappel and Strutchens (2001) also prompted me to develop an expansion box. Their study was on promoting algebraic thinking with concrete models. They used algebra tiles to represent product rectangles of two algebraic expressions. Their use of algebra tiles was focused on multiplication of two expressions, obtaining a product in a quadratic form, linking an algebraic product to the calculation of an area of a square and rectangle. Below is an example of how they structurally used algebra tiles.

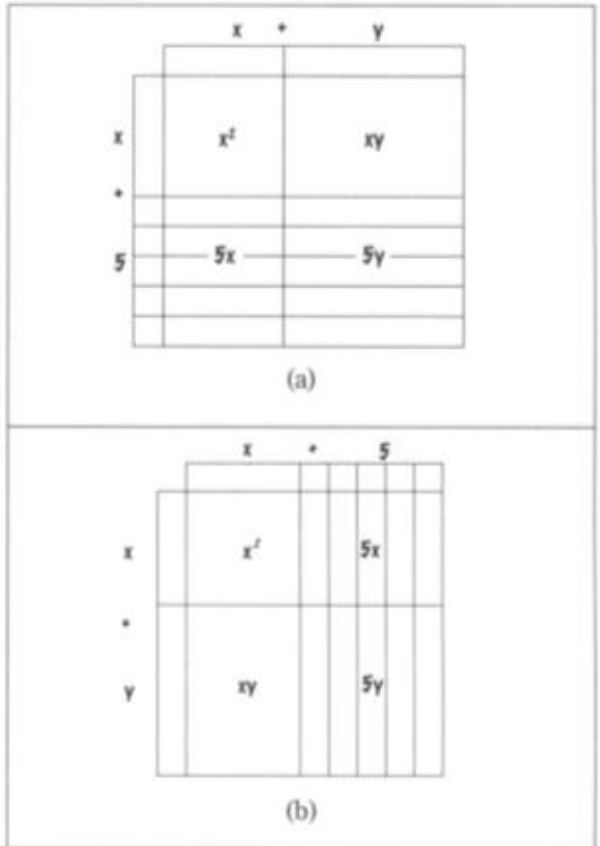


Figure 3.4: **Adapted from Chappel and Strutchens (2001, p. 21)**

This gave me the idea of developing what I termed 'an expansion box'. I used this 'expansion box' in the programme to help learners understand expanding brackets in linear algebra. Below is an example of an expansion box that I developed using Chappel and Strutchens' idea.

	+ x	+ 3
+ 2	+ 2x	+ 6

Figure 3.5: **Demonstrating how to expand 2(x + 3) using an expansion box**

Description of an expansion box:

The (+) signs on the top corners of each square block represents the sign of each term from the original expression.

a) The top right two square blocks take the terms inside the brackets.
b) The bottom left square block takes the multiplier of the brackets.
c) The two bottom right square blocks take the product of c) and b).

Diagrams were also incorporated in the use of an expansion box, representing the terms inside the brackets as a way of maintaining the emphasis of separating terms that are unlike terms.

The use of the balance method in this programme came as a result of readings that indicated that it develops a conceptual understanding of solving problems (as referred to in the literature review chapter, Section 2.6.2).

The table below shows the outline of the teaching programme. The programme has four parts addressing different four linear algebraic components.

Table 3.1: **The teaching programme outline**

Lesson Objectives	**Learners' Activities**	**Teacher's Activities**
Part 1: Understanding Variables (Lessons: 1 & 2)		
- To understand how learners, understand variables - Help learners understand variables as representing quantities	- Share their understanding verbally with the teacher - Do activities in groups and share their solutions	- Probe learners' understanding of variables by asking general questions - Provide activities for the learners - Use examples to explain variables as representing quantities
Part 2: Simplifying expressions by addition/subtraction (Lessons: 3 - 5)		
- Identifying like and unlike terms - Use symbols to simplify expressions by adding/subtracting - Represent expressions in diagrams and simplify the expressions	- Identify like terms from a list of terms - Write activities on simplifying expressions both in symbols and diagrams; 1) in groups and 2) individually - Share their solutions with others and with the teacher	- Provide the lists of terms consisting of like and unlike terms - By using diagrams, demonstrate to the learners how to represent the expressions - Give activities to the learners with expressions to simplify - Give feedback to the learners on their activities
Part 3: Expanding Brackets (Lessons: 6 & 7)		
- Understand how to represent terms in an expansion box	- Try to represent terms and signs from a given expression in an expansion box in their groups	- Demonstrate how to represent terms in an expansion box

- Understand how to represent signs in an expansion box - Expand brackets using an expansion box	- Write group activities on expanding brackets using both symbols and diagrams	- Demonstrate how to represent signs in an expansion box - Demonstrate how to use an expansion box using both symbols and diagrams -Give feedback to the learners on the work they do in class
Part 4: Solving Linear Equations (Lesson 8)		
-Introduce the balance method - Use the balance method to solve equations by means of using diagrams	- Use the balance method to solve some equations in their groups - Share their solutions with the whole class	- Demonstrate how to use the balance method using both diagrams and symbols - Assist and give feedback to the learners on the equations they solved in their groups

3.5 THE PARTICIPANTS

Eight Namibian Grade 8 learners were participants in this study. Learners were approached by the researcher and their teacher, asking them to participate in the study on a voluntary basis bearing in mind that the programme was to be conducted in the afternoon after normal teaching hours. Purposive sampling was employed to select the participants from those volunteering. Devers and Frankel (2000) stated that “purposive sampling strategies are designed to enhance understanding of selected individuals or group’s experiences or for developing theories and concepts”. The study participants included learners with different learning performances (high, medium and low). The learning performances were determined by the performance learner records on the Continuous Assessment (CA) sheet. This helped me find out how the programme could be helpful to all types of learners since it was designed for the system of teaching algebra and not for a specific learning performance group.

3.6 RESEARCH SITE

The research was conducted in one government secondary school. The school is located in the Oshikoto region, in the northern part of Namibia. I chose this school because I am a teacher there. I have observed the performance in the area of algebra among the learners at the junior phase at our school since I started working there. Therefore, I developed an interest in conducting this research at my school to keep track of the participants' performance even after the programme.

3.7 METHODS

3.7.1 Data collection

The data in this study were collected using the following tools:

- Benchmark Tests
- Interviews
- Observation
- Artefacts

3.7.1.1 Benchmark Tests

Benchmark tests were written by the learners before and after the intervention. The pre-test was designed to help me understand how learners made sense of algebra before the intervention and the post-test was given to help me understand how learners made sense of algebra after the intervention. In other words, the benchmark tests were given to help me understand the learners' experiences with regards to understanding algebra before and after the intervention. Furthermore, the benchmark tests were used determine whether the use of the tools used had an influence on the learners' capacity to correctly separate terms, expand brackets and solve linear equations through post-test as compared to pre-test.

3.7.1.2 Interviews

Focus group interviews and individual interviews were conducted in this study. Interviews can be referred to as a dialogue between two or more people. Individual interviews may be used to explore personal experiences, whereas focus groups may be used to examine opinions and beliefs about the phenomenon (Lambert & Loiselle, 2008). Both semi-structured interviews with individual learners and focus groups were employed in this study as these allowed me to probe for clarity. In this study, interviews were used to obtain an in-depth knowledge from the learners on what influence the tools had on their (learners) capacity to handle the algebraic concepts. Kvale, as cited in Cohen, Manion and Morrison (2011, p. 409) remarked that "an interview is an interchange of views between two or more people on a topic of mutual interest, sees the centrality of human interaction for knowledge production, and emphasises the social situatedness of research data".

Focus group interviews were conducted on a regular basis, i.e. at the end of every lesson each day. Kelly (2003) pointed out that focus group interviews allow a researcher to capture the participants' beliefs and understanding on the matter of discussion, that may be missed in individual interviews. Stewart and Shamdasani (2015) asserted that focus group interviews help the researcher to generate data as participants interact. Focus group interviews helped me in my study as learners found it easier to talk freely and explicitly in a group by building on others' points rather than one-on-one. The focus group interviews helped me understand the learners' experiences of a particular lesson as they were requested to share their views on how the programme influenced their thinking. It was during these interviews that learners were able to share what they discovered as they interacted with the programme.

Individual interviews were conducted by the end of the programme. Two learners were selected at random from two groups of learning performances, i.e. medium and low performance. The reason for the sampling from these learning performance groups was that during the focus group interviews, learners in these groups were dominated by the learners with high learning performance. These interviews also provided experiences of individual learner's understanding of algebra after the intervention by probing for their individual understanding. This follows Lambert and Loiselle's (2008) idea that interviews allow participants to discuss their own interpretations of the world they live in.

Learners were asked questions about their interpretation of algebra in both types of interviews. Learners were able to share their experiences on how they understand algebraic concepts before and after the intervention. Learners were also able to share their views about things that excited them and challenged them during the intervention, and this gave me a deep understanding on how the programme influenced the way they make sense of algebra. Audio recordings were used to capture the learners' responses.

3.7.1.3 Observation

Observation means the researcher visits the research site, for example, a school or classroom, to observe what is actually happening at that particular site (Bertram & Christiansen, 2014, p. 84). In this study, I was a participant observer since I was the teacher teaching the learners using the designed programme for teaching algebra. A video camera was used to capture what was taking place during the intervention. After each lesson I wrote a journal reflecting on things relevant to my study which I had observed before, during and after the lesson.

People's utterances may be different from their actions and a reality check can be done through observation (Robson, 2002), thus, observation data were used for triangulation, linking learners' responses in the interviews on how they made sense of algebra to what they actually did in trying to make sense of algebra during the programme. Observation was crucial in this study as it allowed me to collect raw data on how the learners interacted with the programme to develop meaningful understanding of variables through concept models and visualisation. Polkinghorne (2005) concurred with the idea, stating that observation offers a researcher an opportunity to capture 'live' information from the situations of social occurrences. As learners were modelling algebraic problem solving through the use of diagrams, it was through observation that I was able to see how diagrams and expansion box influenced their thinking on separation of terms and expanding brackets respectively.

3.7.1.4 Artefacts

Artefacts are resources that show knowledge, skills and thoughts of the author (Ramdunny-Ellis et al., 2005). Further to this, they stated that artefacts convey the intentions of the author to the viewer. Artefacts referred to in this study are learners' worked activities as they attempted to solve algebraic problems given to them during the intervention. In this study, the learners' worked activities were collected and copies were made from them before handing them back. The relevance of the artefacts in this study was that the learners' worked activities unveiled their implicit knowledge and understanding of the algebraic concepts through their steps of solving problems which informed me how the learners made sense of algebra. The learners' artefacts demonstrated their experiences they got from interacting with the programme and how it influenced their thinking of algebra. Artefacts were also used to see how the use of the balance method influenced the learners' understanding of solving linear equations by applying the method when required to solve linear equations.

3.7.2 Data Analysis

Merriam (1998) defined data analysis as a "process of making sense out of the data" (p. 178). After the data collection process, I started to think of ways to organise the data and how I would prepare it for effective analysis. I made use of two ways to manage and analyse the data. The two ways were: data organisation and the inductive analysis approach.

3.7.2.1 Data Organisation

I found it crucial for me to organize the data in a way that was manageable and made it easier for me to analyse the data. In the first place, I transcribed the observation videos, followed by the focus group recordings and then the unstructured individual interviews' recordings. By then I already had my journals in place that I wrote during data collection, the copies of the learners' work for the benchmark tests and the copies of the learners' work for activities during the programme. I grouped the data as per the data collection tools used. To avoid losing the data, I uploaded the data onto my Google drive account as a way of backing up. After all these, I then embarked on the process of analysing the data.

3.7.2.2 The inductive analysis approach

To analyse the data, I adopted an inductive analytic approach for the qualitative data generated by this study. For qualitative data, the inductive analysis approach is "a process of organising the data into categories and identifying patterns (relationships) among the categories" (Bertram & Christiansen, 2014, p. 117). I strongly agree with Ruona (2005) that the fundamental approach of the analytic approach to qualitative data is inductive analysis which mainly focuses on identifying themes that directly emerge from the data.

In this study, I analysed the data according to the themes that emerged from the raw data of the study. The aim of the inductive approach of analysis is to allow the findings of the study to emerge from those themes which appear most frequently and appear most important in raw data without the restrictions of structured methodologies (Thomas, 2006).

To obtain the themes from the data, firstly, I compared the pre-test and the post-test work of each learner per question. I noted the differences and similarities in the work of the learner. The aim was to see if there were changes in the way they answered the questions because all the test questions were the same for the pre-test and the post-test. Thereafter, I noted common themes based on how similar the learners answered each question in the pre-test and I did the same thing for the post-test. Some themes were also developed based on individual learners' work that appeared to be of serious concern.

Secondly, I started studying the observation transcripts looking for the data that seemed to support the changes and consistencies in the learners' work as they answered the two tests. I used different colours to highlight the texts in the transcripts that were speaking to the themes identified from the tests' results. I then developed the final themes connecting by learners' work from the benchmark tests and their learning engagements in respective areas of learning. The data I looked for from the observation transcripts was aimed at answering all my research questions. The themes were grouped as per the areas of learning, which were: variables, expressions and equations. The themes were developed sequentially in terms of the learners' understanding, e.g. how learners understood the concept before, how they have changed and whether they have achieved a stable understanding.

The table below influenced the way I coded my data for this study.

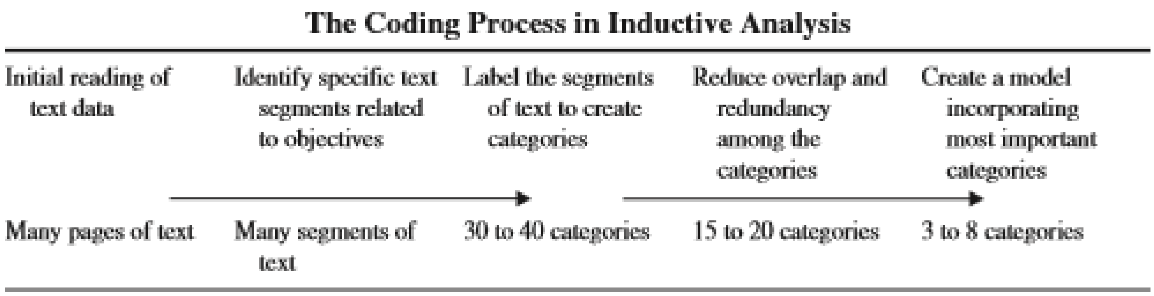


Figure 3.6: **Adapted from Creswell (2002, p. 266, Figure 9.4)**

Below is a snapshot of the process I used to colour-code my data in terms of the level of understanding of the learners in each teaching area.

Key: Red – shows learner is struggling; Yellow – shows the transition; Green – shows the learners' stable understanding.

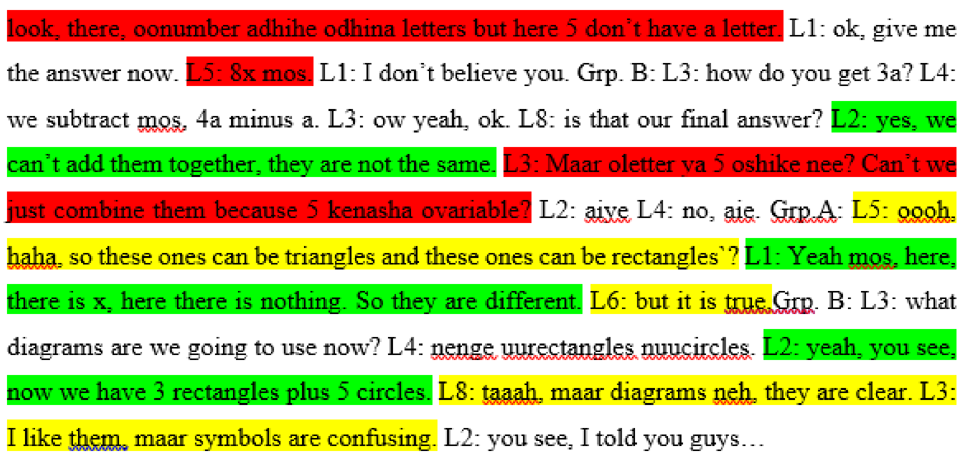

look, there, oonumber adhihe odhina letters but here 5 don't have a letter. L1: ok, give me the answer now. L5: 8x mos. L1: I don't believe you. Grp. B: L3: how do you get 3a? L4: we subtract mos, 4a minus a. L3: ow yeah, ok. L8: is that our final answer? L2: yes, we can't add them together, they are not the same. L3: Maar oletter ya 5 oshike nee? Can't we just combine them because 5 kenasha ovariable? L2: aiye L4: no, aie. Grp A: L5: oooh, haha, so these ones can be triangles and these ones can be rectangles`? L1: Yeah mos, here, there is x, here there is nothing. So they are different. L6: but it is true.Grp. B: L3: what diagrams are we going to use now? L4: nenge uurectangles nuucircles. L2: yeah, you see, now we have 3 rectangles plus 5 circles. L8: taaah, maar diagrams neh, they are clear. L3: I like them, maar symbols are confusing. L2: you see, I told you guys…

Figure 3.7: **Inductive analysis process**

I also extracted data from the focus groups interview transcripts that were relevant to the themes generated from the benchmarks and observations. Most of the focus group, individual interviews and the entire journal data were used for giving the overview of the whole programme and this data was not analysed as the focus was more on the learners' experience of the programme rather than on specific learning.

3.8 VALIDITY

According to Cohen et al., (2011, p. 179) “validity is an important key to research. If a piece of research is invalid, then it is worthless”. In this study, validity was ensured through triangulation of the data collected. Triangulation according to Cohen et al., (2011, p. 195) is “the use of two or more methods of data collection in the study of some aspect of human behaviour”. The view that triangulation has to do with collecting data from a number of different sources is confirmed by Bertram and Christiansen (2014).

This study aligns with the descriptions above as it uses more than one tool for data collection, to ensure the consistency of the type of data collected. I collected the data using the benchmark tests, observations, focus groups information and artefacts. The data from the observations was used to confirm the data from the benchmark tests and the learners’ work. The learners’ interaction during the learning engagement was crucial for me to establish the relationship of their interaction with the way they answered the tests and activity questions. During the learning process, learners engaged with learning the concepts related to the tests’ questions. Furthermore, the focus groups confirmed learners’ experiences captured during the observations. In the focus groups, the learners were given chances to express their dispositions on the understanding of the concepts before and after each lesson. This helped me to gain a full, nuanced understanding of the learners’ position in terms of their understanding of the concepts and eventually, validating the data of the study.

Finally, mechanical tools were used to record data to ensure credibility (Bertram & Christiansen, 2014), for example, using audio and video to record interviews verbatim and lesson observations respectively. The data collected through observations and focus groups were then transcribed.

3.9 ETHICS

According to Bertram and Christiansen (2014, p. 65), ethics is “an important consideration in research”. This study was conducted with the school learners who were ethically considered as minors. A study involving minors requires an ethical clearance from Rhodes University Ethics Standard Committee (RUESC), a committee responsible for ethical approval at the university. To conduct the study, firstly I applied and obtained ethical clearance (see Appendix A) from Rhodes

University, which I submitted together with my proposal. Secondly, I applied for permission from the regional director for the Directorate of Education of the Oshikoto region in which the study was conducted (see Appendix B). I also applied for permission from the principal of the school where I wanted to conduct my study (see Appendix C). Since my study targeted learners, who were minors, I wrote a letter to their parents informing them of the study and asking them to allow their children to participate (see Appendix D). On the letter, I attached a consent form where the parents were required to sign as proof of allowing their children to participate. The teachers of the selected learners assisted me in sending the letters to the parents. After acquiring the parents' consent, I called the learners and explained to them the aim of the study, and also, I indicated to them that their participation was voluntary and it was their right to withdraw from the programme at any time. All the learners agreed and the following day we began with the programme. I ensured that the learners' identities were kept confidential by using codes e.g. (L1, L2) to distinguish between the learners. I kept the audio and video recordings on my electronic equipment which have passwords, so that no one has unauthorised access to them.

3.10 CONCLUSION

This chapter presented the methodology of the study and justified its research design. The research orientation, methodology of the study that includes the three phases of conducting design research, the development of the teaching programme, the research participants and research site have all been described in this chapter. In this chapter, I also described the methods that I used to collect data, which were: benchmark tests, interviews, observations and artefacts. In addition, I discussed the data analysis approach, the validity and the ethical aspects of the study. The following chapter presents the data collected in the above-mentioned methods, while the subsequent chapter offers the analysis and discussion of that data.

CHAPTER FOUR

DATA PRESENTATION

4.1 INTRODUCTION

This chapter presents the data which was collected during the teaching programme as preparation for the more detailed analysis and discussion presented in Chapter Five. A brief detailed experience of the learners about the programme is also presented in this chapter as indicated in chapter one, section 1.4. The data was gathered through the use of four modalities namely, benchmark tests, observations, focus groups and unstructured interviews. In this chapter, the data is presented in two sections: Part A and Part B. The data presented in Part A comprises an overview of the programme. Part B is comprised of the data collected with regards to the specific learning of the three concepts of linear algebra, namely, variables, expressions and equations. The table below shows the sequence of the concepts of linear algebra covered in the programme. The eight lessons were covered in a space of three weeks.

Table 4.2: **Sequence of the concepts taught in the programme**

Lesson sequence	**Concepts covered**
Lesson 1	Variables - what variables are and what they represent
Lesson 2	Variables - what variables represent
Lesson 3	Expressions (simplifying) addition only
Lesson 4	Expressions (simplifying) subtraction only
Lesson 5	Expressions (simplifying) addition and subtraction
Lesson 6	Expanding brackets
Lesson 7	Expanding brackets + Introducing solving equations
Lesson 8	Solving Equations

Below are the keys and pseudonyms used when discussing the data and the findings in Chapter Four and Five:

Focus Group Meeting	FGM (1, 2, 3 …)
Observation	Obs.
Journals	J (1, 2, 3 …)
Interview	Int.
Learner	L (1, 2, 3 …)
Paragraph	Para. (1, 2, 3 …)
Transcribed	Trans.
Group	Grp.
Extract	Ext.
Teacher (Researcher)	T

4.2 PART A: OVERVIEW OF THE IMPLEMENTATION OF THE PROGRAMME

The data sets presented in this section were gathered from the learners through journals, focus groups, unstructured interviews and part of the observations which showed the experience of the learners. It presents the learners' attendance, participation during the programme and their experience of the programme. This data does not answer any of the research questions, however, I felt it is important to share a brief experience of the learners as they interacted with the programme. Thus, the data presented in this section will not be analysed in the next chapter.

4.2.1 Attendance and participation of the learners

Eight learners participated in this programme. The attendance of the learners was good for the first five lessons of the programme, with the learners arriving well on time to the learning venue (J, Lesson 1 - 5). During Lesson 6, only five learners arrived on time and the rest joined later after about 10 minutes. The delay came as a result of the test they had written beforehand. One learner did not attend this lesson because he was away with the school debate club (J, Lesson 6). Half of the learners did not come on time for Lesson 7, and I had to go look for them in their classes. I found some of them wandering around the school and some in their classes. When I asked them why they had not gone to the lesson, some indicated that they were still coming and some did not respond. They all went to the lesson from there. Six learners arrived on time to Lesson 8, and the other two learners joined late as they had a practical assessment with the agriculture teacher in the garden.

Although all attended, most of the learners did not participate well in the beginning. They took a while to start participating and only a few of the learners dominated the input and discussion. After the first two lessons most of the learners started participating. The reason for this might be that the learners were from three different class groups and they were not used to each other. This might have impacted on their participation at the beginning as they were trying to get used to each other and gain confidence. They might have been less free to express themselves in front of other learners who were new to them and also to me, who they had not interacted with before. Learners sat in groups of four for the whole programme. Learners with high performance records in the Continuous Assessment (CA) sheet were the most dominant in their groups for the whole programme. From the fourth lesson, other learners gained confidence and were observed to be fully participating during both the class discussions and group discussions. All learners had a chance to write whenever they were asked to solve a problem in their groups as per my instructions. This was one of the ways to make all learners participate and feel part of the group. In general, all the learners participated well in the programme.

4.2.2 Learners' experience of the programme

The data on the experience of the learners about the programme was drawn from the focus groups, unstructured interviews and parts of the observations. I conducted focus groups after every lesson, except Lesson 2, when learners were called to attend to a meeting with health professionals from the Department of Health. The aim of the focus groups and interviews was to get the experiences of the learners on the learning of each concept and the whole programme at large.

The learners had already covered all the algebraic concepts with their teachers for about two months before the programme, but not all of them had mastered what they had learned. They shared their experiences of learning the concepts during the focus group meetings/interviews. Learners had different difficulties in learning different concepts. Some of them had problems in separating terms in expressions and some expressed that they had challenges with expanding brackets (Obs.).

Generally, learners indicated that they were happy with the programme. They commented that the programme had made an impact on their understanding of algebra, for example, on the expansion of brackets. There were some concepts in which the learners showed they were good at before the programme, such as, solving linear equations. Learners needed less engagement with solving linear equations as they were seen solving them confidently before the programme.

4.2.3 Learners' experiences on working with diagrams

This section presents the learners' experiences of the whole programme, including their experiences with the use of the tools. The experiences differed from one learner to the other.

Some of the learners indicated that they were happy with the use of diagrams. Learners mostly commented on diagrams when they were dealing with simplifying expressions.

Learners often preferred to work with diagrams when working on the problems they were required to do in class during their group discussions. This was observed in Lesson 7 when the learners were asked to solve the same problem in their groups, where one group was required to use diagrams and the other symbols. All learners wanted to use diagrams (Obs., Lesson 7, Line 16-18). This on its own revealed that the learners were confident when using diagrams during the programme.

The use of diagrams brought fun into the programme, particularly in Lesson 5 and 6 where some learners were happy to shift to using diagrams in solving a problem, for example L3 was heard saying, "*my favourite"* (Obs., Lesson 5, para. 38). Some learners further expressed that using diagrams was just like playing a game.

Even when learners did not expressly refer to enjoyment, they were noticeably livelier when engaging with the diagrams in their groups. During the FGM3, L1 indicated that using diagrams was interesting. He was quoted as saying, "*Using diagrams is not the same as using numbers and letters. It is more interesting. Is like you are playing a game*" (FGM3, para. 8). This seemed to suggest that using diagrams motivated learners to become actively involved in the learning, as to them it appeared as if they were playing a game. Most of the learners indicated that using diagrams helped them commit fewer errors especially when simplifying expressions. They commented more on the separation of terms saying, *"diagrams are easy to differentiate"* adding that "*yes it will be difficult just to combine different diagrams"* (FGM3, para. 12 & 14).

However, the comments of the learners about the use of diagrams were not all positive. Some of the learners indicated that though diagrams helped them in getting the correct answers, it took time to draw the diagrams, and they worried whether they would be able to finish an activity within the given time. During FGM3, L3 indicated that diagrams required much time to draw, as she was heard saying "*Yes, they help, but it takes too much time to draw the diagrams"* (para. 10). The same concern also emerged from the interviews when L8 said "*But, diagrams take time to get, like take time to ... diagrams take time for you to finish"* (Int. L8, para. 20).

4.2.4 Learners' experience on working with an expansion box

The use of an expansion box was introduced to the learners during Lesson 6 when they started to deal with expanding brackets. Learners enjoyed making use of the expansion box. Learners indicated that the use of an expansion box helped them when multiplying negative terms and checking their answers if they were correct. (FGM trans. Lesson 6, para. 23 & 27). This seemed to suggest that the learners had confidence in using an expansion box. They went on to indicate that when using an expansion box, it was easier because the signs of the terms were clearly separated. Though it was the first time the learners had made use of an expansion box, it did not take much time to understand the procedure of using it. They were all seen trying to use it during the lessons to expand other brackets they came across.

4.2.5 Learners' experience on learning the balance method

The balance method was the method planned to solve equations in the programme. The method did not appear to have much influence on the learners when solving equations. Some of the learners noted that the method was difficult and some found it confusing. They were really not happy using the method. Below are the responses from some learners:

1. **L3**: I did not really understand how to balance; it is just confusing.
2. **T**: How confusing?
3. **L3**: I just don't know when to add or to subtract.
4. **Ls**: (some) me too.

Working with negative terms in the equations was really a problem to the learning when learners were using the balance method, as they indicated that they did not know when to add or subtract (Obs. trans. Lesson 8).

Nevertheless, some of the learners mentioned that the use of the method had taught them that equations are used in real life activities, something that they had not known (FGM8, Para. 12).

4.3 PART B: SPECIFIC LEARNING ENGAGEMENT

The data presented in this part was gathered through benchmark tests and observations. This data speaks directly to the specific learning experience of the learners in the three areas: variables, expressions and equations. The benchmark tests were comprised of the pre-test and the post-test. The data from the pre-test showed the understanding of the learners in the three areas before the programme and the post-test data showed their understanding after the programme. The data from the observations indicated the learning engagement of the learners in the programme that might have influenced the learners' understanding after the programme in the specific learning areas.

4.3.1 Learning engagement related to variables

The data presented for this area of learning is from the observations only. The benchmark tests did not assess the learners on the understanding of variables.

4.3.1.1 Observations

In Lesson 1, I asked the learners to give an example of variables and what variables represented. The learners gave examples of variables as alphabetical letters and their opinions of what the variables or letters represented. Below is an extract from the lesson:

Extract 1

1. **T**: Good afternoon everyone. In our very first lesson of the programme, I would like us to look at variables. I believe that all of us here have learnt about variables from our algebra lessons. Isn't it?
2. **Ls**: Yes, we have done it.
3. **T**: Who can give me an example of a variable?
4. **Ls**: (randomly one by one) a, x, y, b …

5. **T**: So now, tell me, how do you define variables then?
6. **Ls**: (whispering and not giving their views)
7. **T**: (wrote the examples given by the learners on the board) come on, tell me, what are variables?
8. **L4**: They are representing letters.
9. **T**: Ok, so what do letters represent?
10. **L2**: Unknown numbers.
11. **L1**: They represent unknown objects, anything that we do not know. Like we use letters to represent unknown objects.

The extract above shows how the learners understood the concept of variables. In the same Lesson 1, I gave a scenario as an example for the learners to think about in trying to understand what variables represent. The learners gave their views as they were interacting with me in the extract below:

Extract 2

13. **T**: Now, let's look at the scenario here: In a combined school, the amount of food consumed by primary school learners cost x Namibian dollars. The amount of food consumed by secondary school learners cost twice the cost of food consumed by primary school learners. (repeated the question making it clear to the learners). Now the question is, relate the cost of food for primary learners and for secondary learners using x. How do you relate them using a variable. What variable do we have here?
14. **Ls**: x
15. **T**: So how do we represent these costs using x as a variable? Remember, primary school cost is x, and the secondary cost is twice the primary cost. Can you discuss in your groups?
16. **Ls**: (embarked on discussion)
17. **L1**: We have come up with this, primary cost is x and the sec school cost is twice, so twice means multiply by 2, so we multiplied x by 2 and we get 2x.
18. **T**: (wrote what the learner said on the board) So, meaning primary cost is x and the sec cost is 2x. So, what does this mean to you? In other words, how do you understand this thing generally?
19. **L8**: Secondary cost is 2x greater than the primary cost.
20. **T**: Yes, that's the understanding. So, anyone else? Who wants to give it a try? Come on …
21. **L2**: Secondary learners eat more than the primary learners.
22. **T**: Ok, good, that's what her thought is … yes, anyone else?
23. **L3**: It means that the cost of food for secondary learners is higher that the primary cost.
24. **T**: Good, so the two of you (L2 & L3) have said almost the same thing, that the secondary learners eat more than the primary learners.
25. **Ls**: Yes, (one learner shouted: "yeah, it is true, because they are big").

26. **T**: Now, what does x represent here?
27. **L6**: It represents money.
28. **T**: Represents money? …What money? Nam dollars, Angolan kwachas or what money?
29. **L1**: No, it represents the amount of money…
30. **T**: Ok, so what are we saying now? Is it money or the amount of money?
31. **Ls**: (most of them) The amount of money.
32. **T**: So, what does the amount of money represent then? Is it the unknown things/object or unknown number as you guys told me earlier?
33. **L1**: I think it is unknown number because we still don't know how much. We only know the money is x, but there is nothing like x money. (laughed)
34. **T**: Hahaha, ok. So, let's take an example, if the primary food cost N$100, how much do sec school food cost?
35. **Ls**: (started whispering in their groups and put up their hands to give their answers).
36. **L3**: You need N$200 dollars.
37. **T**: How do you get that?
38. **L3**: You multiply 100 by 2.
39. **T**: Why multiply by 2?
40. **L3**: Because they are saying the cost of sec food is twice the cost of primary food.
41. **T**: So, what represents 100 here?
42. **Ls**: x
43. **T**: Good. Now if next month they say, sec food cost N$1000, how much would the primary food cost?
44. **L7**: 500
45. **T**: Very good. And we are saying, all these amounts are represented by …?
46. **Ls**: x
47. **T**: And what do we call x?
48. **Ls**: Variable

This extract shows the developing understanding of what variables represent through a scenario. The end result of what the x represented was to make learners realise that it represents the *amount* of money and not money as an object.

On a different question, I gave the learners another scenario about the number of teachers in relation to the number of learners in a school. I then asked the learners to give what they thought the letters T and L represented in the equation that contained T and L. Below is an extract of the conversation:

Extract 3

49. **T**: Very good. Let’s look at the other question: in our school, the number of learners (L) is 35 times the number of teachers (T). So, what does L represent again?
50. **Ls**: Learners.
51. **T**: So, it represents learners … ???
52. **L1**: Sorry, it represents the number of learners.
53. **T**: And T?
54. **L2**: The number of teachers.

On the same question, I asked the learners to give the formula or equation that represented the information given. Learners worked out the equations in their groups and gave their solutions verbally to me.

Extract 4

57. **T**: Right, if we have done, let’s discuss. So, what do you get?
58. **L1**: We got like, $L = 35\times T$.
59. **L3**: We got $T\times 35 = L$
60. **T**: Is there a difference between the two?
61. **L1**: No, just the other way round.
62. **T**: Ok, I like that. So, if there are 140 learners in the school, how many teachers are there? Discuss…
63. **Ls**: (Discussion. Some learners in one group seemed excited to have got the answer quickly)
64. **T**: Have you guys done?
65. **Ls**: Yes.
66. **T**: Can anyone tell me how you understand this formula. $L = 35\times T$.
67. **L8**: The number of learners is equal to 35 times the number of teachers.
68. **T**: Very good, that’s what it exactly means. So what answer did you get then and how?
69. **L7**: We say $L = 35T$, and replace L with 140, and we have $140 = 35T$. We then divide both sides by 35, and we get $T = 4$.
70. **T**: So, you got $T= 4$? What does 4 represent again?
71. **Ls** (together): Teachers
72. **T**: Haha , are you sure?
73. **L2**: No sir, I mean number of teachers (laughed).

Towards the end of the lesson, I generally asked the learners to give their views on what they thought variables represented, based on the activities they had done during the lesson. The learners gave their different views to me. The extract below shows the learners’ responses:

Extract 5

74. **T**: Very good, I see you guys are getting it bit by bit. So, let's conclude, what does a variable represent in this context? Remember I am not saying what you have learnt in other areas is wrong, no. But let's use the examples that we have used here to tell me what do you think a variable represents.
75. **L4**: A variable represents an unknown number.
76. **T**: Hmmm, ok. Anyone to give their views again?
77. **L3**: It represents an unknown number and object.
78. **L1**: Sir, let me try. I think it represents an unknown number of the objects. Like, learners are objects, now we are saying how many objects/learners are those? Yeah ... just an idea though…
79. **T**: That's good. Now let's decide, what does it really represent? Is it objects, unknown numbers or number of objects?
80. **Ls**: (randomly): Number of objects…
81. **T**: Good, I agree with you guys. So, the variables represent the number of objects which is the same as quantities. So, what do you think is a quantity?
82. **L1**: Quantity is how much or how many things … Yeah.
83. **T**: Very good. So, a variable represents a quantity of a certain measure. For example, here our measure is teachers and the T represents the number of those teachers, which is a quantity. (I wrote the defn on the board). Quantity means how much or how many? We talk about people, so the people is the measure and the P represents the quantity of those people.

The learners had different views of what variables represented and they expressed their views based on the examples used in the lesson.

In Lesson 2, learners were asked to present their solutions on the homework they had got in the previous lesson. The homework was on the proportion of the amount of money received by Tom and Jerry. Below is the interaction:

Extract 6

3. **T**: Alright, let's look to the homework I gave you yesterday. (Read the homework). Right, how much did Tom get according to the question?
4. **Ls**: P Namibian dollars
5. **T**: And Jerry gets?
6. **L2**: Half of Tom's amount.
7. **T**: So how do we write the amount in terms of P?
8. **L1**: P divided by 2.
9. **T**: Now, let's answer this question. How much does Tom get if Jerry gets N$4?

10. **L7**: 8
11. **T**: Good, so it is 8 Namibian dollars because whatever Tom is getting, Jerry gets half of Tom's amount. So, what did we say variables represent?
12. **L1**: Quantities, amount of something.
13. **T**: What is our variable in this question?
14. **L2**: P
15. **T**: Does it represent a quantity here? And if so, what is the quantity?
16. **L1**: Yes, it represents the amount of money.
17. **T**: Which money?
18. **L1**: Namibian Dollars sir.

4.3.2 Learning engagement related to expressions

The data for learning expressions were drawn from the benchmark tests and observations.

4.3.2.1 Benchmark tests

The data drawn from the benchmark tests are presented in the table form. All test questions are presented with combined/single responses from the learners depending on their similarities or differences in their work. Both responses from pre-test and post-test are presented starting with the pre-test responses.

- ***Separation of terms***

Separation of terms is part of both expressions and equations. In the test, there was only one expression directly aimed at assessing learners' understanding of separating terms. It was discovered that separation of terms was an issue as learners dealt with some expressions and equations in the test. The table below presents selected learners' work that indicated challenges related to the separation of terms.

Table 4.3: **Learners' work in pre-test and post-test on separation of terms**

Questions	**Pre-test**	**Post-test**

Simplify the expression 3x + 2y – x + 5y	• All eight learners had the same solutions. (a) $3x + 2y - x + 5y$ $3x - x + 2y + 5y$ $2x + 7y$	• All eight learners gave the same solution as in pre-test. (a) $3x + 2y - x + 5y$ $3x - x + 2y + 5y$ $2x + 7y$
2(a + 3)	• L3 $2(a+3)$ $2 \times 3a$	• L3 $2 \times a + 2 \times 3$ $2a + 6$
-3(2y + 1) - 4	• L3 $-3 \times 3y - 4$ $-9y - 4$	• L3 (c) $-3(2y + 1) - 4$ $\quad -3 \times 2y + (-3) - 4$ $-6y - 3 - 4$ $-9y - 4$
3x – 2 = x + 5	• L6 (b) $3x - 2 = x + 5$ $3x - 2 = 6x$ ~~3x = x + 5 + 2~~ $: 3x = 6x + 2$ $\frac{3x}{3} = \frac{8x}{3}$ $\quad \therefore x = 2.7x$	• L6 (b) $3x - 2 = x + 5$ $3x - x = 2 + 5$ $\frac{2x}{2} = \frac{7}{2}$ $x = 3.5$ (c) $2a + 3(a - 1) = 13$
y - (2 - 3y) = 2y - 5	• L6, L7 & L8 showed the same work. (d) $y - (2 - 3y) = 2y - 5$ $y \times 2 - y \times 3y = 2y - 5$ $2y - 3y^2 = 2y - 5$ $y = -3y$ $y = -3$ $\quad$ ……$y = -3$……	• L6, L7 & L8 (d) $y - (2 - 3y) = 2y - 5$ $y - 2 - 3y = 2y - 5$ $y - 3y - 2y = +2 - 5$ $\frac{-4y}{-4} = \frac{-3}{-4}$ $y = 0.75$

Table 4.4: **Learners' work in the pre-test and post-test on expanding brackets**

Questions	**Pre-test**	**Post-test**
2(a + 3)	• L1, L2, L4, L5, L6, L7 & L8 showed the same work. (b) 2(a + 3) 2×a + 2×3 2a + 6 • L3 refer to table 1, row 3 column 2	• All learners got the same solution as shown in the pre-test. (b) 2(a + 3) 2a + 6
-3(2y + 1) - 4	• L1, L2, L4, L5, L6 & L8 showed the same work. (c) -3(2y + 1) – 4 -6y + -3 - 4 - 6y + -7 -6y - 7 • L3 refer to table 4.2, row 3 column 2 • L7 (c) -3(2y + 1) – 4 -3×2y + -3(+1) - 4 - 6y + -2 - 4	• L1, L2, L4, L5, L6, L7 & L8 showed the same work. (c) -3(2y + 1) – 4 -6y - 3 - 4 - 6y - 7 • L3 (c) -3(2y + 1) – 4 -3×2y + (-3)1 - 4 -6y - 3 - 4 = 9y - 4
2a + 3(a - 1) = 13	• L4 & L6 showed the same work (c) 2a + 3(a - 1) = 13 = 2a×a + 3×-1 = 13 = 2a² + (-3) = 13 = 2a² - 3 = 13 = 2a² = 13 + 3 = 2a² = 16 = 2a²/2 = 16/2 = a² = 8 • L1, L2, L3, L5, L7 & L8 showed the same work. (c) 2a + 3(a - 1) = 13 2a + 3a - 3 = 13 5a - 3 = 13 5a = 13 + 3 5a/5 = 16/5 a = 3.2	• L1, L2, L4, L5, L6, & L7 showed the same work (c) 2a + 3(a - 1) = 13 2a + 3a - 3 = 13 2a + 3a = 13 + 3 5a = 16 5a/5 = 16/5 a = 3.2 • L3 2a + 3×a - (a×1) = 13 2a + 3a - a = 13 5a - 4a = 13 • L8

		(c) $2a + 3(a - 1) = 13$ $2a + 3a - 1 = 13$ $2a = 13 - 3a + 1$ $\frac{2a}{2} = \frac{11}{2} = a = 5.5$
y - (2 - 3y) = 2y - 5	• L2, L4, L6, L7 & L8 showed the same work (d) $y - (2 - 3y) = 2y - 5$ $y \times 2 - y \times 3y = 2y - 5$ $2y - 3y^2 = 2y - 5$ $2y = 2y - 5 + 3y$ $-2y = 2y + 3y - 5$ $2y = 5y - 5$ $y = \cancel{8} - 1$ • L1 & L5 showed the same work (d) $y - (2 - 3y) = 2y - 5$ $y - (x2 - (-1 \times 3y)) = 2y$ $y - 2 - (-3y) = 2y - 5$ $y + 3y - 2y = 5 + 2$ $3y - 2y = 5 + 2$ $\frac{6y}{6} = \frac{7}{-5}$ $y = -1.5$ • L3 $y - (y - (-1)) = 2y - 5$ $y - 2y = -5 - 6$ $\frac{-1y}{-1} = \frac{-6}{-1}$ $y = 6$	• L1, L2, L4, L5 & L6 showed the work. (d) $y - (2 - 3y) = 2y - 5$ $y - 2 - 3y = 2y - 5$ $y - 3y - 2y = -5 + 2$ $-4y = -3$ $\frac{-4y}{-4y} = \frac{-3}{-4}$ $y = 0.75$ • L3, L7 & L8 showed the same work. $y - 2 \times y - 3y \times y = 2y - 5$ $y - 2y - 3y^2 = 2y - 5$ $-1y - 3y^2 = 2y - 5$ $-4y = 2y - 5$ $4y + 2y = -5$ $\frac{6y}{6} = \frac{-5}{6}$ $y = 0.83'$

4.3.2.2 Observations

- ***Separating terms***

During a group discussion in Lesson 3, learners were given an algebraic statement 2a + 4b + 3a + 3c + a = 5l + 2k + l + p + 3k and asked to reduce it as possible. Learners discussed this in groups.

Below is the extract from one group:

Extract 7

> Discussion: **L7**: Can we put these ones together? **L2**: No, you can't these letters are not the same. **L7**: Haha, true man. **L2**: Let's use diagrams, what shapes can we use? **L3**: rectangle, circle, kite, yeah, those things. **L2**: Now see here, can you add rectangle and triangles together? **L7**: Yeah, kakwali ndatala the letters kaa. **L2**: I told you (Trans. Obs.3, para. 58).

The extract shows how the learners interacted with the diagrams, making choices of what shapes to use to replace the symbols. They were trying to make some connection between symbols and the diagrams.

Also, in Lesson 3, the learners were required to reduce the expression x + 3y + x + y = m + 2n + n + 3m. After the group discussion, I asked for feedback from the learners.

Extract 8

16. **T**: Tell me, how do I combine this side? Yes? (pointing to a learner)
17. **L4**: You collect the like terms like, x + x + 3y + y = m + 3m + 2n + n
18. **T**: From there?
19. **Ls**: (together) 2x + 4y = 4m + 3n
20. **T**: And now, can we move forward?
21. **L2**: No, you can't add them.
22. **T**: Why?
23. **L2**: Because they are not like terms, like x is not the same with y.
24. **T**: Good, so you understand.
25. **Ls**: Yes.
26. **L2**: (pointing at L6) You see, manga wali toti we can add them.

Learners simplified the expressions by collecting the like terms in the expressions.

I asked the learners to present the statement using diagrams. Learners discussed this in groups and gave their views afterwards. See the extract below.

Extract 9

40. **T**: Now, let's represent it in diagrams. Let x be (rectangle), y be (triangle), m be (Circle) and n be (kite). Let's do it in our groups.
41. **Ls**: Discussing: **L4**: No, you can't put kites together with the circles, inayiifa. **L6**: Taa, now I see. I did not know this. **L3**: Maar with diagrams omuwete ngaa kutya oshipu; sometimes with letters you just add together these things without knowing.
42. **T**: Have we done? Who can tell us what they did in their group?
43. **L2**: 1 rect + 3 tri + 1 rect + 1 tri = 1 circ + 2 kites + 1 kite + 3 circ
44. **T**: From there?
45. **L2**: You group them. Rectangles together, triangles together and also the other side group same diagrams together.
46. **T**: So how many rectangles do I have here?
47. **Ls**: 2 rectangles + 4 triangles = 4 circles + 3 kites.
48. **T**: Can we go further?
49. **L4**: No, we can't because the diagrams are not the same.
50. **T**: Good. So, when some people use symbols like here, what mistake do they do?
51. **L1**: They will give 6xy.
52. **Ls**: (Laughed)
53. **L3**: Yes, because the letters are confusing sometimes.
54. **T**: What about diagrams?
55. **L3**: No, they are clear and you can see easily the diagrams are different.

Learners made use of the diagrams of their choice to replace the symbols. They acknowledged the use of diagrams, saying they were clear and indicated some difficulties that came with using the symbols.

In Lesson 5, learners were given an expression different from the ones they did in the last two lessons and were asked to simplify it in their groups. This expression had variables and constant terms. Below is the extract for the group discussion and interaction between me and the learners after the group discussion.

Extract 10

20. **T**: Ok, good. Can we quickly look at this one in our groups, 4a + 3 – a + 2. Let's be faster. Remember you must try it both in symbols and diagrams.
21. Group discussion.
22. (**Grp. A**: **L7**: This is simple, let us correct the like terms. **L1**: Yes, you are right. We say 4a - a + 3 + 2. **L6**: O minus oweyi kuthapo pehala. **L1**: Aiye, minus is for 'a' mos. **L7**: So, our answer is 3a + 5? **L1**: Yeah. **L5**: Are u sure guys. But can we not add them together? **L1**: Haha, you, did we add those ones when we did that with sir. **L6**: But look, there, oonumber

adhihe odhina letters but here 5 don’t have a letter. **L1**: Ok, give me the answer now. **L5**: 8x mos. **L1**: I don’t believe you.

23. **Grp.B**: **L3**: How do you get 3a? **L4**: We subtract 4a minus a. **L3**: Yeah, ok. **L8**: Is that our final answer? **L2**: Yes, we can’t add them together, they are not the same. **L3**: Maar oletter ya 5 oshike nee? Can’t we just combine them because 5 kenasha ovariable? **L2**: Aiye. **L4**: No.
24. **Grp.A**: **L5**: Haha, so these ones can be triangles and these ones can be rectangles? **L1**: Yeah, here, there is x, here there is nothing. So, they are different. **L6**: But it is true.
25. **Grp.B**: **L3**: What diagrams are we going to use now? **L4**: Nenge uurectangles nuucircles. **L2**: Yeah, you see, now we have 3 rectangles plus 5 circles. **L8**: Taaah, maar diagrams neh, they are clear. **L3**: I like them; maar symbols are confusing. **L2**: You see, I told you guys.
26. **T**: Have we done?
27. **Ls**: Yes.
28. **T**: What answer do you get?
29. **L6**: 3a + 5
30. **T**: Good, is it the same here?
31. **L3**: Yes, but…
32. **T**: But what?
33. **L3**: Sir, why can’t we add 3a and 5 so we get 8a.
34. **T**: When you use diagrams, did you use the same diagram for all those terms?
35. **Ls**: (Together) No.
36. **T**: Why? Who can tell me?
37. **L2**: Because, the a, value, no, wait, where there is a, it is not the same like where there is nothing. Yeah.
38. **T**: Good, the constant term is different from the term with a variable, that’s why we cannot use the same diagrams. That means, you can’t add a constant to a term with a variable, they are not the same. It is, just treat it like there is a different letter. Do you get it now?
39. **L3**: Yes, sir. I see now. But diagrams are clear because you can see the difference.

The learners showed different understandings of like terms. Some of them thought the terms were like and some thought they were not. They used some diagrams to see if they could see the difference. Some eventually realised the terms were different.

Still in Lesson 5, learners were given an algebraic statement 3b - 2g - b - 3g = 4w + 2m - 3w - m and asked to simplify the expressions on both sides. The work was done on an individual basis. Below is a sample of the work of some learners.

L2 L6 L7

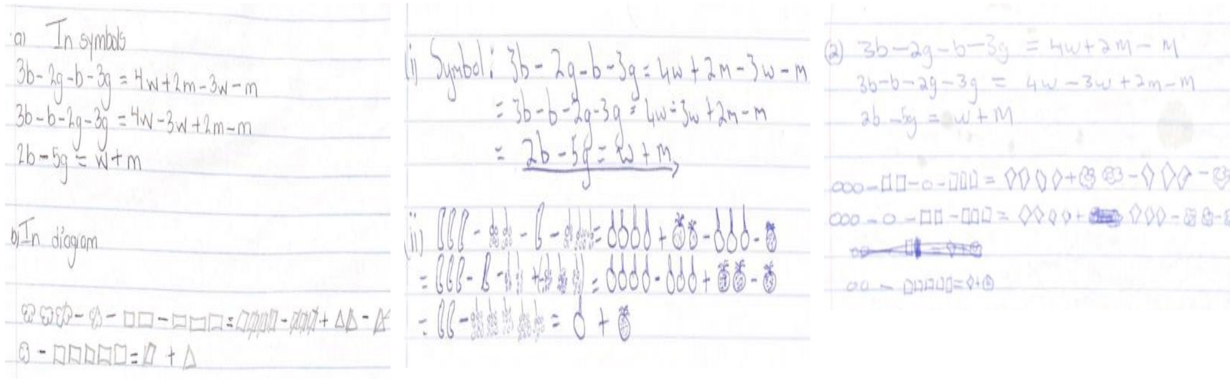


Figure 4.8: **Learners' work on simplifying expressions during a class activity**

The table shows the work for L2, L6 and L7. They simplified the expressions individually.

Expanding brackets

Introducing expanding brackets in Lesson 6, I asked the learners what brackets meant to them given for example the expressions 2(x + 1). Learners shared their views as indicated below:

Extract 11

3. **T**: As I have said today, we are going to look at expansion of brackets. I know you have already covered that only by using symbols. You may not have made any meaning from that but today I want us to do it and make meaning out of it. Right to start, one can say, expand: 2(x + 1). What does the whole expression tell you? What do you think it means to you?
4. **L1**: Brackets mean times, like multiply the number next to the brackets with what is inside the brackets.
5. **T**: A number next to the brackets, like this one? (pointing at 1 inside the brackets)
6. **L1**: No, the one outside the brackets.
7. **T**: Ok, anyone else?
8. **L3**: It also means you work out what is inside the brackets first.
9. **T**: So how can we work these ones out now?
10. **L4**: No sir, I don't think that's possible.
11. **T**: Why?
12. **L5**: Because they are not like terms.
13. **T**: So, we cannot add them together?
14. **Ls**: No.
15. **T**: What we can say here to make meaning, one can say the sum of x and 1 is doubled. Or double the sum of x and 1. Do you understand why we are saying this is a sum?

16. **Ls**: Yes.
17. **T**: Why?
18. **Ls**: Because we are adding.
19. **T**: Good, so the practical example could be, let's say you have a number of sweets here on the RHS and a cake here on the LHS, so can you add them together?
20. **L2**: No, you can't, you will just have 2 apples and 2 oranges.
21. **T**: So, now, if someone tells you to double them, can you double them?
22. **Ls**: Yes.
23. **T**: Good, so, you can double them separately, the number of sweets and you also double the cake. So, what we are saying here now, we are saying, you multiply 2 by x and also multiply 2 by 1. So, we now have 2x + 2, can we add these together?
24. **Ls**: No, they are not like terms.

The learners gave their understandings of the brackets through the use of symbols. We also tried to make sense of the brackets by relating the expressions to real life examples.

I then introduced an expansion box to the learners and explained how it worked using the example 2(x + 1), in the extract below:

Extract 12

46. **T**: (Draws an expansion box on the board and explains to the learners. Shows the learners on selecting the signs to put in the square boxes and also the terms). Let x be a rect and a constant be a circ for example. X and 1 are all positives, so this block is positive and this one is also positive. So, we then have to multiply them (showing on the board) positive and positive?
47. **Ls**: Positive.
48. **T**: Here?
49. **Ls**: Positive.
50. **T**: Good, 2 times a rectangle?
51. **Ls**: 2 rectangles.
52. **T**: 2 times a circle?
53. **Ls**: 2 circles.
54. **T**: So now we only write these ones now. 2 rectangles and 2 circles. So, if you look at this, it is the same as that one in symbols.
55. **L3**: Look, this is just like playing a game. It looks nice.

The extract shows how the learners were being introduced to the use of an expansion box.

Learners were also given an expression that involved negative terms -3(y - 1) and asked to give their views on how to expand it.

Extract 13

30. **T**: Now, (writing on the board) -3(y - 1), what do I get here?
31. **L6**: -3y - -3.
32. **T**: So, have we completed like that?
33. **L1**: No.
34. **T**: Can you continue?
35. **L1**: -3 must be in brackets.
36. **T**: And we done like that?
37. **Ls**: No.
38. **L7**: Now you will add negative and negative.
39. **L2**: No, you will just have - 3.
40. **L1**: (opposing L2) aaawe, ndjono odouble negative, negative and negative is positive. Sir we will just have -3y + 3.
41. **T**: (writing on the board -3y + 3) like this right?
42. **L1**: Yes.
43. **T**: Do we all agree or do we understand?
44. **Ls**: (some) yes. (some are just quiet)

The extract shows the contribution of the learners as they were trying to show how they expanded the brackets and dealt with the negative signs during their discussions.

In the same Lesson 6, learners were given the expression 3n - 2(n - 2) to expand and simplify. Learners were required to work in their groups. The instruction was that they expand and simplify the expression in both symbols and use the expansion box. Below is the extract with the group discussion transcripts and their interactions with me after the group discussions.

Extract 14

64. **Ls**: Discussing: **Grp.A**: **L3**: You guys have you already done multiplying all these ones? **L2**: No, we don't multiply that one, we only multiply the one next to the brackets. **L3**: Are you sure? **L4**: Yes, tala, sir osho ati petameko mos. **L2**: Mani, listen, you guys are, this 3n is far from the brackets, tala opuna no minus ndjika. So, you only multiply 2 by the brackets

opuwo. **L3**: Ok, let’s do it but let’s see what sir will say. **L2**: Even in the box, we only have one block not 2, our scale factor is just -2 not 3n. **L3**: We only have one block for the scale factor shili maan, I see.) **Grp.B**. **L1**: Let’s do it guys. **L6**: This one is difficult. **L1**: Aie, tala, we just multiply like those ones mos. **L5**: Can we multiply 3n by n? **L6**: Ya, they are like terms … **L1**: … maar ne kamuuviteko kaa … We only multiply the one next to the brackets. So, we multiply 2 by n and by -2. **L6**: Ndjino? **L1**: We just bring it down mos. **L6**: Ok. **L1**: (writing) just like this, and yeah, we simplify now. **L7**: Paife taleni nee, our answers are not the same, we have negative here and we have positive here. **L1**: Let’s leave it and see what sir will say. Osho kwali tiikulombwele mpaka. **L1**: Eeno, when you are using diagrams you just put one number in this box, so bring -2 here as a factor. **L7**: But there are like terms there guys, I don’t believe you.

65. **T**: Have you done?
66. **Ls**: Yes.
67. **T**: Now let’s quickly go through it, what is supposed to be multiplied with brackets here?
68. **Ls**: (some) 3n and (some) 2 (some) -2. (some just laughing)
69. **T**: Hahaha, which is which now?
70. **L3**: 2
71. **L1**: -2
72. **T**: Now give reasons.
73. **L6**: I think all of them, 3n and 2 because they are outside the brackets.
74. **L2**: Hahaha, aie … no.
75. **T**: Ok, tell me.
76. **L1**: Sir it is just -2 because we only multiply the number which is really next to the brackets, 3n is far, there is even that negative sign, because even when you use the box you put the number next to the brackets inside the first column.
77. **T**: Ok, do we agree?
78. **L2, L4 & L5**: Yes, I agree.
79. **T**: Good, let’s not waste time. You are right, it is -2.
80. **L6**: Hambaa? Now I see.
81. **T**: So, tell me what I should write.
82. **L5**: 3n - 2n + 4
83. **T**: How did you get positive 4?
84. **L1**: Negative and negative gives you positive. So, we multiply -2 by -2 so is +4.
85. **T**: So, you mean this and that?
86. **L1**: Yes.
87. **T**: Good, so is this our final answer?
88. **L6**: No, we still have those like terms, 3n - 2n give you n then + 4. And that is the final answer.
89. **T**: Do we all agree?
90. **Ls**: Yes.

91. **L1**: (talking to others in a group) hey, tala, I told you already that we can't multiply with that 3n.
92. **L6**: Yeah, I now understand. But we are wrong. This 4 is negative. That one is positive.
93. **L1**: Otwangwangwanithaala oosigns, but next time ngaa nee.
94. **T**: Now, let's use the diagrams, this is our box. Where do I put in this box 3n or -2?
95. **L6**: -2.
96. **T**: Yes, so I can write 3n here away from the box represented by the rectangles like this.
97. **T**: (illustrating on the board with the lead of the learners). So that is how you suppose to do it.
98. **L7**: Taa, now I know, maar shampa tolongitha okaboxa is easy, with numbers otongwangwana.
99. **L4**: But you see, is better you do it with numbers and also with diagrams, just to confirm.
100. **L2**: Yeah, I agree with you,
101. **L3**: But it takes time.

The learners discussed the problem in their groups and shared their views on how to expand the brackets in the given expressions. They had different views: some thought 3n should be multiplied by the brackets and some said not. We discussed the problem together after the group discussions and some learners were beginning to understand the whole expansion.

In Lesson 7, I gave another expression slightly different from the one before to the learners, to work in their groups. The instruction was to expand and simplify the expression 3x - (2x - 3) using both the symbols and an expansion box. Learners embarked on the discussion in their groups and presented their feedback afterwards. See the extract below:

Extract 15

16. **T**: Good. Now in our groups, let's do this. One group will do it in symbols and the other group will do it in diagrams. 3x - (2x - 3). Group A: symbols and Group B: diagrams. Right let's try it.
17. **Ls**: Discussing: **Grp A**: **L7**: Let's multiply now. **L6**: So 3x times 2x what do you get? **L7**: Oh, is that what we are doing? **L6**: Yeah. **L1**: You guys, how can you do that. 3 is far, there is a minus between. **L8**: So, what are we doing now? **L1**: Look, there is 1 between here, so we multiply the brackets with 1. **L7**: Nesiku ndiya sir osho a li tati. So, this 3x just come down. **L1**: Now we have 3x - 2x - 3. And we get, x - 3. You see. **L6**: Mbeya inaya mana, let's use the diagrams also. **L1**: Yeah fast fast. **L7**: Are we not putting 3x here? **L6**: Hey, 3x is out, tala ngwee, opuna oplus mpaka. **L1**: 3x is out, we can't multiply with it. We just put 1 here and … .**Grp B**: **L4**: Let's draw the table, what are we putting here? **L3**: We put 3x.

L2: Is 3x next to the brackets? **L5**: We put 1 there. **L4**: Taleni, 3x is not coming inside the table, oyatetwako ko minus. We just put 1 in box here now. **L2**: Yes, listen guys, there is a 1 here but look, there is this negative, so meaning 3x is out of the box. Let's fill the blocks with the signs first. Obvious here we put 1, this 1 is -, here we put +, and this 3 is -. Let's multiply. **L4**: - + is -, - - is … **L5**: +. **L2**: Yes, good.

18. **T**: Have we done?
19. **Ls**: Yes.
20. **T**: Ok, now can I just have your final answers.
21. **Gr. A**: **L6**: Ours is x - 3.
22. **Gr. B**: **L4**: x + 3.
23. **T**: Good, who is correct now?
24. **Ls**:(randomly) Us, us, our group.
25. **T**: Let's go through it together (go through with the learners and got x + 3)
26. **T**: So where is the mistake that this group did?
27. **Gr. B**: **L2**: Multiplying signs.
28. **T**: Very good. How did you guys get it right?
29. **Gr. B**: **L2**: When we use diagrams, we first put the signs in the small boxes so that we don't get confuse the signs. We multiply them first.
30. **Gr. A**: **L1**: Now I got it, this is tricky!!symbols are confusing man.
31. **Gr. A**: **L6**: Haha, oshili maan, taa, tse otwa multiply just with 1 not -1. Next time ngaa.

The group that used an expansion box got the answer correct and the one that used symbols did not get it right because of the inappropriate use of the signs. Still some learners thought 3x is multiplied by the brackets. Some learners showed signs of understanding during and after the discussions.

In Lesson 8, learners were given an equation to solve that involved expanding brackets and they were required to use an expansion box to expand the brackets. Below is a sample of their work.

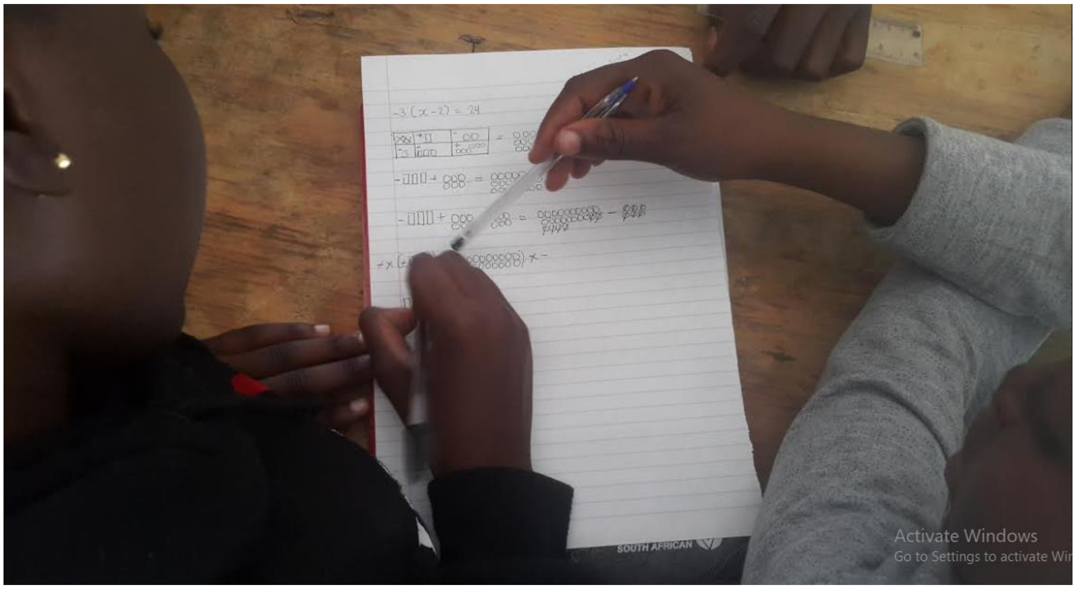


Figure 4.9: **Group A: Learners using an expansion box and diagrams to solve the equations**

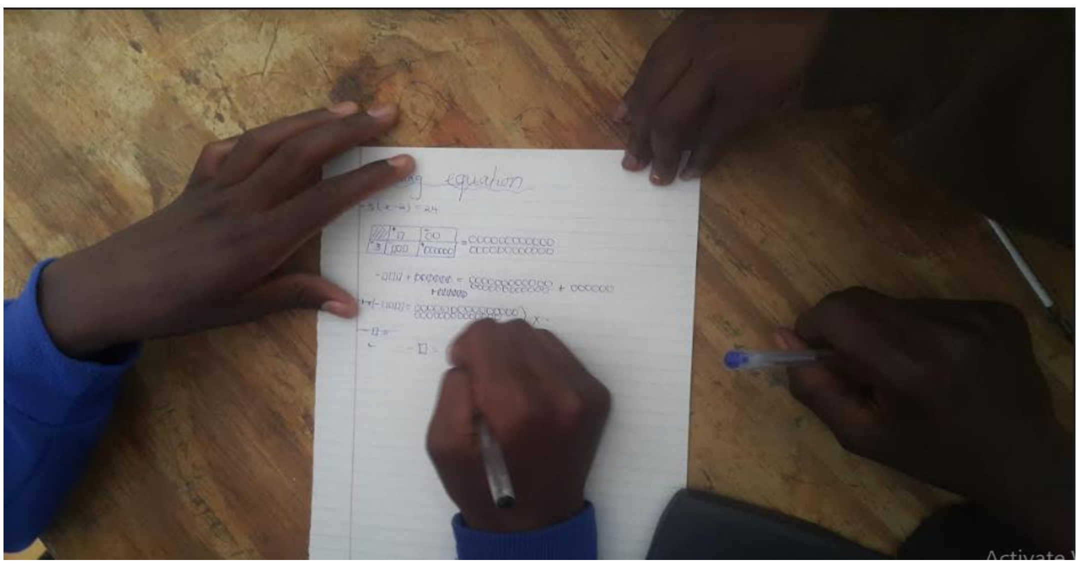


Figure 4.10: **Group B: Learners using an expansion box and diagrams to solve the equations**

Learners worked in groups to solve the given equations.

Towards the end of Lesson 8, I asked the learners some general questions to see if the learners understood the role of the terms in some expressions involving brackets. Learners participated and conveyed what they knew. Below is the extract:

Extract 16

153. **T**: Ok, let me just ask now, if I have 2x - 3(x + 2) = 4, what is the scale factor of the brackets there? In other words, which number should I multiply by the terms inside the brackets?
154. **Ls**: (Shouting) 2x, 3, no, -3.
155. **T**: Calm down, ok, which is which now? One person at a time please.
156. **L3**: Maybe 2x multiply by x and 3 multiply by 2.
157. **Ls**: (some nodding their heads, some were heard saying): Aiye, you just multiply with one number.
158. **T**: Ok, that's her view.
159. **L2**: I think you multiply by 3.
160. **Ls**: (most of them) Yes, true.
161. **T**: Are you sure this is 3?
162. **L7**: No, negative 3.
163. **T**: Good, and you just keep 2x separately, it is very far from the brackets, it cannot jump the negative sign to multiply the brackets.
164. **T**: Lastly, let's have a look at this one: 3a - (2a + 1), what can I multiply the brackets with?
165. **Ls**: (few shouting) 3a.
166. **L6**: Maybe 1.
167. **L2**: No sir, it is -1.
168. **T**: Right, the scale factor of the brackets here is -1. If they do not put a number here, that means there is 1 next to the brackets. So, in this case, it is -1 because there is a negative here. Do you understand?
169. **Ls**: Yes sir.
170. **T**: So, I urge you to go through your textbook and try to solve these types of equations.

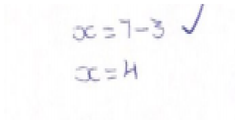
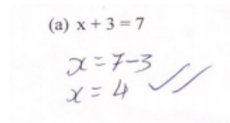
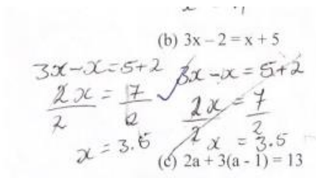
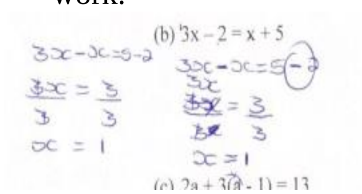
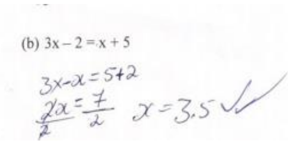
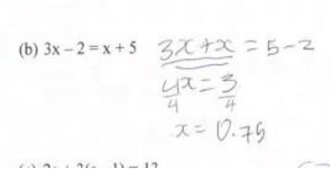
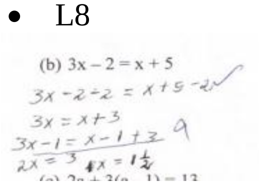
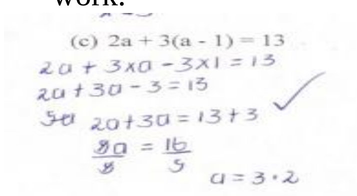
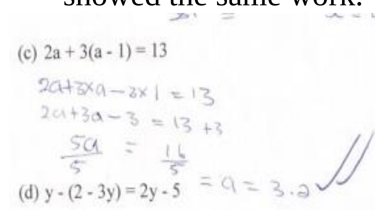
The extract shows how the learners understood the brackets from the lessons. They were still expressing different views about the brackets.

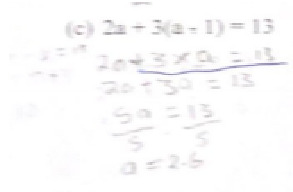
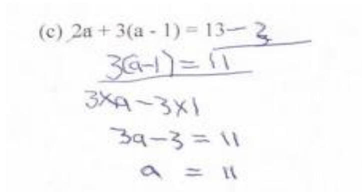
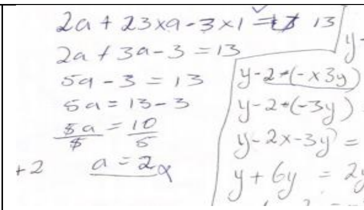
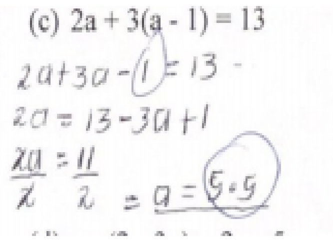
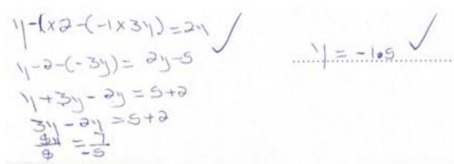
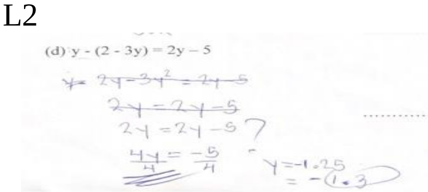
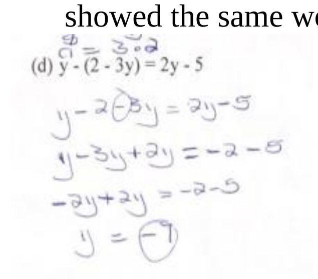
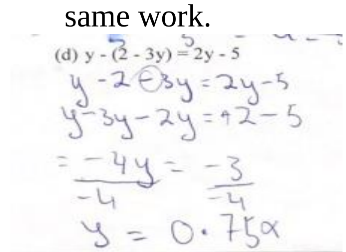
4.3.3 Learning engagement related to equations

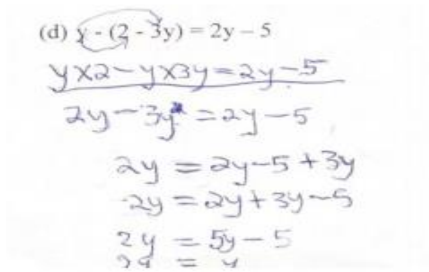
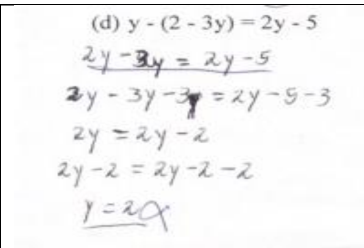
4.3.3.1 Benchmark tests

The table below shows the work of the learners on solving linear equations both in the pre-test and post-test.

Table 4.5: **Learners' work on solving equations in the pre-test and post-test**

Questions	**Pre-test**	**Post-test**
x + 3 = 7	• All learners showed the same work. 	• All learners showed the same work. 
3x - 2 = x + 5	• L1, L2 & L3 showed the same work.  • L4, L5, L7 & L8 showed the same work.  • L6 refer to Table 1, row 4, column 2	• L1, L2, L4, L5 & L6 showed the same work.  • L3 & L7 showed the same work.  • L8 
2a + 3(a - 1) = 13	• L1, L2, L5 & L8 showed the same work.  • L3 	• L2, L4, L5, L6 & L7 showed the same work.  • L1 & L3

	• L4  • L6 refer to table 2, row 3, column 2 • L7 	 • L8 
y - (2 - 3y) = 2y - 5	• L1, L3 & L5 showed the same work.  • L2  • L4 no work shown.	• L1, L2, L3, L5 & L7 showed the same work.  • L4 & L6 showed the same work.  • L8

	• L6, L7 & L8 showed same work.  (d) $y - (2 - 3y) = 2y - 5$ $yx2 - yx3y = 2y - 5$ $2y - 3y = 2y - 5$ $2y = 2y - 5 + 3y$ $2y = 2y + 3y - 5$ $2y = 5y - 5$ $2y = y$	 (d) $y - (2 - 3y) = 2y - 5$ $2y - 3y = 2y - 5$ $2y - 3y - 3y = 2y - 5 - 3$ $2y = 2y - 2$ $2y - 2 = 2y - 2 - 2$ $y = 2x$

4.3.3.2 Observations

The solving of linear equations was introduced in Lesson 7 using an equation similar to the one used in the test, x + 3 = 7. I asked the learners to give their views on how to solve the equation. Learners gave their views and an extract is below:

Extract 17

37. **T**: Let's also look at solving equations. If we have x + 3 = 7, to make meaning of this equation, one would say, if I have a box of apples plus 3 loose apples will give the same weight as 10 loose apples. Now one would want to find the number of apples in the box. So how do we solve this equation?
38. **Ls**: (together) x = 7 - 3, x = 4.
39. **T**: How did you get minus 3?
40. **L5**: Because when you change a number to the other side it changes to negative.
41. **L2**: When you change the number from one side, the sign changes also.

42. **T**: But why?
43. **L1**: It is a rule.
44. **T**: It is a rule?
45. **Ls**: Yes.
46. **T**: Ok, let's look at these diagrams, x is a box, and constants are circles, a box plus 3 apples = 7 apples. How do I do it?
47. **L1**: You bring the box down then = 7 apples minus 3 apples.
48. **T**: Haha, no. (using a ruler as a balance scale) Imagine you have a box this side and 3 loose apples and on the other side you have 7 apples. What will you do so that you remain with a box alone on one side and still have the scale balanced?
49. L1: Just take the apples on the other side.
50. **T**: Hmm, ok, guys, look at this, what I will do I will just start taking loose apples from both sides, I take one here, and also here, I take another 1 and here also, I take 1 from here and here also. Now you see, do we still have loose apples here?
51. **Ls**: A box.
52. **L8**: I see now, you just keep taking until you have a box only there.
53. **T**: So, a box represents?
54. **Ls**: 4 apples.
55. **T**: Good. Now, if I am doing this in numbers and letters, I will just subtract 3 apples from here and also from the other side. So, this gives me 0 and the other 1 gives me 4. Do you understand?
56. **Ls**: (some) Yes.
57. **L7**: Kashona ngaa…
58. **T**: So, we will understand as time goes.

The learners showed what they knew about solving the equations based on their experience.

In Lesson 8, learners were required to write an activity to solve some equations individually. They wrote the activity, solving the two equations both in symbols and using diagrams. Below is a sample of some of their work.

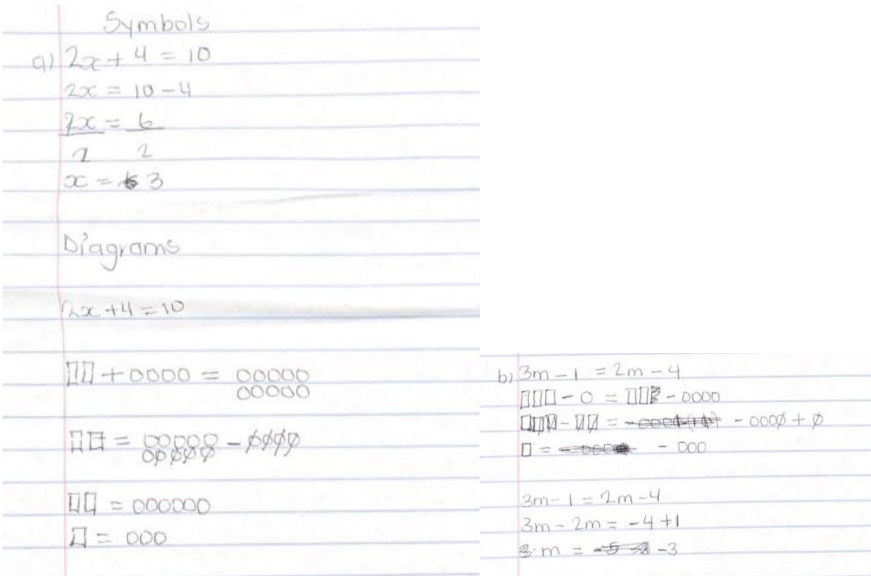


Figure 4.11: **The work for L5 on solving equations**

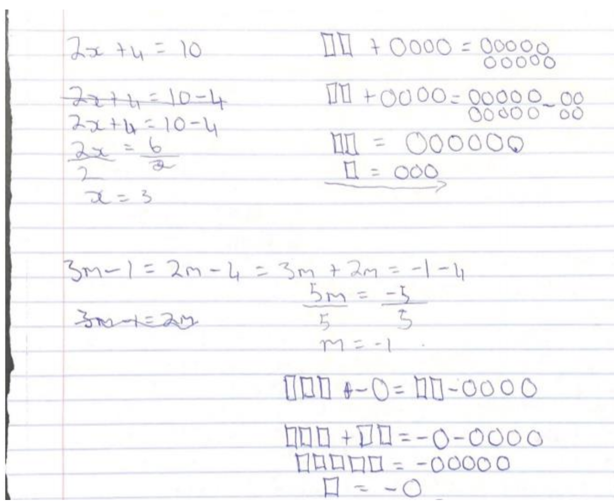


Figure 4.12: **The work for L6 on solving equations**

After the activity, the teacher invited the learners to go through the equations together and learners participated giving their views. See the extract below.

Extract 18

8. **T**: Ok, here, we are solving this equation: 2 times a number of boxes of apples perhaps plus 4 loose apples will give you the same number as 10 loose apples. So of course, how do we solve it procedurally as we have learnt from our classes?
9. **Ls**: (together) $2x = 10 - 4$.
10. **T**: So, you only take 4 to the other side?
11. **Ls**: Yes.
12. **T**: So $2x = 6$, then divide both sides by?
13. **Ls**: 2
14. **T**: x is equal to?
15. **Ls**: 3
16. **T**: So, if I am to use diagrams, (drawing rectangle to represent x and circles to represent constants) let's say x represents the number of apples in the box and constants are loose apples, and I want to find how many apples are in the box. I will say, 2 boxes + 4 apples equals to 10 apples. So, how do I find the number of apples in 1 box?
17. **Ls**: (some) Move them to the other side (some) Remove 4 apples from each side.
18. **T**: Remember, I want to keep the thing balanced. So, if I take 1 from here, I also take 1 from there until one side is finished. If I take all these 4 from here and also from the other side, what do I have here?
19. **Ls**: 2 boxes.
20. **T**: 2 boxes isn't it?
21. **Ls**: Yes.
22. **T**: And on the other side I have how many?
23. **Ls**: 6 apples.
24. **T**: Then from there?
25. **L4**: We find how many apples in 1 box.
26. **L2**: Sir, in 1 box there are 3 apples.
27. **T**: Good. So, if we are saying x was replaced by a box, so if we replace a box by x now here that means x represents what?
28. **L6**: 3 apples.
29. **L4**: Maar oshipu ngaa kashona.
30. **T**: Let's come down to this one ($3m - 1 = 2m - 4$) using diagrams. 3 times the number of boxes - 1 loose apple will give the same results as 2 times the number of boxes and I take 4 loose apples. Now, this is different from the previous one, it has boxes on both sides and loose apples on both sides, also we have negatives now. What can we do now here?
31. **Ls**: Collect like terms.
32. **T**: Remember, we want to balance the pivot, so you mean boxes come one side and loose apples on one side?
33. **Ls**: Yes.
34. **T**: So, how do we do that now?

35. **L3**: To remove is like we are subtracting. So, just remove that 1 loose apple from there and also from the other side. Yeah, I think like that.
36. **T**: Hmm, ok. So, let's look at it, I have 3 times the number of boxes minus 1, now, for me to remove this minus apple, how do I do it?
37. **Ls**: (some saying subtract, some saying add)
38. **T**: Remember when we did expressions, there are those terms that cancel. For me to cancel this minus apple, what do I do?
39. **Ls**: (some saying add and some saying subtract)
40. **T**: Ok, yes, we add. So, we need to add 1 here.
41. **Ls**: (some) Oh aiye, this is just confusing…
42. **T**: Now if we add 1 here then it will be the same as 2 times the number of boxes minus 4. Now because I added 1 here, I also have to add another apple here to balance. You see, if I only add one on this side and I don't add to the other side, the thing will not be balanced. Are we together?
43. **Ls**: (some) Yes. (some are quiet)
44. **T**: So, we are adding because something was subtracted so, I need to add so that we remove it. Now, go back to expressions, if I have negative and positive, what do they do?
45. **Ls**: Cancel.
46. **T**: They cancel isn't it?
47. **Ls**: Yes.
48. **T**: Now here I only have 3 boxes, which is equal to 2 boxes, now look at that, I have only 1 positive and 4 negatives, positive and negative cancel isn't it?
49. **Ls**: Yes.
50. **T**: So, this positive cancels with 1 negative, now how many negatives do I have?
51. **Ls**: Just 3.
52. **T**: Good, ok. Now, loose apples are on one side, I now need to take these boxes to the other side. What do I do?
53. **L1**: Remove them.
54. **T**: If I am removing them, what do I do?
55. **L1**: Subtract.
56. **T**: Good, because they are positive, I need to subtract. So, I take 2 boxes from here and I should also take them from the other side. I cannot just take them from the other side and keep quiet on this side because it will not balance.
57. **T**: So, here we say this minus that and this minus that ... they are just the same thing isn't it? So here, this one and that one and this one and that one. Now I only have one box remaining here.
58. **Ls**: Yes.
59. **T**: And on the other side, we have negative 3.
60. **Ls**: (some) Shuu, this is too confusing … difficult.
61. **L4**: Sir can I not use the old method, just taking from one side to the other?

62. **Ls**: True, true…
63. **T**: Yes, you can. We are just trying to make sense of algebra here. So, if you find the other way easy, then it is fine. How many of you prefer this way?
64. **Ls**: (only 2 raised: L8 and L1). Others: me the old one, me too, me too….
65. **T**: So, meaning we are saying because we have negative apples in the box, that means there are 3 apples missing in the box. There is a need for 3 apples in that box. That's what it says because the answer is negative. Look at this one, one box we said is 3 positive meaning there are 3 apples there, so those apples are there in the box.
66. **T**: So, this is how we make use of algebra in our lives. Some people will just say I hate mathematics, you are just taught x and y where am I going to use it? So, this is it now … did you get it?
67. **Ls**: Yes.

The learners gave their views on solving these equations after they had completed the activity. It showed that some learners were struggling with the negative sign. Relating negative terms to real life examples was also a challenge for some of the learners.

In the same Lesson 8, there was a class discussion on solving the equation involving brackets with the learners led by myself, on what to write on the board as they were solving the equation. An expansion box was also used to expand the brackets. Learners participated and gave their different views on how they understood the equation. See the extract below.

Extract 19

111. **T**: Yes, that's where your challenge is. Now (writing on the board, -2 (2y + 1) = 14) if we have something like that, let us start using diagrams to solve this equation. First is the expansion box (drawing the box and learners leading to fill the square boxes)
112. **Ls**: (indicated the signs of the squares correctly and the boxes and circles to be put in the squares)
113. **T**: Now what do I have here?
114. **L2**: -4 boxes minus 2 circles.
115. **T**: Is the same as?
116. **L2**: 14 circles.
117. **T**: Good, ok ... let's work it out. These are negatives, what do I do first.
118. **Ls**: (whispering) Some were heard saying: you remove (some) subtract. L2 was heard saying: No, you add because that one is negative.
119. **L2**: Sir, first you add 2.

120. **T**: So, I add 2 here, because we are saying when we have negative, it means we are short of something, so we are supposed to fill the gap. And when we do fill the gap on one side, we also do it on the other side.
121. **L2**: Yes, and also to the other side.
122. **T**: Good, for it to remain balanced isn't it?
123. **Ls**: Yes.
124. **L3**: But that side there is no gap there sir.
125. **T**: Where?
126. **L3**: That side where there is 14.
127. **T**: No, what we are saying, if you fill the gap by adding on one side, you should also add the same number on the other side for the whole thing to be balanced. If I don't add here, that means the scale will be like this. But my aim is to keep it balanced. Do you get it?
128. **L3**: Yeah … somehow …
129. **T**: (writing as the learners said). So, look at this, just an expression on one side, positive and negative?
130. **Ls**: Cancel.
131. **T**: Now I remain with -4 boxes is the same as?
132. **Ls**: 16
133. **T**: How do we remove this negative?
134. **L6**: One negative box is equal to 4 circles.
135. **T**: Is that so?
136. **L2**: Somehow yeah … but, I don't know …
137. **T**: Can I have a negative box?
138. **L2**: (debating with others in a group) You see, I told you.
139. **T**: Tell me what to do now, how do I remove a negative sign here?
140. **Ls**: (just whispering)
141. **T**: Ok, procedurally we just multiply by another negative on both sides. Negative and negative cancel. So, we have, 1 box is the same as?
142. **Ls**: -4 boxes.
143. **T**: Very good, meaning there is short of 4 whatever…
144. **L3 & L8**: (seem so depressed)
145. **T**: Just imagine, the way we always do it is, just about when you move a term from one side to the other it becomes negative or positive, but what's the logic? So, this way is trying to make you understand the whole process.
146. **L5**: But it is confusing. Me I just don't get it. The other way is easy.
147. **T**: Ok, I see, and I am not saying it as a master that you should use this method. You can always use the one you are comfortable with.

4.4 CONCLUSION

This chapter presented the data of the study that was gathered through four modalities, namely: benchmark tests, focus groups, observations and journals. The chapter had two sections. Part A consisted of the overview of the programme. In this part, learners shared their views about the programme. Part B focused on the specific learning areas and the data showed how the learners engaged with the learning and the changes in their understanding as they interacted with the tools. After presenting the data I now turn to the analysis and discussion of the findings in Chapter 5.

CHAPTER FIVE

DATA ANALYSIS AND DISCUSSION OF THE FINDINGS

5.1 INTRODUCTION

This chapter is comprised of two sections, Section A and Section B. Section A presents the analysis of the data presented in Chapter Four and Section B presents the discussion of the findings from the analysis.

5.2 PART A: DATA ANALYSIS

This section only presents the analysis of the data presented in Part B of Chapter Four. Part A focused on the learners' experience about the programme and do not have the details for specific learning. This data was collected through benchmark tests and class observations. There were three concepts of linear algebra from which the data was generated, namely; variables, expressions and linear equations. For each concept, the data is analysed according to the emerging themes.

5.2.1 Analysis of the learning related to variables

The data analysed for this area is drawn from the observation data I presented in the previous chapter. Benchmark tests did not include questions that required the understanding of variables. Four themes emerged from the analysis of the data in this area.

5.2.1.1 Learners saw variables as representing unknown numbers

In Lesson 1, it appeared that learners had already known examples of variables as alphabetical letters (Ext. 1, Line 4). Though they had the same understanding of the examples of variables, they had different thoughts of what variables represent. L2 indicated that variables represent *"unknown numbers"* (Ext. 1, Line 10). The response of the learner clearly showed that she understood variables as representing unknown numbers. Toward the end of Lesson 1, L3 and L4 were still thinking that variables represented unknown numbers. This was observed when I asked the learners to reflect on the examples used during the lesson and give their general views of what variables represented based on the lesson experience. L4 responded saying *"a variable represents an unknown number"* (Extract 5, Line 75). L3 also gave her view saying, *"it represents unknown number and object"* (Extract 5, Line 77). This seemed to imply that the examples used in the lesson appeared to have given them an understanding that variables represent unknown numbers. Some other learners also had different views of what variables represented.

5.2.2.2 Learners saw variables as representing objects

In Lesson 1 a general view was given by L1 who seemed to think that variables represented unknown objects that were not known as he said "*they represent ... unknown objects, anything that we do not know. Like we use letters to represent unknown objects"* (Ext. 1, Line 11). This suggested that the learner seemed to think variables were placeholders of objects that were not known in the context presented. It also appeared in the same lesson when we discussed the example on the cost of food for primary and secondary school learners. When I asked what does x represent in the example, L6 responded saying "*it represents money"* (Ext. 2, Line 27). The response of the learner showed that the variable x represented objects, in this case, money.

Still in Lesson 1, on a different example, learners were asked to give their views on what T and L represented. The majority of the learners indicated verbally (call out answers) that L represented *"learners"* (Ext. 3, Line 50). This suggested that learners were still seeing variables as representing objects despite having completed previous examples.

In Lesson 3, during my explanation in trying to help learners make use of diagrams, I also used variables as representing objects, I said "*Let x be a rectangle, y be a triangle, m be a circle and n be a kite"* (Ext. 9, Line 40). My language use also might have had an impact on the learners' understanding of the concept variables.

Even still in Lesson 5, as the learners were working in their groups, L5 seemed to see variables as representing objects. She was heard saying *"... so these ones can be triangles and these ones can be rectangles"* (Ext. 10, Line 24). One can argue that the use of diagrams made the learners interpret variables as representing objects since the learners were supposed to replace the variables by the diagrams. Seeing a diagram in place of a variable might give the meaning that a variable represents objects.

In Lesson 6, L2 was heard saying *"No, you can't, you will just have 2 apples and 2 oranges"* (Ext. 11, Line 20) responding to my question – what if the two terms are added together? This happened because the use of practical examples implied that the terms represent real objects. For this learner, 2x meant 2 apples meaning x was taken to represent an apple. In the same lesson, I instructed the learners to make use of diagrams to expand the expression 2(x + 1): "*let x be a rectangle and a constant be a circle for example"* (Ext. 12, Line 46). This further implied that I also struggled with an understanding of variables as representing quantities and this might have confused the learners.

It also happened that I implied that variables represented objects in Lesson 7 when I was explaining to the learners, giving an example to make sense of the equation x + 3 = 7, where I referred to x as a box (Ext. 17, Line 46). The learners were also heard referring to x as a box (Ext. 17, Line 51). It can be argued that my explanation influenced the learners to see the variable x as representing an object, in this case, a box.

In Lesson 8, as I was going through the equation 2x = 10 - 4 with the learners, relating the equation to a real life situation, I, together with the learners referred to x as a box. We were all reading 2x as 2 boxes (Ext. 18, Lines 16, 19 & 20). This was also observed in the same lesson when the learners saw y representing a box, reading -4y as -4 boxes (Ext. 19, Lines 114, 134 & 142) as they were dealing with the equation -2(2y + 1) = 14). The manner in which learners viewed/interpreted y as a box also speaks to how representation influenced the thinking of the learners. Though some of the learners appeared to view variables as representing unknown numbers or objects, some of them displayed a transition to viewing variables as representing quantities. The next sections present the analysis of the transition.

5.2.2.3 Some learners displayed a transition from seeing variables as representing unknown things (numbers & objects) to seeing variables as representing quantities

In Lesson 1, after discussing the cost of food for primary and secondary school learners, it appeared that L1 started seeing that the variable x used in the equation did not represent money but the amount of money. L1 indicated this when he was responding to my question, asking if L6 was sure that x represented money, saying *"No, it represents the amount of money"* (Ext. 2, Line 29). The learner's response suggested that my question stimulated his thinking, as he appeared to have noticed that I had not responded positively to the answer given by L6. On the same question, most of the learners also indicated that they were grasping that variables represented quantities. This was noticed when I asked them to decide whether x represented money or an amount of money and they responded saying *"Amount of money"* (Ext. 2, Line 31). What this meant is that learners viewed variables differently. Some might have been stimulated by my questions to think about variables as representing quantities.

In a different example that talked about the number of teacher and learners in the same lesson, L1 and L2 appeared gradually to understand that variables represent quantities (from Ext. 3).

49. **T**: Very good. Let's look at the other question: in our school, the number of learners (L) is 35 times the number of teachers (T). So, what does L represent again?
50. **Ls**: Learners.
51. **T**: So, it represents learners … ???
52. **L1**: Sorry, it represents the number of learners.
53. **T**: And T?
54. **L2**: The number of teachers.

This data suggested that my second question to the learners (*so, it represents learners?*) prompted L1 and L2 to think that the collective answer given in the beginning was incorrect. Thus, they seemed to remember what we had done in the previous examples. In the same example, L1 was heard saying *"No sir, I mean number of teachers"* (Ext. 4, Line 73) when I asked if they were sure that T represented teachers as the learners collectively indicated. The response of L1 appeared to have been influenced by my question asking for clarity.

Towards the end of the first lesson, when learners were asked to give their views on variables based on the examples learned during the lesson, L1 responded *"Sir, let me try. I think it represents an unknown number of the objects. Like, learners are objects, now we are saying how many objects/learners are those? Yeah ... just an idea though…*" (Ext. 5, Line 78). The response from L1 suggested that the learner was not confident with his view as he indicated that it was just an idea. However, the learner seemed to have gained an understanding from the examples that variables represent a number of things (quantity).

In other lessons, there was no data that served as evidence of the transition.

5.2.2.4 Learners displayed a stable understanding of variables as representing quantities

It appeared that some learners already had an understanding that variables represented quantities before the programme. In Lesson 1, I asked learners to say what they understood by the formula/equation L = 35 × T. L8 responded saying *"The number of learners is equal to 35 times the number of teachers"* (Ext. 4, Line 67). This seemed to suggest that L8 was confident with the understanding that the variables L and T represented the number of learners and teachers respectively. The learner's response was independent; it did not rely on any view given as the learner was the first to respond to this question. Furthermore, it was the first time that this learner gave her explicit contribution, which was correct and it was in the first lesson.

In Lesson 2, during the revision of the homework from the previous lesson, some learners demonstrated a stable understanding of what variables represented.

3. **T**: Alright, let's look to the homework I gave you yesterday. (Read the homework). Right, how much did Tom get according to the question?
4. **Ls**: P Namibian dollars
5. **T**: And Jerry gets?
6. **L2**: Half of Tom's amount.
7. **T**: So how do we write the amount in terms of P?
8. **L1**: P divided by 2.

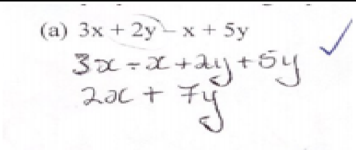
It appeared that L2 was confident with the understanding that P represented an amount of money, thus making the statement that Jerry gets half of the amount of money. This implied that the learner had developed the understanding that variables represented quantities (the number of or amount of something) and seemingly, the learner's understanding was brought about by the programme as she was also observed referring to variables as objects (see Section 5.2.2).

5.2.2 Analysis of the learning related to expressions: Pre-test

The data analysed for this area of learning was drawn from the pre-test, the post-test and observations. Expressions comprised two sub-concepts from which the themes emerged, namely: separating terms and expanding brackets. This section presents the themes that emerged from the pre-test.

5.2.2.1 Learners understood separation of terms involving multiple variables

All learners simplified the expression correctly, keeping the unlike terms separate. Learners seemed to have an understanding before the programme that terms with different variables were different and thus, stayed different.

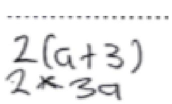


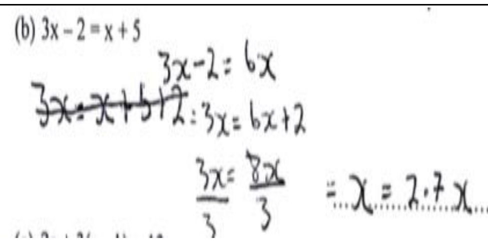
L1

Figure 5.13: **Shows L1 work**

5.2.2.2 Learners battled with separation of terms involving variable and constant terms

Four learners (L3, L6, L7 & L8) combined variable and constant terms by adding and subtracting. Some combined them in expressions that required them to expand brackets, some combined them during equation solving. These actions suggested that the learners perceived variable and constant terms to be like terms that are combined by means of addition or subtraction. Below are the examples of the learners' work:



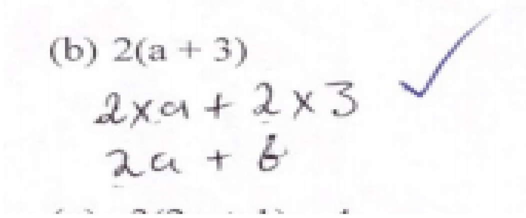


L3 L6

Figure 5.14: **Shows L3 and L6 work**

5.2.2.3 Learners' understanding of expanding brackets in single-term expressions

Seven out of the eight learners (L1, L2, L4, L5, L6, L7 & L8) correctly expanded brackets in the expression 2(a + 3). This suggested that the learners had acquired a stable understanding of expanding brackets in these types of expressions before the programme. In other words, most learners had no problems expanding these brackets. Below is an example of their work.

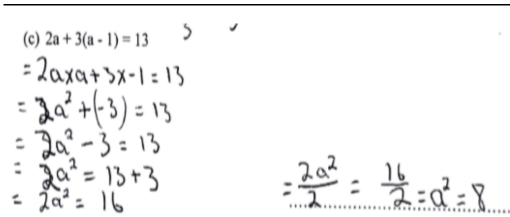


L6

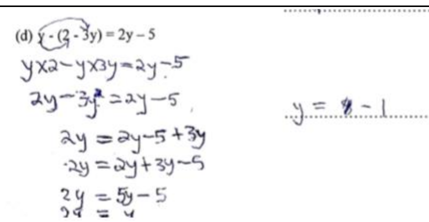
Figure 5.15: **Shows L6 work**

5.2.2.4 Learners saw independent terms as bracket multipliers in expressions with more than one term

Five learners, (L2, L4, L6, L7 & L8) expanded brackets by using the independent terms. For expressions with an explicit multiplier of brackets, e.g. 2a + 3(a - 1), two learners, L4 and L6 multiplied both 2a and 3 by the brackets, i.e. 2a by a and 3 by -1. For expressions with implicit -1 as a multiplier of the brackets e.g. y - (2 - 3y), all five learners multiplied the brackets by y. This seemed to suggest that these learners saw independent terms as multipliers of the brackets, overlooking the additive operations which separate terms. For the expression with the multiplier -1, it appeared that the learners had difficulties in seeing the 1 since it is implicit. See the examples of their work below.



L6



L5

Figure 5.16: **Shows L6 and L5 work**

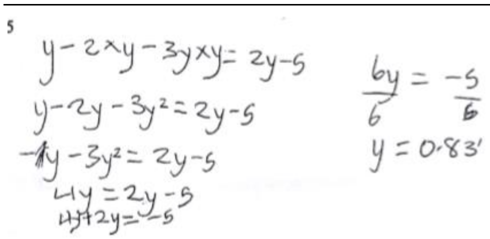
5.2.3 Analysis of the learning related to expressions: Post-test

The themes presented under this section emerged from the post-test. Some themes might have also emerged from the pre-test.

5.2.3.1 Learners saw independent terms as brackets multipliers

This theme also emerged from the pre-test data.

Three learners (L3, L7 & L8) still used an independent term as a multiplier of the brackets. Three learners (L2, L4 & L6) showed improvement as compared to their pre-test work. There was no improvement for L7 and L8 as they still used the independent term to multiply brackets. In this case, the learners used the independent term as a multiplier only for brackets with the implicit multiplier -1. For L3, there was a change, as in the pre-test, she combined the terms inside the brackets first before multiplying with the multiplier. See the example below how this learner expanded the brackets.



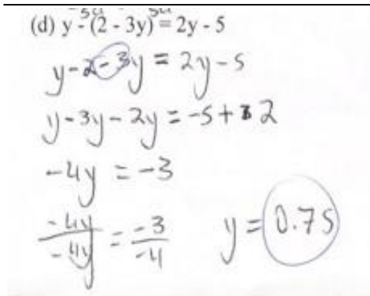
L7

Figure 5.17: **Shows L7 work**

5.2.3.2 Learners used +1 instead of -1 to expand brackets of the form a - (c + d)

This theme only emerged from the post-test data.

Five learners (L1, L2, L4, L5 & L6) used + 1 as a multiplier of the brackets in the expression of this form. This suggested that the learners were struggling to make use of the correct sign for the implicit multiplier.



L5

Figure 5.18: **Shows L5 work**

5.2.4 Observations

This section presents the themes that emerged from the observations. Most of the themes also emerged from the pre-test and post-test. Some of the new themes presented here indicate the process of change in the learners' understanding as they engaged with the learning during the programme, which might have led to the way they answered the post-test questions.

5.2.4.1 Learners understood separation of terms involving multiple variables

During Lesson 3 when learners were simplifying the expressions on both sides in 2a + 4b + 3a + 3c + a = 5l + 2k + l + p + 3k in their groups, L2 was heard saying "*No, you can't these letters are not the same"* (Extract 7) responding to L7 who asked if they can add the terms together which appeared to be unlike. L2 displayed a stable understanding that terms with different variables are kept separate and are not added together. However, L7 also appeared to have the same understanding, but did not look carefully at the terms: *"Yeah, kakwali ndatala the letters kaa (I did not see the letters)"* (Extract 7). I argue that the presence of variables in each term makes it clear for the learners to see the differences and similarities of the terms, thereby noticing like and unlike terms.

In the same lesson, learners demonstrated an understanding of separating terms when they were asked to simplify x + 3y + x + y = m + 2n + n + 3m. Here is part of their exchange:

16. **T**: Tell me, how do I combine this side? Yes? (pointing to a learner)
17. **L4**: You collect the like terms like, x + x + 3y + y = m + 3m + 2n + n
18. **T**: From there?
19. **Ls**: (together) 2x + 4y = 4m + 3n

20. **T**: And now, can we move forward?
21. **L2**: No, you can't add them.
22. **T**: Why?
23. **L2**: Because they are not like terms, like x is not the same with y.
24. **T**: Good, so you understand.

This clearly shows that the learners understood that the terms were different, hence they are kept separate. The response from L2 showed that she was confident that the terms are kept separate as the variables are different. When I asked the learners to represent the same statement using diagrams, they just demonstrated the same thing they did with the symbols, collecting like terms and keeping different diagrams separate. So, the diagrams did not make a contribution to this example, as learners already had an understanding which they had acquired by means of using symbols.

The work of the learners in Lesson 5 served as proof that the learners had a stable understanding in separating terms with variables. All learners separated the terms when they wrote the class activity required to simplify in symbols the expression 3b - 2g - b - 3g = 4w + 2m - 3w - m. They then used the diagrams to simplify the same expression and they were only repeating what they had done in symbols (Figure 4.1).

5.2.4.2 Learners battled with separation of terms in expressions involving variable and constant terms

During Lesson 5, some learners demonstrated that they were struggling to keep unlike terms separate during the group discussion when I asked them to simplify the expression 4a - a + 3 + 2. L7 in Group A seemed to be less confident with the final answer their group got as he asked other learners in the group saying, "*So our answer is 3a + 5?"* (Ext. 10, Line 22). On the same problem, L8 in Group B also asked her fellow learners saying, "*Is that our final answer?"* (Ext. 10, Line 23). Also, in Group A, L5 appeared to be less confident with the answer as well, as she went on to ask for assurance from others and gave her suggestion in a question form saying "*Are u sure guys? But can we not add them together?"* (Ext. 10, Line 22). L5 thought that the two terms are added together to give 8x. This proves that these learners were battling to understand that 3a and 5 are different in the way that 3a is a variable term and 5 is a constant term. L1 referred her fellow learners to the examples that they had done before this expression and L6 counter-responded to L1 (Ext. 10, Line 22).

> **L1**: Haha, you, did we add those ones when we did that with sir. **L6**: But look, there, oonumber adhihe odhina letters but here 5 don't have a letter *(The example you are referring to, all numbers have letters accompanying them but here, 5 does not have a letter).*

The same picture was painted by L3 in Group B when she was heard talking to other learners in the group saying "*Maar oletter ya 5 oshike nee?* (what letter accompanies 5?). *Can't we just combine them because 5 kenasha* (doesn't have) *ovariable?*" (Ext. 10, Line 23).

One can argue that the problem lies with the variable of 5 which was implicit for the learners to see the difference between the two terms. The response of L6 further suggested that if 5 could have been accompanied by a variable, then he would have seen the difference or similarity between the terms. For these types of expressions, it appeared that the learners saw the constant term as the same as the variable term and they perceived these terms as inseparable.

5.2.4.3 Learners showed transition from battling with separating terms to understanding separating terms in expressions involving variable and constant terms

As learners were working in their groups to simplify the expression 4a + 3 - a + 2, L1 explained that the diagrams for constants were different from the ones for the variable terms. L6 reacted, saying, *"But it is true"* (Ext. 10, Line 24). The same learner struggled when they were using symbols to simplify the same expression, but after the use of diagrams, the learner seemed to have gained an understanding that the terms are kept separate. Before the use of diagrams, L6 thought these terms were like and could be combined but, after the explanation with the use of diagrams, the learner appeared to start understanding that terms were different and are kept separate since the diagrams used to represent these terms were different. L8 also commented *"Maar diagrams neh, they are clear"* (Ext. 10, Line 24) after they had used the diagrams in their groups. This suggested that diagrams had facilitated a transition in learning for this learner by explicitly revealing the difference between the variable and constant terms. The use of diagrams also helped L3 in seeing the difference between the variable and constant terms as she expressed *"Yes, sir. I see now. But diagrams are clear because you can see the difference"* (Ext. 10, Line 39). This suggested that, to L3, using symbols alone did not reveal the difference between the terms as the learner thought these terms were the same, and so combined them.

5.2.4.4 Learners showed stable understanding of the separating of terms in expressions involving variable and constant terms

Some learners seemed to have a stable understanding that variable and constant terms are separable. In Lesson 5, when learners were given the expression 4a + 3 - a + 2 to simplify, L1 and L2 demonstrated the understanding that the constant terms are different from the variable terms and thus, are kept separate. L1 strongly disagreed with L5 who indicated that 3a + 5 would give 8x (Ext. 10, Line 22). L2 confidently indicated to other learners in their group saying *"Yes, we can't add them together, they are not the same"* (Ext. 10, Line 23) as she was trying to explain to others who thought they could continue simplifying the expression 3a + 5. After the group discussions, L2 further explained that 3a + 5 is not simplified further. She explained saying *"Because, the a, value, no, wait, where there is a, is not the same like where there is nothing"* (Ext. 10, Line 37) responding to my question when I asked why the expression is not simplified further. All these indicated that these learners (L1 & L2) already had an understanding that variable and constant terms are different and kept different. The use of diagrams seemed to have had very little contribution to the understanding of these learners rather, it just consolidated their existing understanding.

5.2.4.5 Learners understood expansion of brackets in simple expressions

Some learners demonstrated an understanding of expanding simple brackets, i.e. brackets of the form a (b + c). At the beginning of Lesson 6, L1 showed an understanding of expanding these types of brackets by indicating that the brackets 2(x + 1) meant *"multiply the number next to the brackets with what is inside the brackets"* (Ext. 11, Line 4). L4 and L5 also showed an understanding by disagreeing with L3 who thought the terms inside the brackets are combined first, indicating that it was wrong since the two terms are *"not like terms"* (Ext. 11, Lines 10 & 12). L2 on the same problem, also demonstrated that the term outside the brackets is multiplied by the terms inside the brackets (Ext. 11, Line 20). All these contributions from the learners suggested that they already had an understanding of expanding these types of brackets.

5.2.4.6 Learners saw independent terms as brackets multipliers

Expanding brackets in a lengthy expression appeared to be a challenge to some of the learners. In Lesson 6, when learners were required to work on the expression 3n - 2(n - 2) in their groups, L3 was heard asking others if they were not multiplying 3n by the terms inside the brackets (Ext. 14, Line 1). This is an indication that the learner thought 3n should be multiplied by the terms inside the brackets. She appeared to be adamant even after her fellow group members had explained to her that what she thought was incorrect as she suggested *"Ok, let's do it but let's see what sir will say"* (Ext. 14, Line 64). On the same question, L6 thought that 3n should be multiplied by n inside the brackets on the basis that the two terms were like as he answered L5 saying *"Ya, they are like terms"* (Ext. 14, Line 64) when L5 asked if they should multiply 3n by n during their group discussions. This appeared to suggest that these learners were struggling to see that the minus sign separated 3n from the brackets. They also seemed to have overlooked the negative sign for multiplication. As I tried to go through the same expressions together with the learners after their group discussions, some learners still showed that they were struggling to identify the multiplier of the brackets. Some learners still indicated that 3n was the multiplier and some said it was 2 (Ext. 14, Line 68). Those who indicated that the multiplier was 2 seemed to have ignored the negative sign that preceded 2. Answering my question on what term they thought was the multiplier in the same expression, L6 responded saying *"I think all of them, 3n and 2 because they are outside the brackets"* (Ext. 14, Line 73). This indicated that this learner thought that terms that were outside all qualified to be multipliers of the brackets. This demonstrated that the learner lacked a basic understanding of operations as he seemed to fail to recognise the minus sign between 3n and the bracket term.

Also, in Lesson 7, when the learners were working on the expression 3x - (2x - 3), L6 still seemed to be struggling to use the correct brackets multiplier. He still thought the independent term 3x in this expression was the multiplier as their group was working using symbols (Ext. 15, Line 17). In the other group, though they were using an expansion box, L3 and L7 still thought that 3x was the multiplier as they suggested that they put 3x in in the multiplier slot. This indicated that these learners were failing to see the correct multiplier -1.

Towards the end of Lesson 8, I asked the learners some general questions to see if they had developed an understanding of expanding the brackets of the type (a) 2x - 3(x + 2) = 4 and (b) 3a - (2a + 1). For (a), some learners shouted out different answers when I asked them to give the brackets multiplier. They were heard shouting randomly "2x, 3, -3". L3 stood and said, *"Maybe 2x multiply by x and 3 multiply by 2"* (Ext. 16, Line 156). These responses suggested that the learners were still struggling to identify the multipliers of the brackets in these expressions. For (b), there were still a few of the learners who indicated that the multiplier was 3a (Ext. 16, Line 165). It appeared that for these expressions, it was a challenge for the learners to understand the expansion of brackets as they struggled to identify the correct multipliers of the brackets.

5.2.4.7 Learners showed transition from seeing independent terms as multipliers to seeing them as just independent

Even if there were some learners who thought independent terms were brackets multipliers, it appeared that in the process of learning, some displayed changes in their understanding as they interacted. In Lesson 6, L3 thought that both terms 3n and 2 in the expression 3n - 2(n - 2) were used as a multiplier of the brackets. After the explanation by her fellow group members by means of an expansion box, she realised that there was only one multiplier (Extract 14). An expansion box helped this learner to realise that there was only a single multiplier to the brackets since it (an expansion box) only has a single slot (square block) for the multiplier. On the same question, L6 started to understand that the multiplier was -2 as he expressed *"Now I see!"* (Ext. 14, Line 80 & 92) after L1 explained to the class with reference to the use of an expansion box. When I used an expansion box together with the learners to demonstrate how the expression should be worked out, L7 expressed, *"Now I know, maar shampa tolongitha okaboxa is easy, with numbers otongwangwana"* (It is easy when using an expansion box, and confusing when using symbols) (Ext. 14, Line 98). This indicated that the use of an expansion box helped this learner to understand the expansion of brackets of this nature by keeping the independent term outside the box and by only having one square block for a multiplier.

In Lesson 7, as learners were asked to work on the expression 3x - (2x - 3) in their groups, it appeared that L7 was connecting to what they had learnt in the past days relating to expanding these types of brackets as he was heard talking to others saying "Iyaa man, nesiku ndiya sir osho a li tati" (that is even what sir said the other day). "So, this 3x just comes down" (Ext. 15, Line 17). This suggested that the learner was still developing the understanding based on the past experiences of a similar situation. As L2 explained how they got the correct solution using an expansion box, L1 reacted saying "Ooh … now I got it, taa, this is tricky!! Symbols are confusing …" (Ext. 15, Line 30). Such a response showed that the learner was beginning to understand as he noticed where they made the mistake and how an expansion box could help them overcome the challenge.

5.2.4.8 Learners displayed a stable understanding of seeing independent terms as just independent and not brackets multiplier

Although some learners found it difficult to identify the multiplier of the brackets in some complicated expressions, there were some learners who demonstrated an understanding of multiplying brackets in these expressions. This was demonstrated in Lesson 6 as learners were working on the expression 3n - 2(n - 2). L1 and L2 demonstrated confidence in their understanding as they explained to other learners how to expand these types of brackets. They were heard explaining to others in their group (Ext. 14) who thought 3n should also be multiplied by the brackets saying:

> **L2**: No, we don't multiply that one, we only multiply the one next to the brackets … listen, you guys … this 3n is far from the brackets, tala opuna no minus ndjika. So, you only multiply 2 by the brackets opuwo. Even in the box, we only have one block not 2, our scale factor is just -2 not 3n. **L1**: … maar ne kamuuviteko kaa … we only multiply the one next to the brackets. So, we multiply 2 by n and by -2.

One could argue that the learners (L1 & L2) had a stable understanding of what term should be multiplied by the brackets and L2 could demonstrate that with reference to both the symbols and an expansion box. The use of an expansion box helped the learner to see the multiplier in these expressions. L1 persistently demonstrated an understanding of expanding these brackets when we reflected together as a class after the group discussions, as he explained; *"Sir it is just -2 because we only multiply the number which is really next to the brackets, 3n is far, there is even that negative sign, because even when you use the box you put the number next to the brackets inside the first column"* (Ext. 14, Line 76). This explanation indicated that the learner had developed an understanding that independent terms are not multipliers of brackets.

In Lesson 7, L2 was further observed to demonstrate an understanding of expanding brackets when they were dealing with the expression 3x - (2x - 3). She was heard talking to others in the group saying, *"Yes, listen guys, there is a 1 here but look, listen man, there is this negative, so meaning 3x is out of the box. Let's fill the blocks with the signs first. Obvious here we put 1, this 1 is -, here* *we put +, and this 3 is -. Let's multiply"* (Ext. 15, Line 17). This implied that the learners understood that there is a 1 between the minus sign and the brackets. L1 also demonstrated the same understanding as he was confident in his explanation, opposing others who thought 3x was the multiplier (Ext. 15).

In Lesson 8, learners demonstrated that they understood the use of an expansion box to expand the brackets in the expression -3(x - 2). They correctly expanded the brackets (see Figure 4.2 & 4.3).

5.2.4.9 Learners used +1 instead of -1 to expand brackets of the form a - (c + d)

In Lesson 6, when the learners were trying to expand the brackets in the expression 3x - (2x - 3), some of them used the brackets multiplier of just 1 instead of -1. As he was explaining to others, L1 said *"look, there is 1 between here, so we multiply the brackets with 1"* (Ext. 15, Line 17). The product of their brackets was 3x - 2x - 3 that gave the final result as x - 3 (Ext. 15, Line 17 & 21). Even when the learners were using an expansion box, L1 and L4 indicated that they put 1 in the multiplier slot. All of this suggested that L1 seemed to not see the negative sign in front of the 1. It also seemed that the learners only focused on what they saw to be missing, thus they said, *"there is a 1 between here"* and eventually they used 1 instead of -1. In other words, the influence of using just 1 seemed to have come from the verbal language.

5.2.4.10 Learners displayed transition from using +1 to using -1 to expand brackets of the form a - b (c + d)

In Lesson 6, after L2 explained how they got the correct answer in expanding the brackets in the expression 3x - (2x - 3) using an expansion box, L1 expressed *"… now I got it, taa, this is tricky!! Symbols are confusing …"* (Ext. 15, Line 30). L6 in the same group with L1 was also heard saying *"Haha, oshili maan, taa, tse otwa multiply just with 1 not -1. Next time ngaa nee"* (it is true man, for us we only multiplied by 1 and not by -1, so next time) (Ext. 15, Line 31). It appeared that the two learners understood and were aware that the multiplier was not just 1 but -1. This understanding was brought about by the explanation by L2, who explained that they must first put the signs in the expansion box slots before they put in the terms.

5.2.4.11 Learners displayed stable understanding of using -1 to expand brackets of the form a - b (c + d)

In Lesson 6, L2 demonstrated a stable understanding that in the expression 3x - (2x - 3), the brackets multiplier is -1. She demonstrated this when she was explaining to other learners in their group during a group discussion saying *"Yes, listen guys, there is a 1 here but look, listen man, there is this negative, so meaning 3x is out of the box. Let's fill the blocks with the signs first. Obvious here we put 1, this 1 is negative"* (Ext. 15, para. 17). The solution they got was correct, meaning the learners used -1 when they expanded the brackets. L2 was further observed explaining confidently how they got their correct answer by using an expansion box saying: *"When we use diagrams, we first put the signs in the small boxes so that we don't confuse the signs. We multiply them first, ja"* (Ext. 15, Line 29). This indicates that the use of an expansion box helped L2 to make use of the negative sign by firstly allocating the signs in the slots of the box.

5.2.5 Analysis of the learning related to equations: pre-test

This section presents the analysis of the learners' work on how they solved linear equations in both the benchmark tests and during the programme. The section below presents the themes that emerged from the pre-test.

5.2.5.1 Learners solved equations by the transfer method

All eight learners demonstrated the use of the transfer method to solve the equations. This indicated that the learners had learned the transfer method to solve equations before the programme. For the equation with a single term on the RHS of the equal sign, all learners got the correct solution. For the equation with two terms on the RHS of the equal sign, three learners (L1, L2 & L3) got the correct solution. Below are the examples of their work.

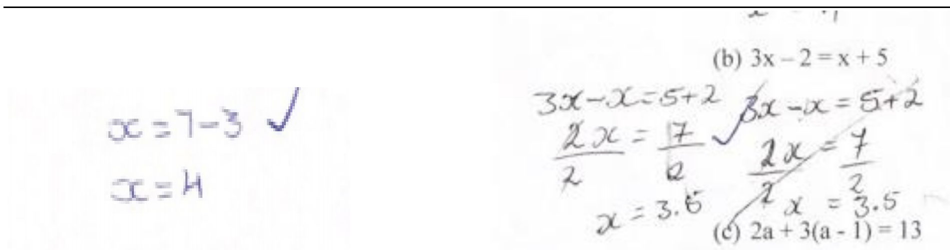
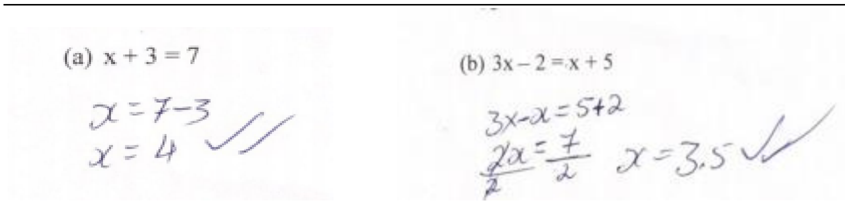


Figure 5.19: **Shows L7 & L1 work**

5.2.6 Analysis of the learning related to equations: post-test

5.2.6.1 Learners solved equations by the transfer method

After the programme, seven out of eight learners (L1, L2, L3, L4, L5, L6 & L7) still demonstrated the use of the transfer method to solve the equations. This on its own indicated that the learners had a strong preference for the transfer method. For the equation with a single term on the RHS of the equal sign, all learners got the correct solutions, including L8 who also used the transfer method for this equation. For the equation with two terms on the RHS of the equal sign, among the seven learners listed above, five of them (L1, L2, L4, L5 & L6) got the correct solutions.



L4 L6

Figure 5.20: **Shows the work of L4 & L6**

5.2.6.2 Learners solved equations by the balance method

One learner (L8) used the balance method to solve the equation with two terms on the RHS of the equal sign. However, the learner could not get the correct solution to the equation as she made the mistake of subtracting rather than adding. This implied that the learner did not grasp how the balance method works in terms of using the signs. Below is the example of how she did it.

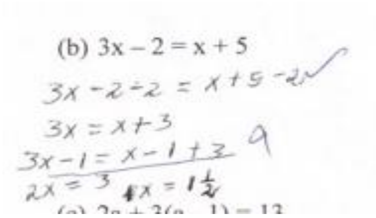


Figure 5.21: **L8 tried to use the balance method**

5.2.7 Observations

The themes presented here emerged from the observations during the learning of solving equations.

5.2.7.1 Learners solved equations by the transfer method

Most of the learners demonstrated a stable understanding of using the transfer method. During the introduction of equations in Lesson 7, I asked the learners how we could solve the equation x + 3 = 7. All learners together responded saying "*x = 7 - 3, x = 4*" (Extract 17, Line 38). L2 also indicated that when the term changes the side, the sign changes as well, when I asked them how they got the minus on the other side (Ext. 17, Lines 39 & 41). This appeared to suggest that the learners had a stable understanding of solving linear equations by the transfer method before the programme. It further appeared that the learners understood this method procedurally as they indicated that changing the sign after changing the side *"Is a rule"* (Ext. 17, Lines 43 & 45). Even when I introduced the use of diagrams, L1 applied the same method as he said, *"You bring the box down then = 7 apples minus 3 apples"* (Ext. 17, Line 47). As I tried to make them aware of the balance method through explanations, the learner persisted in using the transfer method.

In Lesson 8, the learners' work on solving some equations during individual classwork showed that the learners used the transfer method correctly both in symbols and in diagrams.

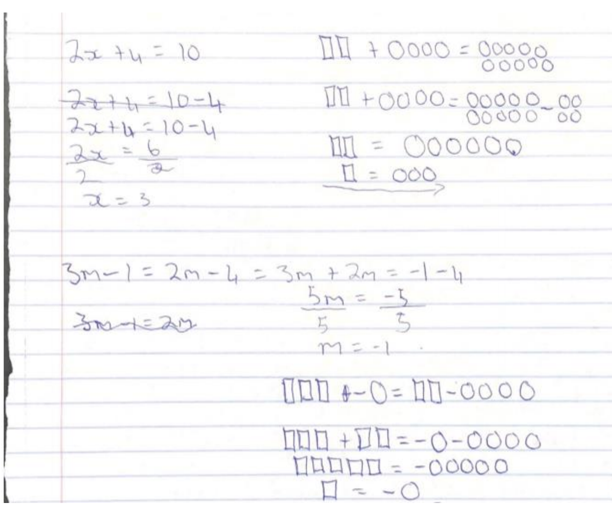


Figure 5.22: **Shows the work of L6 applying the transfer method using diagrams**

This suggested that the learners already had an understanding of the method before the programme. They further demonstrated the method when we reflected on the activity after they had done with the writing.

5.2.7.2 Learners struggled to use addition to get rid of negative terms

In Lesson 8, as we were solving the equation 3m - 1 = 2m - 4, I asked the learners how we solve the equation by the balance method using diagrams. L3 responded saying, *"To remove is like we are subtracting. So, just remove that one loose apple from there and also from the other side. Yeah, I think like that"* (Ext. 18, Line 35). This appeared to suggest that when solving equations by the balance method, one would remove terms, and by removing them, mathematically it means to subtract. The use of this word "remove" earlier in the lessons, seemed to be misguided, as to the learners, it might have given them the incorrect understanding. Thus, the learner could only think of subtracting instead of adding to get rid of the -1. Some learners were also observed struggling in deciding on what to do to get rid of the negative terms, as they shouted out: *"Subtract!"* (Extract 18, Line 37).

5.2.7.3 Learners found balance method difficult and confusing

In Lesson 8, when we used the balance method to solve the equation 3m - 1 = 2m - 4, the learners expressed that the method was "*confusing and difficult"* (Ext. 18, Line 41). When I asked the learners if they understood the example through the balance method, some of the learners were just quiet (Line 43). This seemed to suggest that the learners did not understand as they might have found it difficult. Learners also commented that the method was confusing later in the same lesson when I asked them if they understood after solving another equation. See the extract (from Ext. 18) below:

59. **T**: And on the other side, we have negative 3.
60. **Ls**: (some) Shuu, this is too confusing … difficult.
61. **L4**: Sir can I not use the old method, just taking from one side to the other?
62. **Ls**: True, true…
63. **T**: Yes, you can. We are just trying to make sense of algebra here. So, if you find the other way easy, then it is fine. How many of you prefer this way?
64. **Ls**: (only 2 raised: L8 and L1). Others: me the old one, me too, me too….

65. **T**: So, meaning we are saying because we have negative apples in the box, that means there are 3 apples missing in the box. There is a need for 3 apples in that box. That's what it says because the answer is negative. Look at this one, one box we said is 3 positive meaning there are 3 apples there, so those apples are there in the box.

Most of the learners found the balance method difficult as they went on to suggest that they wanted to rather use the usual method which is the transfer method.

5.2.8 Summary of the findings

Overall, learners had different understandings towards the three concepts, i.e. variables, expressions and equations.

The data suggests that some of the learners had known variables as representing unknown numbers rather than quantities. Also, apart from seeing variables as representing unknown numbers, some of the learners interpreted variables as representing objects. Only few learners demonstrated a transition from their initial interpretations to the understanding that variables represent quantities during the programme. A stable understanding of variables as representing quantities was displayed by very few learners and this suggests that the programme had less impact on the learners' understanding of variables.

Separation of terms had two versions both using additive operations. The first version looked at terms with explicit distinct variables, e.g. 2x + 3y. It was noted during the pre-test and during the programme that the learners had a stable understanding that terms with different variables are kept separate. The second version looked at terms with variables and constant terms, e.g. 2x + 3. It emerged from the pre-test and during the programme that some of the learners thought these terms are combined and reduced for example to 5x. The programme appeared to have assisted these learners in this regard as no further misconceptions were detected after the programme, i.e. from the post-test.

On expanding brackets, it was noted that most of the learners had difficulties with expanding brackets in expressions with independent terms before the programme. The use of an expansion box appeared to have aided some of the learners in understanding the expansion of brackets in these types of expressions. However, not all learners demonstrated the same competence after the programme. Some of the learners were further observed using an independent term as a multiplier to the brackets.

In terms of equation solving, it appeared that all learners understood the use of the transfer method to linear equations before the programme. The use of the balance method seemed to have been less considered/received by many learners as they were consistently observed to be using the transfer method during and after the programme. It is worth noting that only one learner attempted to use the balance method during the post-test.

5.3 PART B: INTRODUCING THE DISCUSSION OF THE FINDINGS

This section presents the discussion of the findings of the study. The aim of this study was to determine whether and how the specific visual and kinaesthetic teaching tools (diagrams, expansion box and balance method) used may have contributed to learners' understanding of algebraic concepts and techniques (variables, expressions and equations).

There is little data about learners' interpretation of variables in this study, because the research was more focused on technique than interpretation. For this reason, learners interpretation of variables will not be included in the discussion.

The research generated insight into a number of technical issues relating to learners' manipulation of expressions, including:

- The separation of terms having distinct variables;
- The separation of the constant term;
- Expanding brackets;
- Solving of linear equations.

5.3.1 Diagrams appeared to have little impact on separating terms involving multiple variables as variables are explicit

In this study, it appeared that the learners were confident with separating terms in these types of expressions. Learners could use the symbols to notice the algebraic structure (Pre-test analysis, 5.2.2.1). Paying attention to the structure helps learners make connections, note similarities and differences in an expression, and to understand the characteristics of algebraic expressions and equations presented in any format (symbolic, numeric, verbal, diagrammatic or graphic) (Star et al., 2015).

The use of diagrams appeared to not be necessary as learners were confident in separating these terms based on the variables (Pre-test analysis, 5.2.2.1). The variables/letters accompanying the numbers appeared to be sufficient for the learners to make a distinction between these terms (Obs. analysis, 5.2.4.1 & Pre-test analysis, 5.2.2.1). The data agrees with Davis (1989) who indicated that learners choose like terms based on the sameness of the letters accompanying each number. Tirosh et al. (1998) also indicated that explicit variables help learners understand that like terms are kept separate and not combined because their letters are different. Therefore, the use of diagrams does not have much impact on helping learners to separate terms as variables on their own are clear.

5.3.2 Use of diagrams enabled explicit separation of terms involving variable and constant terms

Learners initially found it difficult to separate this type of expression (Pre-test analysis, 5.2.2.2). Learners thought the constant terms were combined with the variable terms because they had no variables/letters accompanying them (Obs. analysis, 5.2.4.2). After the use of diagrams during the programme, learners were able to separate the terms (Obs. analysis, 5.2.4.2).

Tirosh et al. (1998) suggested that learners see this type expression as a process rather than an object, indicating that learners see the expression as requiring an action to be performed (Kieran, 1992). Tirosh et al. (1998) also suggested that learners perceive the expression $2x + 3$ as incomplete and that it needs to be completed into a single solution product. By this Tirosh et al. meant that the learners see the need to reduce the expression into a single term, for example, $2x + 3$, learners seek to reduce it to $5x$. A different view came from Tall and Thomas (1991) who suggested that this misconception occurs as a result of what they termed as 'parsing obstacle' where learners find it difficult to deal with the order of operation.

This data appears to support the idea of incompleteness where learners felt that there was a need to reduce the expression to a single term solution. Learners appeared to have difficulty in identifying a pure coefficient (3) as a separable algebraic term, therefore, they combined it in the expressions $2x + 3$ to get $5x$. As discussed above, learners had no problem in separating expressions of the form $2x + 3y$ because the numbers are accompanied by explicit variables or letters that are distinct. In $2x + 3$, this study suggests that the lack of a variable multiplying the 3 may have made the conception/perception of separation less explicit to the learners.

The use of diagrams appeared to have helped the learners in separating these terms (Obs. analysis, 5.2.4.2). Giving a shape for the pure coefficient, different from the variable term, appeared to overcome the challenge of keeping the terms separate. It made the algebraic structure more explicit in the manner variables or letters do in the expressions 2x + 3y. This supports the idea of Panasuk (2010) who stressed that the use of diagrams and pictorial representations gives the learners an external view of abstract representation of ideas. English and Sharry (1996) on the same note also asserted that the use of concrete or pictorial representation adds value to conceptual learning as it mirrors the structure of the concept and thus enables the learner to use the structure of the representation to build a mental model of the concept.

5.3.3 An Expansion Box appeared to have little impact on expanding simple brackets e.g. a (b + c)

This data revealed that the learners did not have difficulty in expanding these types of expressions (Pre-test analysis, 5.2.2.3). The use of an expansion box appeared to have not made a notable impact on helping learners understand the expansion of these types of brackets, as the learners clearly demonstrated the understanding in the pre-test. As discussed in 1(a), the algebraic structure appeared to be sound enough which made these expressions comprehensible to the learners.

5.3.4 An expansion box enabled correct choice and use of the multiplier in expressions of the form; a + b (c + d) as it had a single multiplier slot

Some learners had difficulty in expanding these types of brackets where they used the independent term (2a) as a multiplier of a and multiplied 3 by 1 in the expression 2a + 3(a + 1) (Pre-test analysis, 5.2.2.4 and Post-test analysis, 5.2.3.1). It further appeared during the lessons that some of the learners demonstrated the same challenges in their group discussions (Obs. analysis, 5.2.4.6).

Drijvers and Jupri (2014) suggested that this inappropriate expansion of brackets is rooted in the use of distributive property of multiplication over addition. Learners overlook the additive operations viewing them as multiplication, thus, end up multiplying terms over addition (Bokhove, 2011). This data appears to support the idea of seeing addition as multiplication. Learners overlooked the additive operation, failing to realise that it separated terms.

For instance, the use of an expansion box appeared to have helped the learners to see that the independent terms 2a in the expression 2a + 3(a + 1) is separate from 3(a + 1) by keeping it outside the box. For example, below: let a be a rectangle and the constant be a circle.

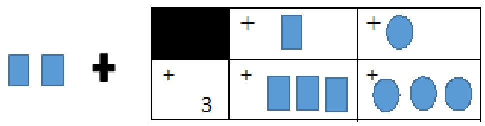


Figure 5.11: **Expansion box example**

This was observed from the post-test results as learners kept the independent terms separate. Larbi and Mavis (2016) expressed the same idea that tables and tiles assist learners to separate terms in algebraic expressions, a suggestion which this study confirmed. An expansion box only absorbs the term involving brackets, leaving other terms outside, and this helps learners not to overlook the additive operation. On the same note, Russel (2011) indicated that the use of tables which appeared to have the same functionality with an expansion box used in this study, enables learners to split terms apart when dealing with algebraic expressions and helps them with mental processes. The study of Ntsohi (2013) that explored the use of spreadsheets in algebra, appeared to have the same idea as an expansion box, and indicated that spreadsheets enabled learners to identify terms and their roles in a given algebraic expression.

5.3.5 An expansion box enabled seeing the implicit multiplier -1 in expressions of the form; a - (b - c) as it keeps the independent term outside the box

Some of the learners in this study demonstrated difficulty in seeing the brackets' multiplier (-1). Instead they used the independent term 'y' (Pre-test analysis 5.2.2.4 & Post-test analysis, 5.2.3.1). The same challenge was further observed during the lessons as learners were required to expand the same types of expressions in their groups (Obs. analysis, 5.2.4.6).

A study by Seng (2010) suggested that when given an expression x, learners see the coefficient of x as 0 instead of 1, just because the coefficient 1 is implicit and zero means nothing, implying that x does not have a coefficient. Corresponding to Seng's view, Macgregor and Stacey (1996) also discussed the ignorance to recognise the expression 'a' as a product of a and 1.

This study does not seem to support the idea of seeing the coefficient of x as 0 as it does not fit the data – if the learners saw the coefficient of (2 - 3y) as 0, then they could have interpreted y - (2 - 3y) as y + 0(2 - 3y) ending up getting y rather than y × (2 - 3y). The idea of learners' unawareness is partly supported by the pre-test data on the basis of hypothesis, but the observation data highlighted that learners could not see the multiplier (-1). The coefficient (-1) of the brackets is invisible, thus learners made use of the only term outside the brackets which was explicit without considering the additive operation. Therefore, this study suggests that the problem here is the implicitness of the multiplier (-1) that needs to be made explicit for the learners to see it and make use of it.

The use of an expansion box appeared to have assisted the learners to see the multiplier (-1) (Post-test analysis, 5.2.3.1 & Obs. analysis, 5.2.4.7). As discussed in the previous point: (2a + 3(a + 1), that an expansion box separates terms, in this expression it reminded the learners of the multiplier (-1) as they filled the slots in the box, realising that there was nothing in the multiplier slot and the independent term y was already outside the box. Using an expansion box as part of multiple representations, Ntsohi (2013) indicated that the use of different representations reveals information for a particular situation as opposed to a single way of representation. It implies in this study that the expansion box revealed the implicit multiplier (-1) that learners were failing to see through using the formal algebraic way of expanding brackets. An expansion box seems to be one of the representations alluded to by Ainsworth (2006) as having unique superfluous features that encode aspects of a domain, in this case revealing the implicit multiplier.

5.3.6 Learners used the implicit multiplier in expressions of the form a - (b - c) as + 1 when they verbalised the terms in calculation

Though an expansion box appeared to have helped the learners see the implicit multiplier (-1) of the brackets, learners demonstrated difficulty using it as -1 when expanding the brackets. They used it instead as 1 (Post-test analysis, 5.3.2.2.). Misunderstanding of the use of -1 was also observed in the group discussions during the programme (Obs. analysis, 5.3.3.9).

A study by Seng (2010) associated this problem with ignorance, stating that learners tend to ignore the negative sign when it precedes a number or term. For instance, given -3, learners read it as 3, leaving out the negative sign, and this also applied in their writing. The other view is that learners tend to detach the signs (+) or (-) from the terms that come after them, treating the sign as mere operations rather than as signs too (Ayres, 2000b; Seng, 2010), for example, in 2x - 3, learners treat the second term as 3 rather than -3. The reason is that learners see the (-) sign only as an operation that separates the two terms rather than a sign for the number it precedes.

The data of this study appears to disagree with Seng's view of ignorance on the basis that learners correctly expanded the expressions -3(x + 1), using -3 as a multiplier and not 3. Learners could have ignored the negative sign in this expression if ignorance was the issue. The data appears to support the idea of detaching the sign and treating it as a mere operation. Given that in the expression y - (2 - 3y), y precedes the minus sign, learners appeared to have treated the (-) as just an operation rather than as a negative sign to the multiplier. For example, in the post-test, in y - (2 - 3y), they got y - 2 - 3y after expansion. This implied that the learners only multiplied the brackets by 1 instead of -1. Observation data supported this as some learners were heard during their group discussion saying that the multiplier is 1, leaving the negative sign. It was also noted that when the learners were working on the expressions, for example, 3n - 2(n - 2), some of them indicated that the multiplier was a 2 rather than a -2 (Obs. analysis, 5.3.3.9).

Though an expansion box seemed to have assisted the learners during their group discussions in using the multiplier with the correct sign, it appeared that learners did not grasp this concept probably due to the short time available for the teaching programme.

5.3.7 Most learners could solve equations before using the transfer method

It appeared that most of the learners already understood solving linear equations before the programme. Learners also demonstrated their understanding of solving linear equations during class discussions and through participation (Obs. analysis, 5.4.1.1). The programme appeared to have not made an impact on the learners in terms of equation solving through the transfer method (Post-test analysis, 5.4.2.1).

Since the learners had already covered algebra before the programme, data from both the pre-test and observations showed that the learners were taught solving linear equations by the transfer method. Most of the learners could correctly solve linear equations during the pre-test. Their participation showed a high level of mastery of solving linear equations by the transfer method (Obs. analysis, 5.4.3.1). Therefore, teaching equation solving was not necessary as learners could already solve the equation.

5.3.8 Most learners could represent their solution processes using diagrams but, diagrams did not appear to add anything to the way they performed their solutions

Most of the learners were able to represent algebraic terms with diagrams as they had learned from the expressions. Learners were able to represent different terms (variables and constants) with different diagrams to show the differences between terms. However, the use of diagrams did not appear to provide any understanding in terms of solving equations. Learners used diagrams to solve equations the same way they used symbols (Obs. analysis, 5.4.3.1). It appeared that the use of diagrams seemed to have not added capacity to the learners' ability to solve linear equations. In other words, learners only repeated the same procedure of change side-change sign by grouping diagrams of the same shapes on one side of the equal sign, similar to their operations with abstract symbols.

5.3.9 Most learners struggled with the balance method

In this study, the use of the balance method was not necessary to the learners as they could already solve linear equations by the transfer method (Obs. analysis, 5.4.3.2 and Post-test, 5.4.2.2). It appeared that the balance method was new to the learners and they were reluctant to learn or use it. The learners also found the balance method to be difficult and confusing (Obs. analysis, 5.4.3.3).

This study suggests that there is no need to learn a new method of solving a problem if learners can solve the same problem using the other method that they have mastered. The transfer method is a formal, rule-based method that is used to solve linear equations (Kieran, 1992) thus, learners did not need another method and it appeared to just confuse them. Learners were persistently using the transfer method to solve equations though they were required to use the balance method. The reason is that that they could already solve the equations correctly using the transfer method. Therefore, there was no need to teach what was already known.

5.3.10 The use of tools enabled learners to deal with implicit terms explicitly

This is a general theme that appeared from the use of diagrams to separate terms and an expansion box to expand brackets. The findings of this study revealed that there was a problem of implicitness in learning some algebraic concepts that needed to be made explicit for the learners. The use of the tools (diagrams and an expansion box) used in this study revealed hidden information in learning some of the concepts. Diagrams helped the learners see the difference between the variable and constant terms as they (learners) initially combined them by adding or subtracting. An expansion box assisted the learners to see the implicit brackets multiplier 1. The discussion on the aspect of implicit information made explicit by these tools is found under 5.4.2 and 5.4.3.3 of this section.

This theme appeared to align well with the ideas of implicit knowledge in algebra. However, there is no literature on implicit knowledge in the literature section of this thesis. The literature used here is drawn from the use of tools and how they assist in learning mathematics and algebra in particular. The use of tools aligns well with Vygotsky's views of social learning through the use of tools. The use of diagrams as tools in this study concurs with the ideas of English and Sharry (1996) who asserted that the use of concrete or pictorial representations adds value to conceptual learning as it mirrors the structure of the concept, thus enabling the learner to use the structure of the representation to build a mental model of the concept. Ntsohi (2013) indicated that the use of different representations revealed information for a particular situation as opposed to a single way of representation.

5.4 CONCLUSION

In this chapter I presented two sections, Section A for data analysis and Section B for the discussion of the findings. The data analysed in section A was gathered from the learners through benchmark tests and classroom observations. The study found out that there were some areas where learners had difficulties in learning the concepts of algebra, and that there were some areas where the learners were aware and showed understanding. Furthermore, it emerged that the learners had been learning algebra through the inherent use of symbols and the use of diagrams and the expansion box was new to them. Generally, the use of an expansion box and diagrams was positively commented on by the learners and it appeared to have made a positive impact on the learners' understanding of some of the concepts. The post-test results of the learners indicated a sign of improvement as compared to the pre-test results. There were also some positive changes observed during the observations as learners interacted among each other using the newly introduced tools.

Section B summarised the findings of this study by discussing them using the views of algebra learning from the literature as a lens.

The next chapter is the conclusion.

CHAPTER SIX

CONCLUSION

6.1 INTRODUCTION

This study explored the use of visual and kinesthetic tools to develop the understanding of the linear algebraic concepts with Grade 8 learners. These tools were diagrams (geometrical plane shapes), an expansion box and the balance method. I developed a teaching programme that I used to teach a sample of eight Grade 8 learners. I implemented the programme and evaluated it at the end. The focus of the programme was to determine whether and how the specific visual and kinaesthetic teaching tools (diagrams, expansion box and balance method) used may have contributed to learners' understanding of algebraic concepts and techniques (variables, expressions and equations). To notice the changes made by the use of the tools, I first analysed the results of the learners from the pre-test and compared it to the post-test. The pre-test provided the initial understanding of the learners before the programme and the post-test provided their understanding after the programme. I then used the observation data which I collected by video recording to establish the possible influence of the learning process that might have led to the learners' understanding after the programme.

The analysis chapter of this study shared the learners' initial understanding of the algebraic concepts that included the challenges they faced before the programme and the opportunities provided by the use of the tools to develop the desired understanding of the concepts. The analysis also provided the area in which the tools did not have influence on the learners' understanding.

I conclude this chapter by discussing the themes that emerged from the analysis of the study. The following elements structure this chapter:

- A summary of the study;
- Importance of the programme;
- Limitations of the study;
- Recommendations, and
- Areas for further research.

6.2 SUMMARY OF THE RESULTS OF THE STUDY

The following findings emerged from this research:

- **The use of diagrams had little influence on the learners' understanding to separate terms with multiple variables, e.g. 2x + 3y.**
  The learners could already separate the terms in expressions involving multiple variables. This implied that the learners had already mastered the separation of terms based on the variables.

- **The use of diagrams had a great influence on the learners' understanding to separate variables and constant terms, e.g. 2x + 3.**
  Different shapes of diagrams used to represent terms made it clear for the learners to see the difference between the terms, hence they kept them separate.

- **The use of an expansion box had little impact on the learners' understanding to expand brackets in simple expressions, e.g. 2(a + 1).**
  An expansion box was not necessary for these types of expressions since the learners appeared to have already mastered the expansion of these brackets before the programme.

- **The use of an expansion box had an impact on the learners' understanding to expand brackets in expressions of the type a + b (c + d).**
  An expansion box was helpful for the learners to make use of the correct multiplier because it kept the independent terms outside the brackets.

- **The use of an expansion box had an impact on the learners' understanding to see the implicit brackets' multiplier in the expressions of the type a - (b + c).**
  Because an expansion box keeps the independent term outside, it helped the learners to remember the multiplier -1 of the brackets as the multiplier slot inside an expansion box remained blank after the learners had filled all the other slots.

- **The use of an expansion box had no influence on the learners' understanding to use -1 as a multiplier of the brackets in the expressions of the type a - (b + c).**
  Learners used 1 as a multiplier of the brackets in these types of expressions instead of -1. Though an expansion box was useful to help the learners to see the multiplier, learners could not use the multiplier with its negative sign.

- **Learners did not like the use of the balance method to solve linear equations.**
  Learners indicated that the balance method was difficult and confusing.

- **The learners preferred the use of the transfer method to solve linear equations.**
  It appeared from the observations that the learners were comfortable with using the transfer method to solve the linear equations. Also, in the post-test, most of the learners made use of the transfer method to solve linear equations.

- **Generally, the use of tools made the implicit information explicit to the learners.**
  Some learners struggled to see the difference between a variable and a constant term and the use of diagrams to represent these terms made the difference that seemed implicit, explicit to the learners. Likewise, learners battled to see the implicit brackets multiplier -1 and the use of an expansion box made it explicit for the learners to see and make use of it.

6.3 SIGNIFICANCE OF THE STUDY

This study explored the use of visual and kinesthetic tools in teaching linear algebra to a sample of eight Grade 8 learners from Namibia. To find how effective these tools were to the learning of the algebraic concepts, the study investigated the type of understanding the learners had before and after the programme.

The findings of the study indicated that the learners had difficulties in separating variable and constant terms before the programme. After the programme, the findings indicated that the use of visual tools had helped learners see the difference between a variable and a constant term, thereby leading them to separate these terms. The findings of the study also noticed the impact made by the use of an expansion box when expanding brackets. These findings will be useful to mathematics teachers, curriculum planners and textbook authors to incorporate the use of these tools in their teaching, designs and writing, respectively.

6.4 LIMITATIONS OF THE STUDY

The exploratory nature of this research does not allow the findings to be generalised. The findings of this study are only based on the perspective of the eight Grade 8 learners sampled for the study. Nevertheless, they can be useful to bring understanding to different classroom situations by any teachers who wish to bring improvements to their teaching of linear algebra.

The findings of the study are also limited to my own observations, perceptions and my own interpretations of the research data. The design of the teaching programme and its implementation were also limited to what I thought of including in the programme as a novice researcher. However, my experience in the field of teaching mathematics guided me on how to go about the programme and its implementation. The guidance from my research supervisor also played a role in ensuring the trustworthiness of the design experiment and the research instruments in accordance with my research goal. I have also tried to confine myself within the boundaries of the ethical guidelines to avoid biased information.

6.5 RECOMMENDATIONS

From this study, I recommend the following based on the research findings in order to enhance understanding in the learning of linear algebra.

- Mathematics teachers need to employ the use of diagrams when teaching simplifying expressions by addition and subtraction, to help learners clearly see the differences and similarities of the terms.
- The curriculum developers should include in algebraic expressions, simplifying expressions by addition and subtraction with both variables and constants terms, to establish the understanding of like and unlike terms between variable and constant terms.

- Mathematics teachers should familiarise with and make use of an expansion box when teaching expanding brackets to emphasise the correct use of the brackets multiplier and help learners avoid multiplying independent terms by the brackets.
- Textbook writers should include the use of both diagrams and expansion boxes in the concepts of simplifying expressions by addition/subtraction and expanding brackets respectively, to encourage learners to use these tools on their own in their free time.
- Mathematics teachers should provide room for learners to choose the method of solving linear equations deemed appropriate by themselves (learners).
- Mathematics teachers need to incorporate the use of necessary tools in teaching algebra to enhance explicit knowledge among learners.

6.6 AREAS FOR FURTHER RESEARCH

The findings of this study left me with these suggestions to be taken into account for any future research:

- Research of a similar nature needs to be carried out on a large scale with a larger number of learners across the country to be able to generalise the findings.
- A study needs to be carried out on how to make use of the tools in dealing with further algebraic concepts, e.g. expanding brackets that give a quadratic product.
- The use of an improved expansion box needs to be considered, that would enable learners to make use of negative one (-1) as a bracket's multiplier.
- Further exploration of the use of tools to make implicit knowledge explicit in other concepts of algebra, should be considered.

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APPENDICES

APPENDIX A: ETHICAL CLEARANCE

RHODES UNIVERSITY
Where leaders learn

Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics

28 August 2018

Bruce Brown
B.Brown@ru.ac.za

Dear Bruce Brown,

Re: HUMAN SUBJECTS ETHICS APPLICATION
Researching Programme Design for Developing Meaningful Understanding of Grade 8 Algebra.
Reference number: 10522551
Submitted: 6/8/2018

This letter confirms that the above research proposal has been reviewed by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee.

The committee's decision is **Provisional approval pending gatekeeper permission**. Please ensure an assent letter is included for participating learners.

Ethics approval is valid until 31 December 2018. An annual progress report is required in order to renew approval for the following year.

Please ensure that the ethical standards committee is notified should any substantive change(s) be made, for whatever reason, during the research process. This includes changes in investigators. Please also ensure that a brief report is submitted to the ethics committee on completion of the research. The purpose of this report is to indicate whether the research was conducted successfully, if any aspects could not be completed, or if any problems arose that the ethical standards committee should be aware of. If a thesis or dissertation arising from this research is submitted to the library's electronic theses and dissertations (ETD) repository, please notify the committee of the date of submission and/or any reference or cataloguing number allocated.

Sincerely,

Prof Jo Dames
Chair: Human Ethics sub-committee, RUESC- HE

Note:

1. The ethics committee cannot grant retrospective ethics clearance.

APPENDIX B: PERMISSION FROM THE REGIONAL DIRECTOR

REPUBLIC OF NAMIBIA

OSHIKOTO REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE

REGIONAL COUNCIL
OSHIKOTO

Tel (065) 281900
Fax (065) 240315
Enq: Ms H Tende

Private Bag 2028
ONDANGWA

04 September 2018

Ref: 12/3/10/1

Mr Enos Kalua
PO Box 3452
Ondangwa
Email: enosn88@gmail.com

Dear Mr Kalua

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT SECONDARY SCHOOL

Our office acknowledges your letter, seeking for permission from the Director to conduct a research study on design for developing meaningful understanding of grade 8 Algebra. Kindly be informed that permission has been granted to you to carry out your research SS.

It is very important that your research does not interfere with the normal teaching and learning process at school, any participation either by learners or teachers/parents should be on a voluntary basis and the information to be gathered should be treated confidential and only for research purposes. Please consult the school principal well in advance to ensure a proper co-ordination of other school activities.

Thank you for showing interest to do research in Oshikoto Region. It is our sincere hope that the information you are going to get will be useful towards the completion of your qualification.

Yours faithfully

OSHIKOTO REGIONAL COUNCIL
Director
04 SEP 2018
PRIVATE BAG 2028
ONDANGWA
DIRECTORATE OF EDUCATION, ARTS AND CULTURE

....
MR LAMEK T. KAFIDI
DIRECTOR OF EDUCATION
OSHIKOTO REGION

APPENDIX C: PERMISSION FROM THE SCHOOL PRINCIPAL

REPUBLIC OF NAMIBIA

REGIONAL COUNCIL

OSHIKOTO

OSHIKOTO REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS & CULTURE

04 September 2018

Inq:

Tel: (

Mr E. Kalua

P.O. Box 3452

Ondangwa

Dear Sir

Re: Request for permission to conduct research

I acknowledge the receipt of your letter dated 30th August 2018 which was received on the 3rd September 2018, requesting for permission to conduct your research with the grade 8 learners at our school. Kindly be informed that your request to conduct a study at our school on design for developing meaningful understanding of grade 8 algebra has been granted permission.

I am pleased that your request states that the programme will be conducted after teaching hours not to interfere with the normal teaching programme at the school. Please note that the information involving identities and/or personal details of the participants should be treated with confidentiality and be abided by research ethics.

I am thankful that you chose our school as your research institution. It is my wish that the information you obtain will be useful towards the completion of your qualification.

Yours faithfully

....

School Principal

MINISTRY OF EDUCATION
OFFICE OF THE PRINCIPAL
04 SEP 2018
ONDANGWA
REPUBLIC OF NAMIBIA

APPENDIX D: LETTER TO PARENTS/ GUARDIANS

Enos Kalua 30 August 2018

Rhodes University

Grahamstown

6139

Dear Parent

Re: Invitation for your child to participate in the research study

You are invited to authorise your child to participate in the study entitled, Researching Programme Design for Developing Meaningful Understanding of Grade 8 Algebra, which will be conducted at the same school your child attends by Enos Kalua, a Mathematics teacher and a student at Rhodes University. The aim of this research is to understand the use and the effectiveness of the algebra teaching programme to develop the understanding of algebraic concepts of grade 8 learners. Children's participation is important to see how the programme may be of any assistance in the understanding of the learners in algebra.

The research will be undertaken through writing tests, interviews, observations and collecting learners' written work (artefacts). Your child's participation in the research is anonymous and her/his identity will not be revealed. The collection of this data will require between 1 to 2 weeks. The programme is planned to be conducted in the afternoon not to interrupt the normal teaching.

Should you agree for your child to participate, I will explain in more detail what would be expected of he/she, and provide her with the information he/she needs to understand the research at school before we begin with the programme. These guidelines would include benefits and rights to participate. Once this study has been approved by the Ethics Committee of the Faculty of Education you will be sent the letter of ethical approval.

Participation in this research is voluntary and a positive response to this letter of invitation does not oblige your child to take part in this research. For your child to participate, you will be asked to sign a consent form to confirm that you understand and agree to the conditions, prior to any test, interview, observation or collection of artefacts commencing. Please note that your child has the right to withdraw at any given time during the study.

Thank you for your time and I hope that you will respond favourably to my request.

MOshiwambo

Oto indilwa nesimaneko, opo wu pitike omunona gwoye, ngoka a hogololwa a kuthe ombinga momapekapeko gopailongo, ngoka taga ka ningilwa posekundosikola yaHans Daniel Namuhuya moshilongwa shOmwaalu.Omapekapeko otaga ningwa kuEnos Kalua, omulongi gwOmwaalu posikola opo tuu mpoka, ye oku li wo omwiilongi koshiputudhilo sha Rhodes University moSouth Africa. Elalakano lyomapekapeko ngaka, okukwathela aanona okuuva ko oshitopolwa sho (Algebra) mOmwaalu shoka shi li eshongo enene meilongo lyaalongwa.

Omapekapeko oga thanekwa okutameka momasiku 10 gaSeptember 2018 sigo 22 gaSeptember 2018, okutameka aluhe pombali sigo opondatu omasiku gosikola kakele komatitano. Uukwatya womunona otawu kala oshiholekwa nedhina lye itali ka holoka momishangwa dheilongo ndika. Onga omuvali owa pumbwa okushaina ofooloma ya kwatelwa ko kombapila huka, onga ezimino okupitika omunona a kuthe ombinga.

Tangi unene kelongelokumwe lyoye.

Yours sincerely/ Gwoye

…………………………………….

Enos Kalua (Student Number: 16K7503)

Declaration:

I --- (**full names of the parent/guardian/Class teacher**) hereby confirm that I understand the content of this document and the nature of this research study and that I have permitted --- ----------------------------- (**learner's name**) to participate. I understand that I reserve the right to withdraw my child/learner from this study at any time.

Signature: **Date:**

--- **---**

Parent / Guardian/ Class teacher

APPENDIX E: THE TEACHING PROGRAMME

LESSON PLANS FOR THE PROGRAMME

LESSON 1

Concept: Variables

Phases	**Teacher's Activities**	**Activities**	**Learners' Activities**
Phase 1: Introduction (15 Minutes)	-Introduces the concept of variables by asking learners to give their views on how they understand variables		-Answer the teacher's questions verbally
Phase 2: Core Learning (25 Minutes)	-Give learners activities to do in groups. He gives question 1 first and discusses it with the learners -Gives the second question and discuss again with the learners as to what variables in their solutions represent.	***Q.1.*** *A catering company feeds two schools in Oshigambo circuit, one primary and one secondary school. Both schools have the same number of learners. The catering company knows that in general, secondary school learners eat twice as much as primary school learners.* *-Relate the cost of food for primary and secondary school learners.* *-Use variables to describe this.* *-Given that the cost of food for primary school learners is N$ 6000 a month, what is the cost of food for secondary*	-Do the activities in groups -Share their solutions with the whole class -Discuss what the variable represents in their solution together with the teacher

		school learners a month? *-If the total budget is N$ 1000, how much is the budget for primary school?* ***Q.2.*** *In our school, the number of learners (L) is 35 times the number of teachers (T).* *-Write this information in algebraic form* *-How many teachers are needed for 70 learners?*	
Phase 3 Conclusion (10 Minutes)	-Explain to the learners what variables represent relating to the solutions of the two activities the learners did in class -Gives homework to the learners		-Listen and ask questions if they have -Take the homework

LESSON 2

Concept: Variables

Phases	**Teacher's Activities**	**Activities**	**Learners' Activities**
Phase 1: Introduction (15 Minutes)	-Asks learners to present their solutions to the homework questions -Write the solutions on the board	-Presentation of the learners' solutions	-Present their solutions verbally
Phase 2:	-Gives feedback to the learners on the two	***Q.1.*** *Tom and Jerry are two brothers. Their mother decides to give*	-Discuss the feedback with the teacher

Core Learning (25 Minutes)	questions of the homework -Use some other related examples to explain the concept of variables to the learners	*them money to buy sweets. Tom gets N\$ P and Jerry gets half of Tom's amount. Write an expression to represent Jerry's amount. Hence, find Tom's amount, given that Jerry gets N\$ 4.* ***Q.2*** *The weight of apples is M kg which is 1/3 of the weight of grapes N kg. in terms of M and N, write an equation to represent the information and determine the weight of grapes if the weight of apples is 12kg.*	-Ask questions where they need clarity
Phase 3: Conclusion (10 Minutes)	-Conclude the concept of variables		-Listen and ask questions if they have

LESSON 3

Concept: Combining/Separating terms by addition

Phases	Teacher's Activities	Activities	Learners' Activities
Phase 1: Introduction (15 Minutes)	-Asks general questions about what do learners know about like and unlike terms		-Share their views of what they know about like and unlike terms
Phase 2: Core Learning (25 Minutes)	-Write lists of different terms on the board -Asks learners to identify like terms -Asks learners to simplify/reduce some expressions in their groups	*-Identify the like terms from the list:* *2a; 4b; 2b; 4x; y; 3a; 4b; 2d* *-In your groups, reduce the following expressions* * $x + 3y + x + y = m + 2n + n + 3m$ *$2a + 4b + 3a + 3c + a = 5l + 2k + l + p + 3k$ *-Simplify the following expression;*	-Identify the like terms from the list -Simplify the expressions both in symbols and in diagrams -Share and discuss their

	-Introduce the use of diagrams to represent terms in the expressions. -Asks learners to make use of the diagrams to simplify the expressions in their groups	□□ + ○○ + △ △ + □ + ○○○ + △△△△	solutions with the teacher
Phase 3 Conclusion (10 Minutes)	-Conclude the lesson by highlighting how to make use of the diagrams		-Listen and ask questions if they have

LESSON 4

Concept: Combining/Separating terms by subtraction (Negation)

Phases	**Teacher's Activities**	**Activities**	**Learners' Activities**
Phase 1: Introduction (15 Minutes)	-Introduces the lesson by asking learners to work out the expression: -2 – 3; 3 – 7; etc, without using a calculator		-Quickly try to work out what they are required to do individually -Share their solutions with the teacher
Phase 2: Core Learning (25 Minutes)	-Explore some examples related to real life examples that demonstrate the concept of negation in real life situations -Asks learners to simplify/reduce some expressions in their groups -Introduce the use of diagrams to represent terms in the expressions -Asks learners to make use of the diagrams to simplify the expressions in their groups	-Example; what does it mean 'account is credited/debited? What basic operation can be linked with those terms - in your groups, simplify the the expression; 4m – 3n – 3m – 5n = 6x + 4y – 2x – y, without using a calculator. -Use diagrams to represent the expression and simplify it using diagrams	-Simplify the expressions both in symbols and in diagrams -share and discuss their solutions with the teacher.
Phase 3 Conclusion (10 Minutes)	-Conclude the lesson by highlighting how to make use of the diagrams -Gives homework to the learners		-Listen and ask questions if they have -Take the homework

LESSON 5

Concept: Simplifying expressions by adding and subtracting

Phases	**Teacher's Activities**	**Activities**	**Learners' Activities**
Phase 1: Introduction (15 Minutes)	-Recap the previous lesson by giving feedback on the previous homework given		-Present their solutions to the homework -Work on the feedback from the teacher

Phase 2: Core Learning (25 Minutes)	-Present an expression that involves both addition and subtraction and demonstrates how to simplify it both in symbols and in diagrams. E.g. 4y + 3x - 2y + x -Asks learners to simplify some expressions in their groups both in diagrams and symbols -Gives learners an activity to work individually.	Groupwork -simplify the following: a) 5a - 2b + a + 3b - 3a b) 2m + 3n - 3m - 5n c) 4a + 3 - a + 2 Do them both in symbols and in diagrams Indidvidual work Simplify the expression on both sides of the equal sign. In symbols and in diagrams 3b - 2g - b - 3g = 4w + 2m - 3w - m	-Simplify the expressions both in symbols and in diagrams -Share and discuss their solutions with the teacher -Write the individual class activity and submit their work
Phase 3 Conclusion (10 Minutes)	-Conclude the lesson by giving emphasis on what he saw was a challenge to the learners during the lesson		-Listen and ask questions if they have

LESSON 6

Concept: Expanding brackets

Phases	**Teacher's Activities**	**Activities**	**Learners' Activities**
Phase 1: Introduction (15 Minutes)	-Introduces lesson by giving simple expressions with brackets and ask learners to give their views of what brackets mean and what can be done to expand them		-Give their views verbally
Phase 2: Core Learning (25 Minutes)	-Introduces the use of an expansion box to learners -Demonstrate how to use it with different forms of expressions that involve brackets	-Expand the the expressions in your groups. Try them both the usual way and by using an expansion box a) 2(2x - 3)	-Work out the activity in their groups -Share their solutions with the whole class

	-Gives an activity to the learners to work in groups using an expansion box -Discuss the solutions from the learners with them and gives feedback	b) -3(2m + 1) c) 3n - 2(n - 2)	-Discuss their solutions together with the teachers
Phase 3 Conclusion (10 Minutes)	-Conclude the lesson by giving emphasis on what he saw was a challenge to the learners during the lesson		-Listen and ask questions if they have

LESSON 7

Concept: Expanding brackets + solving equations

Phases	Teacher's Activities	Activities	Learners' Activities
Phase 1: Introduction (15 Minutes)	-Refreshes the learners on the use of an expansion box by asking them to demonstrate how it works with some given examples of expressions.		-Learners to demonstrate on the chalk board (some).
Phase 2: Core Learning (25 Minutes)	-Demonstrates the expansion of a slightly different expression e.g. 2a – (a + 3) on the board both in a usual way and by an expansion box -Gives an activity to the learners to do in groups **Introduce Equations** Asks learners to explain how to solve the equation x + 3 = 7	-Expand 3x - (2x - 3). Also use an expansion box.	-Work out the activity in their groups -Share their solutions with the whole class -Discuss their solutions together with the teachers -Give their views verbally based on their understanding
Phase 3: Conclusion (10 Minutes)	-Ask learners to represent the equation with diagrams and solve it -Gives feedback by demonstrating how to solve it using diagrams	-Individual work on representing the equation diagrammatically	-Represent and solve -Listen to the feedback

LESSON 8

Concept: Expanding brackets

Phases	Teacher's Activities	Activities	Learners' Activities
Phase 1: Introduction (15 Minutes)	-Continues with solving equations. Asks learners what they remember about solving equations from the previous lesson		-Learners to give their views verbally. -Ask questions where they may need clarity

	-Demonstrate how to use the balance method using diagrams.		
Phase 2: Core Learning (35 Minutes)	-Asks the learners to solve the equations $2x + 4 = 10$ and $3m – 1 = 2m – 4$ in their individually using the balance method. -Gives feedback after the learners have completed. -Give the learners another equation to work in their groups -Gives feedback	-Solve the equations a) $2x + 4 = 10$ and b) $3m - 1 = 2m - 4$ both in symbols and in diagrams. - Solve the equation $-3(x - 2) = 24$ in your groups	-Work out the activity and submit their papers -Discuss their solutions together with the teachers Learners to work in their groups. -Share their solutions with the class and discuss the feedback with the teacher
Phase 3: Conclusion (10 Minutes)	-Highlights the use of diagrams to solve equations using balance method.		-Ask questions where they may need clarity

APPENDIX F: BENCHMARK TESTS

Mathematics test.

Name: …………………………………………….

Instruction

Answer all questions and show your working clearly

1. Simplify the following expressions:

   (a) $3x + 2y - x + 5y$

   (b) $2(a + 3)$

   (c) $-3(2y + 1) – 4$

2. Solve the following equations

   (a) $x + 3 = 7$

   (b) $3x – 2 = x + 5$

   (c) $2a + 3(a - 1) = 13$

   (d) $y - (2 - 3y) = 2y - 5$

APPENDIX G: FOCUS GROUPS QUESTIONS

The following are questions designed for a focus group interview.

(a) What did you find interesting in the lesson?

(b) Do you think working with the (tools) will help you understand the components of algebra we discussed today? How?

(c) What challenges did you encounter in this lesson?

APPENDIX H: INDIVIDUAL INTERVIEWS QUESTIONS

The following are questions designed for individual interviews.

(a) Do you think the programme has helped you in developing understanding of algebra? If yes, how, and if no, why?

(b) What was your experience in relating symbols and diagrams to model algebraic concepts?

(c) What was your experience in using the expansion box to expand brackets?

(d) How do you relate your understanding of algebra before and after the programme?

APPENDIX I: OBSERVATION TRANSCRIPTS

Lesson Observations Transcripts

Lesson 1 (Variables)

1. **T**: Good afternoon everyone. In our very first lesson of the programme, I would like us to look at variables. I believe that all of us here have learnt about variables from our algebra lessons. Isn't it?
2. **Ls**: Yes, we have done it.
3. **T**: Who can give me an example of a variable?
4. **Ls**: (randomly one by one) a, x, y, b …
5. **T**: So now, tell me, how do you define variables then?
6. (learners were sort of whispering and not giving their views. Teacher wrote the examples given by the learners on the board)
7. **T**: Come on, tell me, what are variables?
8. **L4**: They are representing letters.
9. **T**: Ok, hmm, so what do letters represent?
10. **L2**: Unknown numbers.
11. **L1**: (stood and said) They represent hmmm, unknown aah objects, anything that we do not know. Like we use letters to represent unknown objects.
12. **T1**: Ok, we use letters to represent unknown objects? Ok … is there anyone with a different idea? Ok (showing the examples on the board) so we are saying these are the examples, letters used to represent variables, some are saying variables represent unknown numbers, some are saying they represent unknown objects or things. Good. So, no one is wrong in their views. Because we are all correct depending on our interpretations. However, we are going to look at the variables in our context. We want to understand what do they represent in the context we are going to look at.
13. **T**: Now, let's look at the scenario here: In a combined school, the amount of food consumed by primary school learners cost x Namibian dollars. The amount of food consumed by secondary school learners cost twice the cost of food consumed by primary school learners. (repeated the question making it clear to the learners). now the question is, relate the cost of food for primary learners and for secondary learners using x. how do you relate them using a variable. What variable do we have here?
14. **Ls**: x
15. **T**: So how do we represent these costs using x as a variable? Remember, primary school cost is x, and the secondary cost is twice the primary cost. Can you discuss in your groups?
16. **Ls**: (embarked on discussion)
17. **L1**: (rose and said) We have come up with this, primary cost is x and the sec school cost is twice, so twice means multiply by 2, so we multiplied x by 2 and we get 2x.
18. **T**: (wrote what the learner said on the board) So, meaning primary cost is x and the sec cost is 2x. So, what does this mean to you? In other words, how do you understand thing generally?
19. **L8**: Secondary cost is 2x greater than the primary cost.
20. **T**: ... Yes, that's her understanding. So, anyone else? Who wants to give it a try? Come on…
21. **L2**: Secondary learners eat more than the primary learners.
22. **T**: Ok, good, that's what her thought is … yes, anyone else?

23. **L3**: It means that the cost of food for secondary learners is higher that the primary cost.
24. **T**: Good, so the two of you (L2 & L3) have said almost the same thing, that the secondary learners eat more than the primary learners.
25. **Ls**: Yes. (one learner shouted: ‘yeah, it is true, because they are big’).
26. **T**: Now, what does x represent here?
27. **L6**: It represent money.
28. **T**: Represent money? Mmmm, what money? Nam dollars, Angolan kwachas or what money?
29. **L1**: No, it represent the amount of money…
30. **T**: Ok, so what are we saying now? Is it money or amount of money?
31. **Ls**: (most of them) Amount of money.
32. **T**: So, what does the amount of money represent then? Is it the unknown things/object or unknown number as you guys told me earlier?
33. **L1**: I think it is unknown number because we still don’t know how much. We only know the money is x, but there is nothing like x money. (laughed)
34. **T**: Hahah, ok. So, let’s take an example, if the primary food cost N$100, how much do sec school food cost?
35. **Ls**: (start whispering in their groups and raise their hands to give their answers)
36. **L3**: You need N$200 dollars.
37. **T**: How do you get that?
38. **L3**: You multiply 100 by 2.
39. **T**: Why multiply by 2?
40. **L3**: Because they are saying the cost of sec food is twice the cost of primary food.
41. **T**: So, what represents 100 here?
42. **Ls**: x
43. **T**: Good. Now if next month they say, sec food cost N$1000, how much would the primary food cost?
44. **L7**: 500
45. **T**: Very good. And we are saying, all these amounts are represented by….?
46. **Ls**: x
47. **T**: And how do we call x?
48. **Ls**: Variable.
49. **T**: Very good. Let’s look at the other question: in our school, the number of learners (L) is 35 times the number of teachers (T). So, what does L represent again?
50. **Ls**: Learners
51. **T**: So, it represents learners????
52. **L1**: Ow sorry, it represents the number of learners
53. **T**: And T?
54. **L2**: The number of teachers.
55. **T**: So now, let’s represent this info in algebraic equation: the number of learners (L) is 35 times the number of teachers (T).
56. **Ls**: (discussion)
57. **T**: Right, if we have done, let’s discuss. So, what do you get?
58. **L1**: We got like , $L = 35\times T$.
59. **L3**: We got $T\times 35 = L$
60. **T**: Is there a difference between the 2?
61. **L1:** No, just the other way round.

62. **T**: Ok, I like that. So, if there are 140 learners in the school, how many teachers are there? Discuss….
63. **Ls**: (discussed. Some learners in one group were seen excited to have got the answer quickly)
64. **T**: Have you guys done?
65. **Ls**: Yes
66. **T**: Can anyone tell me how you understand this formula. L = 35×T.
67. **L8**: The number of learners is equal to 35 times the number of teachers.
68. **T**: Very good, that's what it exactly means. So what answer did you get then and how?
69. **L7**: We say L = 35T, and replace L with 140, and we have 140 = 35T. we then divide both sides by 35, and we get T = 4.
70. **T**: So, you got T= 4? What does 4 represent again?
71. **Ls** (together): Teachers.
72. **T**: Haha, are you sure?
73. **L2**: No sir, I mean number of teachers (laughed).
74. **T**: Very good, I see you guys are getting it bit by bit. So, let's conclude, what does a variable represent in this context? Remember I am not saying what you have learnt in other areas is wrong, no. but let's use the examples that we have used here to tell me what do you think a variable represents.
75. **L4**: Aa variable represents unknown number.
76. **T**: Hmm, ok. Anyone to give their views again?
77. **L3**: It represents unknown number and object.
78. **L1**: Sir, let me try. I think it represents unknown number of the objects. Like, learners are objects, now we are saying how many objects/learners are those? Yeah….just an idea though…
79. **T**: That's good. Now let's decide, what does it really represent? Is it objects, unknown numbers or number of objects?
80. **Ls**: (randomly): Number of objects…
81. **T**: Good, I agree with you guys. So, the variables represent number of object which is the same as quantities. So, what do you think is a quantity?
82. **L1**: Quantity is how much or how many things…. Yeah
83. **T**: Very good. So, a variable represents a quantity of a certain measure. For example, here our measure is teachers and the T represents the number of those teachers, which is a quantity. (he wrote the defn on the board). Quantity means how much or how many? We talk about people, so the people is the measure and the P represents the quantity of those people.
84. **T**: (gave homework)

Lesson 2 (Variables continued … and introduction to diagrams)

1. **T**: Yes, good afternoon.
2. **L**: Good afternoon sir
3. **T**: Alright, let's look to the homework I gave you yesterday. (Read the homework). Right, how much did **T**om get according to the question?
4. **Ls**: P Namibian dollars
5. **T**: And Jerry gets?
6. **L2**: Half of **T**om's amount
7. **T**: So how do we write the amount in terms of P?
8. **L1**: P divided by 2
9. **T**: Now, let's answer this question. How much does **T**om get if Jerry gets N$4?

10. **L7**: 8
11. **T**: Good, so it is 8 Namibian dollars because whatever Tom is getting, Jerry gets half of Tom's amount. So, what did we say variables represent?
12. **L1**: Quantities, amount of something
13. **T**: What is our variable in this question?
14. **L2**: P
15. **T**: Does it represent a quantity here? And if so, what is the quantity?
16. **L1**: Yes, it represent the amount of Money.
17. **T**: Which money?
18. **L1**: Namibian Dollars sir.
19. **T**: Good. That is correct. Now let's look at something new. I want us to look at how we can use diagrams in algebra. Right, let me say we have 2x, this means 2 times x isn't?
20. **Ls**: Yes
21. **T**: How do we call 2 and how do we call x?
22. **L2**: 2 is an umm, cos… umm ... cofsent … ow and x is a variable.
23. **L1**: 2 is a coefficient sir.
24. **T**: Good, coefficient. We can a**ls**o say, 2 is a multiplier and x is a variable. Now coming to the diagram, we can only replace the multiplier by diagram, we only replace a variable. Let's say x is represented by a rectangle, then we have 2 times the rectangle. So how many rectangles do I have now?
25. **L2**: 2 rectangles.
26. **T**: Good, if I have this, what will I have here?
27. **L4**: 3 triangles.
28. **T**: Good, so if I have this? 2 times 3 rectangles?
29. **L1**: 6 rectangles.
30. **T**: Now, let's go back to Tom and Jerry example. If Tom gets 8 of these (drawing 8 rectangles) how many of these does Jerry get? Anyone to come and draw…
31. **L5**: (stood and drew on the board. 8 rectangles for Tom and 4 rectangles for Jerry)
32. **T**: Yes, this is correct. So, we can use diagrams to represent variables. Let's do this quickly. If I have 5x = 10. If we are to use rectangles to represent x and circles to represent the constant, now we will have 5 open brackets then a rectangle here close brackets is the same as 10 circles. So, how many circles does a rectangle represent ?
33. **L3**: 50
34. **L1**: No sir. Just 2.
35. **T**: 50 or 2? Hahah.
36. **L1**: 2, because if 5 of those rectangles is the same as 10 circles, each rectangle takes 2 circles so they can all balance.
37. **T**: Good, you are right, just two. Look, this 1 takes two of these, this 1 takes another two, just like this. So, one rectangle takes 2 circles.
38. **T**: Good, this is the end of our lesson today. Thank you very much.

Lesson 3 (Combining terms)

1. **T**: Alright , good afternoon.
2. **Ls**: Afternoon sir.
3. **T**: Today I want us to look at combining terms. We want to see which terms are combined and which ones we cannot combine. You might have seen when you did simplify terms, some people add these together some subtract that and sometimes it is wrong but this is happening because people do not understand. Right, let me ask you, if I add a pen to this ruler, do I get a single thing?
4. **Ls**: (some) yes (some) no.
5. **L3**: Tala, this is just one thing … you put them together mos.
6. **L6**: Ooo, u mean this? But is true.
7. **T**: So, what do you end up having when you combine a pen and a ruler?
8. **L5**: Ruler-pen.
9. **Ls**: (laughed)
10. **L7**: Oow, yeah but you can't shili kaa.
11. **L1**: You see, you can't add cats to dogs.
12. **T**: Remember, last time we talked about, let's say we have $x + 3y + x + y = m + 2n + n + 3m$, something like that, and they are saying make this or find which terms you can combine and what will you have at the end? So, if we are to combine these, can we quickly do that in our groups, just in a piece of paper.
13. **Ls**: (discussing)
14. **T**: Right we all done isn't?
15. **Ls**: Yes.
16. **T**: Tell me, how do I combine this side? Yes? (pointing to a learner)
17. **L4**: You collect the like terms like, $x + x + 3y + y = m + 3m + 2n + n$
18. **T**: From there?
19. **Ls**: (together) $2x + 4y = 4m + 3n$
20. **T**: And now, can we move forward?
21. **L2**: No, you can't add them.
22. **T**: Why?
23. **L2**: Because they are not like terms, like x is not the same with y … and yeah.
24. **T**: Good, so you understand.
25. **Ls**: Yes
26. **L2**: (pointing at **L6**) You see, manga wali toti we can add them.
27. **T**: Ok, looking at these symbols or letters, do you know what might they represent in real life?
28. **Ls**: No.
29. **T**: So, you are not making sense of them you can only combine them?
30. **Ls**: Yes.
31. **T**: But can you think on your own of what they may represent, like assuming you telling someone to make sense of the whole thing.
32. **L1**: I think because we have different letters, they can represent different things. For example, x can be a lion, y can be a pig, m be a person and n a snake.
33. **T**: Right, quite interesting. Good, you are almost there. So, to make sense, one would say, you take 1 x no. of lions + 3 x no. of pigs + 1 x no. lions + 1 x no. pigs they will give you the same weight as 1 x no. of people + 2 x no. snakes + 1 x no. of people + 3 x no. of snakes. So, that may make sense to a person. You can always try to relate to real life situations for you to

make sense. Coming down to combining, u combine the lions, u combine the pigs but can you combine lions with the pigs?

34. **Ls**: (some) yes, (some) no, you cannot.
35. **T**: Why or why not?
36. **L1** and **L2**: Because they are not the same.
37. **T**: So, when we talked about variables, we said they represent quantities of certain measures, so are our measures here the same?
38. **Ls**: No.
39. **T**: So, we cannot combine measures that are not the same.
40. **T**: Now, let's represent it in diagrams. Let x be (rectangle) y be (triangle) m be (Circle) and n be (kite). Let's do it in our groups.
41. **Ls**: (discussing) **L4**: No, you can't put kites together with the circles, inayiifa. **L6**: Ooo, taa, now I see. I did not know this. **L3**: Maar with diagrams omuwete ngaa kutya oshipu. (sometimes with letters you just add together these things without knowing)
42. **T**: Have we done? Who can tell us what they did in their group?
43. **L2**: 1 rect + 3 tri + 1 rect + 1 tri = 1 circ + 2 kites + 1 kite + 3 circ
44. **T**: From there?
45. **L2**: You group them. Rectangles together, triangles together and also the other side group same diagrams together.
46. **T**: So how many rectangles do I have here?
47. **Ls**: 2 rectangles + 4 triangles = 4 circles + 3 kites.
48. **T**: Can we go further?
49. **L4**: No, we can't because the diagrams are not the same.
50. **T**: Good. So, when some people use symbols like here, what mistake do they do?
51. **L1**: They will give 6xy.
52. **Ls**: (laughed)
53. **L3**: Yes, because the letters are confusing sometimes.
54. **T**: What about diagrams?
55. **L3**: No, they are clear and u can see easily the diagrams are different.
56. **T**: So, it is important to compare or do it in diagrams or relate with real life things to make the distinctions.
57. **T**: Ok, can we try this one here: 2a + 4b + 3a + 3c + a = 5l + 2k + l + p + 3k in your groups. Also try to tell a story first to make sense of the whole thing.
58. **Ls**: (Discussing) **Grp. A**: **L7**: Can we put these ones together? **L2**: No, you can't these letters are not the same. **L7**: Haha, true man. **Grp. B**: **L1** Guys, we need just to collect like terms only, how can you add the a and b together? Those letters are different you. **L6**: Ooh, oh, is true, I just confuse. Me I like diagrams. **Grp. A**: **L2**: Let's use diagrams, what shapes can we use? **L3**: Rectangle, circle, kite, yeah, those things. **L2**: Now see here, can you add rectangle and triangles together? **L7**: Yeah, kakwali ndatala the letters kaa. **L2**: I told you.
59. **T**: Give me a story that you came up with.
60. **Ls**: **Grp A**:(**L2**): Ok, in Kalenga's lunch box there are 2 apples, 4 bananas, 3 apples again, 3 coconuts and 1 apple which is the same as Toini's lunch with 5 lemons, 2 grapes, 1 lemon, 1 peanut and 3 grapes.
61. **Ls**: **Grp. B**: (**L1**): Us we make like, you go to a shop to buy school stuff. You buy 2 books, 4 pens, another 3 books, 3 calculators, and 1 book again which cost the same as 5 pencils , 2 glues, another pencil, plus a table and another 3 glues.

62. **T**: So, meaning, the cost of these stuff is the same as the cost of these stuff?
63. **Ls**: Yes.
64. **T**: Let's reflect back to variables again. What did we say variables represent?
65. **Ls**: Quantity.
66. **T**: Quantity means?
67. **Ls**: How much or how many.
68. **T**: So, let's use apple , banana example., so here we are say, 2…?
69. **Ls**: Apples……!!!
70. **T**: Hahaha, ok, if I say 2a reads as 2 apples, what does 'a' represent to you?
71. **Ls**: (some) Apples, some, ai hee?
72. **L6**: Apples yes.
73. **T**: Apples, isn't? ok, so meaning the variable here stands for the name of the object but not the quantity. But if we want it to represent a quantity, how do we read it?
74. **L1**: 2 times the number of apples.
75. **T**: Of course, yes … that's correct. Same apply to your example, it is not 2 books but 2 times the number of books. So that's what I have observed in the two groups.
76. **T**: In 2a, if a is equal to 4, what will I end up having?
77. **L5**: Two four.
78. **Ls**: (laughed)
79. **L1**: 2 times 4 sir.
80. **T**: Sure, it is 2 times 4, not 24. Haha, because 2a is the same as 2 x a.
81. **T**: Now, what is our final answer?
82. **Ls**: (Together) 6a + 4b + 3c = 6l + 5k + p
83. **T**: I know with the diagrams you have done it the same way as in the first example, isn't ?
84. **Ls**: Yes.
85. **T**: (writing a question in words on the board) right, let's represent the information both in symbols and in diagrams. Also simplify the information where possible. Please do it in your groups.
86. **Ls**: (Discussing)
87. **T**: Right, can we have someone come and present their solution on the board.
88. **L6**: (stood and wrote what they did in their group)
89. **T**: Good, that looks correct hey.
90. **Ls**: Yes.
91. **T**: (explained)
92. **T**: (used diagrams and presented the information on behalf of the learners)

Lesson 4 (Negation)

1. **T** : Good afternoon.
2. **Ls**: Afternoon sir.
3. **T**: Let's look at something slightly different from what we did last time. So, you guys do Entrepreneurship, right?
4. **Ls**: Yes.
5. **T**: Right, if we are saying you have credit, what does it mean?
6. **Ls**: Hee? (asking in confusion)
7. **T**: Ok if we say your account is credited with 10 nam dollars, what does that mean?
8. **L2**: N$ 10 is transferred in the account.
9. **Ls**: (some) is taken from your account.

10. **T**: Now, is it taken from or transferred into to the account?
11. **L2**: Transferred into.
12. **T**: So, 10 dollar has been put into your account, meaning you get some money. So, debited?
13. **Ls**: (randomly) taken from.
14. **T**: Alright, if I have to use the two operations, if it is credited, what operation do I use?
15. **L1**: Aah aah, u add.
16. **T**: Good , is like you buy credit to put on your phone, so you are adding isn't?
17. **Ls**: Yes.
18. **T**: Debiting?
19. **Ls**: (randomly) subtract.
20. **T**: So, if I have 4m - 3n - 3m - 5n = 6x + 4y - 2x - y, can we simplify that quickly in our groups.
21. **Ls**: (discussed in their groups)
22. **T**: Now, tell me, what do I do?
23. **Ls**: Collect like terms.
24. **L4**: Like, 4m minus 3m minus 3n minus 5n equal to 6x minus 2x plus 4y minus y.
25. **T**: Ok, this is correct. Then?
26. **L1**: We then say 4m - 3m you get m.
27. **T**: (writes m)
28. **T**: Then?
29. **L1**: Then -3n - 5n you get … umm ...
30. L8: -2n
31. **T**: (writes -2n)
32. **L1**: Equals to 4x + 3y
33. **T**: So, is that correct or perhaps there is someone doubting something?
34. **L1**: Yes, me, aah, -3n - 5n is not -2n, is -8n.
35. **L5**: Is just because you used a calculator.
36. **L1**: Not really?
37. **Ls**: (some) laughed and said is true.
38. **L1**: Aah, I was trying to tell you.
39. **T**: So, the right answer is -8n right?
40. **Ls**: Yes.
41. **T**: Let me use the diagrams (drew diagrams) now I have this, so rectangles can start taking each other through subtraction, this takes that, this and that and this and that. Now I have 1 rectangle, now look at these with negatives. What do we do?
42. **L1**: You are going to take the subtraction sign and write positive.
43. **T**: How? What do you mean and where?
44. **Ls**: Down there.
45. **T**: How? Why do I do that?
46. **L1**: Let me try sir. You see, you just take 3 of those and 5 of those and add them together and you get negative 8.
47. **T**: Ok, you are making sense anyway … so meaning this is the same as adding together?
48. **L1**: Yes.
49. **T**: Let's look at this: if I have (drawn -1 rect + (-2 recs) what do I get?
50. **Ls**: (some) 3, (some) -3, (some) -1.
51. **T**: The right answer is -3.

52. **Ls**: (some) celebrated.
53. **T**: But explain why it is -3?
54. **Ls**: (just quiet)
55. **T**: ok, look, this means that I am just adding the negatives together. Even here, of course there are no addition signs, but it means these terms are being added together. So, there are addition signs here though we are not seeing them. So here it is the same as adding these negative terms together thus, our answer remains negative.
56. **T**: So, at the end, this is the result we will end up getting.

Lesson 5 (simplifying expressions)

1. **T**: Let's look at this one. What do we mean by simplify?
2. **Ls**: (some) collect like terms.
3. **T**: Collect like terms?
4. **L1**: Making the expression shorter.
5. **L2**: You make it simple.
6. **T**: So how do we do this? Tell me.
7. **Ls**: Collect the like terms.
8. **T**: What do we look at to say these are like terms?
9. **Ls**: To the variables.
10. **T**: So, tell me what to write…
11. **Ls**: 5x - 2x - 3y - y
12. **L3**: We subtract, we get 3x … (then quiet)
13. **L1**: -3y - y it will give you -4y, so the final answer is 3x negative 4y. Like 3x - 4y
14. **T**: Very good, yes (pointing to **L6**) do you understand how we got -4y?
15. **L6**: Yes, we say -3 + -1.
16. **T**: Good, I see you guys now understand. So, can we continue from here?
17. **Ls**: No.
18. **T**: Why?
19. **L4**: Because those variables are different.
20. **T**: Ok, good. Can we quickly look at this 1 in our groups, 4a + 3 - a + 2. Let's be faster. Remember you must try it both in symbols and diagrams.
21. (Group discussion) **Grp. A**: **L7**: This is simple, let us correct the like terms. **L1**: Yes, you are right. We say 4a - a + 3 + 2. **L6**: O minus oweyi kuthapo pehala. **L1**: Aiye, minus is for a mos. **L7**: So, our answer is 3a + 5? **L1**: w. **L5**: Are u sure guys. But can we not add them together? **L1**: Hahah, you, did we add those ones when we did that do that with sir. **L6**: But look, there, oonumber adhihe odhina letters but here 5 don't have a letter. **L1**: Ok, give me the answer now. **L5**: 8x mos. **L1**: I don't believe you. **Grp. B**: **L3**: How do you get 3a? **L4**: We subtract mos., 4a minus a. **L3**: Ow yeah, ok. **L8**: Is that our final answer? **L2**: Yes, we can't add them together, they are not the same. **L3**: Maar oletter ya 5 oshike nee? Can't we just combine them because 5 kenasha ovariable? **L2**: Aiye **L4**: No, aie. **Grp.A**: **L5**: Ooh, haha, so these ones can be triangles and these ones can be rectangles`? **L1**: Yeah mos, here, there is x, here there is nothing. So, they are different. **L6**: But it is true. **Grp. B**: **L3**: What diagrams are we going to use now? **L4**: Nenge uurectangles nuucircles. **L2**: Yeah, you see, now we have 3 rectangles plus 5 circles. **L8**: Taaah, maar diagrams neh, they are clear. **L3**: I like them, maar symbols are confusing. **L2**: You see, I told you guys…
22. **T**: Have we done?
23. **Ls**: Yes.

24. **T**: What answer do you get?
25. **L6**: 3a + 5
26. **T**: Good, is it the same here?
27. **Ls**: Yes.
28. **L3**: But…
29. **T**: But what?
30. **L3**: Sir, why can't we add 3a and 5 so we get 8a.
31. **T**: When you use diagrams, did you use the same diagram for all those terms?
32. **Ls**: (together) no.
33. **T**: Why? Who can tell me?
34. **L2**: Because, the a, value, no, wait, where there is a, is not the same like where there is nothing. Yeah.
35. **T**: Good, the constant term is different from the term with a variable, that's why we cannot use the same diagrams. That means, you can't add a constant to a term with a variable, they are not the same. It is, just treat it like there is a different letter. Do you get it now?
36. **L3**: Yes sir. I see now. But diagrams are clear because you can see the difference.
37. **T**: Very good. now can you do this in our groups. 2a – b – 4a – 2b
38. **Ls**: (Discussing) **Grp. A**: **L1**: Let us collect like terms. **L6**: So, everything here is minus. **L7**: Yeah, so, 2a - 4a - b - 2b. then we get 2a, Shuu, what do we get here? **L1**: Aie, we don't get 2, is negative 2. Just use a calculator. And here we have , hmm, negative b., no wait, is negative 3b. **L6**: How do you get that? **L1**: -1 minus 2 mos. **L1**: You see, look at the diagrams. 2 rectangles minus 4 rectangles, cancel cancel, you remain with 2 but they are negative. **L7**: Shuu, maar owuuviteko ngaa. **Grp. B**: **L3**: This one I can do it. Collect like terms, 2a - 4a - b - 2b. **L2**: Then? Hahah. **L3**: Ow, this minus nee, aie. **L4**: Let's just use a calculator. **L8**: Then you get -2a here. **L2**: Yeah. Here what do you get? **L4**: Negative 3b. **L8**: Now diagrams. **L3**: Iyaa, my favourite. We just repeat mos. **L2**: 2 triangles minus 4 triangles, give us -2 triangles. **L4**: Minus 3 circles. **L3**: Yes, opuwo otwamana. We cannot subtract those things. **L8**: Paife onduuviteko nee.
39. **T**: Finished?
40. **Ls**: Yes.
41. **T**: Which group got a different answers between symbols and diagrams?
42. **Ls**: Quite
43. **T**: So, you all got the same answer in both?
44. **Ls**: Yes
45. **T**: What is that answer?
46. **L2**: -2a - 3b. and 2 negative triangles minus 3 circles.
47. **T**: Is that the same as this group?
48. **L6**: Symbols yes, but diagrams, no. We have negative 2 rectangles minus triangles.
49. **T**: Good, different diagrams are not a problem. So, meaning we have now understood right?
50. **Ls**: Yes sir.
51. **T**: We just need to practice a lot.

Lesson 6 (Expansion of Brackets)

1. **T**: Afternoon everyone
2. **Ls**: Afternoon sir
3. **T**: As I have said today, we are going to look at expansion of brackets. I know you have already covered that only by using symbols. You may not have made any meaning from that but today I want us to do it and make meaning out of it. Right to start, one can say, expand: 2(x + 1). What does the whole expression tell you? What do you think it means to you?
4. **L1**: Brackets mean times, like multiply the number next to the brackets with what is inside the brackets.
5. **T**: A number next to the brackets, like this one? (pointing at 1 inside the brackets)
6. **L1**: No no, the one outside the brackets
7. **T**: Ok, anyone else?
8. **L3**: It also means you work out what is inside the brackets first.
9. **T**: So how can we work these ones out now?
10. **L4**: No sir, I don't think that's possible.
11. **T**: Why?
12. **L5**: Because they are not like terms
13. **T**: So, we cannot add them together?
14. **Ls**: No
15. **T**: Umm, what we can say here to make meaning, one can say the sum of x and 1 is doubled. Or double the sum of x and 1. Do you understand why we are saying this is a sum?
16. **Ls**: Yes.
17. **T**: Why?
18. **Ls**: Because we are adding
19. **T**: Good, so the practical example could be, let's say you have a number of sweets here on the RHS and a cake here on the LHS, so can you add them together?
20. **L2**: No, you can't, you will just have 2 apples and 2 oranges.
21. **T**: So, now, if someone tells you to double them, can you double them?
22. **Ls**: Yes
23. **T**: Good, so, you can double them separately, the number of sweets and you also double the cake. So, what we are saying here now, we are saying, you multiply 2 by x and also multiply 2 by 1. So, we now have 2x + 2, can we add these together?
24. **Ls**: No, they are not like terms.
25. **T**: Ok, now let's look at this one, 2(2x - 3). What does this tell you now?
26. **L2**: It says doubling the difference of 2x and 3 because we are subtracting there. Like you say 2 times 2x and 2 times 3, and you get 4x - 6
27. **T**: Ok, good. So, can you then think of an example that you would use to make sense of this expression.
28. **L2**: Maybe, aah, you double 2 times the number of tables minus 3 chairs.
29. **T**: So, this means we now have 4x - 6
30. **T**: Now, (writing on the board) -3(y - 1), what do I get here?
31. **L6**: -3y - -3.
32. **T**: So, have we completed like that?
33. **L1**: No
34. **T**: Can you continue
35. **L1**: -3 must be in brackets

36. **T**: And we done like that?
37. **Ls**: No
38. **L7**: Now you will add negative and negative.
39. **L2**: No, you will just have - 3
40. **L1**: (opposing **L2**) aaawe, ndjono odouble negative, negative and negative is positive. Sir we will just have -3y + 3.
41. **T**: (writing on the board-3y + 3) like this right?
42. **L1**: Yes.
43. **T**: Do we all agree or do we understand?
44. **Ls**: (some) yes, (some just quiet)
45. **T**: Ok, let's make use of the diagrams to see if they can help us understand this expansion.
46. **T**: (draws an expansion box on the board and explained to the learners. he shows the learners on selecting the signs to put in the square boxes and also the terms) Let x be a rect and a constant be a circ for example." X and 1 are all positives, so this block is positive and this one is also positive. So, we then have to multiply them, (showing on the board) positive and positive?
47. **Ls**: Positive
48. **T**: Here?
49. **Ls**: Positive
50. **T**: Good, 2 times a rectangle?
51. **Ls**: 2 rectangles
52. **T**: 2 times a circle?
53. **Ls**: 2 circles
54. **T**: So now we only write these ones now. 2 rectangles and 2 circles. So, if you look at this, it is the same as that one in symbols.
55. **L3**: Shuu, look, this is just like playing a game. It looks nice.
56. **T**: Let's look at that other one.
57. **T**: Let's use the box quickly to expand this -3(y - 1) (drawing the box). What do I put here? **Ls**: -, **T**: here? **Ls**: +, and -. **T**: ok, let's multiply. What do I get here? **Ls**: -, and +. **T**: Good, so here? **Ls**: 3 **T**: What diagrams can I use in this blocks? **L1**: Rectangle and circle. **T**: What do I get here? **Ls**: 3 rectangles, and 3 circles. **T**: So, our final answer is? **Ls**: Negative 3 rectangles plus 3 circles. **T**: Good, so I hope we understand.
58. **T**: Now, do this one in your groups -3(2m + 1). Discuss.
59. **Ls**: (Discussing) **Grp. A**: **L5**: Aaawe, that one will be positive 3 **L6**: No, is negative, **L5**: Ok, let's see.
60. **L1**: The answer is -6m - 3.
61. **L4**: You see, I told you the results are all negative, positive and negative is always negative.
62. **L3**: Eewa nee, onduuvako, so you win.
63. **T**: So, can we try this one. 3n – 2(n – 2)
64. **Ls**: (discussing) **Grp. A**: **L3**: you guys have you already done multiplying all these ones? **L2**: no, we don't multiply that one, we only multiply the one next to the brackets. **L3**: are you sure? **L4**: yes, tala, sir osho ati petameko mos. **L2**: mani, listen, you guys are, this 3n is far from the brackets, tala opuna no Minus ndjika. So, you only multiply 2 by the brackets opuwo. **L3**: ok, let's do it but let's see what sir will say. **L2**: even in the box, we only have one block not 2, our scale factor is just -2 not 3n. **L3**: Iyaa man, we only have one block for the scale factor shili maan, I see.) **Grp. B**: **L1**: let's do it guys, **L6**: taa, this one is difficult. **L1**: aie,

tala, we just multiply like those ones mos. **L5**: can we multiply 3n by n? **L6**: ya, they are like terms mos … **L1**: haha … maar ne kamuuviteko kaa…. We only multiply the one next to the brackets. So, we multiply 2 by n and by -2. **L6**: ndjino? **L1**: we just bring it down mos. **L6**: ok. **L1**: writing: just like this, and yeah, we simplify now. **L7**: paife taleni nee, our answers are not the same, we have negative here and we have positive here… **L1**: let's leave it and see wat sir will say. Osho kwali tiikulombwele mpaka. **L1**: eeno, when you are using diagrams you just put one number in this box, so bring -2 here as a factor. **L7**: but there are like terms there guys, I don't believe you.).

65. **T**: Have you done?
66. **Ls**: Yes
67. **T**: Now let's quickly go through it, what is it supposed to be multiplied with brackets here?
68. **Ls**: (some) 3n and 2 (some) 2 (some) -2. (Some just laughing)
69. **T**: Hahaha, which is which now?
70. **L3**: 2
71. **L1**: -2
72. **T**: Now give reasons
73. **L6**: I think all of them, 3n and 2 because they are outside the brackets.
74. **L2**: Hahaha, ie ooh… no.
75. **T**: Ok, tell me .
76. **L1**: Sir it is just -2 because we only multiply the number which is really next to the brackets, 3n is far, there is even that negative sign, because even when you use the box you put the number next to the brackets inside the first column.
77. **T**: Ok, do we agree?
78. **L2**, **L4** & **L5**: Yes, I agree.
79. **T**: Good, let's not waste time. You are right, it is -2
80. **L6**: Ooh, Hambaa? Now I see.
81. **T**: So. Tell me what I should write.
82. **L5**: 3n - 2n + 4
83. **T**: How did you get positive 4?
84. **L1**: Negative and negative gives you positive. So, we multiply -2 by -2 so is +4
85. **T**: So, you mean this and that?
86. **L1**: Yes.
87. **T**: Good, so is this our final answer?
88. **L6**: No, we still have those like terms, 3n – 2n give you n then +4. And that is the final answer.
89. **T**: Do we all agree?
90. **Ls**: Yes.
91. **L1**: (talking to others in a group) hey, tala, I told you already that we can't multiply with that 3n, is.
92. **L6**: Ow yeah, I now understand. But we are wrong. This 4 is negative. that one is positive.
93. **L1**: Otwangwangwanithaala oosigns, but next time ngaa nee
94. **T**: Now, let's use the diagrams, this is our box. Where do I put in this box 3n or -2?
95. **L6**: -2
96. **T**: Yes, so I can write 3n here away from the box represented by the rectangles like this.
97. **T**: (illustrating on the board with the lead of the learners). So that is how you suppose to do it.

98. **L7**: Taa, now I know, maar shampa tolongitha okaboxa is easy, with numbers otongwangwana.
99. **L4**: but you see, is better you do in with numbers and also with diagrams, just to confirm.
100. **L2**: yeah, I agree with you,
101. **L3**: but it takes time nee,
102. **T**: Ok ok guys… I see most of you have learned something, you have seen where your mistake was.
103. **Ls**: Yes.
104. **T**: So, if you have time, is better you also do it in diagrams.
105. **L3**: Our group got it right, both with symbols and with diagrams.
106. **T**: Yes, so let's clap hands for this group.
107. **Ls**: (clapped hands)

Lesson 7 (Expanding expressions)

1. **T**: The sum of male and female learners is tripled. Write an expression and expand it. Think of your own variables to use.
2. **L1**: Sir, I am done.
3. **T**: So, tell me what you think
4. **L1**: I think you say m be the number of male learners and f the number of female learners, you add them because it is the sum like m + f and you put it in the brackets and times 3 because triple is 3.
5. **T**: Like this (m + f) × 3?
6. **L1**: No, like the 3 you write it first before the brackets.
7. **T**: Like this 3(m + f)?
8. **L1**: Yes
9. **T**: Is there a difference between this and that?
10. **L1**: I think so…aah…. I am not so sure
11. **L3**: I think so, that first one, 3 is far from the brackets, so we can't multiply…
12. **L8**: Maybe the first one, 3 is just multiplied by f.
13. **L2**: Sir, I think those are just the same. Multiplication is not separating the 3 from the brackets… so they are the same.
14. **T**: Good, your answer is correct, these are just the same. Is like we are saying 2 times 3 and 3 times 2, we just get the same answer. So, we are talking about the sum of male and female learners, and this is correct to write it this way. If we expand it, what will we get?
15. **L4**: 3m + 3f
16. **T**: Good. Now in our groups, let's do this. One group will do it in symbols and the other group will do it in diagrams. 3x – (2x – 3). Which group is going to do it in diagrams?
17. **Ls**: Ours, ours…
18. **T**: Ok, let's vote, gr. A will use symbols and B will use diagrams. If u take A, your group will do it in symbols. So, Group A: symbols and Group B: diagrams. Right let's try it.
19. **Ls**: (discussing) **Grp A**: **L7**: let's multiply now. **L6**: So 3x times 2x what do you get? **L7**: oh, is that what we are doing? **L6**: yeah mos, **L1**: haha, you guys, how can you do that. 3 is far, there is a minus between. **L8**: so, what are we doing now? **L1**: look, there is 1 between here, so we multiply the brackets with 1. **L7**: Iyaa man, nesiku ndiya sir osho a li tati. So, this 3x just come down. **L1**: Iyaa, ish. Now we have 3x - 2x - 3. And we get, x – 3. You see.) **L6**: mbeya inaya mana, let's use the diagrams also. **L1**: yeah fast fast. **L7**: are we not putting 3x here? **L6**: hey, 3x is out, tala ngwee, opuna oplus mpaka. **L1**: eeno, 3x is out, we can't multiply

with it. We just put 1 here and … **Grp B**: **L4**: let's draw the table, what are we putting here? **L3**: we put 3x mos. **L2**: lol, is 3x next to the brackets? **L5**: Ooh, we put 1 there. **L4**: Iyaa, taleni, 3x is not coming inside the table, oyatetwako ko minus. We just put 1 in box here now. **L2**: yes, listen guys, there is a 1 here but look, listen man, there is this negative, so meaning 3x is out of the box. let's fill the blocks with the signs first. Obvious here we put 1, this 1 is - , here we put +, and this 3 is -. Let's multiply. **L4**: - + is -, - - is **L5**: +. **L2**: yes, Good.)

20. **T**: Have we done?
21. **Ls**: Yes
22. **T**: Ok, now can I just have your final answers
23. **Grp. A: L6**: ours is x – 3
24. **Grp. B**: **L4**: x + 3
25. **T**: Good, who is correct now?
26. **Ls** (randomly) us, us, our group
27. **T**: Let's go through it together. (go through with the learners and got x + 3)
28. **T**: So where is the mistake that this group did?
29. **Grp. B**: **L2**: multiplying signs.
30. **T**: Very good. How did you guys get it right?
31. **Grp. B: L2**: When we use diagrams, we first put the signs in the small boxes so that we don't get confuse the signs. We multiply them first, ja .
32. **Grp. A**: **L1**: Ooh, iyaa man, now I got it, taa, this is tricky!!symbols are confusing maan... oh
33. **Grp. A**: **L6**: Haha, oshili maan, taa, tse otwa multiply just with 1 not -1. Next time ngaa nee,
34. **T**: Aright guys, good that you now know where your problem came from.
35. **T:** We do not have time anymore, so, I urge you to make use of your textbooks and try similar problems.
36. **Equations**
37. **T**: Let's also look at solving equations. If we have x + 3 = 7, to make meaning of this equation, one would say, if I have a box of apples plus 3 loose apples will give the same weight as 10 loose apples. Now one would want to find the number of apples in the box. So how do we solve this equation?
38. **Ls**: (together) x = 7 – 3, x = 4.
39. **T**: How did you get minus 3?
40. **L5**: Because when you change a number to the other side it changes to negative.
41. **L2**: When you change the number from one side, the sign change also.
42. **T**: But why?
43. **L1**: It is a rule
44. **T**: It is a rule?
45. **Ls**: Yes
46. **T**: Ok, let's look at these diagrams, x is a box, and constants are circles, a box plus 3 apples = 7 apples. How do I do it?
47. **L1**: You bring the box down then = 7 apples minus 3 apples.
48. **T**: Haha, no (using a ruler as a balance scale), imagine you have a box this side and 3 loose apples and on the other side you have 7 apples. What will you do so that you remain with a box alone on one side and still have the scale balanced?
49. **L1**: Just take the apples on the other side.

50. **T**: Hmm, ok, guys, look at this, what I will do I will just start taking loose apples from both sides, I take one here, and also here, I take another 1 and here also, I take 1 from here and here also. Now you see, do we still have loose apples here?
51. **Ls**: A box
52. **L8**: Ooo, I see now, u just keep taking until you have a box only there.
53. **T**: So, a box represents?
54. **Ls**: 4 apples
55. **T**: Good. Now, if I am doing this in numbers and letters, I will just subtract 3 apples from here and also from the other side. So, this gives me zero and the other 1 gives me 4. Do you understand?
56. **Ls**: (some) yes
57. **L7**: Kashona ngaa…
58. **T**: So, we will understand as time goes.
59. **T**: Now let's look at this in our groups. $2y + 3 = 13$. Let us discuss, let's say y represents a box of apples and we want to find how many apples are in the box.
60. **Ls**: (discussing) **Grp. A: L1**: taleni, we can draw like this, 2 boxes plus 3 apples equal to 13 apples. **L8**: Aaawe, tala, we are just removing one by one. **L6**: removing owafaala tosubtract mos. **L7**: Ooh, taa, tashiti you take here and here? **L1**: Iyaa mos. L8: wait, tala, 1 box is having 5 apples. **L6**: one, two, three, four, five. Ooo, owuli mondjila. There are 5 apples Grp. 2: **L2**: Aaawe man, tala, ngao owafaala toningi ngaashi shito. Look at that example, we take 1 from there and from there. **L3**: now what can we do? **L4** iyaa man. Oh! How many apples in one box now? **L2**: 5 apples… **L3**: ow yeah… maar oshipu ngaa.
61. **T**: Done?
62. **Ls**: Yes
63. **T**: How many apples are in 1 box?
64. **L3**: 5 apples
65. **T**: That's what this group got. This group?
66. **L1**: 5 apples also.
67. **T**: How do you find it?
68. **L2**: I think is fine.

Lesson 8 (solving equations)

1. **T**: Yes, good afternoon.
2. **Ls**: Good afternoon sir
3. **T**: Solving equations continue. Right we also use diagrams to show how we can solve these equations with understanding and also to help us not to make mistakes with the signs. The other thing is also to make meaning of the equations. Like to know why, where and how do I make use of equations in our daily lives.
4. **T**: Now, can we refresh by solving this equation: $2x + 4 = 10$. I know most of us can solve that equation using symbols. So, can we try it using diagrams only and see whether you can get the right answer. Let me give you some papers so that you can solve it individually. Also, try this $3m - 1 = 2m - 4$. You can do it both in symbols and diagrams. The first one you can solve it in symbols first and the 2^{nd} one in diagrams first.
5. **Ls**: (busy writing)
6. **T**: Yes, can I have the papers. Bring what you have written. Yes, do not worry, even if you have not done. So, how did you find it? Easy, difficult or confusing?
7. **Ls**: Somehow

8. **T**: Ok, here, we are solving this equation: 2 times a number of boxes of apples perhaps plus 4 loose apples will give you the same number as 10 loose apples. So of course, how do we solve it procedurally as we have learnt from our classes?
9. **Ls**: (together) $2x = 10 - 4$
10. **T**: So, you only take 4 to the other side?
11. **Ls**: Yes
12. **T**: So $2x = 6$, then divide both sides by?
13. **Ls**: 2
14. **T**: x is equal to?
15. **Ls**: 3
16. **T**: So, if I am to use diagrams, (drawing rectangle to represent x and circles to represent constants) let's say x represents the number of apples in the box and constants are loose apples, and I want to find how many apples are in the box. I will say, 2 boxes, + 4 apples equals to 10 apples. So, how do I find the number of apples in 1 box?
17. **Ls**: (some) move them to the other side (some) remove 4 apples from each side.
18. **T**: Remember, I want to keep the thing balanced. So, if I take 1 from here, I also take one from there until one side is finished. If I take all these 4 from here and also from the other side, what do I have here?
19. **Ls**: 2 boxes
20. **T**: 2 boxes isn't?
21. **Ls**: Yes
22. **T**: And on the other side I have how many?
23. **Ls**: 6 apples
24. **T**: Then from there?
25. **L4**: We find how many apples in 1 box.
26. **L2**: Sir, in 1 box there are 3 apples.
27. **T**: Good. so, if we are saying x was replaced by a box, so if we replace a box by x now here, that means x represents what?
28. **L6**: 3 apples
29. **L4**: Maar oshipu ngaa kashona
30. **T**: Let's come down to this one ($3m - 1 = 2m - 4$) using diagrams. 3 times the number of boxes – one loose apple will give the same results as 2 times the number of boxes 'and I take 4 loose apples. Now, this is different from the previous one, it has boxes on both sides and loose apples on both sides, also we have negatives now. What can we do now here?
31. **Ls**: Collect like terms
32. **T**: Remember, we want to balance the pivot, so u mean boxes come one side and loose apples on one side?
33. **Ls**: Yes
34. **T**: So, how do we do that now?
35. **L3**: To remove is like we are subtracting. So, just remove that 1 loose apple from there and also from the other side. Yeah, I think like that…
36. **T**: Hmm, ok. So, let's look at it, I have 3 times the number of boxes minus 1, now, for me to remove this minus apple, how do I do it?
37. **Ls**: Some saying subtract, some saying add.
38. **T**: Remember when we did expressions, there are those terms that cancel. For me to cancel this minus apple, what do I do?

39. **Ls**: Some saying, add and some saying subtract.
40. **T**: Ok, yes, we add. So, we need to add 1 here.
41. **Ls**: (Some) oh aaaye, this is just confusing… (some), you see,
42. **T**: now if we add 1 here then it will be the same as 2 times the number of boxes minus 4. Now because I added 1 here, I also have to add another apple here to balance. You see, if I only add one on this side and I don't add to the other side, the thing will not be balance. Are we together?
43. **Ls**: (some) yes (some) quiet
44. **T**: So, we are adding because something was subtracted so, I need to add so that we remove it. Now, go back to expressions, if I have negative and positive, what do they do?
45. **Ls**: Cancel
46. **T**: They cancel isn't?
47. **Ls**: Yees
48. **T**: Now here I only have 3 boxes, which is equal to 2 boxes, now look at that, I have only one positive and 4 negatives, positive and negative cancel isn't?
49. **Ls**: Yes
50. **T**: So, this positive cancels with one negative, now how many negatives do I have?
51. **Ls**: Just 3
52. **T**: Good, ok. Now, loose apples are on one side, I now need to take these boxes to the other side. What do I do?
53. **L1**: Remove them.
54. **T**: If I am removing them, what do I do?
55. **L1**: Subtract
56. **T**: Good, because they are positive, I need to subtract. So, I take 2 boxes from here and I should also take them from the other side. I cannot just take them from the other side and keep quiet on this side because it will not balance.
57. **T**: So, here we say this minus that and this minus that … they are just the same thing isn't? so here, this one and that one and this one and that one. Now I only have one box remaining here.
58. **Ls**: Yes.
59. **T**: And on the other side, we have negative 3.
60. **Ls**: (some) Shuu, this is too confusing … ooh aaaye, difficult.
61. **L4**: Sir can I not use the old method, just taking from one side to the other?
62. **Ls**: True, true…
63. **T**: Yes, you can. We are just trying to make sense of algebra here. So, if you find the other way easy, then it is fine. How many of you prefer this way?
64. **Ls**: (only 2 raised: **L8** and **L1**. Others: me the old one, me too, me too…
65. **T**: So, meaning we are saying because we have negative apples in the box, that means there are 3 apples missing in the box. There is a need for 3 apples in that box. That's what it says because the answer is negative. look at this one, one box we said is 3 positive meaning there are 3 apples there, so those apples are there in the box.
66. **T**: So, this is how we make use of algebra in our lives. Some people will just say I hate mathematics, you are just taught x and y where am I going to use it? So, this is it now… did you get it?
67. **Ls**: Yes

68. **T**: Now, let's look at something like this. 2(x +1) = 8. What comes into your mind when you see these brackets in terms of diagrams?
69. **L2**: Umm, everything in the brackets are multiplied with the number outside the brackets.
70. **T**: So, everything inside is multiplied by that (pointing the scale factor). Good, so for example, if you have a number of apples on your RHS and may be 1 orange on your LHS and you are told to double them, so u multiply the number of apples and the orange each by 2, isn't it?
71. **Ls**: Yes
72. **T**: So, this times that?
73. **Ls**: 2x
74. **T**: This times that?
75. **Ls**: 2
76. **T**: Is the same as 8. So, this one has already come there that straight form of equation. So procedurally, how do we solve it?
77. **L4**: You say, 2x equals 8 minus 2.
78. **T**: Ok, so meaning when this go on the other side we subtract?
79. **Ls**: yes
80. **T**: So, if it was positive it becomes negative?
81. **Ls**: Yes
82. **T**: So, 2x equals to 6, so now?
83. **Ls**: Divide both sides by 2.
84. **T**: And we get?
85. **Ls**: 3
86. **T**: With diagrams now, we talked about an expansion box, (drawing the box on the board and learners leading him on what sign to put on each square box) is equal to eight circles.
87. **T**: our scale is?
88. **Ls**: 2
89. **T**: What do we have there?
90. **Ls**: A box
91. **T**: Here?
92. **L2**: One circle
93. **T**: Let's start with the signs, positive positive?
94. **Ls**: Positive
95. **T**: Positive positive?
96. **Ls**: Positive
97. **T**: 2 times the box?
98. **Ls**: 2 boxes
99. **T**: 2 times a circle?
100. **Ls**: 2 circles
101. **T**: So, what do I write down now?
102. **L5**: 2 boxes and 2 circles.
103. **T**: Good, So meaning we only write these ones. 2 boxes plus 2 circles will give you 8 circles. So that's what we have now. This is simple isn't?
104. **Ls**: Yes
105. **T**: So, you just remove, this one and this one x2 with learners repeating after him. So now we have 2 boxes which is the same as?

106. **Ls**: 6 apples
107. **T**: Meaning in one box, there are how many apples?
108. **Ls**: 3 apples.
109. **T**: Good, I know this is easy to you and I know where you find it difficult. So, when do you usually face challenges?
110. **L2**, L4 & L8: when there is negatives.
111. **T**: Yes, that's where your challenge is. Now, (writing on the board, -2(2y + 1) = 14) if we have something like that, let's us start using diagrams to solve this equation. First is the expansion box, (drawing the box and learners led him to fill the square boxes)
112. **Ls**: (indicated the signs of the squares correctly and the boxes and circles to be put in the squares)
113. **T**: Now what do I have here?
114. **L2**: -4 boxes minus 2 circles.
115. **T**: Is the same as?
116. **L2**: 14 circles
117. **T**: Good, ok ... let's work it out. These are negatives, what do I do first.
118. **Ls**: (whispering) some were heard saying: you remove, (some) subtract, **L2** was heard saying: no, you add because that one is negative.
119. **L2**: Sir, first you add 2
120. **T**: So, I add 2 here, because we are saying when we have negative, it means we are short of something, so we suppose to fill the gap. And when we do fill the gap on one side, we also do it on the other side.
121. **L2**: Yes, and also to the other side.
122. **T**: Good, for it to remain balanced isn't?
123. **Ls**: Yes
124. **L3**: but that side there is no gap there sir.
125. **T**: where?
126. **L3**: that side where there is 14.
127. **T**: no, what we are saying, if you fill the gap by adding on one side, you should also add the same number on the other side for the whole thing to be balanced. If I don't add here, that means the scale will be like this. But my aim is to keep it balanced. Do you get it
128. **L3**: yeah… somehow ngaa…
129. **T**: (wrote as the learners said). So, look at this, just an expression on one side, positive and negative?
130. **Ls**: cancel
131. **T**: now I remain with -4 boxes is the same as?
132. **Ls**: 16
133. **T**: How do we remove this negative?
134. **L6**: One negative box is equal to 4 circles.
135. **T**: Is that so?
136. **L2**: Somehow yeah… but, I don't know….
137. **T**: Can I have a negative box?
138. **L2**: (debating with others in a group) ish, you see, I told you.
139. **T**: Tell me what to do now, how do I remove a negative sign here?
140. **Ls**: (just whispering)

141. **T**: Ok, procedurally we just multiply by another negative on both sides. Negative and negative cancel. So, we have, 1 box is the same as?
142. **Ls**: -4 boxes.
143. **T**: Very good, meaning there is short of 4 whatever…
144. **L3** & **L8**: (seem so depressed)
145. **T**: Just imagine, the way we always do it is just about when you move a term from one side to the other it becomes negative or positive, but what's the logic? So, this way is trying to make you understand the whole process.
146. **L5**: But it is confusing. Me I just don't get it. The other way is easy.
147. **T**: Ok, I see, and I am not saying it is a master that you should use this method. you can always use the one you are comfortable with.
148. **T**: Now, can we try this, -3(x – 2) = 24. First let's do diagrams.
149. **Ls**: Trying it in their groups
150. **T**: Right, because of time, let me just quickly go through without using an expansion box. (The teacher solved the equation on the board with learners giving their views at random.) which group got it right.
151. **Ls**: (from 1 group) ours.
152. **T**: Good, what about this group? Let me see … Ooo, ok, so instead of subtracting here on both sides, or just removing , they added.
153. **T**: Ok, let me just ask now, if I have 2x – 3(x + 2) = 4, what is the scale factor of the brackets there? In other words, which number should I multiply by the terms inside the brackets?
154. **Ls**: (Shouting) 2x, 3, no, -3.
155. **T**: Come down, ok, which is which now? One person at a time pls.
156. **L3**: Aah … maybe 2x multiply by x and 3 multiply by 2.
157. **Ls**: (some nodding their heads; some were heard saying: Aiye, you just multiply with one number)
158. **T**: Ok, that's her view.
159. **L2**: I think you multiply by 3.
160. **Ls**: (most of them) Yes, true.
161. **T**: Are you sure this is 3?
162. **L7**: No, negative 3.
163. **T**: Good, and you just keep 2x separately, it is very far from the brackets, it cannot jump the negative sign to multiply the brackets.
164. **T**: Lastly, let's have a look at this one: 3a - (2a + 1), what can I multiply the brackets with?
165. **Ls**: (few shouting) 3a
166. **L6**: Maybe 1
167. **L2**: No sir, it is -1.
168. **T**: Right, the scale factor of the brackets here is -1. If they do not put a number here, that means there is 1 next to the brackets. So, in this case, it is -1 because there is a negative here. Do you understand?
169. **Ls**: Yes sir.
170. **T**: So, I urge you to go through your textbook and try to solve these types of equations.
171. **Ls**: Ok sir
172. **T**: Thank you, see you on Thursday.

APPENDIX J: FOCUS GROUP TRANSCRIPTS

Focus group interview

Lesson 1 (variables)

1. **T**: We have known variables and we have explained it. Did this lesson taught you, have this lesson taught you something that perhaps you did not understand, or did it make any change or no change? … anyone is free, just raise and tell me what you think. For example , yes, it did, this way and this and that … or no, it did not because of this and that … ja, let's just discuss. Ow yes (pointing at a learner who raised his hand)
2. **L1**: Yes, it made some changes, because we were only taught that variables represent things that we don't know like …
3. **T**: It made some change?
4. **L1**: Jaaa … because we were just told that we use variables or letters to represent numbers that we do not know, ja, unknown numbers.
5. **T**: So, because, initially you were only taught that variables represent unknown numbers?
6. **L1**: Yeah
7. **T**: Uhm, ok. So, what do you know now about variables? What is the new thing that you have just learnt now? Because you said yes, the lesson made a difference, so what's that difference? Yes…
8. **L2**: I now know that variables represent a quantity of a certain measure.
9. **T**: Do we all have the same understanding now?
10. **Ls**: Yes
11. **T**: It represents a certain quantity of a certain measure? So that's what you know now?
12. **Ls**: Yees
13. **T**: Ok, now, so meaning in this lesson you have come to know more about variables, is that so?
14. **Ls**: Yes
15. **T**: And now you have discovered that it represents a quantity of a certain measure. And what is a quantity again?
16. **Ls**: (randomly) A quantity refers to how many or how much.
17. **T**: (repeats after learners) – a quantity refers to how many or how much. Very good. Is there anyone who has another contribution to relate what we have known before and what we just learnt today? Maybe something new that you have learnt, something that was not clear to you or something or something that you cannot agree with perhaps. Yes (points to a learner).
18. **L3**: Since we were not given numbers in the first and second questions, how did you, like when it comes in examination, can you just guess the number the way we did or how?
19. **T**: Alright, here do not be worried much about examination. What we are trying to do is to understand the meaning of the variable, you understand that?
20. **Ls**: Hmm, yes.
21. **T**: Now of cause if you are worrying about examination, worry about something like this, you can be given a question like that in words but you should know, when you are working with those variables to find the quantity of the other measure, of course you should know that the variable you are using represents a number, and that number is a quantity. So, if you come

across some of the questions like examination questions, they would come in words and they would require you to write something like this and of course they will also ask you to find either this or that, giving you one of the two. If I give you this, find that, if I give that, find this, the same thing that we did here. But the first question here, what we did here, it was more of making us to understand a variable as representing a quantity. 100, is a quantity, isn't? we talked about Dollars here, it is a measure and we tried to find the quantity of that measure. Our measure is dollar, we are talking about the cost in dollars. So, how many of those dollars, that is the quantity we are talking about.

22. **L1**: Ooo, so I see now. Ok … good.
23. **T**: You got it right?
24. **L1**: Yes.
25. **T**: So, we are saying, variables do not only represent objects, or names of objects but can also represent quantities (how many or how much) of certain measures. Is there any other question?
26. **T**: Good, now we have learned more about variables, and this was our first lesson and we had to start with variables and the next thing we will look at something connected to variables again. You might be finding this hard but as we move on, it will become more exciting to learn about.
27. **T**: Thank you very much, you may enjoy the rest of your day.

Lesson 3 (Combining terms)

1. **T**: What did you find interesting in today's lesson?
2. **L8**: Making the letters as the name of animals/objects.
3. **T**: Really? What did that teach you?
4. **L8**: It teach me that I cannot combine different things, like animals I cannot combine cats with dogs.
5. **T**: Really?
6. **Ls**: Yes
7. **T**: Ok, anyone else?
8. **L1**: Umm, ah, using diagrams is not the same as using numbers and letters. It is more interesting. Is like you are playing a game, you can see easily the difference when using diagrams.
9. **T**: Wow, ok that's good to know. So, what do you want to say?
10. **L3**: Yes, they help, but it takes too much time to draw the diagrams.
11. **T**: It takes time hey?
12. **Ls**: Yes, very much.
13. **T**: But then what did the diagrams teach you?
14. **L1**: Not to combine things that are not the same.
15. **T**: So, it indicated to you clearly what things are not combined?
16. **Ls**: Yes.
17. **T**: Apart from time consuming when using diagrams, what other challenge did you face?
18. **L1**: When you use letters, sometimes you end up combining them wrongly. You just add up the numbers without looking at the letters.
19. **T**: So that's the other challenge, right?
20. **Ls**: Yes.
21. **T**: So, good. We meet next time.

Lesson 4 (negation)

1. **T**: Yes, can we have a brief reflection of our lesson of today.
2. **T**: Tell me guys, how did you find today's lesson? Was there anything interesting in this lesson? What is that that was interesting if there was? If you picked up something, just raise and tell me. Yes, tell us.
3. **L1**: Umm, I found out that when using diagrams, we add the negative numbers even the is no addition sign.
4. **T**: So, what you found interesting was, even if things seem to be subtracting, in actual fact we are adding them? So that is what you found interesting. Just like this and say ohoo, things look like we are subtracting but to understand them, let's add them for us to avoid making this mistake and get wrong answers, like you told me that -2 – 1 = -1. Ok. So, is there anyone else who has discovered something interesting apart from this?
5. **T**: Do you think that working with diagrams helped you understand these types of activities, adding and subtracting terms? Do you think diagrams have helped you in a way? Just tell me, there is no harm in telling me.
6. **L1**: Yes, diagrams, umm, me no, because when you are dealing with negatives numbers when it comes to diagrams is confusing but with numbers you just do it straight but with diagrams, no.
7. **T**: So, with diagrams, they are not really helping when you are working with negatives?
8. **L1**: Not at all.
9. **T**: So, it doesn't, to you.
10. **L1**: Yes, to me, yes.
11. **T**: Alright, anyone else? What challenges did you encounter today in our lesson?
12. **L2**: Dealing with negatives and diagrams.
13. **T**: Dealing with negatives and diagrams?
14. **L2**: Yeah.
15. **T**: How was that a challenge to you?
16. **L2**: Like it is difficult to add two negative terms or subtract them. For example, when you have -2a + a, with diagrams you sometimes just give -3a. But with a calculator, you get –a.
17. **T**: So, you couldn't really understand it well?
18. **L2**: No.
19. **T**: Ok, I hope as time goes, we will get to understand all these things.

Lesson 5 (Simplifying expressions)

1. **T**: Ok, let's have our short review of our today's lesson.
2. **T**: Did you find something interesting today?
3. **Ls**: Yes.
4. **T**: Tell me what you found interesting today?
5. **L4**: The diagrams.
6. **T**: The diagrams?
7. **L4**: Yeah.
8. **T**: What about them?
9. **L4**: When you use diagrams, you get a good answer, is not like when umm when you use letters or symbols.
10. **T**: So, when you use the diagrams, you always get the correct one.
11. **Ls**: Yeees … sure
12. **T**: So, it is not the same when you use symbols?
13. **Ls**: No, is not the same.

14. **T**: Is that case with everyone or perhaps there is anyone who has a different opinion?
15. **L1**: What if I get a different answer when I use symbols and a different answer when I use diagrams? How can you tell which one is correct and which one is not correct?
16. **T**: Yes, he has a question, to say, he gets a different answer here and a different one here. How can he tell which one is correct? That is a question that he is asking. Right, what you need to do, go back to your working both in symbols and in diagrams. Have a clear sheet, go through and see whether you can still get the same answers.
17. **T**: For you, which one did you find to be easy?
18. **L1**: The diagrams were easy, not like the symbols.
19. **T**: Ok, how many of you have made mistakes when using symbols than when using diagrams?
20. **Ls**: (raise their hands)
21. **T**: So almost everyone is saying the same thing? Meaning you get the correct answers when you use diagrams?
22. **Ls**: Yes
23. **T**: What was the challenges that you encounter in today's lesson? What challenges did you face just in this lesson? Are you telling me you did not meet any challenge in this lesson?
24. **Ls**: No…
25. **T**: So, everything was fine with everybody?
26. **Ls**: Yes.
27. **L7**: But sir, I did not understand when you use those examples of taking money from the bank.
28. **T**: Ow, you mean when we tried to make meaning of expression?
29. **L7**: Yes.
30. **L4**: I think I don't understand also.
31. **T**: Ok, so who understood?
32. **L2**: I think I did. I think it was just like to tell us that expressions can be used everywhere. Like positive term is like you deposit, negative term, you withdraw.
33. **L1**: Yeah, sir, and different variables is like we are saying the money is in different currencies.
34. **T**: Good. Did you get it?
35. **L4**: I think I get it.
36. **L7**: Somehow ngaa … but it is fine.
37. **T**: Ok. So, if everything was fine then, that's fine. What you suppose to do, when you are trying to work out on your activities, just try to be careful and make sure you know what you are doing.
38. **T**: Otherwise, thanks very much for you time, we will meet again for the next lesson.

Lesson 6 (Expanding Brackets)

1. **T**: Let's use these last minutes to reflect on our lesson. The same questions as always. What did you learn from this lesson of today? Did you learn something new, something exciting or whatever? Let's share our experience. Starting from the expressions, I gave you the other activities isn't't? we are not only saying you give the positive views, but also the negatives.
2. **L3**: When you are dealing with symbols, no I mean when you are dealing with diagrams, without solving the, without hmmm, solving the expression, I mean when you are doing the expression and you are doing it in symbols no, I mean in diagrams first, without solving it into symbols, is much, is not, is very hard because it takes time and you might make mistakes

because just calculating without knowing is this right or wrong because yeah you are not sure. Yeah…

3. **T**: So, meaning when you are trying to expand an expression just using diagrams without symbols it wastes time isn't it?
4. **Ls**: Yes, it does.
5. **T**: And it is a bit hard. Is that what you trying to say?
6. **L3**: Yes sir, it is difficult.
7. **T**: Is there anyone of you who have enjoyed using diagrams to expand expressions? Because you may have different opinions here. Yes, you did?
8. **L1**: Ahm, ok. Only when you have something to compare your answer with. Like you do it in both symbols and diagrams.
9. **T**: So, what do you think needs to done? Do you think you need to work with both perhaps and confirm your answers or, what is that that you think needs to be done?
10. **Ls**: Both, Both… Yeah both is better.
11. **T**: So, you prefer working with both?
12. **Ls**: Yeees…
13. **T**: Even like in class you should be taught like ok, let's use a diagram here and symbols here to compare our answers. Is that what you are saying?
14. **Ls**: Yes.
15. **T**: Anyone else?
16. **L6**: I enjoy using the expansion box. It helped me to see the number that is multiplied by the numbers inside the brackets.
17. **T**: You mean the scale factor?
18. **L6**: Yes, and also, it is clear, you can see that the factor is multiplied with all the numbers inside, but when you are using … umm … letters, I mean symbols only, sometimes you just times with the first number in the brackets.
19. **T**: So, you mean the use of the box really help you to see everything?
20. **L6**: Yes sir.
21. **L3**: Is true, me I … umm … I used to think that you put together the numbers inside the brackets and times nee ando … Now I see the box is clear.
22. **T**: That's good to hear. Yes, did the diagrams also help you?
23. **L2**: Yes, by checking our answers in symbols if it is correct.
24. **T**: To check if your answer is correct? Where is the technicality in the … haha. Ok, is there something that you usually make mistake about or something?
25. **L3**: Eee, yes.
26. **T**: What is that?
27. **L5**: When you are multiplying negative and negative,
28. **L7**: With symbols is different, with diagrams is much easier…
29. **L5**: Yes, much easier because you will be separating the signs one by one in the small boxes.
30. **T**: So, you mean there is that problem that you normally make … to say when you are using diagrams, multiplying negative numbers, diagrams help you not to make that mistake?
31. **Ls**: Yes.
32. **T**: But with symbols you end up making mistakes?
33. **Ls**: Yes
34. **T**: So that is what you are saying?

35. **Ls**: Yes
36. **T**: So, it is better that we use both symbols and diagrams … good.
37. **T**: So, this is the end of our lesson today. We meet on Monday.

Lesson 8 (Solving Equations)

1. **T**: Do we now get to know what equations are?
2. **Ls**: Yes
3. **T**: Are we now getting the real way of how to solve equations? Not just to be told when it comes that side it becomes negative or positive…
4. **Ls**: yes.
5. **T**: Is it? Are we now getting to know how we end up subtracting or adding on the other side of the equal sign?
6. **Ls**: Yeees.
7. **T**: In fact, what was so interesting in this lesson? Or what was the challenge that you encountered here? Share your views.
8. **L3**: Me I did not really understand how to balance, it is just confusing.
9. **T**: How confusing?
10. **L3**: I just don't know when to add or to subtract.
11. **Ls**: (some) me too.
12. **L2**: For me, what I found interesting was that you can use the equations in real life and I did not know that.
13. **T**: So, now you understand that equations are used in real life?
14. **Ls**: Yes
15. **T**: Challenge? Yes…
16. **L3**: When solving equations with brackets, symbols are a bit challenging when it comes to signs. Sometimes you confuse like you end up putting a negative instead of positive. But when you are using the expansion box, it is easy and help you not to confuse the signs.
17. **T**: So, the expansion box helps you not to make mistakes with the signs?
18. **L3**: Yes... because even when you are just using diagrams without expansion box, you can make mistakes also.
19. **T**: Ok, so in terms of solving these types of equations, what will you recommend to a friend?
20. **Ls**: Oow, expansion box of course… (some)it is the answer to our mistakes. (some) it is easy.
21. **T**: Ok, that is good. So, this is the end of our lesson. Enjoy your day.

APPENDIX K: INDIVIDUAL INTERVIEW TRANSCRIPTS

Individual interview

Learner 6

1. **T**: Good afternoon
2. **L**: Good afternoon sir
3. **T**: Welcome to our interview session, feel free to answer the questions that I will ask you.
4. **L**: Ok sir
5. **T**: Alright, as you have seen we have been doing all these activities from the beginning. The thing I want you to tell me is your experience about the programme.
6. **L**: Hmm, ok
7. **T**: Ok, my first question is, do you think the programme has helped you in developing a meaningful understanding of algebra? if yes, how and if no, what was the problem?
8. **L**: Aah, I think it helped me, aah, is like when we were taught before in class, is like we didn't really understand what is going on, we only taught to find the answer for you to get marks. But now I can use algebra in real life, like you can use it when you are at home to calculate your own things not getting any mark.
9. **T**: So, you mean you now you have developed understanding that algebra can be used in our daily lives?
10. **L:** Yeah
11. **T**: Alright the next question is, what was your experience in relating symbols to diagrams to model the concepts? Meaning in other words, I am saying what did you find, was it difficult for you to relate symbols to diagrams or you found it easy?
12. **L**: Umm, actually it was kind of easy but yes, it is easy, you know diagrams and symbols are different things. So, when you are using diagrams you are more likely to get the correct answer than when you are using symbols. Diagrams are easy to differentiate, but symbols or numbers or letters are not easy to differentiate. Like a square and a circle, they can't be mixed or put together when you are adding.
13. **T**: You mean they can't be combined?
14. **L**: Yes, you can't combine them. Yes, it will be difficult just to combined different diagrams, not like the letters. Letters you can confuse and just end up combining them or putting them together without knowing.
15. **T**: Now, how do you compare your understanding of algebra after the program and before? Meaning how do you rate your understanding of algebra that you had before the programme and the understanding you have now after the programme?
16. **L**: Ooh, ok. Aahm, like before I got this programme, I only know the method, the method to get the correct answer. It doesn't matter whether you will get the correct answer or not because you can't prove that. But now with the diagrams you can now prove it by doing it both in symbols and diagrams and you can even use it in examination if there is enough time.
17. **T**: So, meaning are you saying your understanding of algebra is better …….?
18. **L**: Yeah yeah, it is better than before. Sure
19. **T**: Or the programme has confused you or something?

20. **L**: No no, not at all. The programme it make it easier to get the correct answer. Like before, you only, you really not sure whether this is the correct answer or not, you only wait for the teacher to mark and give you a tick or wrong answer.
21. **T**: So, meaning your understanding now is better than the one you had before?
22. **L**: Of cause yes, it is better.
23. **T**: Ok, thank you very much
24. **L**: You are welcome
25. **T**: You may have a good day

Learner 8

1. **T**: Good afternoon
2. **L**: Good afternoon sir
3. **T**: Umm … we have come to the end of our programme, as we started way long back and only today that we are finishing.
4. **L**: Ok
5. **T**: Ok, I believe you will be able to remember and you have gained an experience from the programme. So, I will just ask you of the experience that you have gained during the programme by asking you the following questions, just to share your experience with me also.
6. **L**: Ok sir
7. **T**: Do you think the programme has helped you in developing a meaningful understanding of algebra? If yes, how did it help you and if no, maybe why are you saying no? something like that.
8. **L**: Umm … may you please repeat again
9. **T**: Do you think the programme has helped you to understand algebra?
10. **L**: Ooh yes, it did, because I never knew I never knew algebra will be like that easy as such, maybe I, like I thought algebra is just like for example 2 times a number of…. Ya, but now I know that algebra is anything that we do in everyday life and I also know and now and now, I experienced like now I know how to do a certain equation or expression without using a calculator instead I can use diagrams.
11. **T**: Hmm, so so, meaning the programme has helped you to know how to do some calculations without using a calculator?
12. **L**: Yes
13. **T**: And you can do that by using symbols or diagrams……
14. **L**: Yeah, both yeah ... symbols and diagrams. But sometimes diagrams are hard, but symbols I know take, aah, ok. Diagrams take time but they are very easy to get a correct answer that quickly except like diagrams, I mean symbols can make you, can make you … umm … confused sometimes.
15. **T**: So sometimes you mean symbols can confuse you?
16. **L**: Yes
17. **T**: But you think when you are using diagrams then it becomes more easy for you?
18. **L**: Yes
19. **T**: Alright, aah, good. The next question is, what was your experience in relating symbols to diagrams to model the concepts? To say how did you find it to compare diagrams, sorry not to compare but to relate diagrams to symbols? Like you have symbols here, and you have diagrams here. Did you find it hard, a challenge, was it easy or something? Just your experience that I am asking for.

20. **L**: Ok, about symbols, it is a challenge when you dividing, wait no, when no, when there are 2 like when there is subtraction, but in diagrams, you can, you can get the answer even when there is a subtraction sign you can get it just using diagrams. But diagrams take time to get, like take time to. diagrams take time for you to finish but symbols take aahm… just little bit of time but sometimes challenging again just as diagrams. Well, diagrams are challenging, but they can help you in a lot of ways.
21. **T**: So, you mean diagrams they can help you, but they can be more challenging?
22. **L**: Yeah... sure
23. **T**: So, meaning relating diagrams to symbols like you said, for example if you have x then I use what diagram so it wasn't difficult to find which diagram to use for which symbol? So, it wasn't that difficult?
24. **L**: Yes
25. **T**: Aah, ok now. How do you compare your understanding algebra now and your understanding before the programme? Do you think you have now understood these concepts of algebra a bit better than before or there is no change or now it has even confused you?
26. **L**: Well, at first it confused me, but now like at first when we use it in class, it's confusing. But when I start using new methods and techniques now, I get that. Now, I understand it more better than before.
27. **T**: So, the programme has helped you to get more understanding of these concepts of algebra?
28. **L**: Yeah , yes.
29. **T**: So, you mean now you understand better than before?
30. **L**: Yes sir.
31. **T**: Ok, so would you recommend it perhaps for some other learners to try it or maybe not?
32. **L**: Yes, ooh, aah, more especially for the learners who do not understand algebra, that's the technique I would recommend for them.
33. **T**: So, you can recommend it for those who struggle with algebra?
34. **L**: Yes
35. **T**: Thank you very much, this was just a short time interview, and thank you. Have a good day.

APPENDIX L: JOURNALS

Lesson 1

All learners were present and arrived on time for the lesson. It was observed that the participation of the learners was quite minimal to most of the learners as only few of them were free to participate explicitly during the lesson. Learners were at the same time observed to be willing to learn as they all looked listening both to the teacher and to other learners as they interacted among themselves.

The lesson was focusing on variables and some learners gave their experiences of what they know about variables. As said earlier, not all learners were fully participating in terms of giving their views and answering to the teacher's questions. This implies that the views shared during the class discussion were mostly coming from the few participating learners.

Learners were required to do some class activities in their groups and shared their solutions with the whole class. It was observed that the learners who were participating sharing their views during the lesson, were the same learners dominating in their group discussions.

Lesson 2

All the learners were recorded present to the lesson and they all arrived on time to the lesson. The participation of the learners during the lesson was still observed low with no difference to the participation from the previous lesson. The lesson was still dominated by the same learners who dominated in lesson 1.

The lesson looked first at the homework that was given to the learners from lesson 1. The correction was done on the board by the teacher with the involvement of the learners giving their views and what they have done. Some learners looked so tired and were not seen contributing to the learning. Although some learners have answered the homework questions correctly, they were still struggling to show the understanding of variables as representing quantities.

This lesson did not go up to the end because the learners were called to go attend a meeting with some health professionals from the department of health who came to conduct an awareness.

Lesson 3

All learners arrived on time and the lesson commenced as planned. The participation of the learners was observed to be improving as compared to the first two lessons. It appeared that most of the learners were gaining confidence during the lessons. Almost every learner was observed to be contributing to the learning more especially during group discussions. However, learners who were active from lesson 1 were still observed dominating during the group discussions in this lesson.

The focus of this lesson was more on identifying like terms from a list consisting of like and unlike terms. They started working with symbols. When the learners were required to combine terms, most of the learners demonstrated the understanding of like and unlike terms.

The introduction of using diagrams to represent terms appeared to have brought fun in the lesson. Learners were seen excited as they were using diagrams. Some were heard as saying it is like a game. The learners were observed active during their discussions in their groups as they were working with the diagrams.

Lesson 4

All learners were present and they all arrived on time to the lesson. The focus of the lesson was on negation and the participation of the learners was satisfactory though it was observed that they had some difficulties with the calculations involving negative terms. Learners used diagrams as well to simplify the expressions including the negative sign. it appeared that the diagrams did not help as such, as learners seemed to experience the same problems in symbols.

Lesson 5

All the learners were present in this lesson. The arrival was fine also though 2 learners joined the class about 5 minutes later, but they had a reason that they were called by their class teacher. The participation of the learners was good and learners were observed active. In this lesson, the focus was on simplifying algebraic expressions by adding and subtracting. Learners did some activities in their groups.

The use of diagrams appeared to have activated the learners' interests as learners were observed to be more active, arguing with reference to the use of diagrams. Learners presented their solutions of the activities they did in their group to the whole class. Learners commented to their solutions, with some asking for clarity as they seemed unsure for some of the solutions.

Lesson 6

For this lesson, only 5 learners arrived on time and the other 2 learners were delay due to the test that they were required to write in other subject. I only got to know about the test when I went to find the learners' whereabouts. The 2 learners joined the lesson about 10 minutes later after finishing writing their test. One learners did not attend this lesson as he was away with the school debate club.

It was observed that the two learners were reluctant to attend the lesson as the I had to go back to get them from their respective classroom to come for the lesson despite the fact that they were informed to join the class soon after they finish writing the test.

This lesson was on expanding brackets. it was in the same lesson when I introduced the use of an expansion box to expand brackets. Most of the learners were observed to be excited to use an expansion box. The participation of the learners was very good in this lesson. Learners were observed active when they were doing some group activities, with some arguments observed from the groups.

Lesson 7

The beginning of this lesson was disrupted by the change of venue as the usual venue was occupied by the English Debate Club. Only half of the learners arrived on time and the rest joined later. I had to go look for these learners. I found some of them wandering around the school and the other 2 I found them in their classrooms. Some indicated that they were still coming to join the class when I asked them why they have not gone for the lesson. Some just remained quite without giving any reason.

The participation of the learners was satisfactory as learners were heard interacting during their group discussions. The lesson started with the previous topic of expanding brackets and towards the lesson, learners were introduced to solving equations. In the first example, the learners were asked to develop meaning of the equation by giving an examples that could be represented by the equation given. It was difficult for the learners to present the examples suit for the equation.

Lesson 8

Six of the learners arrived well on time and the other two joined about 15 minutes later as they indicated that they were at the garden with the teacher for practical assessment. Learners looked happy and some of them were heard saying, 'iyaloo shilii, today is the last day' [meaning, thanks very much]. Learners' interaction was not really good as usual but, however, they were heard debating during their group discussions.

It appeared that most of the learners were not getting the method I introduced to them to solve equations. They were heard insisting that they want to use the old method, referring to the transfer method that they had learned earlier.