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Abstract

We consider a newsvendor who earns a revenue from the sales of her product to end users as well as from multiple advertisers paying to obtain access to those end users. We study the optimal decisions of a price-taking and a price-setting newsvendor when the advertisers have private information about their willingness to pay. We focus on the impact of the number of advertisers on the newsvendor's optimal decisions. We find that regardless of the number of advertisers, the newsvendor may exclude advertisers with a low willingness to pay and distort the price and inventory from their system-efficient levels to screen the advertisers. Moreover, the newsvendor's decision to exclude an advertiser is based exclusively on that advertiser's characteristics, and the newsvendor's optimal decision thus reveals independence among the advertisers. Nonetheless, the profits of the newsvendor and the advertisers also display network effects as both increase in the number of advertisers. Finally, our numerical results show that the newsvendor prefers an equivalent single advertiser to multiple advertisers due to the pooling effect.

Key words: newsvendor model; inventory; pricing; mechanism design; revenue management

1. Introduction

In some industries, a seller collects revenue not only from her product sales, but also from selling access to the product's end users to third parties. This revenue pattern exists in the publishing industry, where advertisers pay newspapers and magazines for the right to advertise within their publications. This secondary revenue can represent a large share of a seller's revenue. Advertising income, for instance, made up between 19% and 83% of the total revenue of top selling magazines in 2007, including National Geographic, The Economist, and Newsweek (Magazine Publishers of America, 2008). Newspapers similarly derive a large percentage of income from advertising. For example, in 2009, The New York Times reported revenues of \$683 million from circulation and \$797 million from advertising (The New York Times Company, 2009).

Our paper looks at this business setting in which a newsvendor obtains revenue from multiple advertisers. A similar model was introduced by Wu et al. (2011) who formulate a newsvendor problem in which a second source of revenue from a single advertiser is considered. Magazines and newspapers, however, carry many advertisements. In fact, the number of pages of a magazine issue can vary with its popularity with advertisers, the September issue of major fashion magazines being a well-known example. Indeed, fashion houses want to run their advertisements to coincide with the launch of their new collections at the major fashion shows held in September. Publishers have even started to report the number of ad pages in their September issues, with Vogue well in front with the 532 pages of advertisements in their September 2010 issue (Peters, 2010).

We study how the multiplicity of advertisers affects the newsvendor's decisions. Besides operational decisions such as product pricing, inventory (also referred to as production quantity), and advertising rate, Wu et al. (2011) also showed that the newsvendor's optimal policy may exclude the advertiser if his contribution to the newsvendor is insufficient. In addition, in their exogenous price model, the newsvendor may set an even higher threshold on the advertiser's contribution for the newsvendor to produce anything. With multiple advertisers, however, it is conceivable that the decisions regarding an advertiser's exclusion might be influenced by the presence of other advertisers, even when we restrict the model to the case where there is no competition between advertisers. Note that this assumption of low or no competition for access is reasonable for magazines that can expand their advertising linage by increasing the number of pages.

We consider the exogenous and endogenous price models and characterize the optimal decisions of the newsvendor (she), taking into account the fact that each advertiser (he) has private information about his willingness to pay (or benefit type) for advertisements. To fully understand the impact of multiple advertisers, we set up a model in which the newsvendor chooses her product inventory (and price in the endogenous price model) to maximize her expected profit by charging multiple advertisers who desire to obtain access to the product's end users. The newsvendor sets the fee individually for each advertiser, taking into consideration the benefit the advertiser derives from each product sold, or benefit type. Within the principal-agent framework, we model the newsvendor as the principal, i.e., the leader, and the advertisers as the agents. An advertiser's benefit type is his private information, which is unknown to the newsvendor and other advertisers. Note that advertisers belong to two different categories: individuals or small advertisers placing classifieds or small ads at a pre-determined rate card price per line, and large corporate clients who run regular advertising campaigns with the newsvendor over half or even full pages. Our model focuses on the latter category of clients. Although the newsvendor also publishes a rate card for her corporate clients, it is common industry practice to negotiate individual discounts with each advertiser (Nicholson, 2001). Sass (2009) writes that "behind closed doors, many big publishers give substantial discounts to media buyers off their official rate cards" and reports that Time Inc.'s magazines gave discounts of 56% on average in 2009. Indeed, flexible pricing benefits the newsvendor as it enables her to gain value with an inclusive pricing paradigm that links the quantity produced and advertisement prices to extract maximum profit from her heterogenous advertisers.

Our work is closely related to the literature on the newsvendor problem and principal-agent models. The newsvendor problem originally emerged as a cost minimization problem. We refer the readers to Porteus (1990) and Khouja (1999) for excellent reviews of the early development of this problem and its various extensions. The extension most closely related to our paper is the introduction of endogenous pricing, which transforms the newsvendor's problem into a profit maximization problem. Petruzzi and Dada (1999) summarize the early work on the joint inventory and pricing problem. Ha (2001) characterizes the joint concavity of the objective function directly. To the best of our knowledge, little research has been done to explore the newsvendor's decision problem when her profit is enhanced by the opportunity to earn secondary revenue, and our work attempts to fill this gap. More recently, the newsvendor literature has been further expanded along several dimensions. Granot and Yin (2007), Özer et al. (2007), and Liu and Özer (2010) consider the interaction of parties in a supply chain. Keren and Pliskin (2006) and Sévi (2010) deal with a risk-averse newsvendor. Qin et al. (2011) provide an up-to-date survey of the newsvendor literature since Khouja (1999).

The principal-agent setting has been extensively researched (see Hartmann-Wendels, 1993, for a review) and widely applied in the operations literature. Studies have investigated incentives in new product development projects (Mihm, 2010), issues in outsourcing (Ren and Zhou, 2008; Kaya and Özer, 2009; Özer and Raz, 2011), procurement and capacity contracting (Cachon and Zhang, 2006; Xu et al., 2010), disruptions and contracts in supply chains (Corbett et al., 2005; Iyer et al., 2005; Ahn et al., 2008), incentives and individual motivation (Radhakrishnan and Ronen, 1999; Forno and Merlone, 2010), and information sharing in a newsvendor setting (Özer and Wei, 2006; Wang et al., 2009; Gan et al., 2010).

Our newsvendor model with multiple advertisers presents two slightly atypical features. First, we choose to allow a cutoff policy, i.e., we dispense with the assumption common to the majority of the literature that all agent types participate in the contract. Moorthy (1984) argues that the cutoff policy is optimal based on numerical results. Corbett et al. (2004) assume a cutoff policy and then find the optimal cutoff level. Ha (2001) formally establishes the optimality of the cutoff policy by exploiting structural similarities between the complete information and asymmetric information cases. More recently, Lutze and Özer (2008) make use of the monotonicity of the agent's payoff function to prove the optimality of the cutoff policy when agent types are discrete. We consider continuous agent types and our approach complements that of Lutze and Özer (2008).

Second, we allow multiple agents to participate in the same contract. Demski and Sappington (1984) discuss contracts with multiple agents, and reveal the importance of the correlation among advertisers. Correlation allows the principal to infer information from one agent's type about the other agents' types. In the absence of correlation, i.e., the agents' types are independent of each other, the authors find that each agent should be treated as an individual. However, uncorrelated agents might nonetheless be linked by a reward function based on a variable which is common to all agents (McAfee and McMillan, 1991; Li and Balachandran, 1997). In a setting where the principal's revenue depends on the common output of several agents, these studies show that even uncorrelated agents cannot be treated independently. When a contract is written on joint output, each individual agent's revenue will be influenced by the other agents' efforts, thus linking their effort levels. Similarly, in our setting, the sales of the newsvendor's product to end users will be common to all advertisers, even though it is possible to charge a different fee to each advertiser. Our paper, however, differs in that the newsvendor (principal) collects revenue from two sources, her own product sales and the advertisers' (agents) fees.

Our results and contributions are summarized as follows. First, we show that each advertiser is considered for inclusion in the contract based on his individual contribution to the newsvendor, and the advertisers are independent in that sense. An advertiser is cut off based exclusively on his net contribution to the newsvendor, and this cutoff level is determined in the same way irrespective of the number of advertisers. Second, we find that there is interdependence among the advertisers when they are included in the contract. In the exogenous price case, for an unprofitable product, the newsvendor determines a break-even boundary that the advertisers' aggregate contribution must exceed to make production and sales profitable for the newsvendor. Therefore, the contribution of one advertiser affects whether the newsvendor will produce and thus whether there will be a real contract (a contract with positive inventory) for all the advertisers. More generally speaking, in both the endogenous and exogenous price cases, we find that there exist network effects and that the advertisers benefit from the presence of additional advertisers. Indeed, for each advertiser that is added, the newsvendor's inventory will increase, which benefits all the advertisers in the relationship. Third, we find that the newsvendor's pricing power may benefit the newsvendor, the advertisers and the consumers, contrary to the expectation that it would only favor the newsvendor at the expense of the advertisers and consumers. Finally, our numerical results show that the newsvendor prefers an equivalent single advertiser to multiple advertisers due to the pooling effect.

The remainder of the paper is organized as follows. Section 2 describes the model framework. Sections 3 and 4 examine the exogenous and endogenous price models, respectively. Section 5 presents a numerical analysis to complement our analytical results. The paper concludes in Section 6. All proofs in the paper are provided in the Appendix.

2. Model Framework

The notation we use is summarized below. In our model formulation and analysis, the subscripts and arguments of functions are omitted for brevity, unless this results in confusion. Variables with upper case superscripts indicate optimal solutions.

Parameters:

p – unit selling price of the newsvendor's product (a decision variable in the endogenous price model)

c – unit cost of the newsvendor's product (note that we do not require that $c < p$)

X – random demand for the newsvendor's product, nonnegative

$f(x)$ – probability density function (pdf) of demand, positive

$F(x)$ – cumulative distribution function (cdf) of demand

n – number of advertisers, index $i \in \{1, 2, ..., n\}$, $-i$ refers to all but the ith advertiser

θ_i – advertiser i's benefit type, nonnegative, with $\mathbf{\Theta} = (\theta_1, \theta_2, ..., \theta_n)$

$\check{\theta}_i$ – advertiser i's self-reported benefit type, nonnegative, with $\check{\mathbf{\Theta}} = (\check{\theta}_1, \check{\theta}_2, ..., \check{\theta}_n)$

$\hat{\theta}_i$ – advertiser i's cutoff benefit type, nonnegative

θ_i^b – advertiser i's break-even benefit type, nonnegative

$[\underline{\theta}_i, \overline{\theta}_i]$ – support of θ_i and $\check{\theta}_i$, $\underline{\theta}_i \geq 0$

$\psi_i(\theta_i)$ – probability density function of θ_i

$\Psi_i(\theta_i)$ – cumulative distribution function of θ_i

Decisions of the newsvendor:

$t_i(\mathbf{\Theta})$ – transfer payment from the advertiser, contingent on his benefit type

$q(\mathbf{\Theta})$ – newsvendor's inventory decision, contingent on the advertisers' benefit types

Decision of the advertisers:

$\delta_i(\theta_i)$ – advertiser i's participation decision; 1 means participation and 0 otherwise.

As we are including the newsvendor's second source of revenue, the unit selling price p need not necessarily be bounded below by the unit cost c. This allows for the possible scenario where $p \leq c$, i.e., the newsvendor sells below cost, as exemplified by the recent proliferation of free newspapers. Our model considers the exogenous and endogenous price cases, i.e., a price-taking and price-setting newsvendor, respectively. We focus on the exogenous price model in the following description of our model framework for the convenience of exposition.

We define the expected sales $S(q) = E_X[\min(q,x)] = q - \int_0^q F(x)dx$, which can be verified as concave increasing in q. The newsvendor sells a product and receives a sales revenue $pS(q)$. The newsvendor's product, however, can yield a benefit to the advertisers and the newsvendor can thus charge each advertiser i a payment t_i for the right to access this benefit, e.g., for the right to place an advertisement in a magazine/newspaper. This creates a second source of revenue for the newsvendor who receives $\sum_{i=1}^{n} t_i$, the total payment from all advertisers.

An advertiser i's total benefit from accessing the newsvendor's product is proportional to his benefit type θ_i and the expected number of products sold $S(q)$. The type indicates the profit per consumer who has been exposed to the advertisement, whereas sales matter because the higher the circulation, the more consumers are aware of the advertiser's product. We define advertiser i's total benefit as $\theta_i S(q)$ (see Wu et al., 2011, for more detail). As the newsvendor does not know advertiser i's benefit type, she faces a typical principal-agent problem in which the uninformed principal (the newsvendor) offers a menu of contracts to informed agents. In such a setting, the newsvendor decides on the contract terms, the common inventory and the individual transfer payment for each advertiser, under information asymmetry.

The advertisers' benefit types θ_i are independently drawn from cumulative distributions $\Psi_i(\theta_i)$ (also called prior beliefs) on the support $[\underline{\theta}_i, \overline{\theta}_i]$. The realization of each advertiser's benefit type is only known to himself, and the distributions $\Psi_i(\theta_i)$ are the common prior beliefs of the newsvendor and the other advertisers (than advertiser i). These prior beliefs are assumed to be independent of each other because each advertiser is a different company with unrelated marketing campaigns and there is no competition for advertising space. Thus, each advertiser's type is unaffected by the other advertisers.

The sequence of events in our model is depicted in Figure 1. At time $T = 1$, each advertiser discovers his benefit type θ_i. Then, at $T = 2$, the newsvendor designs a menu of contracts $\{q(\mathbf{\Theta}), t_1(\mathbf{\Theta}), ..., t_n(\mathbf{\Theta})\}$ and announces $\{q(\mathbf{\Theta}), t_i(\mathbf{\Theta})\}$ to advertiser i. The advertisers decide whether to participate and simultaneously report a type $\check{\theta}_i \in [\underline{\theta}_i, \overline{\theta}_i]$ at $T = 3$. The newsvendor chooses the inventory based on the collectively reported $\check{\mathbf{\Theta}}$ at $T = 4$. At $T = 5$, the demand is realized and finally, at $T = 6$, the newsvendor collects the transfer payments from the advertisers.

The above timeline follows standard modeling assumptions for principal-agent models with multiple agents (see Chapter 7.4 of Fudenberg and Tirole, 1991). The menu of contracts $\{q(\mathbf{\Theta}), t_1(\mathbf{\Theta}), ..., t_n(\mathbf{\Theta})\}$ specifies an inventory $q(\mathbf{\Theta})$ together with an individual trans-

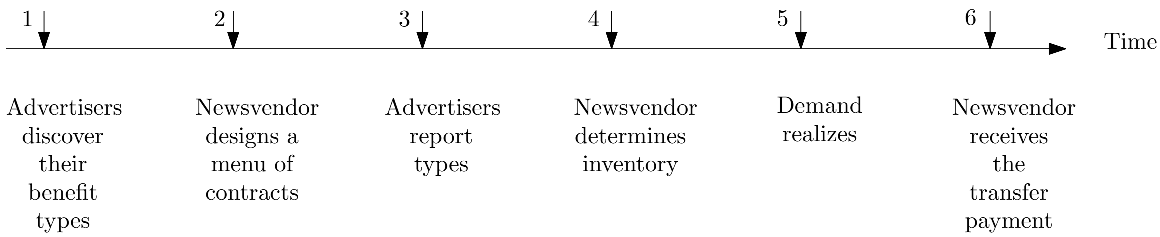


Figure 1: Timeline of Events

fer payment $t_i(\mathbf{\Theta})$ for each advertiser i, both determined for each possible combination of advertisers' types. The implementation of the contract merits some further discussion. The contracting game is based on the revelation principle, which states that there exists an optimal mechanism that induces truth telling. Accordingly, in negotiating the contract the principal may without loss of generality use a communication game with a direct mechanism such that the agent truthfully reports his type (Fudenberg and Tirole, 1991). Bester and Strausz (2000) confirm that the revelation principle is applicable to the multi-agent case and ensures that all agents simultaneously announce their types truthfully. In our problem context, the direct mechanism implies that advertiser i does not choose the exact inventory or transfer fee he will pay, as that will depend on the types of the other advertisers. Instead, he directly reports a type $\check{\theta}_i$. The advertisers simultaneously report their types, which jointly determine the inventory of the newsvendor and the transfer fee each advertiser has to pay.

3. The Exogenous Price Model

As frequent price adjustments are uncommon in the newspaper industry, we start by taking price as given and analyze how the second source of revenue affects the newsvendor's inventory decision.

The newsvendor could potentially choose from a wide range of contract designs, making this problem hard to solve. However, as discussed above, the direct revelation principle enables us to restrict our search to incentive compatible contracts, i.e., contracts that make it optimal for the advertisers to reveal their true benefit types $\mathbf{\Theta}$.

In anticipation of the advertisers' rational responses $\delta_i(\theta_i)$, the newsvendor designs a menu of contracts $\{q(\mathbf{\Theta}), t_1(\mathbf{\Theta}), \dots, t_n(\mathbf{\Theta})\}$ to maximize her expected profit as follows.

$$\max_{q(\mathbf{\Theta})\geq 0,\ t_i(\mathbf{\Theta})} E_{\mathbf{\Theta}} \left\{ \sum_{i=1}^{n} [\delta_i(\theta_i) t_i(\mathbf{\Theta})] + pS(q(\mathbf{\Theta})) - cq(\mathbf{\Theta}) \right\}, \tag{1}$$

s.t.

$$E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\mathbf{\Theta})) - t_i(\mathbf{\Theta})]\right\} \geq 0, \quad \forall i,\ \forall \theta_i, \tag{2}$$

$$E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\mathbf{\Theta})) - t_i(\mathbf{\Theta})]\right\} \geq E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})]\right\}, \quad \forall i,\ \forall \theta_i,\ \forall \check{\theta}_i. \tag{3}$$

In the above formulation, the expectation in the objective function is taken over all the possible types of all advertisers, as the types are unknown to the newsvendor when she designs the contract. Equations (2) and (3) are the individual rationality (IR) constraint and incentive compatibility (IC) constraint, respectively, commonly found in the standard mechanism design problem. The IR constraint ensures that the participating advertisers make at least their reservation profit, which is normalized to 0 without loss of generality. The IC constraint follows from the revelation principle. It guarantees that a participating advertiser achieves the highest expected profit by reporting his benefit type truthfully. It is worth noting that in the constraints for advertiser i, the expectation is taken over the possible types of the other advertisers $-i$. Thus, our formulation uses a Bayesian mechanism, where the constraints are required to hold only on average over the other advertisers' reports θ_{-i}. Also, this Bayesian formulation for each advertiser supposes that all other players report truthfully. See Chapter 7 of Fudenberg and Tirole (1991) for a more detailed discussion.

In what follows, we take the backward induction approach to solve this leader-follower game. We start by characterizing the advertisers' participation decisions $\delta_i(\theta_i)$ for a given menu $\{q(\mathbf{\Theta}), t_1(\mathbf{\Theta}), \ldots, t_n(\mathbf{\Theta})\}$ designed by the newsvendor, and then substitute these optimal participation decisions into the newsvendor's decision problem to find her optimal menu of contracts.

3.1 The Advertisers' Optimal Decisions

Note that the IR and IC constraints can be viewed as the advertisers' optimization problem. The IR constraint determines whether an advertiser participates in the game. If he does, then the IC constraint determines which benefit type is optimal for him to report.

Advertiser i with type θ_i will decide not to participate in the game if and only if he incurs an expected loss for all items in the menu of contracts offered by the newsvendor. That is, $\delta_i(\theta_i) = 0$ if and only if

$$E_{\theta_{-i}}[\theta_i S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})] < 0, \quad \forall i,\ \forall \check{\theta}_i. \tag{4}$$

The following lemma characterizes the advertisers' participation decision.

Lemma 1 *If advertiser i does not participate in the game when his benefit type is θ_i , then he will also not participate when his type is lower than θ_i.*

Lemma 1 implies that each advertiser's participation decision is of a threshold type: for any advertiser i, there exists a cutoff level $\hat{\theta}_i$ such that if $\theta_i \geq \hat{\theta}_i$, the advertiser will participate in the contract, otherwise he will not. That is, $\delta_i(\theta_i) = 1, \forall\theta_i \geq \hat{\theta}_i$; and 0 otherwise. The value of the cutoff level $\hat{\theta}_i$ will be determined by the newsvendor's optimal contract design. Our characterization of the optimal cutoff policy is similar in spirit to the approach taken by Lutze and Özer (2008), because we both make use of the monotonicity of the agent's payoff function in his type, despite different problem structures. More specifically, Proposition 5(b) of Lutze and Özer (2008) shows that the retailer's cost function is increasing in his type. As a result, if type i retailer is cut off (i.e., his cost reverts to the reservation level U_r^{max}), then type j ($j \geq i$) retailer will have an even higher (in a weak sense) cost and will also be cut off. Our characterization of the cutoff policy employs a similar logic.

In view of Lemma 1, we can now limit our attention to the case $\theta_i \geq \hat{\theta}_i$ (for all i), and replace $\delta_i(\theta_i)$ with 1 in advertiser i's optimization problem described by his IC constraint. The IC constraint essentially means that a type θ_i advertiser i maximizes his expected profit $G(\check{\theta}_i) = E_{\theta_{-i}}[\theta_i S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})]$ by choosing to report an appropriate $\check{\theta}_i$. If his optimal decision is to report θ_i, then the contract is truth revealing. The following lemma presents a condition for the advertisers to report their types truthfully.

Lemma 2 *An incentive compatible contract $\{q(\mathbf{\Theta}), t_1(\mathbf{\Theta}), \ldots, t_n(\mathbf{\Theta})\}$ must satisfy the following necessary condition:*

$$E_{\theta_{-i}}[t_i(\mathbf{\Theta})] = E_{\theta_{-i}}[\theta_i S(q(\mathbf{\Theta}))] - \int_{\hat{\theta}_i}^{\theta_i} E_{\theta_{-i}}[S(q(\tilde{\theta}_i, \theta_{-i}))] d\tilde{\theta}_i, \ \ \forall i, \ \forall\theta_i \geq \hat{\theta}_i. \tag{5}$$

Furthermore, if $q(\mathbf{\Theta})$ is monotonic, i.e., $dq(\mathbf{\Theta})/d\theta_i \geq 0$, $\forall i$, the above condition (5) is also sufficient for incentive compatibility.

This lemma gives the conditions that must be met for the optimal menu of contracts designed by the newsvendor to be an optimal, truth telling mechanism. After solving the newsvendor's optimization problem, we verify that the optimal transfer payment and inventory satisfy the above conditions.

3.2 The Newsvendor's Optimal Contract Menu

Based on Lemmas 1 and 2, we can substitute the expected transfer payments defined in Equation (5) into Equation (1) and then integrate by parts to reformulate the newsvendor's problem as follows (see proof of Theorem 1 in the Appendix for details).

$$\max_{q(\mathbf{\Theta})\geq 0,\ \hat{\mathbf{\Theta}}} E_{\mathbf{\Theta}} \left\{ \left(p(\mathbf{\Theta}) + \sum_{i=1}^{n} \left[\delta_i(\theta_i) V_i\left(\theta_i\right) \right] \right) S(q(\mathbf{\Theta})) - cq(\mathbf{\Theta}) \right\}, \tag{6}$$

where $V_i(\theta_i)$ is defined as follows.

Definition 1 *A virtual benefit type is* $V_i(\theta_i) = \theta_i - \frac{1-\Psi_i(\theta_i)}{\psi_i(\theta_i)},\ \forall i.$

Similar to the single advertiser case studied by Wu et al. (2011), the above definition of advertiser i's virtual type can be viewed as his net contribution to the newsvendor per unit of sales of the newsvendor's product. Indeed, in the absence of information asymmetry, the newsvendor would set advertiser i's fee at his commonly known benefit type θ_i per unit of sales. Information asymmetry, however, introduces an information cost $\left[1 - \Psi_i(\theta_i)\right]/\psi_i(\theta_i)$ as the newsvendor cannot charge advertiser i the full extent of his benefit type. Instead, the newsvendor has to provide an incentive to induce advertiser i to report his type truthfully. The virtual benefit type nets the information cost from the benefit type, and represents advertiser i's contribution per unit of sales under information asymmetry.

The above reformulation merits some discussion. First, the newsvendor has to determine the optimal cutoff levels for all advertisers, and an additional decision $\hat{\mathbf{\Theta}}$ is introduced into the newsvendor's problem. Once the optimal cutoff levels are determined, the advertisers' participation decisions $\delta_i(\theta_i)$ can then be replaced by either 0 or 1, depending on the value of θ_i compared to the cutoff level $\hat{\theta}_i$. Second, condition (5) is used to substitute the transfer payment decision out of the newsvendor's objective function without verifying whether it is sufficient at this point. Later, we will show that the optimal solution to $q(\mathbf{\Theta})$ is indeed monotonically non-decreasing, thus assuring the sufficiency of condition (5). Finally, the above reformulation indicates that the newsvendor will receive no transfer payment only if all the advertisers are simultaneously below their respective cutoff levels (i.e., $\delta_i(\theta_i) = 0$ for any i). In that case, her problem reduces to the traditional newsvendor problem.

To facilitate further discussion, we define the following notation.

Definition 2 *A virtual price is* $\hat{p}(\mathbf{\Theta}) = p + \sum_{i=1}^{n} V_i^+(\theta_i)$*, where* $V_i^+(\theta_i) = max(0, V_i(\theta_i))$.

This definition indicates that under information asymmetry, the role of the second source of revenue is to augment the selling price p by the advertisers' aggregate contribution to the newsvendor. The intuition behind the "+" operator in the definition is that all types with negative net contribution will be cut off, as will be formally shown in Theorem 1 below.

In the mechanism design literature, the increasing failure rate (IFR) assumption, namely, the assumption that $\psi_i(\theta_i)/\left[1 - \Psi_i(\theta_i)\right]$ is increasing in θ_i, $\forall i$, is usually made to ensure incentive compatibility of the optimal menu of contracts. Our problem requires a similar technical condition, as stated below. One can verify that it is weaker than the IFR assumption.

Assumption 1 $dV_i(\theta_i)/d\theta_i \geq 0$.

Let a decision with superscript A denote its optimal solution. We can now write the newsvendor's optimal menu of contracts in the following theorem.

Theorem 1 *Under Assumption 1, the optimal cutoff level* $\hat{\theta}_i^A$ *for each advertiser is such that* $V_i(\hat{\theta}_i^A) = 0$, $\forall i$, *and the optimal menu of contracts is:*

$$q^A(\mathbf{\Theta}) = F^{-1}\left[\left(\frac{\hat{p}(\mathbf{\Theta}) - c}{\hat{p}(\mathbf{\Theta})}\right)^+\right], \tag{7}$$

$$E_{\theta_{-i}}[t_i^A(\mathbf{\Theta})] = \begin{cases} E_{\theta_{-i}}[\theta_i S(q^A(\mathbf{\Theta}))] - \int_{\hat{\theta}_i^A}^{\theta_i} E_{\theta_{-i}}[S(q^A(\tilde{\theta}_i, \theta_{-i}))]d\tilde{\theta}_i, & \forall i, \forall \theta_i \geq \hat{\theta}_i^A, \\ E_{\theta_{-i}}[\hat{\theta}_i^A S(q^A(\hat{\theta}_i^A, \theta_{-i}))], & \forall i, \forall \theta_i \leq \hat{\theta}_i^A. \end{cases} \tag{8}$$

The newsvendor's decision to include an advertiser depends on the advertiser's virtual benefit type $V_i(\theta_i)$, as a measure of his net contribution to the newsvendor. The newsvendor only includes the advertisers that make a nonnegative net contribution in the game. If none of the advertisers merit inclusion in the newsvendor's contract, the newsvendor produces the optimal inventory without secondary revenue. This ensures that the optimal profit (inventory) with secondary revenue will never be less than the optimal profit (inventory) without secondary revenue.

The assumption that $V_i(\theta)$ is non-decreasing implies that if $V_i(\underline{\theta}_i) > 0$, then the equation $V_i(\hat{\theta}_i^A) = 0$ will have no root and the optimal cutoff level reduces to $\underline{\theta}_i, \forall i$. We assume that $V_i(\underline{\theta}_i) < 0$ to exclude that trivial case.

Theorem 1 specifies that the expected transfer payment from advertiser i should be set at $E_{\theta_{-i}}\left[\hat{\theta}_i^A S(q^A(\hat{\theta}_i^A, \theta_{-i}))\right]$ for all types of advertiser i below his cutoff level $\hat{\theta}_i^A$. However, multiple solutions to this problem exist, as the newsvendor can set any arbitrary transfer payment for those low types that is large enough to guarantee that advertiser i prefers not to participate in the game. The chosen value offers the convenience of continuity of $E_{\theta_{-i}}[t_i^A(\mathbf{\Theta})]$ at $\hat{\theta}_i^A$.

The optimal inventory decision $q^A(\mathbf{\Theta})$ specified in Equation (7) resembles that in the traditional newsvendor problem, with the virtual price $\hat{p}(\mathbf{\Theta})$ in place of the unit selling price p. The solution can be interpreted in terms of the newsvendor critical fractile, with the underage cost being $\hat{p}(\mathbf{\Theta}) - c$, and the overage cost being c. It follows from Assumption 1 that $q^A(\mathbf{\Theta})$ is increasing in θ_i, $\forall i$, thus validating the incentive compatibility of the optimal contract. The "+" operator in the solution is due to the fact that we do not require $p > c$, unlike the traditional newsvendor model. This generalization makes sense because in many cases, the newsvendor sells below cost when there exists a second source of revenue. In those cases (i.e., when $p < c$), if the pooled contribution from all advertisers is still insufficient to help the newsvendor break even, then the newsvendor is better off not producing anything at all. This observation leads us to define the break-even boundary such that $\sum_{i=1}^{n} V_i^{+}(\theta_i) = c - p > 0$, where the sum of net contributions of the participating advertisers covers the newsvendor's loss per unit. We define the individual break-even level θ_i^b, determined by $V_i(\theta_i^b) = c - p > 0$, as advertiser i's type for which the newsvendor breaks even when he is the only advertiser participating in the contract. Given Assumption 1, we know that $\theta_i^b > \hat{\theta}_i^A$.

Note that the newsvendor's problem with multiple advertisers reduces to a problem with a single advertiser if all other advertisers $-i$ are below their cutoff levels. The following corollary shows that the results in the multiple advertisers case can be reduced to those obtained in Wu et al. (2011) for the single advertiser case.

Corollary 1 *Under Assumption 1 and with only one advertiser (or $n = 1$), the optimal solution reduces to a cutoff level $\hat{\theta}_1^A$ such that $V_1(\hat{\theta}_1^A) = 0$ and the following optimal menu of contracts:*

$$q^A(\theta_1) = F^{-1}\left[\left(\frac{\hat{p}(\theta_1) - c}{\hat{p}(\theta_1)}\right)^{+}\right], \tag{9}$$

$$t_1^A(\theta_1) = \begin{cases} \theta_1 S(q^A(\theta_1)) - \int_{\hat{\theta}_1^A}^{\theta_1} S(q^A(\tilde{\theta}_1))d\tilde{\theta}_1, & \forall \theta_1 \geq \hat{\theta}_1^A, \\ \hat{\theta}_1^A S(q^A(\hat{\theta}_1^A)), & \forall \theta_1 \leq \hat{\theta}_1^A. \end{cases} \tag{10}$$

We would like to draw attention to the fact that the cutoff level is the same regardless of the number of advertisers. The structure of the optimal contract is similar, but now the expectation (over all other advertisers) operator for the transfer payment disappears. Finally, when $p < c$, the break-even boundary reduces to the individual break-even level defined above, i.e., θ_1^b, determined by $V_1(\theta_1^b) = c - p > 0$.

3.3 An Example

To illustrate the optimal menu of contracts analyzed above and its implementation, let us consider a numerical example with two advertisers. We assume a selling price $p = 2$ with a cost $c = 1$. The demand is uniformly distributed on $[0, 100]$, and benefit type distributions of advertisers 1 and 2 are uniform on $[10, 20]$ and $[20, 30]$ respectively. Note that both advertisers' virtual benefit types are nonnegative on the entire support, i.e., $V_i(\theta_i) = \theta_i -$

$[1 - \Psi_i(\theta_i)] / \psi_i(\theta_i) \geq 0$, for $i = 1, 2$; hence the optimal menu of contracts will not cut off any advertiser type.

We insert these parameters into the expected sales function, the virtual price definition, and the optimal contract terms defined in Theorem 1, which gives us the following equations:

$$S(q(\theta_1, \theta_2)) = q(\theta_1, \theta_2) - \frac{1}{200}[q(\theta_1, \theta_2)]^2,$$

$$\hat{p}(\theta_1, \theta_2) = 2\theta_1 + 2\theta_2 - 48,$$

$$q^A(\theta_1, \theta_2) = 100 - \frac{50}{\theta_1 + \theta_2 - 24},$$

$$E_{\theta_2}[t_1^A(\theta_1, \theta_2)] = 500 + \frac{5}{4}\left[\frac{\theta_1}{\theta_1 + 6} - \frac{\theta_1}{\theta_1 - 4} + \ln\frac{8(\theta_1 - 4)}{3(\theta_1 + 6)}\right], \tag{11}$$

and

$$E_{\theta_1}[t_2^A(\theta_1, \theta_2)] = 1000 + \frac{5}{4}\left[\frac{\theta_2}{14 - \theta_2} + \frac{\theta_2}{\theta_2 - 4} + \ln\frac{8(\theta_2 - 4)}{3(\theta_2 - 14)}\right]. \tag{12}$$

Let us briefly expand on the optimal expected transfer payment. Any $t_1^A(\theta_1, \theta_2)$, such that its expectation over θ_2 equals the right-hand side of Equation (11), induces advertiser 1 to report his type truthfully. Such $t_1^A(\theta_1, \theta_2)$ could be independent of θ_2 and equal to the right-hand side, or any function of θ_2 which preserves the equality, e.g., by adding a term $(\theta_2 - 25)$ whose expectation (over θ_2) is 0 to the right-hand side.

The menu of contracts is implemented as follows: each advertiser is only apprised of the minimum necessary information to make his truthful choice, namely the optimal production quantity function and his own transfer payment. This means that the newsvendor presents $\{q^A(\theta_1, \theta_2), t_1^A(\theta_1, \theta_2)\} = \left\{100 - \frac{50}{\theta_1+\theta_2-24},\ 500 + \frac{5}{4}\left[\frac{\theta_1}{\theta_1+6} - \frac{\theta_1}{\theta_1-4} + \ln\frac{8(\theta_1-4)}{3(\theta_1+6)}\right] + (\theta_2 - 25)\right\}$ and $\{q^A(\theta_1, \theta_2), t_2^A(\theta_1, \theta_2)\} = \left\{100 - \frac{50}{\theta_1+\theta_2-24},\ 1000 + \frac{5}{4}\left[\frac{\theta_2}{14-\theta_2} + \frac{\theta_2}{\theta_2-4} + \ln\frac{8(\theta_2-4)}{3(\theta_2-14)}\right] + (\theta_1 - 15)\right\}$ to advertisers 1 and 2 respectively. Note that the transfer payments from advertisers 1 and 2 are arbitrary functions that fulfill Equations (11) and (12), respectively.

3.4 Discussion of Modeling Results

Our analysis shows that while the advertisers are independent with regards to their cutoff levels, the newsvendor's inventory decision creates interdependence among them.

The advertisers are independent of each other in the sense that the optimal individual cutoff level is the same as in the single advertiser case, without being affected by the presence of other advertisers. In other words, each advertiser is judged according to his own contribution to the newsvendor on whether he should be induced to participate in the contract. Furthermore, the optimal contract implies that an advertiser's actual profit can be made independent of the ex-post realizations of the other advertisers' types.

Our results also reveal that the advertisers are interdependent. The advertisers complement each other because it is their aggregate virtual benefit that determines the newsvendor's optimal inventory specified in Equation (7). Hence, an increase in the number of potential advertisers is beneficial to the system. Our problem displays network effects, i.e., each advertiser is better off for the presence of other advertisers, as the optimal inventory $q^A(\mathbf{\Theta})$ increases and, consequently, the advertisers' expected profit (their expected informational rent) $\int_{\hat{\theta}_i^A}^{\theta_i} E_{\theta_{-i}}[S(q^A(\tilde{\theta}_i, \theta_{-i}))]d\tilde{\theta}_i$ also increases.

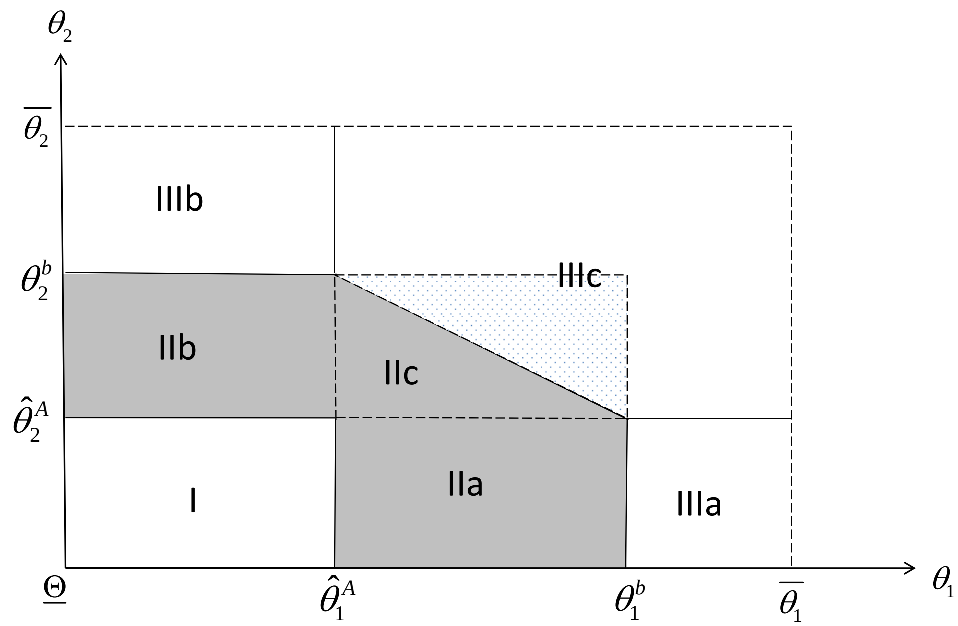


Figure 2: Participation Decisions of Two Advertisers when $p < c$

The network effects are most salient when the newsvendor sells below cost, i.e., when $p < c$. Figure 2 depicts the advertisers' participation decisions for the two-advertiser case. In region I, both advertisers are below their individual cutoff level, and the newsvendor prefers not to produce. Region II (the whole shaded area) is below the break-even boundary, but at least one advertiser exceeds his individual cutoff level. Nonetheless, the joint contribution of both advertisers to the newsvendor's profit is not sufficient to induce production, or $\hat{p}(\mathbf{\Theta}) < c$, and no transaction occurs. Finally, in region III, the newsvendor and at least one advertiser (in IIIa: advertiser 1, in IIIb: advertiser 2, and in IIIc: both advertisers) sign a contract with strictly positive inventory. It is interesting to note that due to network effects, in the dotted portion of region IIIc, even though both advertisers are individually below the break-even benefit level (θ_i^b), their pooled contribution to the newsvendor's profit is enough to encourage production.

In sum, we find that if the advertisers collectively exceed the break-even boundary, then they are independent of each other in the sense that advertiser i's participation decision and expected profit are unaffected by advertisers $-i$'s benefit type realizations. Nevertheless, there is still some interdependence among the advertisers, because the newsvendor's inventory decision depends on the aggregate contribution of all advertisers, and an increase in the number of advertisers raises the expected profit for each advertiser.

Moreover, an increase in the number of advertisers increases inventory $q^A(\boldsymbol{\Theta})$ and, thus, also benefits both the customers and the newsvendor. To elaborate, an increased inventory $q^A(\boldsymbol{\Theta})$ implies a higher in-stock probability, thus benefiting customers. The higher in-stock probability, in turn, means the newsvendor can sell her product to more customers. As a result, she can improve her primary revenue and her secondary revenue, as the more customers she serves, the larger the expected transfer payments from advertisers.

4. The Endogenous Price Model

In some settings, the newsvendor may be able to set the price of her product. In that case, we need to endogenize the price decision, and consider the contract design problem of the newsvendor when facing price-dependent demand.

Following Ru and Wang (2010) and Wu et al. (2011), we assume the demand to be $y(p)x$, where a price-independent noise x is scaled by a deterministic downward sloping function $y(p) = \xi e^{-\tau p}$, for positive scalar coefficient ξ and price sensitivity coefficient τ. We choose this particular demand function for two reasons. First, it has been shown to fit empirical data well (Hoch et al., 1995). Second, it allows for closed-form solutions and guarantees the monotonicity of the optimal price and inventory decisions (see Wu et al., 2011, for more discussion). Note that now the newsvendor can choose to make the price contingent on the advertisers' types $\boldsymbol{\Theta}$. Also note that the expected sales are dependent on both inventory and price, and are defined as $S(q(\boldsymbol{\Theta}), p(\boldsymbol{\Theta})) = \mathrm{E}_X[\min(q(\boldsymbol{\Theta}), y(p(\boldsymbol{\Theta}))x)]$. The newsvendor designs a menu of contracts $\{q(\boldsymbol{\Theta}), p(\boldsymbol{\Theta}), t_1(\boldsymbol{\Theta}), \dots, t_n(\boldsymbol{\Theta})\}$ to optimize her expected profit as follows, assuming the advertisers' rational responses $\delta_i(\theta_i)$.

$$\max_{q(\boldsymbol{\Theta})\geq 0,\ p(\boldsymbol{\Theta})\geq 0,\ t_i(\boldsymbol{\Theta})} E_{\boldsymbol{\Theta}} \left[\sum_{i=1}^{n} [\delta_i(\theta_i) t_i(\boldsymbol{\Theta})] + p(\boldsymbol{\Theta}) S(q(\boldsymbol{\Theta}), p(\boldsymbol{\Theta})) - c q(\boldsymbol{\Theta}) \right] \tag{13}$$

s.t.

$$E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))-t_i(\mathbf{\Theta})]\right\} \geq 0, \quad \forall i, \forall \theta_i, \tag{14}$$

$$E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))-t_i(\mathbf{\Theta})]\right\} \geq E_{\theta_{-i}}\left\{\delta_i(\theta_i)[\theta_i S(q(\check{\theta}_i,\theta_{-i}),p(\check{\theta}_i,\theta_{-i}))-t_i(\check{\theta}_i,\theta_{-i})]\right\}, \quad \forall i, \forall \theta_i, \forall \check{\theta}_i. \tag{15}$$

This formulation resembles its counterpart in the exogenous price model. In the following solution procedure, we take the same backward induction approach, starting with the advertisers' rational responses.

4.1 The Advertisers' Optimal Decisions

The characterization of the advertisers' optimal decisions is similar to that in the exogenous price case. One can verify that the endogenous price case inherits the cutoff policy characterized in Lemma 1, enabling us to limit our attention to the case $\theta_i \geq \hat{\theta}_i$ (for any i) only. As such, we can replace $\delta_i(\theta_i)$ with 1 when solving advertiser i's optimization problem described in the IC constraint (Equation (15)).

The following lemma mirrors its counterpart in Lemma 2, with the changes reflecting the endogenized pricing decision.

Lemma 3 *An incentive compatible contract $\{q(\mathbf{\Theta}), p(\mathbf{\Theta}), t_1(\mathbf{\Theta}), \ldots, t_n(\mathbf{\Theta})\}$ must satisfy the following necessary condition:*

$$E_{\theta_{-i}}[t_i(\mathbf{\Theta})] = E_{\theta_{-i}}[\theta_i S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))] - \int_{\hat{\theta}_i}^{\theta_i} E_{\theta_{-i}}[S(q(\tilde{\theta}_i,\theta_{-i}),p(\tilde{\theta}_i,\theta_{-i}))]d\tilde{\theta}_i, \;\; \forall i, \; \forall \theta_i \geq \hat{\theta}_i. \tag{16}$$

Furthermore, if $dq(\mathbf{\Theta})/d\theta_i \geq 0$ and $dp(\mathbf{\Theta})/d\theta_i \leq 0$, $\forall i$, the above condition (16) is also sufficient for incentive compatibility.

4.2 The Newsvendor's Optimal Contract Menu

The newsvendor's problem can now be reformulated as follows, by substituting the expected transfer payments in Equation (16) into Equation (13), followed by an integration by parts.

$$\max_{q(\mathbf{\Theta})\geq 0, p(\mathbf{\Theta})\geq 0, \hat{\mathbf{\Theta}}} E_{\mathbf{\Theta}}\left\{\left(p(\mathbf{\Theta}) + \sum_{i=1}^{n}[\delta_i(\theta_i)V_i(\theta_i)]\right)S(q(\mathbf{\Theta}),p(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\}. \tag{17}$$

We expand the definition of the virtual price to $\hat{p}(\mathbf{\Theta}, p(\mathbf{\Theta})) = p(\mathbf{\Theta}) + \sum_{i=1}^{n} V_i^{+}(\theta_i)$, and let a decision with the superscript E denote its optimal solution. Then the newsvendor's optimal menu of contracts can be summarized in the following theorem.

Theorem 2 *Let p^0 be the optimal price for the newsvendor in the absence of the secondary revenue. If the distribution of the random shock x satisfies IFR, namely, if $f(x)/[1-F(x)]$ is increasing, then under Assumption 1, the optimal cutoff level for each advertiser is such that $V_i(\hat{\theta}_i^E)=0$, $\forall i$, and the optimal menu of contracts is:*

$$p^E(\mathbf{\Theta}) = \max\left(p^0, \sum_{i=1}^n V_i^+(\theta_i)\right) - \sum_{i=1}^n V_i^+(\theta_i), \tag{18}$$

$$q^E(\mathbf{\Theta}) = y\left(p^E(\mathbf{\Theta})\right) F^{-1}\left(\frac{\hat{p}\left(\mathbf{\Theta}, p^E(\mathbf{\Theta})\right) - c}{\hat{p}\left(\mathbf{\Theta}, p^E(\mathbf{\Theta})\right)}\right), \tag{19}$$

$$E_{\theta_{-i}}\left[t_i^E(\mathbf{\Theta})\right] = \begin{cases} E_{\theta_{-i}}\left[\theta_i S\left(q^E(\mathbf{\Theta}), p^E(\mathbf{\Theta})\right)\right] - \int_{\hat{\theta}_i^E}^{\theta_i} E_{\theta_{-i}}\left[S\left(q^E(\tilde{\theta}_i, \theta_{-i}), p^E(\tilde{\theta}_i, \theta_{-i})\right)\right] d\tilde{\theta}_i, \\ \qquad \forall i, \forall \theta_i \geq \hat{\theta}_i^E, \\ E_{\theta_{-i}}\left[\hat{\theta}_i^E S\left(q^E(\hat{\theta}_i^E, \theta_{-i}), p^E(\hat{\theta}_i^E, \theta_{-i})\right)\right], \quad \forall i, \forall \theta_i \leq \hat{\theta}_i^E. \end{cases}$$

The above results mimic those in the exogenous price case. By definition, p^0 is completely independent of the advertisers' benefit types $\mathbf{\Theta}$. As shown in the proof, and as expected from intuition, $p^0 > c$, because the newsvendor is better off setting a price above the unit cost c to make a profit in the absence of the secondary revenue. As a result, the virtual price $\hat{p}\left(\mathbf{\Theta}, p^E(\mathbf{\Theta})\right) = \max\left(p^0, \sum_{i=1}^n V_i^+(\theta_i)\right) > c$, leading to the disappearance of the + operator in the optimal inventory $q^E(\mathbf{\Theta})$ in Equation (19). The optimal price $p^E(\mathbf{\Theta})$ explains why the newsvendor may choose to offer her product for free. As we can see, when the combined contributions from the multiple advertisers are sufficiently large, i.e., when $\sum_{i=1}^n V_i^+(\theta_i) > p^0$, the increase in demand and corresponding transfer fees from the advertisers more than compensate for the lost revenue from her own product sales.

Theorem 2 suggests that the newsvendor's decision to include an advertiser depends on the advertiser's virtual coefficient $V_i(\theta_i)$, as a measure of his net contribution to the newsvendor. If none of the advertisers merit inclusion in the newsvendor's contract, the newsvendor will set the price to p^0 and produce the optimal inventory without the secondary revenue. In addition, the advertisers' individual cutoff level $V_i(\theta_i)$ is the same as in the exogenous price model. This is because the newsvendor bases the cutoff level solely on each advertiser's virtual benefit type $V_i(\theta_i)$, which consists of the benefit type θ_i net of the information cost $[1-\Psi_i(\theta_i)]/\psi_i(\theta_i)$, regardless of whether price is a decision variable or not.

We next examine the impact of the advertisers' benefit types on the optimal price, inventory, and associated expected sales in the following corollary.

Corollary 2 *For any i, the optimal price $p^E(\mathbf{\Theta})$ decreases in θ_i, but the optimal inventory $q^E(\mathbf{\Theta})$ increases in θ_i. The optimal expected sales $S(q(\mathbf{\Theta}), p(\mathbf{\Theta}))$ increase in θ_i.*

The characteristics of the optimal solution listed in this corollary ensure that condition (16) is sufficient for incentive compatibility, and thus the newsvendor's optimal menu of contracts presented in Theorem 2 is truth revealing. Based on the direct revelation principle, this guarantees the optimality of the menu of contracts.

4.3 Discussion of Modeling Results

While the endogenous price model mostly preserves the general results discussed in the exogenous price case, let us point out three subtle changes.

First, in the endogenous price case, without advertising revenue, the newsvendor always sets a price above the unit cost c, and the newsvendor's product is always profit-making. Therefore, the break-even type analysis for advertisers and the impact of the multiplicity of advertisers on the newsvendor's break-even boundary become irrelevant, as the newsvendor can always break even, regardless of the advertisers' participation decisions. This contrasts with the exogenous price case where the break-even boundary is determined by the pooled net contribution from all advertisers, thus creating interdependence among advertisers.

Second, our results confirm that the inclusion of additional advertisers creates a win-win situation for all parties involved: the consumers, the advertisers, and the newsvendor. Not only does the optimal inventory $q^E(\mathbf{\Theta})$ increase – similar to the exogenous price case – but also the optimal price $p^E(\mathbf{\Theta})$ decreases, as suggested by Theorem 2. This evidently benefits the consumers because they pay a reduced price and enjoy improved service due to the enhanced fill rate. It also benefits the advertisers because a reduced price and an increased inventory together imply increased expected sales, leading to increased advertiser expected profit $\int_{\hat{\theta}_i^E}^{\theta_i} E_{\theta_{-i}}\left[S\left(q^E(\tilde{\theta}_i, \theta_{-i}), p^E(\tilde{\theta}_i, \theta_{-i})\right)\right] d\tilde{\theta}_i$. As for the newsvendor, it is impossible for her to be worse off because she can otherwise set a high enough transfer payment for the new advertiser to exclude him from the contract while maintaining all other decisions to reap the same expected profit as before.

A third issue of interest concerns the impact of the newsvendor's pricing power on the advertisers' expected profit, assuming the newsvendor holds the same belief $\psi_i(\theta_i)$. It is reasonable to think that in the endogenous price case the newsvendor's power to control both price and inventory might enable her to squeeze more benefit from the advertisers, thereby reducing their informational rent. However, we can find exceptions to this intuition, for example, in the case where the exogenous price is below cost. Indeed, this means that the newsvendor is better off producing nothing when each advertiser is above his cutoff level but their pooled virtual benefit type is insufficient to cover the newsvendor's loss (see region IIc in Figure 2). In this case, each advertiser will be offered a null contract and thus earn no profit, even though he makes a positive contribution to the newsvendor. In the endogenous price case, however, this null contract possibility disappears, implying a higher level of advertiser participation and thus a higher expected profit for the advertisers. This is because the newsvendor will always choose a price, inventory and transfer payment that includes all advertisers with a positive virtual benefit type. Therefore, advertisers with relatively low aggregate virtual benefit types will prefer a price-setting newsvendor to a price-taking one. An increase in the number of advertisers, however, raises the probability of the aggregate virtual benefit type exceeding the break-even boundary required by the price-taking newsvendor, and thus reduces the advertisers' preference for the price-setting newsvendor. Similarly, the newsvendor's pricing power benefits herself through the increased profitability of her own product and the increased flexibility in writing contracts. In addition, the newsvendor's pricing power allows her to increase her reach and reap revenue from the advertisers whose aggregate contribution would otherwise have been insufficient to cover her losses in the exogenous price case.

5. Numerical Analysis

When introducing multiple advertisers, the multiplicity of advertisers can be studied in two ways. Thus far, we have discussed the impact of including additional advertisers on the newsvendor's profit and optimal decisions, which has allowed us to discover network effects among the advertisers, in the exogenous and endogenous cases. Another way of looking at multiplicity, however, is to assume that multiple advertisers collapse into a single advertiser, i.e., to investigate what happens when the newsvendor faces a single advertiser with a distribution of benefit types that is the convolution of the benefit types of the individual advertisers. In this section, we take the second path, and perform numerical analyses to understand the difference between the single and multiple advertiser cases with comparable benefit type distribution.

In the representative example reported below, we illustrate our findings by comparing the equivalent single advertiser case with a two-advertiser case. To create a fair comparison between the two cases, the distribution of the equivalent single advertiser benefit type is constructed as the convolution of the benefit types in the two-advertiser case, i.e., $\theta = \theta_1 + \theta_2$, where θ is the benefit type in the single advertiser case, whereas the independent and identically distributed θ_1 and θ_2 are the benefit types in the two-advertiser case. In our analysis, we focus our attention on the effect of the benefit type standard deviation, because this parameter connotes the degree of information asymmetry between the newsvendor and the advertiser(s), which is a central feature of our problem. Moreover, Wu et al. (2011) have observed the intricate effects of this parameter in their numerical experiments.

The system parameters of the single advertiser case are described as follows: the cost $c = 2$ and the exogenous price $p = 3$. In the endogenous price model, we set the price sensitivity coefficient $\tau = 1.5$ and the parameter $\xi = \exp(1.5 \times 3) = 90$ so that $y(p = 3) = 1$. That is, the endogenous model has the same demand distribution as the exogenous price model. We use a gamma distribution for both the demand and the benefit types. The mean demand E[X]=10, and the standard deviation of demand $\sigma(X) = 10$. The mean benefit type of the single advertiser is fixed at E[θ]=1, and his benefit type standard deviation $\sigma(\theta)$ varies from 0.5 to 1.5 with step size 0.1. In the two-advertiser case, all the parameters remain the same, except that the benefit type distribution is split into two. Note that splitting the gamma distribution halves the shape parameter and skews the distribution to the left. In other words, compared to the single advertiser case, the chance of there being low benefit types is higher in the two-advertiser case.

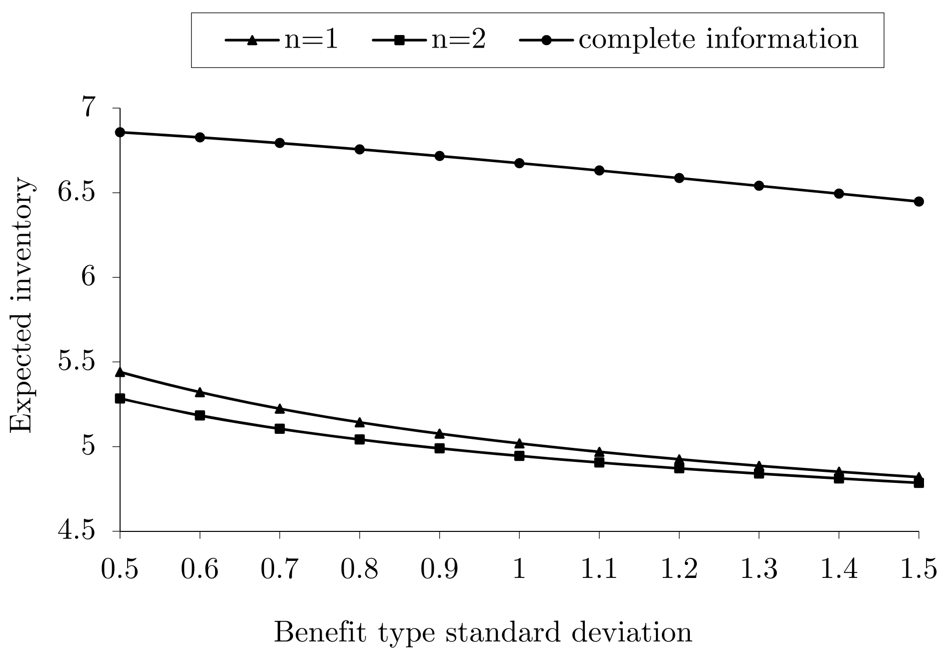
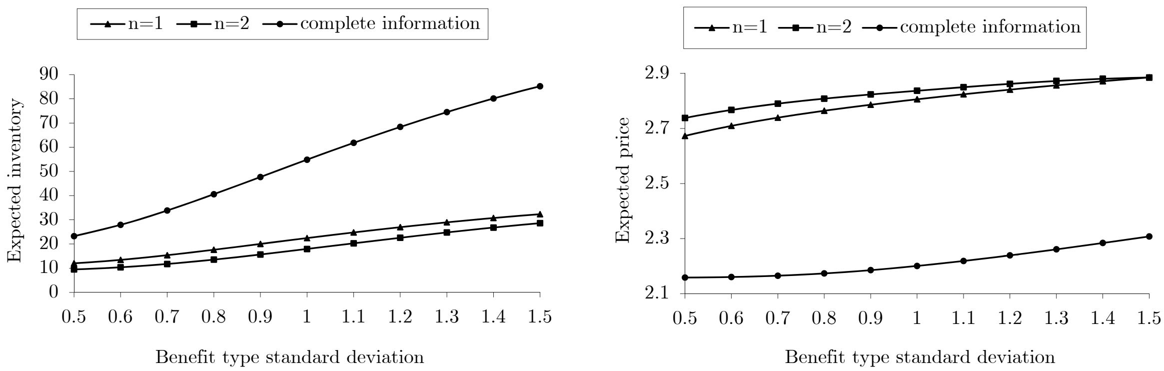


Figure 3: Expected Inventory in the Exogenous Price Model

We first look at the impact of the benefit type standard deviation on the optimal decisions, which are shown in Figures 3 and 4. We observe that when the benefit type standard deviation varies, the expected inventory and price change in the same direction in both the single and the two-advertiser cases. The direction of change is consistent with the results of Wu et al. (2011). Moreover, the inventory and the price are less distorted from their system-efficient levels (i.e., when complete information is available) in the single advertiser case than in the two-advertiser case. This may be attributable to the pooling effect, because the newsvendor faces less uncertainty (as measured by the coefficient of variation) with a single advertiser than with two equivalent advertisers.



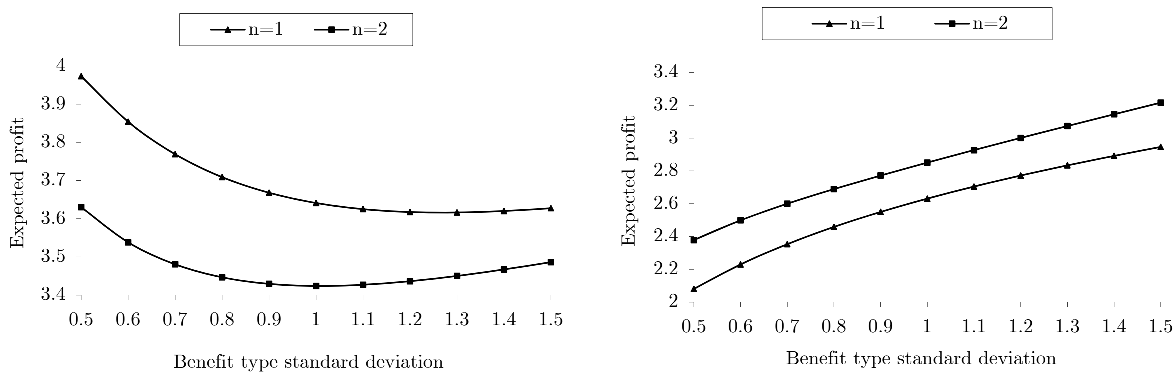
(a) Expected inventory

(b) Expected price

Figure 4: Expected Inventory and Price in the Endogenous Price Model

We next turn our attention to the impact of the benefit type standard deviation on the expected profits of the newsvendor and the advertisers. In the exogenous price model, we observe from Figure 5 that an increase in the standard deviation of the benefit type distribution has the same effect on the newsvendor's and advertiser's expected profit regardless of the number of advertisers, thus confirming the results of Wu et al. (2011) for the single advertiser problem. Second, we see that the newsvendor is better off with one advertiser than with two advertisers, which follows from the aforementioned pooling effect. The total advertisers' expected profit, however, shows the reverse and is higher in the two-advertiser case because the newsvendor has to pay more informational rent to the advertisers to induce truth-telling when facing higher uncertainty.

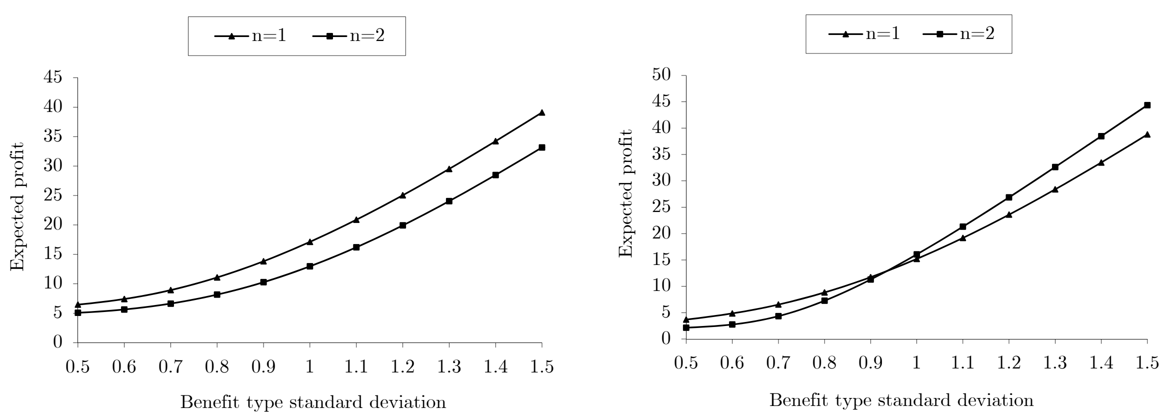
The results for the endogenous price case are not as clear cut, as shown in Figure 6. First, the newsvendor's expected profit behaves similarly to the exogenous case and, in particular, benefits from the pooling effect. The advertisers' expected profit, however, behaves slightly differently and the joint advertisers' expected profit may be less than the single advertiser's expected profit for low standard deviations. One possible explanation for this could be rooted in the screening instruments that the newsvendor can use for different standard deviations for the single versus the multiple advertisers case. Our analytical results in Section 4.2 show that when the benefit type is high, the price charged to the end consumers falls to zero. This means that the newsvendor can only use inventory as a tool to screen advertisers with high benefit types, rather than inventory and price. We expect that when the newsvendor is able to use both inventory and price for screening, she will be able to do so more efficiently and



(a) Newsvendor's expected profit (b) Advertiser's expected profit

Figure 5: Expected Profits in the Exogenous Price Model

give away less rent to the advertiser(s). Now recall that the two-advertiser case is more likely to see low benefit types, as discussed earlier. As a result, at low standard deviations, the newsvendor is more likely to be able to use both inventory and price as screening instruments in the two-advertiser case than in the single advertiser case, thus counteracting the fact that more uncertainty benefits the advertiser. However, as the standard deviation increases, which enlarges the right tail of the benefit type distribution, the single and two-advertiser cases both tend to have more instances in which the optimal price is set to zero, and only inventory is available to screen the advertisers, thus allowing the pooling effect to dominate.



(a) Newsvendor's expected profit (b) Advertiser's expected profit

Figure 6: Expected Profits in the Endogenous Price Model

6. Conclusion and Future Research

In this paper, we extend the newsvendor problem to include the secondary revenue generated by multiple advertisers and focus on the impact of the multiplicity of advertisers on the newsvendor's optimal decisions. The second source of revenue often forms a significant part of the newsvendor's profit, as evidenced by the magazine and newspaper industry. The presence of the second source of revenue, however, raises the question of how to charge the advertisers for the privilege of accessing the newsvendor's end users. The prices that the advertisers will be willing to pay are determined by the advertisers' benefit types, which are not known to the newsvendor. In this case, to maximize her expected profit, we show that the newsvendor benefits from a more inclusive pricing contract that links the production quantity and prices. We use a principal-agent framework to model the newsvendor's decision problem, and find the newsvendor's optimal inventory, price, transfer payment and cutoff policies.

We confirm that the newsvendor may prefer not to include an advertiser with a low benefit type and decides to follow a cutoff policy. Interestingly, this cutoff level is unaffected by the number of advertisers that are interested in the newsvendor's product, and each advertiser is included based on his own contribution to the newsvendor's profit. Second, in the exogenous price model, we characterize a break-even boundary, which is relevant in the case in which the primary profit is negative, e.g., in the case of a free newspaper. The break-even boundary requires that the advertisers' joint contribution exceed the newsvendor's loss per unit, and in that case a contract may be signed even though all participating advertisers are below their individual break-even benefit level. This creates interdependence among the advertisers as the newsvendor will offer a real contract based on the pooled contribution of all advertisers above their cutoff levels. The break-even boundary will disappear if the newsvendor can set the optimal product price, as intuitively it is always suboptimal to set a price that yields a negative profit. This implies a higher level of advertiser participation, which benefits both the newsvendor and the advertisers. Therefore, some advertisers, notably those whose benefit type is below their individual break-even level, may prefer a price-setting newsvendor over a price-taking newsvendor. The newsvendor will always prefer to have pricing power as this increases both her primary and secondary revenue. Third, in the exogenous and endogenous price cases, the newsvendor problem with secondary revenue displays network effects, and the entire system is better off if the number of advertisers increases. Finally, however, our numerical results show that for a given total benefit contribution by all advertisers, the newsvendor prefers an equivalent single advertiser over multiple advertisers.

Extending the analysis of the newsvendor's problem with a second source of revenue to a setting with multiple advertisers reveals further avenues of research. The first extension would be to model the different product classes, e.g., advertisement size and placement, explicitly to introduce a choice model for the advertisers' demand. Another promising avenue of future research would be to acknowledge the scarcity of advertising space and introduce competition among advertisers. This would require the newsvendor to craft an optimal allocation rule or auction mechanism to maximize her revenue from the limited advertising space. Intuitively, we expect that this will reduce or even remove the network effects we have observed in our setting. Finally, we have ignored the impact of advertising on the demand for the newsvendor's product. However, the end users' utility could be influenced by the amount, quality or content of the advertisements inserted in the publication, as these factors may affect the end users' perception of the publication's content.

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Appendix

Proof of Lemma 1.

If advertiser i does not participate when his type is θ_i, then Inequality (4) must hold. It then follows that $E_{\theta_{-i}}[\theta_l S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})] \leq E_{\theta_{-i}}[\theta_i S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})] < 0$, for any $\underline{\theta}_i \leq \theta_l \leq \theta_i$, and for any $\check{\theta}_i$. Therefore, advertiser i will also not participate if his type is θ_l, otherwise he will always incur a loss.

Proof of Lemma 2.

If advertiser i reports $\check{\theta}_i$, then his expected profit is $G(\check{\theta}_i) = E_{\theta_{-i}}[\theta_i S(q(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})]$. The first and second order derivatives of this function are as follows:

$$\frac{dG(\check{\theta}_i)}{d\check{\theta}_i} = E_{\theta_{-i}}\left[\theta_i \frac{dS(q(\check{\theta}_i, \theta_{-i}))}{d\check{\theta}_i} - \frac{dt_i(\check{\theta}_i, \theta_{-i})}{d\check{\theta}_i}\right],$$

$$\frac{d^2G(\check{\theta}_i)}{d\check{\theta}_i^2} = E_{\theta_{-i}}\left[\theta_i \frac{d^2S(q(\check{\theta}_i, \theta_{-i}))}{d\check{\theta}_i^2} - \frac{d^2t_i(\check{\theta}_i, \theta_{-i})}{d\check{\theta}_i^2}\right].$$

For the contract to be incentive compatible (i.e., for advertiser i's optimal report to be his true type θ_i), the first order condition (FOC) must be met at θ_i. That is,

$$\left.\frac{dG(\check{\theta}_i)}{d\check{\theta}_i}\right|_{\check{\theta}_i=\theta_i} = E_{\theta_{-i}}\left[\theta_i \frac{dS(q(\mathbf{\Theta}))}{d\theta_i} - \frac{dt_i(\mathbf{\Theta})}{d\theta_i}\right] = 0. \tag{A-i}$$

Total differentiation of this FOC with respect to θ_i yields

$$E_{\theta_{-i}}\left\{[1 - F(q(\mathbf{\Theta}))]\frac{dq(\mathbf{\Theta})}{d\theta_i} + \theta_i \frac{d^2S(q(\mathbf{\Theta}))}{d\theta_i^2} - \frac{d^2t_i(\mathbf{\Theta})}{d\theta_i^2}\right\} = 0,$$

which is then substituted into the second order derivative at θ_i to obtain the following:

$$\left.\frac{d^2G(\check{\theta}_i)}{d\check{\theta}_i^2}\right|_{\check{\theta}_i=\theta_i} = E_{\theta_{-i}}\left[\theta_i \frac{d^2S(q(\mathbf{\Theta}))}{d\theta_i^2} - \frac{d^2t_i(\mathbf{\Theta})}{d\theta_i^2}\right] = -E_{\theta_{-i}}\left\{[1 - F(q(\mathbf{\Theta}))]\frac{dq(\mathbf{\Theta})}{d\theta_i}\right\}.$$

Therefore, the second order sufficient condition is satisfied if $dq(\mathbf{\Theta})/d\theta_i \geq 0$. Later, we will verify that our optimal solution $q^A(\mathbf{\Theta})$ meets this monotonicity condition.

Solving the differential equation $E_{\theta_{-i}}[dt_i(\mathbf{\Theta})/d\theta_i] = E_{\theta_{-i}}[\theta_i dS(q(\mathbf{\Theta}))/d\theta_i]$ in Equation (A-i) yields $E_{\theta_{-i}}[t_i(\mathbf{\Theta})] = E_{\theta_{-i}}\left[\theta_i S(q(\mathbf{\Theta})) - \int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i, \theta_{-i}))d\tilde{\theta}_i\right] + K$, where the right hand side follows from integration by parts, and K is a constant. The envelope theorem implies that advertiser i's expected profit $E_{\theta_{-i}}[\theta_i S(q(\mathbf{\Theta})) - t_i(\mathbf{\Theta})]$ in an incentive compatible contract is increasing in his type θ_i, for all $\theta_i \geq \hat{\theta}_i$. As a result, the IR constraint must bind at

$\hat{\theta}_i$. Otherwise, the newsvendor can always improve her objective function by increasing $E_{\theta_{-i}}[t_i(\hat{\theta}_i, \theta_{-i})]$ until the IR constraint is binding at $\hat{\theta}_i$ without violating any other types' participation constraint. This further implies that the constant $K = 0$.

Proof of Theorem 1.

In this proof, we first show how the reformulation in Equation (6) is obtained, and then use a sequential optimization approach to solve it for the optimal solutions:

$$
\begin{aligned}
&\max_{q(\mathbf{\Theta})\geq 0,\ t_i(\mathbf{\Theta})} E_{\mathbf{\Theta}}\left\{\sum_{i=1}^{n}[\delta_i(\theta_i)t_i(\mathbf{\Theta})] + pS(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\} \\
&= \max_{q(\mathbf{\Theta})\geq 0,\ t_i(\mathbf{\Theta})} \sum_{i=1}^{n}\left\{E_{\theta_i}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[t_i(\mathbf{\Theta})\right]\right\}\right\} + E_{\mathbf{\Theta}}\left\{pS(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\} \\
&= \max_{q(\mathbf{\Theta})\geq 0,\ \hat{\mathbf{\Theta}}} \sum_{i=1}^{n}\left\{E_{\theta_i}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[\theta_i S(q(\mathbf{\Theta})) - \int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i, \theta_{-i}))d\tilde{\theta}_i\right]\right\}\right\} \\
&\quad + E_{\mathbf{\Theta}}\left\{pS(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\} \\
&= \max_{q(\mathbf{\Theta})\geq 0,\ \hat{\mathbf{\Theta}}} E_{\mathbf{\Theta}}\left\{\sum_{i=1}^{n}[\delta_i(\theta_i)\theta_i S(q(\mathbf{\Theta}))] + pS(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\} \\
&\quad - \sum_{i=1}^{n}\left\{E_{\theta_i}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[\int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i, \theta_{-i}))d\tilde{\theta}_i\right]\right\}\right\} \\
&= \max_{q(\mathbf{\Theta})\geq 0,\ \hat{\mathbf{\Theta}}} E_{\mathbf{\Theta}}\left\{\left(p + \sum_{i=1}^{n}[\delta_i(\theta_i)V_i\left(\theta_i\right)]\right) S(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right\},
\end{aligned}
$$

where the last step follows from the derivation below using integration by parts:

$$
\begin{aligned}
&\sum_{i=1}^{n}\left\{E_{\theta_i}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[\int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i,\theta_{-i}))d\tilde{\theta}_i\right]\right\}\right\}\\
=&\sum_{i=1}^{n}\left\{\int_{\underline{\theta}_i}^{\bar{\theta}_i}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[\int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i,\theta_{-i}))d\tilde{\theta}_i\right]\right\}d\Psi_i(\theta_i)\right\}\\
=&\sum_{i=1}^{n}\left\{\delta_i(\theta_i)E_{\theta_{-i}}\left[\int_{\hat{\theta}_i}^{\theta_i} S(q(\tilde{\theta}_i,\theta_{-i}))d\tilde{\theta}_i\right]\Psi_i(\theta_i)\Bigg|_{\theta_i=\underline{\theta}_i}^{\theta_i=\bar{\theta}_i}\right\}\\
&-\sum_{i=1}^{n}\left\{\int_{\underline{\theta}_i}^{\bar{\theta}_i}\Psi_i(\theta_i)\delta_i(\theta_i)E_{\theta_{-i}}\left[S(q(\boldsymbol{\Theta}))\right]d\theta_i\right\}\\
=&\sum_{i=1}^{n}\left\{E_{\theta_{-i}}\left[\int_{\hat{\theta}_i}^{\bar{\theta}_i}\delta_i(\theta_i)S(q(\boldsymbol{\Theta}))d\theta_i\right]\right\}-\sum_{i=1}^{n}\left\{\int_{\underline{\theta}_i}^{\bar{\theta}_i}\Psi_i(\theta_i)E_{\theta_{-i}}\left[\delta_i(\theta_i)S(q(\boldsymbol{\Theta}))\right]d\theta_i\right\}\\
=&E_{\boldsymbol{\Theta}}\left\{\sum_{i=1}^{n}\left[\delta_i(\theta_i)\frac{1-\Psi_i(\theta_i)}{\psi_i(\theta_i)}S(q(\boldsymbol{\Theta}))\right]\right\}.
\end{aligned}
$$

It is worth noting the fact that $\delta_i(\theta_i)=0$ for $\theta_i \le \hat{\theta}_i$ is used in the above derivation. We now use a sequential optimization method to solve the problem. Let $\sum_{-i}$ denote the summation of all advertisers other than i. It is instrumental to rewrite the newsvendor's objective function in Equation (6) as $\max_{\hat{\theta}_i} U(\hat{\theta}_i)$, where

$$
\begin{aligned}
U(\hat{\theta}_i) = \max_{q(\boldsymbol{\Theta})\ge 0,\ \hat{\theta}_{-i}} &\int_{\underline{\theta}_i}^{\hat{\theta}_i} E_{\theta_{-i}}\left[\left(p+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S(q(\boldsymbol{\Theta}))-cq(\boldsymbol{\Theta})\right]\psi_i(\theta_i)d\theta_i\\
&+\int_{\hat{\theta}_i}^{\bar{\theta}_i} E_{\theta_{-i}}\left[\left(p+V_i(\theta_i)+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S(q(\boldsymbol{\Theta}))-cq(\boldsymbol{\Theta})\right]\psi_i(\theta_i)d\theta_i\ .
\end{aligned}
$$

The envelope theorem implies that

$$
\begin{aligned}
\frac{dU(\hat{\theta}_i)}{d\hat{\theta}_i} =&E_{\theta_{-i}}\left[\left(p+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S(q(\hat{\theta}_i,\theta_{-i}))-cq(\hat{\theta}_i,\theta_{-i})\right]\psi_i(\hat{\theta}_i)\\
&-E_{\theta_{-i}}\left[\left(p+V_i(\hat{\theta}_i)+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S(q(\hat{\theta}_i,\theta_{-i}))-cq(\hat{\theta}_i,\theta_{-i})\right]\psi_i(\hat{\theta}_i)\\
=&-E_{\theta_{-i}}\left[V_i(\hat{\theta}_i)S(q(\hat{\theta}_i,\theta_{-i}))\right]\psi_i(\hat{\theta}_i).
\end{aligned}
$$

Setting $dU(\hat{\theta}_i)/d\hat{\theta}_i$ to 0 yields $V_i(\hat{\theta}_i^A)=0$, where $\hat{\theta}_i^A$ is the optimal cutoff level for advertiser i. The optimality of $\hat{\theta}_i^A$ stems from the monotonicity of $V_i(\hat{\theta}_i)$ assumed in Assumption

1. Due to this assumption, on the left (right) of $\hat{\theta}_i^A$, the newsvendor's expected profit is increasing (decreasing), implying that it is unimodel in $\hat{\theta}_i$. Therefore its maximizer is the stationary point $\hat{\theta}_i^A$. By the same token, the optimal cutoff levels of all other advertisers can be characterized in the same fashion.

Now it is immediate that $\delta_i(\theta_i)V_i(\theta_i) = V_i^+(\theta_i)$. Therefore, Equation (6) reduces to

$$\max_{q(\mathbf{\Theta})\geq 0} E_{\mathbf{\Theta}}\left[\hat{p}(\mathbf{\Theta})S(q(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right] \ ,$$

which mimics the classic newsvendor problem and can be shown to be concave in $q(\mathbf{\Theta})$. The optimal inventory stated in Equation (7) follows from the first order condition in conjunction with the nonnegativity constraint $q(\mathbf{\Theta}) \geq 0$.

Proof of Corollary 1.

When there is only a single advertiser, i.e., when $n = 1$, the menu of contracts reduces to $\{q(\theta_1), t_1(\theta_1)\}$. We can now formulate the newsvendor's problem in a way similar to Equations (1)–(3), dropping the expectation over all other advertisers' types in both the objective function and the constraints. The results in Corollary 1 are derived by following exactly the same solution procedure used in Section 3. Not surprisingly, those results are identical to Theorem 1 of Wu et al. (2011).

Proof of Lemma 3.

Advertiser i attempts to maximize his expected profit $G(\check{\theta}_i) = E_{\theta_{-i}}[\theta_i S(q(\check{\theta}_i, \theta_{-i}), p(\check{\theta}_i, \theta_{-i})) - t_i(\check{\theta}_i, \theta_{-i})]$ by choosing to report an appropriate $\check{\theta}_i$. If his optimal decision turns out to be θ_i, then the contract is incentive compatible. The first and second order derivatives are as follows:

$$\frac{dG(\check{\theta}_i)}{d\check{\theta}_i} = E_{\theta_{-i}}\left[\theta_i \frac{dS(q(\check{\theta}_i, \theta_{-i}), p(\check{\theta}_i, \theta_{-i}))}{d\check{\theta}_i} - \frac{dt_i(\check{\theta}_i, \theta_{-i})}{d\check{\theta}_i}\right],$$

$$\frac{d^2G(\check{\theta}_i)}{d\check{\theta}_i^2} = E_{\theta_{-i}}\left[\theta_i \frac{d^2S(q(\check{\theta}_i, \theta_{-i}), p(\check{\theta}_i, \theta_{-i}))}{d\check{\theta}_i^2} - \frac{d^2t_i(\check{\theta}_i, \theta_{-i})}{d\check{\theta}_i^2}\right].$$

For the contract to be incentive compatible (i.e., for advertiser i's optimal report to be his true type θ_i), the first order condition must be met at θ_i. That is,

$$\left.\frac{dG(\check{\theta}_i)}{d\check{\theta}_i}\right|_{\check{\theta}_i=\theta_i} = E_{\theta_{-i}}\left[\theta_i \frac{dS(q(\mathbf{\Theta}), p(\mathbf{\Theta}))}{d\theta_i} - \frac{dt_i(\mathbf{\Theta})}{d\theta_i}\right] = 0. \tag{A-ii}$$

Total differentiation of this FOC with respect to θ_i yields

$$E_{\theta_{-i}}\left\{\frac{dS(q(\mathbf{\Theta}),p(\mathbf{\Theta}))}{d\theta_i}+\theta_i\frac{d^2S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))}{d\theta_i^2}-\frac{d^2t_i(\mathbf{\Theta})}{d\theta_i^2}\right\}=0,$$

which is then substituted into the second order derivative at θ_i to obtain the following:

$$\left.\frac{d^2G(\breve{\theta}_i)}{d\breve{\theta}_i^2}\right|_{\breve{\theta}_i=\theta_i}=E_{\theta_{-i}}\left[\theta_i\frac{d^2S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))}{d\theta_i^2}-\frac{d^2t_i(\mathbf{\Theta})}{d\theta_i^2}\right]=-E_{\theta_{-i}}\left\{\frac{dS(q(\mathbf{\Theta}),p(\mathbf{\Theta}))}{d\theta_i}\right\}.$$

Therefore, the second order sufficient condition is satisfied if $dS(q(\mathbf{\Theta}),p(\mathbf{\Theta}))/d\theta_i\geq 0$, which is true if $dq(\mathbf{\Theta})/d\theta_i\geq 0$ and $dp(\mathbf{\Theta})/d\theta_i\leq 0$, as can be seen from the definition of $S(q(\mathbf{\Theta}),p(\mathbf{\Theta}))$. Later, we will verify that our optimal solutions $q^E(\mathbf{\Theta})$ and $p^E(\mathbf{\Theta})$ meet this monotonicity condition.

Solving the differential Equation (A-ii) using a similar argument to that in the proof of Lemma 2 yields the result in Equation (16).

Proof of Theorem 2.

Following similar steps to those in the proof of Theorem 1, we can obtain the objective function stated in Equation (17), which is then rewritten as $\max_{\hat{\theta}_i}U(\hat{\theta}_i)$, where

$$\begin{aligned}U(\hat{\theta}_i)=&\max_{q(\mathbf{\Theta})\geq 0,\ p(\mathbf{\Theta})\geq 0,\ \hat{\theta}_{-i}}\int_{\underline{\theta}_i}^{\hat{\theta}_i}E_{\theta_{-i}}\left[\left(p(\mathbf{\Theta})+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S\left(q(\mathbf{\Theta}),p(\mathbf{\Theta})\right)-cq(\mathbf{\Theta})\right]\psi_i(\theta_i)d\theta_i\\&+\int_{\hat{\theta}_i}^{\bar{\theta}_i}E_{\theta_{-i}}\left[\left(p(\mathbf{\Theta})+V_i(\theta_i)+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S\left(q(\mathbf{\Theta}),p(\mathbf{\Theta})\right)-cq(\mathbf{\Theta})\right]\psi_i(\theta_i)d\theta_i\ .\end{aligned}$$

The envelope theorem implies that

$$\begin{aligned}\frac{dU(\hat{\theta}_i)}{d\hat{\theta}_i}=&E_{\theta_{-i}}\left[\left(p(\hat{\theta}_i,\theta_{-i})+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S(q(\hat{\theta}_i,\theta_{-i}),p(\hat{\theta}_i,\theta_{-i}))-cq(\hat{\theta}_i,\theta_{-i})\right]\psi_i(\hat{\theta}_i)\\&-E_{\theta_{-i}}\left[\left(p(\hat{\theta}_i,\theta_{-i})+V_i(\hat{\theta}_i)+\sum_{-i}\delta_j(\theta_j)V_j(\theta_j)\right)S\left(q\left(\hat{\theta}_i,\theta_{-i}\right),p(\hat{\theta}_i,\theta_{-i})\right)-cq(\hat{\theta}_i,\theta_{-i})\right]\psi_i(\hat{\theta}_i)\\=&-E_{\theta_{-i}}\left[V_i(\hat{\theta}_i)S(q(\hat{\theta}_i,\theta_{-i}),p(\hat{\theta}_i,\theta_{-i}))\right]\psi_i(\hat{\theta}_i).\end{aligned}$$

Setting $dU(\hat{\theta}_i)/d\hat{\theta}_i$ to 0 yields $V_i(\hat{\theta}_i^E)=0$, where $\hat{\theta}_i^E$ is the optimal cutoff level for advertiser i. The optimality of $\hat{\theta}_i^E$ can be argued in the same fashion as that of $\hat{\theta}_i^A$. The optimal cutoff levels of all other advertisers can be characterized in the same way.

Now it is immediate that $\delta_i(\theta_i)V_i(\theta_i) = V_i^+(\theta_i)$. Therefore Equation (17) reduces to

$$\max_{q(\mathbf{\Theta})\geq 0,\ p(\mathbf{\Theta})\geq 0} E_{\mathbf{\Theta}}\left[\hat{p}(\mathbf{\Theta}, p(\mathbf{\Theta}))S(q(\mathbf{\Theta}), p(\mathbf{\Theta})) - cq(\mathbf{\Theta})\right].$$

We introduce two transformations of variable. The first transformation defines the stocking factor $z(\mathbf{\Theta}) = q(\mathbf{\Theta})/y(p(\mathbf{\Theta}))$, and the second treats $\hat{p}(\mathbf{\Theta}, p(\mathbf{\Theta}))$ as a decision variable in place of $p(\mathbf{\Theta})$. As a result, the original joint optimization over $p(\mathbf{\Theta})$ and $q(\mathbf{\Theta})$ is transformed into the joint optimization over $\hat{p}$ and z (the arguments of $\hat{p}(\mathbf{\Theta}, p(\mathbf{\Theta}))$ and $z(\mathbf{\Theta})$ are omitted for brevity):

$$\max_{\hat{p}\geq\sum_{i=1}^n V_i^+(\theta_i),\ z\geq 0} E_{\mathbf{\Theta}}\left\{y\left(\hat{p} - \sum_{i=1}^n V_i^+(\theta_i)\right)\{\hat{p}\mathrm{E}_X[\min(z,x)] - cz\}\right\}.$$

We next follow the sequential optimization approach to characterize the optimal solution. We find it convenient to first optimize the stocking factor z for a given $\hat{p}$, and then substitute the optimal stocking factor as a function of $\hat{p}$ back into the objective function to turn it into a single variable optimization problem:

$$\max_{\hat{p}\geq\sum_{i=1}^n V_i^+(\theta_i)} E_{\mathbf{\Theta}}\left\{y\left(\hat{p} - \sum_{i=1}^n V_i^+(\theta_i)\right)R(\hat{p})\right\}, \tag{A-iii}$$

where $R(p)$ defines the maximum profit of a standard newsvendor problem for a given price p:

$$R(p) = \max_{z\geq 0}\{p\mathrm{E}_X[\min(z,x)] - cz\}.$$

The well-known critical fractile solution yields

$$z^*(p) = F^{-1}\left[\frac{(p-c)^+}{p}\right], \text{ and } R(p) = p\int_0^{z^*(p)} xf(x)dx.$$

The above results indicate that we can restrict our attention to the case $p > c$ only, because otherwise $z^*(p) = R(p) = 0$ for whatever price p, i.e., the newsvendor will always produce nothing and thus make 0 profit. Noting that $dy(p)/dp = -\tau y(p)$, we can write the first order derivative of the newsvendor's objective function in Equation (A-iii) with respect to $\hat{p}$ as follows.

$$E_{\mathbf{\Theta}}\left\{y\left(\hat{p} - \sum_{i=1}^n V_i^+(\theta_i)\right)\left(R'(\hat{p}) - \tau R(\hat{p})\right)\right\}. \tag{A-iv}$$

Next we will show that the function $R'(p) - \tau R(p)$ is strictly concave with two roots c and p^0, where $p^0 > c$. We first note that $dz^*(p)/dp = 1/\left[ph(z^*(p))\right]$, where $h(x) = f(x)/[1-F(x)]$

is the failure rate of the random shock x. Furthermore, we have:

$$R'(p) = \frac{cz^*(p)}{p} + \int_0^{z^*(p)} xf(x)dx, \; R''(p) = \frac{c}{p^2 h(z^*(p))}, \; R'''(p) = -\frac{c}{p^3 h(z^*(p))}\left[\frac{h'(z^*(p))}{h^2(z^*(p))} + 2\right].$$

As the failure rate $h(x)$ is non-negative and its derivative $h'(x)$ is also non-negative by the IFR assumption, $R''(p) \geq 0$ and $R'''(p) < 0$. Therefore, the second order derivative of $R'(p) - \tau R(p)$, given by $R'''(p) - \tau R''(p)$, is strictly negative. This implies the concavity of $R'(p) - \tau R(p)$. As a result, it has at most two roots.

We next show that $R'(p) - \tau R(p)$ has exactly two distinct roots c and p^0, and $p^0 > c$. As $R'(c) = R(c) = 0$, c is a root. Furthermore, $R'(p) - \tau R(p)$ strictly increases at $p = c$ because its first order derivative at c is $1/[ch(0)] > 0$. This implies that there exists some $p > c$ such that $R'(p) - \tau R(p) > 0$. On the other hand, $\lim_{p\to\infty}[R'(p) - \tau R(p)] < 0$ because by L'Hopital's rule, this limit is $\lim_{p\to\infty} c/[ph(z^*(p))] + E[X] - \infty$, but the first term goes to 0 by the IFR assumption. Therefore, in the region (c, ∞), $R'(p) - \tau R(p)$ has another root p^0, as it must cross 0 once from positive to negative.

The above analysis reveals that the first order derivative in Equation (A-iv) has two roots c and p^0, though root c is a local minimizer. Furthermore, the derivative is strictly positive in (c, p^0) and strictly negative in (p^0, ∞). This implies that p^0 is the unique maximizer of the unconstrained problem. For the constrained problem, the optimal virtual price is $\hat{p}^E = \max(p^0, \sum_{i=1}^n V_i^+(\theta_i))$. Then, the optimal price $p^E(\mathbf{\Theta})$ and optimal inventory $q^E(\mathbf{\Theta})$ are derived from the definitions of the virtual price $\hat{p}$ and stocking factor z, respectively. The optimal expected transfer payment $E_{\theta_{-i}}[t_i(\mathbf{\Theta})]$ follows from substituting the optimal price $p^E(\mathbf{\Theta})$ and optimal inventory $q^E(\mathbf{\Theta})$ into equation (16).

Proof of Corollary 2.

(i). The optimal price $p^E(\mathbf{\Theta}) = p^0 - \sum_{i=1}^n V_i^+(\theta_i)$, if $\sum_{i=1}^n V_i^+(\theta_i) \leq p^0$; and 0 otherwise. Therefore it is (weakly) decreasing in θ_i due to Assumption 1.

(ii). The optimal inventory $q^E(\mathbf{\Theta}) = y\left[p^0 - \sum_{i=1}^n V_i^+(\theta_i)\right] F^{-1}\left[(p^0 - c)/p^0\right]$, if $\sum_{i=1}^n V_i^+(\theta_i) \leq p^0$; and $y(0)F^{-1}\left[\left(\sum_{i=1}^n V_i^+(\theta_i) - c\right) / \sum_{i=1}^n V_i^+(\theta_i)\right]$ otherwise. In the former case, $F^{-1}\left[(p^0 - c)/p^0\right]$ is independent of θ_i, while $y\left[p^0 - \sum_{i=1}^n V_i^+(\theta_i)\right]$ increases in θ_i. Hence, $q^E(\mathbf{\Theta})$ increases in θ_i. In the latter case, $y(0)$ is a constant while $F^{-1}\left[\left(\sum_{i=1}^n V_i^+(\theta_i) - c\right) / \sum_{i=1}^n V_i^+(\theta_i)\right]$ increases in θ_i. Hence, $q^E(\mathbf{\Theta})$ increases in θ_i.

(iii). Parts (i) and (ii) above together imply that the optimal expected sales $S(q(\mathbf{\Theta}), p(\mathbf{\Theta})) = \mathrm{E}_X\left\{\min\left[q^E(\mathbf{\Theta}), y(p^E(\mathbf{\Theta}))x\right]\right\}$ increase in θ_i, due to the decreasing property of the $y(\cdot)$ function.

Response to Comments on

"The Newsvendor's Optimal Incentive Contracts for Multiple Advertisers"

(Manuscript No. EJOR-D-10-01865R2)

Response to Editor

Dear Prof Dyson,

Thank you for your decision to accept our paper to *European Journal of Operational Research* with minor revisions. We have addressed all the remaining points either in the text or in the letter to the referee.

We hope that you and the referees will find that the new version of the paper is now ready for publication. We look forward to hearing from you soon in the hope of positive news.

Once more, thank you for your time and effort as an editor for this paper.

Best regards.

Response to Referee 2

We thank you for your further detailed feedback on the paper. We were glad to read that you generally approved of the revision, and hope that this version will address all remaining issues.

Content

1. *Defend the case where the sales do not depend on p. Why would this be reasonable?*

   The sales function $S(q) = E_X[\min(q, x)]$ refers to the sales volume not the revenue. The sales revenue is $pS(q)$. However, the sales volume function $S(q)$ itself is independent of p in the exogenous price case: as the price p is assumed to be given and constant in the exogenous case, it can be implied in the demand function without having to be made explicit. In order to reduce the complexity of notation to a minimum, we have chosen to do so in the paper for the exogenous price case. After solving the newsvendor's optimization problem, we confirm that the optimal inventory q^A is a function of the exogenous price p.

   Obviously, in the endogenous case, price becomes variable and influences demand, and has to be expressed explicitly in the demand and sales function.

2. *Is it reasonable for the newsvendor and all advertisers to have same distribution of prior beliefs $\psi_i(\theta_i)$?*

   We apply standard economic theory that assumes the prior beliefs, i.e., the distribution of θ_i, are shared and common knowledge. For example, Fudenberg and Tirole (1991) note that "*[...] types are drawn from independent distributions. [...] the distributions are common knowledge*" (page 268).

   As our focus in this paper is to obtain insights into the newsvendor's problem in the presence of secondary revenue from multiple advertisers, we adopted standard methodologies and assumed the same prior beliefs in our model, although we agree that in reality the newsvendor and advertisers may hold different prior beliefs. We also note that even if different prior beliefs are used, the qualitative insights of our paper will remain unchanged.

3. *You say, "if any advertiser's benefit type is above his cutoff level, the newsvendor will have to solve the mechanism design problem." Why would the newsvendor know this in advance of solving the problem?*

   We agree that this sentence in the previous version was not clear. We did not mean to imply that the newsvendor would know in advance when to solve the mechanism design problem or not — your understanding that she cannot do so is correct. We meant to say that the mechanism design problem will only be relevant and its constraints binding when at least one advertiser exceeds his cutoff level. However, as this sentence caused confusion and essentially duplicated the previous two sentences by presenting the complement of what happened when at least one advertiser was above his cutoff level, we have decided to delete it.

4. *When you get to Figure 2, you have not yet defined θ_i^b?*

   As per your recommendation in the points relating to the writing style, we have expanded our list of notation to include the cutoff level θ_i^b. Furthermore, we now explicitly refer to the cutoff level in the discussion of Figure 2:

   "It is interesting to note that due to network effects, in the dotted portion of region IIIc, even though both advertisers are individually below the break-even benefit level (θ_i^b), their pooled contribution to the newsvendor's profit is enough to encourage production."

5. *Can you defend why the exogenous case is important to work with? Give me an example of when this would happen in the newspaper/magazine context you are using?*

   In some regions, prices in the publishing industry are relatively stable over time. For example, the *Time* magazine in the US changed its newsstand price to $4.95 in 2006 and still sells at that price now. The *New York Times* last hiked its price in 2009. Our exogenous case is applicable in these examples.

   Furthermore, the exogenous price case is a stepping stone to study the more interesting endogenous price case. Analyzing both cases allows us to understand the different roles inventory and price play as instruments to discriminate between advertisers' benefit types.

Writing

1. *Do not begin sentences with "there are", "it is", etc. Doing so represents poor writing style.*

   We have eliminated many of those instances.

2. *You appear to have a missing "E" both in Corollary 1 and at the bottom of page 13*

   We do not require an expectation E_{-i} in Corollary 1, since there is only one advertiser and $-i$ is empty.

On page 13, we present one out of many possible payment schemes that would respect the optimality conditions. The optimality conditions are only specified up to an expectation over $-i$: thus the transfer fee can be determined as a function of a given $\mathbf{\Theta}_{-i}$, as long as *in expectation* that function will satisfy the optimality conditions.

3. *Better describe what you mean by "pooled, single advertiser", and please justify why you only consider the benefit type $\theta_1 + \theta_2$.*

   We have changed the sentence in the introduction which referred to a "pooled, single advertiser" since it has not been defined yet at that moment. It now reads:

   "Finally, our numerical results show that the newsvendor prefers an equivalent single advertiser to multiple advertisers due to the pooling effect."

   In the numerical section, we define that the equivalent single advertiser has a probability distribution of benefit types that is the convolution of the benefit types of two advertisers. In order to have comparable situations for the multiple and single advertiser case, the equivalent single advertiser's benefit type is the sum of the individual benefit types because the total transfer fee earned and the total benefit accrued are additive.

4. *Provide a more comprehensive list of notation, perhaps broken into categories, because this paper is extremely dense in notation. The result is that the reader must go back and search for what each incarnation of θ means.*

   We have taken your advice to expand our list of notation and have included the cutoff and break-even benefit type in that list.

5. *Set off the single advertiser case on page 12 by beginning a new paragraph at : "Note that the newsvendor's problem..." Throughout, you should set this off as a special case, to keep your thoughts organized and to aid the reader.*

   We have taken your advice to break that paragraph and highlight the comparison with the results from the single advertiser case, allowing us to preface Corollary 1 with a more explicit introduction.

   "Note that the newsvendor's problem with multiple advertisers reduces to a problem with a single advertiser if all other advertisers $-i$ are below their cutoff levels. The following corollary shows that the results in the multiple advertisers case can be reduced to those obtained in Wu et al. (2011) for the single advertiser case."

6. *Second paragraph under "Discussion of Modeling Results" on page 18: Add comma after 'c' to break up two independent clauses.*

We have done so.

7. *Discussions after results are not written from a managerial perspective. They restate the result. Can you explain in a more effective way?*

   We have opted to keep all the managerial discussion within the (sub)sections titled "Discussion of Modeling Results" (3.4 and 4.3) and "Numerical Analysis" (5). The discussion immediately after the theorems and corollaries express in words the meaning of the obtained results. Following your suggestion that we write from a more managerial perspective, we have made some changes to add more interpretation to our discussion though. For instance, after Theorem 2, we have added the discussion of the phenomenon of free newspapers.

Many thanks for your contributions to the improvement of this paper.

Response to Referee 3

We are very grateful for your comments throughout the several rounds of revision which have helped improve the paper. We thank you for your time and effort, as well as for your kind congratulations on our last manuscript.