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Modeling myopia: Application to non-renewable resource extraction ☆

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HIGHLIGHTS

- A model of myopia is developed in this study.
- The model is analytically and numerically convenient.
- The model is applied to a non-renewable extraction problem.
- Extraction permits may be a useful policy option.

ABSTRACT

We develop a parsimonious model of myopia with an infinitesimal period of commitment as an extension to a standard dynamic optimization in a continuous-time environment. We clearly distinguish the processes of planning future controls and choosing the current control, which makes the model both analytically and numerically convenient. In its application to a simple non-renewable resource extraction problem, we show that whether the terminal time is free or fixed determines the appropriateness of the approximation to myopic agents by constant discounting. We also show that the expiry of extraction permits may be useful in the presence of myopia.

1. Introduction

Like other animals, human beings have been observed to make *impulsive* choices. That is, they exhibit a preference for smaller–sooner rewards over larger–later rewards, but the preference ordering may change when the reward is delayed (Ainslie, 1975). For example, we may prefer \$100 today to \$120 next year while preferring \$120 in 21 years to \$100 in 20 years at the same time, even though the latter choice is equivalent to the former choice with a 20 year delay. However, such preference changes are not consistent with constant discounting. In fact, behaviors that cannot be explained by constant discounting have been found consistently in various laboratory experiments and field studies (see, for example, Ainslie, 1991, Frederick et al., 2002, Kirby and Herrnstein, 1995, Loewenstein and Thaler, 1989, DellaVigna, 2009 and the studies cited therein).

To explain this type of behavior, we can consider an agent who "overemphasizes" short-run gratification relative to long-run gains. A conventional way to model it is to introduce a parameter for "present-biasedness". For example, the quasi-hyperbolic discounting, which was proposed by Phelps and Pollak (1968) and popularized by Laibson (1997, 1998), uses a sequence of discount factors $\beta\delta, \beta\delta^2, \beta\delta^3, \ldots$, where β and δ respectively reflect the agent's present-biasedness and time preference. This model has become a standard model of non-constant discounting and been applied to a number of issues, including job search (Paserman, 2008), retirement (Diamond and Kőszegi, 2003), and addiction (Gruber and Kőszegi, 2001).

In this model, the agent at each point in time is modeled as a separate self (agent), who cares about her present and future selves. Each agent can choose her current control (action). While she cannot directly choose future controls, she can influence them by changing the state variable through her current control. The actual control chosen by each agent depends not only on her present-biasedness but also on to what degree the agent is aware of her present-biasedness. For example, a (completely) naïve agent, who is unaware of her present-biasedness, would choose her current control assuming she can make a full commitment over the entire planning time horizon. On the other hand, a (completely) *sophisticated* agent, who is fully aware of her present-biasedness, would correctly predict her future behavior and choose her current control taking the strategies of her future agents as given. As O'Donoghue and Rabin (2001) have shown, it is also possible to consider a more general type of agent, who is partially aware of her present-biasedness.

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The quasi-hyperbolic discounting model is characterized by two important features. First, each agent can commit to her current control for one period, which affects the state variable in an non-negligible manner. Second, (partially) sophisticated agents take future maximization problems as given. In a discrete-time environment with a predetermined finite time horizon, this can be solved by backward induction. However, this method is often analytically inconvenient. Further, as elaborated later, the numerical solution may also be difficult to obtain, particularly when the time horizon is free or infinite.

In this study, we propose a model of myopia, which also aims to explain the behaviors that cannot be explained within the framework of constant discounting. However, instead of taking the future maximization problem as given, we take the future control rule as given and distinguish between the decision discount rate ρ (the discount rate used for deciding the current control) and the planning discount rate $\hat{\rho}(\leq\rho)$ (the discount rate used for determining the future control rule). The former reflects the agent's belief about how she will behave in the future, whereas the latter reflects how she actually weighs the future utilities.

These two discount rates may also differ from the rational discount rate $\gamma\,(\leq\hat{\rho})$, which reflects the agent's true (or "moral") time preference and affects the agent's long-run utility.[1] This distinction may arise, for example, when the incentives that the agents (e.g., politicians) face are not aligned to the maximization of the objective functions (e.g., social welfare) that they are expected to solve. Hence, agents in our model generally choose their actions sub-optimally. Because the planning, decision and rational discount rates are the same in the standard dynamic programming equation, our model can considered its generalization. As elaborated later, our model is analytically and numerically convenient, because we can use the standard dynamic optimization technique for both predicting future controls and determining the current control.

Unlike most other models of non-constant discounting, we construct our model in a continuous-time environment. We assume that each agent can commit to her action for only an infinitesimal period of time. Therefore, unlike the quasi-hyperbolic discounting model, the current agent's action gives only a negligible impact on the stock (state) variable in our model.[2] Therefore, relaxing the constraint that the agent's current action affects the stock variable in a non-negligible manner is one important contribution of this study.

The situation considered in this study is particularly relevant in environmental and resource management problems, in which the relevant time horizon is much longer than the commitment period of the current agent. Hence, this paper is closely related to the body of literature on non-constant discounting applied to environmental and resource management problems, such as Duncan et al. (2011), Fujii and Karp (2008), Hepburn et al. (2010), Groom et al. (2005), Karp (2005) and Weitzman (2001).

We apply our model to a simple non-renewable resource extraction problem. We show that both the agent's myopia and her awareness of myopia affect her control rule and lifetime utility. We also show that their effects are crucially dependent on whether the planning time horizon is fixed or free. A second important contribution of this study is the finding that constant discounting provides a reasonably good approximation for non-constant discounting when the time horizon is fixed but this is not the case when the time horizon is free in our application.

[1] Formal definition of these discount rates are presented in the next section.

[2] We mostly use the term "stock variable" instead of "state variables" in this study because the primary example of the state variable in this study is stock of a non-renewable resource.

This paper is organized as follows: in the next section, we formally introduce our model. In Section 3, we apply our model to a simple non-renewable extraction problem and derive some analytical results for both free and fixed terminal time. In Section 4, we provide some numerical examples of the non-renewable extraction problem to illustrate the behavior of myopic agents. Section 5 provides some discussion and conclusions.

2. Model setup

Let us now formally introduce our model of myopia. Let the calendar time start at $\tau = 0$. The terminal time T may be fixed or free, but we fix T for the time being. The planning time horizon is $\mathcal{T} \equiv [0, T]$ and the stock variable (e.g., oil reserve) and control variables (e.g., oil extracted from the field) at time $\tau \in \mathcal{T}$ are $S(\tau) \in \mathcal{S}$ and $x(\tau) \in \mathcal{X}$, where $\mathcal{S}(\subset \mathbb{R})$ and $\mathcal{X}(\subset \mathbb{R})$ are the state space and the control space, respectively. The initial stock is $S_0(\in \mathcal{S})$.

The instantaneous utility function is $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$, whereas the transition function is $g : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$, which gives the time-derivative of the stock variable. There may be I $(<\infty)$ inequality constraints on the control and stock variables such that we must have $h^i(x(t), S(t)) \geq 0$ for $\forall i \in \mathcal{I} (\equiv \{1, \dots, I\})$ and $\forall t \in \mathcal{T}$, where $h^i : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$. We use the subscript to denote partial derivatives (e.g., $f_S(x, S) \equiv \partial f(x, S)/\partial S$). To simplify the presentation, we denote the set of all constraints from t_1 to t_2 (i.e., $h^i(x(t), S(t)) \geq 0,\ \forall t \in [t_1, t_2],\ \forall i \in \mathcal{I}$) by $\mathcal{H}_{t_1}^{t_2}$ and the transition equation from t_1 to t_2 (i.e., $\dot{S}(t) = g(x(t), S(t)),\ \forall t \in [t_1, t_2)$) by $\mathcal{G}_{t_1}^{t_2}$.

At time $\tau \in \mathcal{T}$, the agent takes as given the value of the stock variable S_τ. The agent does not have a commitment technology, and thus her decision is binding only for the current control. The agent's utility has two components, one coming from the stream of utility until the terminal time T and the other from leaving the stock $S(T)$ at time T. We write her utility $U : \mathcal{X}^{\mathcal{T}} \times \mathcal{T} \times \mathcal{S} \times \mathbb{R}_{++} \to \mathbb{R}$ in the following way:

$$U(x, \tau, S_\tau, \gamma) \equiv \left(\int_\tau^T f(x(t), S(t)) e^{-\gamma(t-\tau)} \mathrm{d}t + \phi(S(T)) e^{-\gamma(T-\tau)} \right), \tag{1}$$

where $\phi : \mathcal{S} \to \mathbb{R}$ is the salvage value function and γ is the *rational* discount rate, which a rational agent would use for predicting future controls and choosing the current control. Hereafter, when the last argument of U is γ, U reflects the hedonic utility that the agent actually experiences. We shall call the hedonic utility evaluated at $\tau = 0$ the lifetime utility. We shall use this measure only to compare the hedonic utility across different types at a given point in time.[3]

To keep the analysis straightforward, we assume three regularity conditions throughout this study. First, we assume that f, g, h, and ϕ are twice continuously differentiable in all the arguments. Note that we do not exclude the possibility that the control is not continuous. Second, we assume $g_x(x, S) \neq 0$ for any (x, S). That is, the control always affects the changes in the stock variable. Finally, we assume that the set of permissible controls $\mathcal{C}(S) \equiv \{\xi | h^i(\xi, S) \geq 0,\ \forall i \in \mathcal{I}\}$ is compact for any $S \in \mathcal{S}$.

Now, let us consider the behavior of the rational agent at calendar time τ. She maximizes her hedonic utility subject to relevant constraints. Therefore, a rational agent plans her control $x^R(t; \tau, S_\tau, \gamma)$ for $t \in [\tau, T]$ by maximizing Eq. (1) subject to

[3] Note that we can always "reset" the calendar time to make $\tau = 0$ and to compare the hedonic utility across different types.

the initial condition $S(\tau) = S_\tau$, the transition equation $\mathcal{G}_\tau^T$, and other constraints $\mathcal{H}_\tau^T$. Because this is just a standard dynamic optimization problem, we can define the present-value Hamiltonian $H : \mathcal{T} \times \mathcal{X} \times \mathcal{S} \times \mathbb{R} \to \mathbb{R}$ as follows:

$$H(t, x(t), S(t), \lambda(t)) \equiv f(x(t), S(t))e^{-\gamma t} + \lambda(t)g(x(t), S(t)),$$

where $\lambda(t) \in \mathbb{R}$ is a costate variable. Letting $\mu_i : \mathcal{T} \to \mathbb{R}_+$ for $\forall i \in \mathcal{I}$ be the Lagrange multiplier and $\mathrm{Ind}(\cdot)$ be the indicator function, the Lagrangian $L : \mathcal{T} \times \mathcal{X} \times \mathcal{S} \times \mathbb{R} \times \mathbb{R}_+^I \to \mathbb{R}$ can be defined as follows:

$$\begin{aligned} &L(t, x(t), S(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{I}}) \\ &\quad\equiv H(t, x(t), S(t), \lambda(t)) + \sum_{i=1}^{I} \mu_i(t)h^i(x(t), S(t)) \\ &\qquad + \mathrm{Ind}(t = T)\phi(S(T)). \end{aligned}$$

Let $x^*(t)$ for $t \in [\tau, T]$ be the argument that maximizes U and its associated trajectory of the stock be $S^*(t)$. Then, there exist λ and $\{\mu_i(t)\}_{i\in\mathcal{I}}$ that satisfy the set of conditions C1–C7 in Appendix A, which is just a variant of standard results (Kamien and Schwartz, 1991; Seierstad and Sydsaeter, 1987). To avoid unnecessary complications, we assume A1–A2 in Appendix A. Under these assumptions, both the control rule for a rational agent $x^R(t; \tau, S_\tau, \gamma) = x^*(t)$ and the set $\mathcal{J}$ of junction times in A1 are unique, and the differentiability condition C8 in Appendix A is satisfied. Because the control for a rational agent for time t planned at time τ is exactly the same as the control actually chosen at time t (i.e., $x^R(t; t, S_t, \gamma) = x^R(t; \tau, S_\tau, \gamma)$ for $\forall t \geq \tau$), we hereafter simply write $x^R(t, S_t, \gamma)$ to mean the actual control at time t.

Now, we turn to the behavior of a myopic agent. We assume that a myopic agent uses the *decision* discount rate ρ to choose the current control. That is, her control at time τ is based on the maximization of her decision utility $U(x, \tau, S_\tau, \rho)$ instead of the maximization of the hedonic utility $U(x, \tau, S_\tau, \gamma)$.

Let us begin with a completely naïve agent. She predicts her future controls based on the assumption that she will behave as a rational agent in the future. A naïve agent knows how a rational agent behaves. However, she makes a sub-optimal decision because she is not solving the right problem; she evaluates the stream of future instantaneous utility using ρ instead of γ over the planning horizon when choosing her current control.

To describe the behavior of a completely naïve agent, let us suppose for now that τ is not a junction time. Then, there exists $\omega > 0$ such that the set of binding constraints do not change between $t = \tau$ and $t = \tau + \omega$. Now, consider a very short period of time $(0 <)\Delta_\tau (\leq \omega)$ during which time she can commit her control. We shall later let $\Delta_\tau \downarrow 0$, so that her commitment is instantaneous.

We define the value function $V(\tau, S_\tau, \rho, \gamma) \equiv U(x^R(\cdot; \tau, S_\tau, \gamma), \tau, S_\tau, \rho)$, which is the decision utility that evaluates the stream of instantaneous utility using the decision discount rate ρ. Then, she will choose her current control x_τ by solving the following maximization problem:

$$\begin{aligned} W(\tau, S_\tau, \rho, \gamma, \Delta_\tau) &\equiv \max_{x_\tau}\left[\int_\tau^T f(x(t), S(t))e^{-\rho(t-\tau)}\mathrm{d}t \right.\\ &\quad \left. + \phi(S(T))e^{-\rho(T-\tau)}\right] \quad \text{s.t. } S(\tau) = S_\tau,\ \mathcal{G}_\tau^T,\ \mathcal{H}_\tau^T,\ x(t) \\ &= \begin{cases} x_\tau & \text{for } t \in [\tau, \tau + \Delta_\tau] \\ x^R(t, S(t), \gamma) & \text{for } t \in (\tau + \Delta_\tau, T] \end{cases} \\ &= \max_{x_\tau}\left[\int_\tau^{\tau+\Delta\tau} f(x_\tau, S(t))e^{-\rho(t-\tau)}\mathrm{d}t \right.\\ &\quad \left. + e^{-\rho\Delta_\tau}V(\tau + \Delta_\tau, S(\tau + \Delta_\tau), \rho, \gamma)\right] \quad \text{s.t. } S(\tau) \\ &= S_\tau,\ \mathcal{G}_\tau^{\tau+\Delta_\tau},\ \mathcal{H}_\tau^{\tau+\Delta_\tau} \\ &= V(\tau, S_\tau, \rho, \gamma) + \Delta_\tau \cdot \Big[\max_{x_\tau}\big[f(x_\tau, S_\tau) \\ &\quad + V_S(\tau, S_\tau, \rho, \gamma)\cdot g(x_\tau, S_\tau) + O(\Delta_\tau)\big] - \rho V(\tau, S_\tau, \rho, \gamma) \\ &\quad + V_\tau(\tau, S_\tau, \rho, \gamma)\Big] \quad \text{s.t. } S(\tau) = S_\tau,\ \mathcal{G}_\tau^{\tau+\Delta_\tau},\ \mathcal{H}_\tau^{\tau+\Delta_\tau}. \end{aligned} \tag{2}$$

There are seven points worth noting here. First, we can interpret the discount rate γ as a "moral" discount rate that reflects the behavior of the naïve agent's ideal self. However, the agent permits herself to behave as she likes and evaluates the expected future outcomes with her "actual" discount rate ρ, even though she knows what the ideal self would do. While the term naïve is used in this paper to be consistent with the literature, one might call our version of naïve agents ultra-optimistic agents in the sense that they presume that the future selves will behave exactly like their ideal selves. Alternatively, the naïve agents may be considered paranoiac, because the naïve agents believe that their future behavior will be restricted to prevent them from optimizing their decision utility even though there is no such restriction whatsoever.

Second, the control $x(t)$ may be discontinuous at $t = \tau + \Delta_\tau$. This discontinuity does not pose a problem because the future control is just a plan, whereas the current control is the action taken by the agent. Thus, the control $x(t)$ for time $t\ (>\tau)$ planned at time τ is generally different from the control actually chosen when time t comes.

Third, the process of planning future controls (i.e., maximizing the hedonic utility) is clearly distinguished from that of choosing the current control (i.e., maximizing the decision utility). The naïve agent is influenced by myopia only in the latter because her future planned controls are identical to those of the rational agents. Note that the naïve agent essentially solves two maximization problems; she first maximizes the hedonic utility to find the planned future controls. Using them, she maximizes the decision utility to determine the current control. This approach is analytically convenient because we can use the standard optimization techniques in each step. As we shall show in the next section, we can obtain an analytical result for the control variable. In contrast, the standard quasi-hyperbolic discounting requires us to solve only one maximization but the standard techniques of dynamic programming are not directly applicable.

Fourth, the constraints $\mathcal{G}_{\tau+\Delta_\tau}^T$ and $\mathcal{H}_{\tau+\Delta_\tau}^T$ are automatically satisfied by the construction of x^R. Thus, we do not need to explicitly include them in Eq. (2). Fifth, in the maximand of Eq. (2), V and V_τ are independent of x_τ, and thus can be removed from the maximization operator. Sixth, V is differentiable with respect to S_τ (i.e., V_S exists) on $[\tau, \tau + \Delta_\tau]$ for small enough $\Delta_\tau (< \omega)$ and that V_S is continuous because of condition C8 in Appendix A. Seventh, x_τ implicitly depends on Δ_τ, but we are ultimately only interested in the limit as $\Delta_\tau \downarrow 0$. Therefore, we do not explicitly include Δ_τ in its argument.

To find the limit, we can use an argument similar to the derivation of the dynamic programming equation. Notice first that $W(\tau, S_\tau, \rho, \gamma, 0) = V(\tau, S_\tau, \rho, \gamma)$ holds. Hence, subtracting $V(\tau, S_\tau, \rho, \gamma)$ from both sides of equality in Eq. (2), dividing by Δ_τ and letting $\Delta_\tau \downarrow 0$, we have the following:

$$\begin{aligned} &W_{\Delta_\tau}^+(\tau, S_\tau, \rho, \gamma, 0) + \rho V(\tau, S_\tau, \rho, \gamma) - V_\tau(\tau, S_\tau, \rho, \gamma) \\ &\quad = \max_{x_\tau}[f(x_\tau, S_\tau) + V_S(\tau, S_\tau, \rho, \gamma)\cdot g(x_\tau, S_\tau)] \\ &\qquad \text{s.t. } S(\tau) = S_\tau,\ h^i(x_\tau, S_\tau) \geq 0 \quad \text{for } \forall i \in \mathcal{I}, \end{aligned} \tag{3}$$

where $W_{\Delta_\tau}^+$ is the right-derivative of W with respect to Δ_τ.

The maximization problem on the right-hand side is a standard problem and therefore can be solved with the standard method using the Lagrangian. Letting the maximizing argument be x_τ^{**}, the (planned) control schedule of a naïve agent is as follows:

$$\tilde{x}^N(t;\tau,S_\tau,\gamma,\rho) = \begin{cases} x_\tau^{**} & \text{for } t=\tau \\ x^R(t,S(t),\gamma,T) & \text{for } t\in(\tau,T]. \end{cases}$$

Note that $\tilde{x}^N(t;\tau,S_\tau,\gamma,\rho)$ for $t>\tau$ is simply a planned future control. We distinguish this from the actual control of the naïve agent at time τ, which we denote by $x^N(\tau,S_\tau,\gamma,\rho)$. In general, we have $x^N(\tau,S_\tau,\gamma,\rho) = \tilde{x}^N(t;\tau,S_\tau,\gamma,\rho)$ at $t=\tau$, but $x^N(\tau,S_\tau,\gamma,\rho) \neq \tilde{x}^N(t;\tau,S_\tau,\gamma,\rho)$ for $t>\tau$.

In a special case where there is no binding constraint (i.e., $\mu_i^N = 0$ for $i\in\mathcal{I}$), the first order condition for the maximization problem in Eq. (3) is as follows:

$$f_x(x_\tau,S_\tau) + V_S(\tau,S_\tau,\rho,\gamma)g_x(x_\tau,S_\tau) = 0. \quad (4)$$

Eq. (4) permits the usual interpretation. That is, marginal gains from increasing one unit of control are equal to the marginal cost (as measured by the marginal decrease in the decision utility) via marginal changes in the stock variable.

The difference between Eq. (4) and the first order conditions in the standard dynamic programming model is the presence of the decision discount rate ρ. As with the standard setting, the myopic agent plans her future controls with the rational discount rate γ because she believes she will behave like a rational agent. However, the stream of utility generated from her control schedule is discounted with ρ when choosing her current control.

Several observations and cautions are in order. First, one should note that the hedonic utility for the naïve agent is given by $U(x^N(t,S_t,\gamma,\rho),\tau,S_\tau,\gamma)$, and it is different from the decision utility. Because the hedonic utility of the naïve agent is maximized by the rational agent, the myopic agent generally takes a suboptimal control.

Second, note that the agent can make a binding decision only for the current control when $\Delta_\tau \downarrow 0$. Hence, the current control has no first-order effect on S or U. This contrasts with other discrete-time models of non-constant discounting, in which the current control has a first-order impact on these variables and thus the present-value of the stream of future instantaneous utility is directly altered by the current control.

Third, the future plans for the rational and completely naïve agents are identical. However, the control x^R for the rational agent is different from the control x^N for the naïve agent because the discount rates in their decision utility are different. Of course, they coincide when $\rho=\gamma$ and thus we have $x^R(\tau,S_\tau,\gamma) = x^N(\tau,S_\tau,\gamma,\gamma)$.

Fourth, the above discussion holds for autonomous problems without any major modifications. One notable difference is that the arguments τ in x, W and U will be unnecessary and the salvage value function disappears from the equations.

Thus far, we have assumed that the completely naïve agent is completely unaware of the myopia. This assumption may be too strong, because the agent may be aware her myopia. Let us therefore consider another extreme case in which the agent knows exactly how she will behave in the future. Such a completely sophisticated agent would, in principle, take her future behavior as given, realizing that the future agent will choose the action so as to maximize the decision utility instead of the hedonic utility.

Hence, the completely sophisticated agent would use ρ both to plan her future controls and to choose the current control. In effect, her behavioral pattern is the same as that of a rational agent, except the relevant discount rate for choosing her behavior is ρ instead of γ. In other words, a completely sophisticated agent with $\rho^S = r$ (and $\gamma^S \neq r$) is observationally equivalent to a rational agent with a *rational* discount $\gamma^R = r$ in our model, unless we can directly measure their hedonic utility. Thus, the control x^S of a completely sophisticated agent is given by $x^R(\tau,S_\tau,\rho)$, which does not have γ in its argument.

Note that the hedonic utility of the sophisticated agent is $U(x^R(t;\tau,S_\tau,\rho^S),\tau,S_\tau,\gamma^S)$, which obviously depends on γ^S. In contrast, the hedonic utility for an observationally equivalent rational agent is $U(x^R(t;\tau,S_\tau,\rho^R),\tau,S_\tau,\rho^R)$. Because $r=\rho^R\neq\gamma^S$, the hedonic utility of the sophisticated agent is not the same as that of the observationally-equivalent rational agent. For a given rational discount rate, the rational agent always achieves the highest utility.

Completely naïve and sophisticated agents both take future behavioral patterns as given. The difference is the discount rate used in the decision utility to plan their future controls. The completely naïve agent incorrectly assumes that she would use γ to choose her future controls, whereas the completely sophisticated agent correctly uses ρ for that purpose. It would be natural to consider a generalized myopic agent who uses the decision discount rate ρ to choose the current control but uses the *planning* discount rate $\hat{\rho}\in[\gamma,\rho]$ to plan her future controls. The rational, completely naïve and completely sophisticated agents are subsumed into this generalized agent; a rational agent is the generalized agent with $\gamma=\rho=\hat{\rho}$, because she correctly predicts the discount rate she uses to plan her future controls, which corresponds to her rational discount rate. Similarly, the completely naïve agent corresponds to the one with $\gamma=\hat{\rho}$, and the completely sophisticated agent corresponds to $\rho=\hat{\rho}$. Note that the current control chosen by the generalized agent is based on the limit of $W(\tau,S_\tau,\rho,\hat{\rho},\Delta_\tau)$ as $\Delta_\tau\downarrow 0$. Thus, the maximization problem that the generalized agent uses to find her control is obtained by replacing γ with $\hat{\rho}$ in Eq. (3).

Now, let us drop the assumption that τ is not a junction time and consider the problem for a generalized agent when τ is a junction time. The set of binding constraints changes at time τ, so V_S does not exist in general. When V_S does not exist, τ will not be a junction time once S_τ is perturbed by a small amount (otherwise A1 in Appendix A would be violated). This in turn means that the right-derivative V_S^+ and the left-derivative V_S^- exist. Therefore, Eq. (3) holds even when τ is a junction time once V_S is replaced by a one-sided derivative. That is, we can use V_S^+ [V_S^-] when $S_{\tau+\Delta\tau}$ approaches S_τ from above [below] as $\Delta_\tau\downarrow 0$. Thus, when $g(x_\tau,S_\tau)$ is strictly positive [negative], we can replace V_S in Eq. (3) by V_S^+ [V_S^-]. When $g(x_\tau,S_\tau)=0$, the term $V_S\cdot g\cdot\Delta_\tau$ drops out as $\Delta_\tau\downarrow 0$. We can now rewrite Eq. (3) for a generalized agent allowing for the possibility of junction time as follows:

$$\begin{aligned} &W_{\Delta_\tau}^+(\tau,S_\tau,\rho,\hat{\rho},0) + \rho V(\tau,S_\tau,\rho,\hat{\rho}) - V_\tau(\tau,S_\tau,\rho,\hat{\rho}) \\ &\quad = \max_{x_\tau}\Big[f(x_\tau,S_\tau) + g(x_\tau,S_\tau) \\ &\qquad \cdot\big\{V_S^+(\tau,S_\tau,\rho,\hat{\rho})\cdot\mathrm{Ind}(g(x_\tau,S_\tau)>0) \\ &\qquad + V_S^-(\tau,S_\tau,\rho,\hat{\rho})\cdot\mathrm{Ind}(g(x_\tau,S_\tau)<0)\big\}\Big] \\ &\qquad \text{s.t. } h^i(x_\tau,S_\tau)\geq 0 \quad \text{for } \forall i\in\mathcal{I} \end{aligned} \quad (5)$$

when there is no binding constraint and τ is not a junction time, we have the following generalization of the first order condition in Eq. (4), which is applicable to the case of a generalized agent:

$$0 = f_x(x_\tau,S_\tau) + V_S(\tau,S_\tau,\rho,\hat{\rho})\cdot g_x(x_\tau,S_\tau). \quad (6)$$

By solving Eq. (5), we can again find the control $x^G(\tau,S_\tau,\rho,\hat{\rho})$ for the generalized agent at time τ. Note here that the rational discount rate γ does not enter into x^G. Plugging x^G into the definition of hedonic utility, we have the hedonic utility $U(x^G(t,S_t,\hat{\rho},\rho),\tau,S_\tau,\gamma)$.

In principle, solving for x^G is straightforward, especially when there is no binding constraint. We can first find the control for the rational agent whose rational discount rate is $\hat{\rho}$. We can then calculate the value function V and solve the first order condition Eq. (6). By plugging into the transition function the control expressed as a function of the stock and time, we obtain a differential equation for the stock. Solving this, we find the time-evolution of the stock, which in turn allows us to find the time-evolution of the control. It is also straightforward to calculate the hedonic utility.

It is often the case, however, that an analytic solution does not exist. In this case, we need to find the solution numerically. Our model is convenient for numerical simulation as well because we can simulate the time-evolution of state and control in a forward-looking manner.

That is, we first identify (an approximant of) the control for each state for the completely sophisticated agent whose planning discount rate is $\hat{\rho}$ using a standard numerical method. Second, we numerically calculate the value function V for the myopic agent at $\tau = 0$. Third, we find the numerical derivative of V with respect to the (initial) stock. Fourth, we solve for the control at time $\tau = 0$ by plugging V_S in Eq. (6). Fifth, substituting the initial control in the transition function, we calculate the new stock after a sufficiently small period of time. Sixth, taking the new stock as the initial stock, we advance the time by one time step. Repeating the same process until T, we can find the time-evolution of the stock and control.

Notice that a solution method like this is invalid in the standard quasi-hyperbolic discounting model, because the control for each time period must be calculated by backward induction and the hedonic utility must be recalculated each time we go back in time by one period. Further, there is no obvious method of computation in the quasi-hyperbolic discounting model when the time horizon is infinite. If the time horizon is finite but free, backward induction may be computationally demanding, if not impossible, because there may be multiple candidate terminal time periods, in which case we need to conduct backward induction for each candidate. In contrast, the time horizon can be free or arbitrarily long in our model, though the computational errors tend to increase when the time horizon is longer.

While we have taken T as fixed so far, our discussion holds only with a minor modification even when T is free and there is a terminal condition for the management problem. For example, the agent's resource management problem may disappear once the resource is exhausted (i.e., $S(T) = 0$). In this case, the terminal time is a choice variable, and the Lagrangian would include a term for the terminal condition. The agent plans (predicts) her controls and the terminal time T at the same time. Rational and sophisticated agents correctly predict their future controls and the terminal time. However, the planned (predicted) terminal time $\hat{T}$ is generally different from the actual terminal time T for (partially) naïve agents. $\hat{T}$ depends on $\hat{\rho}$, τ and S_τ, whereas T depends on $\hat{\rho}$, ρ and S_0. Also note that the decision utility at time τ is dependent on $\hat{T}$ but not on T. The argument we used to derive Eq. (5) is still valid because the current control x_τ has no first-order effect on V through the changes in $\hat{T}$. In the next section, we demonstrate the application of our model to a non-renewable resource extraction problem and highlight the difference between free T and fixed T cases.

3. Application to non-renewable resource extraction

As an illustration, let us consider the simple non-renewable resource extraction problem first considered by Hotelling (1931). We use this example because it is well known and analytically tractable. Further, both fixed T and free T are plausible in this problem. In this section, we analytically derive various agents' behavior. We provide a numerical example in the next section.

Suppose that an agent in the public sector (a social planner) is responsible for managing a non-renewable resource (e.g., oil). She wants to extract the resource over time to maximize the social surplus. To simplify the problem, we assume the marginal cost of extraction is constant at $c(\geq 0)$ and that the agent observes the stock (e.g., oil in the ground) at each point in time. She faces a linear demand curve $d_0 - b \cdot (p(t) + c)$, where $p(t)(\geq 0)$ denotes the economic rent on one unit of the resource at time t, and d_0, b and c are positive constants. The agent chooses a non-negative amount of extraction $x(t)$ at each point in time. We also assume that there is no salvage value of the stock and thus $\phi(\cdot) = 0$.

The instantaneous utility of the agent, or the social surplus at time t, is given by $f(x(t)) = (2ax(t) - x^2(t))/2b$, where $a \equiv d_0 - bc$ is assumed to be positive. The stock is depleted by the amount extracted so that $g(x(t)) = -x(t)$, where the control must satisfy $0 \leq x(t) \leq a$ by the non-negativity assumptions for the control and the price. Further, we require that the stock variable is always non-negative (i.e., $S(t) \geq 0$). In what follows, we only consider the cases where these constraints are not binding except at the end of the planning horizon. When these constraints are not binding, the stock must be scarce so that $S_0 < aT$. Further, the stock must be depleted at the end of the planning horizon and thus the terminal condition is $S(T) = 0$. Note that f and g are independent of the stock level in this problem.

As in the previous section, we begin with a fixed T and a rational agent. The rational agent's problem is a standard non-renewable resource extraction problem. That is, she wants to solve the following maximization problem.

$$V(\tau, S_\tau, \gamma, \gamma) \equiv \max_{x(t)} \int_\tau^T \frac{2ax(t) - x^2(t)}{2b} e^{-\gamma(t-\tau)} \mathrm{d}t$$

$$\text{s.t. } \dot{S} = -x(t),\ S(\tau) = S_\tau. \quad (7)$$

Hence, we can use the standard technique of optimal control to solve the problem. The solution to this problem is as follows:

$$x^R(t; \tau, S_\tau \gamma) = a - \frac{\gamma(a(T-\tau) - S_\tau)}{e^{\gamma(T-\tau)} - 1} e^{\gamma(t-\tau)} \quad \text{and} \quad (8)$$

$$V(\tau, S_\tau, \gamma, \gamma) = \frac{1}{2b\gamma}\left(a^2(1 - e^{-\gamma(T-\tau)}) - \frac{\gamma^2(a(T-\tau) - S_\tau)^2}{e^{\gamma(T-\tau)} - 1}\right). \quad (9)$$

Now, let us consider the problem of the generalized agent with the rational discount rate γ, decision discount rate ρ and planning discount rate $\hat{\rho}$. Her planned control at time $t > \tau$ is given by $x^R(t; \tau, S_\tau, \hat{\rho})$. Hereafter, we assume that $\rho \neq 2\hat{\rho}$ to keep our presentation simple.[4] The decision of the generalized agent is based on the following present-discounted value of the stream of future instantaneous utility evaluated under the decision discount rate:

$$\begin{aligned} V(\tau, S_\tau, \rho, \hat{\rho}) &= \int_\tau^T f(x^R(t; \tau, S_\tau, \hat{\rho})) e^{-\rho(t-\tau)} \mathrm{d}t \\ &= \frac{1}{2b}\left(\frac{a^2(1 - e^{-\rho(T-\tau)})}{\rho} + \frac{\hat{\rho}^2(a(T-\tau) - S_\tau)^2(1 - e^{(2\hat{\rho}-\rho)(T-\tau)})}{(e^{\hat{\rho}(T-\tau)} - 1)^2(2\hat{\rho} - \rho)}\right). \end{aligned}$$

One can easily verify that this is an extension of Eq. (9) by setting $\gamma = \rho = \hat{\rho}$. The first order condition Eq. (6) gives us $(a - x)/b = V_S(\tau, S_\tau, \rho, \hat{\rho})$. Therefore, taking the partial derivative of V with

[4] Even when $\rho \neq 2\hat{\rho}$, our results hold as a limiting case when $\rho - 2\hat{\rho} \to 0$.

Table 1
The lifetime utility $U(x^G(0, S_0, \hat{\rho}, \rho), 0, S_0, \gamma)$ for various types of agents. In each cell, the label for each agent is given in the top row, the lifetime utility for fixed T is given in the bottom left row and the lifetime utility for free T is given in the bottom right row.

	$\hat{\rho} = 0.01$		$\hat{\rho} = 0.07$		$\hat{\rho} = 0.20$		$\hat{\rho} = 0.50$	
$\rho = 0.01$	R		–		–		–	
	4.7106	6.7164						
$\rho = 0.07$	N1		S1		–		–	
	3.5948	3.1445	3.5613	3.9953				
$\rho = 0.20$	N2		P1		S2		–	
	2.3247	1.5860	2.2267	2.0788	2.1460	2.1861		
$\rho = 0.50$	N3		P2		P3		S3	
	1.3952	1.1120	1.2833	1.1450	1.1119	1.0470	0.9916	0.9918

respect to S_τ and plugging this in the first order condition, we have:

$$x^G(\tau, S_\tau, \rho, \hat{\rho}) = a + \frac{\hat{\rho}^2(1 - e^{(2\hat{\rho}-\rho)(T-\tau)})(a(T-\tau) - S_\tau)}{(2\hat{\rho} - \rho)(e^{\hat{\rho}(T-\tau)} - 1)^2}. \quad (10)$$

By the transition equation, we know that $dS(\tau)/d\tau = -x^G(\tau, S(\tau), \rho, \hat{\rho})$ holds. This differential equation can be solved analytically, and we can express the time-evolution of the stock variable as a function of $t, S_0, \hat{\rho}$ and ρ. The derivation of the analytic solution is given in Appendix B. Plugging the stock variable back in Eq. (10), we obtain the control as a function of time $t, S_0, \hat{\rho}$ and ρ, which in turn allows us to calculate the hedonic utility $U(x^G(t, S_t, \rho, \hat{\rho}), 0, S_0, \gamma)$. Thus, in the fixed T case, a closed-form analytic solution can be obtained. As shown in Appendix C, x^G is increasing in $\hat{\rho}$, a point which we get back to in the next section.

Now, let T be free and consider once again a rational agent. In this case, T depends on her rational discount rate and the initial stock. Further, at each point in time, the remaining time until the end of the planning horizon predicted by a rational agent depends only on the rational discount rate and the remaining stock. Hence, let the remaining time predicted by the rational agent be $r : \mathcal{S} \times \mathbb{R}_+ \to \mathbb{R}$. r does not have τ in its argument because the predicted remaining time does not depend on the calendar time at which the prediction is made. For a rational agent, the predicted remaining time is the same as the actual remaining time such that the predicted terminal time $\hat{T}$ satisfies $\hat{T} \equiv r(S_\tau, \gamma) + \tau = T$ for $\forall\tau \in \mathcal{T}$. Noting that the Hamiltonian evaluated at time T along the optimal solution for Eq. (7) is zero, we have the following results:

$$x^R(t; S_{\tau,}\gamma) = a(1 - e^{-\gamma(r(S_\tau,\gamma)-t)}) \quad \text{and}$$

$$V(S_\tau, \gamma) = \frac{a^2}{2b\gamma}(1 - e^{-\gamma r(S_\tau,\gamma)})^2,$$

where $r(S_\tau, \gamma)$ satisfies

$$S_\tau = ar(S_\tau, \gamma) - \frac{a}{\gamma}(1 - e^{-\gamma r(S_\tau,\gamma)}). \quad (11)$$

Now, let us consider a generalized agent. We know that the control plan for the general agent is given by $x^R(t; S_\tau, \hat{\rho})$. Hence, we can simply plug this in the definition of V to find the value function:

$$V(S_\tau, \hat{\rho}, \rho) = \frac{a^2}{2b} \cdot \left(\frac{1 - e^{-\rho r(S_\tau,\hat{\rho})}}{\rho} + \frac{e^{-2\hat{\rho} r(S_\tau,\hat{\rho})} - e^{-\rho r(S_\tau,\hat{\rho})}}{2\hat{\rho} - \rho} \right).$$

Hence, using the first order condition Eq. (6), we obtain

$$x^G(S_\tau, \rho, \hat{\rho}) = a\left(1 - \frac{\hat{\rho}(e^{-\rho r(S_\tau,\hat{\rho})} - e^{-2\hat{\rho} r(S_\tau,\hat{\rho})})}{(2\hat{\rho} - \rho)(1 - e^{-\hat{\rho} r(S_\tau,\hat{\rho})})}\right). \quad (12)$$

We do not have a closed-form solution for the time-evolution of x or S because we do not have an analytic expression for r. However, noting that $dS(\tau)/d\tau = -x^G(S(\tau), \rho, \hat{\rho})$, we have the differential equation of S_τ with respect to τ. This and the initial condition $S(0) = S_0$ allow us to follow the time-evolution of r in a forward-looking manner.

It is important to keep in mind that r is the predicted remaining time horizon based on the planning discount rate $\hat{\rho}$. The actual terminal time $T(S_0, \hat{\rho}, \rho)$ at which the agent finishes extracting the resource can be found by solving for T in $S(T, S_0, \rho, \hat{\rho}) = 0$. In the next section, we demonstrate this distinction with a numerical example.

4. Numerical example

In this section, we provide a numerical example of the non-renewable resource extraction problem discussed in the previous section. The purpose of this section is to show that a surprising behavioral pattern can emerge under our simple model. In particular, we find the extraction path is very different between the fixed T and free T cases, a finding that is unexpected from the constant discounting ("rational") case.

We can set $a = b = 1$ without loss of generality by changing the units for the price and the stock. The ratio S_0/a measures the length of time for which the resource lasts under open access, where we take the unit of time to be a decade. We set $S_0 = 9$ in this example. The rational discount rate is set at $\gamma = 0.01$. We take 0.01, 0.07, 0.20 and 0.50 as the values of the decision and planning discount rates to demonstrate the effects of various degrees of myopia.[5] For the fixed T case, we let T be 10 decades.

To compute the lifetime utility of the agent, we first discretize the time, and compute the stock and control at each point in time starting from $\tau = 0$. We compute the time-evolution of the stock using the fourth-order Runge–Kutta method for each type of agent. We then evaluate the stream of instantaneous utility in the middle of the time step and sum over the entire planning horizon. We set the time step to be small enough so that we have sufficiently accurate numbers.[6]

In Table 1, we report the lifetime utility for various agents. In the upper row of each cell, we give the label for each agent. For example, the agent with $\rho = 0.20$ and $\hat{\rho} = 0.07$ is referred to as P1 agent. We use the prefix R, N, S and P for rational, completely naïve, completely sophisticated and partially naïve (and partially sophisticated) agents, respectively. In the lower row of each cell, the lifetime utility for the fixed T (left) and free T (right) are reported. For example, P1's lifetime utility is 2.2267 when T is fixed and 2.0788 when T is free.

There are three points worth noting here. First, we are not interested in the magnitude of the lifetime utility *per se*, but we

5 These values are 0.10, 0.68, 1.84, and 4.14% per annum, which are similar to the discount rates used in various other studies.

6 The time step we use is at most 10^{-6} decades for all of the cases we considered. We compare the calculated figures with the analytic solution whenever possible and the comparison suggests that the reported figures are sufficiently accurate.

can take it as a convenient summary statistic. We know that the rational agent's behavior maximizes the hedonic utility by construction. Further, the control chosen by the rational agent is Pareto-efficient in the sense that it is not possible to increase the hedonic utility for any self without decreasing the hedonic utility of at least one other self. Thus, the lifetime utility provides a useful benchmark for welfare comparison.

Second, the agent's lifetime utility depends on whether T is fixed or free. The difference is particularly large for the rational agent. R agent takes 45.651 decades to deplete the resource when T is free, but she is forced to deplete the resource in 10 decades when T is fixed. As a result, her lifetime utility is much lower when T is fixed. However, it is not always the case that the free T gives a higher lifetime utility. For N3 agent, fixed T is better because her myopia is so strong that she would deplete resources too quickly if T is free. Hence, she is better off when she is forced to deplete the resource within a fixed amount of time because the fixed T provides an implicit commitment device for myopic agents.

Third, given the level of myopia, it is not obvious whether the sophisticated agents perform better than the naïve agents. In fact, whether T is fixed or free also changes whether *knowing thyself* (i.e., being sophisticated) is a good thing. When T is fixed, naïve agents do better than sophisticated agents. For example, for agents with $\rho = 0.20$, N2 has the highest lifetime utility followed by P1 and S2 under the fixed T. When T is free, however, the converse is true. For agents with $\rho = 0.50$, the situation is not as simple. When T is fixed, N3 does better than P2, P3 and S3. However, when T is free, $P2$ has the highest utility followed by N3, P3 and S3.

To make sense of this result, let us first consider a sophisticated agent in the fixed T environment. Her future controls are chosen to maximize the stream of instantaneous utility evaluated at her planning discount rate $\hat{\rho} = \rho$. Now, suppose that she suddenly becomes a naïve agent (i.e. $\hat{\rho}$ changes from ρ to $\gamma(< \rho)$). Then, her future planned controls will no longer maximize the stream of instantaneous utility discounted to the present-value by ρ. As a result, the marginal decision utility V_S of stock will become higher. This in turn leads to a slower resource extraction by the (naïve) agent in the current period as formally shown in Appendix C. This helps the naïve agent to extract the resources more smoothly over time than the sophisticated agent. As a result, the naïve agent enjoys a higher lifetime utility than the sophisticated agent.

The above argument may still hold when T is free. However, we also need to take the opposite effect into consideration. When the agent is naïve, her predicted remaining time r is much longer than for the sophisticated agent with the same ρ. Because the rational agent extracts resources very slowly, the naïve agent also plans to do so. This means that a marginal increase in the stock will translate into a small increase in extraction averaged over a long time horizon. Hence, the current marginal value of stock V_S may be smaller for a naïve agent than a sophisticated agent. Therefore, the naïve agent may extract more in the current period and decrease her lifetime utility as a result.

Let us now look at the time evolution of the control and stock variables for both cases. As Fig. 1(a) shows, the differences in the trajectory of $S(\tau)$ across different types of agents are small when T is fixed. This is not very surprising given that the planning period is relatively short, even though the discount rate varies among agents. Fig. 1(b) shows the time evolution of $S(\tau)$ when T is free. We restricted the time domain for this graph to clearly present the differences in controls among the agents. Unlike the fixed T case, the trajectories look very different across different types. When we look at the control $x(\tau)$, the differences among agents are sharper. For the fixed T case, when ρ is close to γ, S1 and N1 agents behave in a similar manner. However, when ρ gets larger, their controls are strikingly different. N3 agents start with less resource extraction because $S(\tau)$ does not decline as fast as S3, as shown in Fig. 2(a).

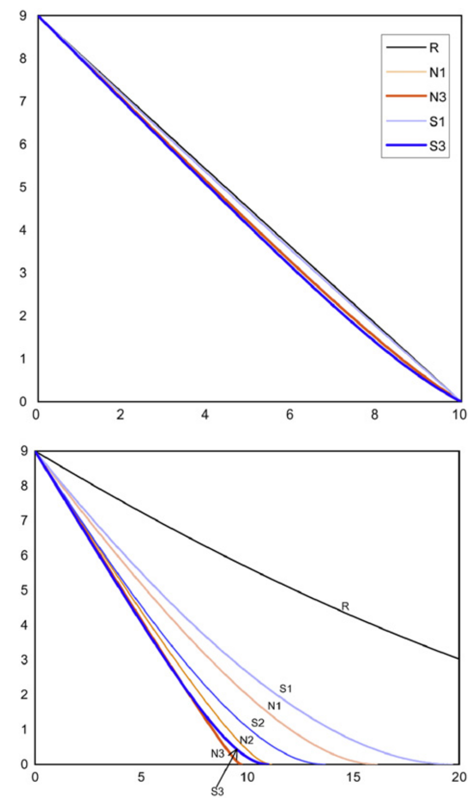

Fig. 1. Graph of $S(\tau)$ for selected types of agents (a) when T is fixed (top) and (b) when T is free (bottom).

When T is free, the controls for the naïve and sophisticated agents are quite different even when ρ is small as shown in Fig. 2(b). The control for naïve agents declines much more rapidly than for sophisticated agents. This is due to the time-inconsistency of naïve agents. For example, N3's control starts below S3's control because N3 thinks that she will extract the resources very slowly. In reality, however, N3 keeps $x(\tau)$ over 0.9 until just before the stock is depleted. The degree of myopia is so high that the graphs of $x(\tau)$ for S3 and N3 cross each other twice. The time-inconsistency of myopic agents can clearly be seen in Fig. 3. The horizontal axis measures the actual time τ elapsed, whereas the vertical axis measures the predicted terminal time $\hat{T}(\tau, S_\tau, \hat{\rho})$ at time τ. When the graph hits the diagonal 45° line, the extraction ends. The rational and sophisticated agents are time-consistent and thus have a horizontal graph of $\hat{T}$. In contrast, $\hat{T}$ declines over time for naïve agents because they extract resources faster than they planned. Table 2 gives the predicted extraction time $r(S_0, \rho)$ at time $\tau = 0$ (left) and the actual duration of extraction (right). These results indicate that the myopia considered in this study cannot be simply assumed away, particularly when T is free.

To underscore this point, we also look at the problem from a different angle. Assume that we can observe the control x^O_τ and stock S^O_τ at time τ. Then, we can find the observationally equivalent rational discount rate $\tilde{\gamma}(\tau)$ for time τ that is consistent with these observations such that $x^O_\tau = x^R(t, S^O_\tau, \tilde{\gamma}(\tau))$. One may expect that $\tilde{\gamma}(\tau)$ should be somewhere between γ and ρ for $\forall t \in \mathcal{T}$. This is indeed the case when τ is fixed as shown in Fig. 4(a). Surprisingly, however, this is not the case when τ is free as shown in Fig. 4(b). $\tilde{\gamma}(\tau)$ can be much higher than ρ towards the end of the extraction, which implies that the agent would show increasingly impulsive behavior.

Table 2
The predicted terminal time $\hat{T}$ at time $\tau = 0$ and the actual terminal time for all types of agents.

	$\hat{\rho} = 0.01$	$\hat{\rho} = 0.07$	$\hat{\rho} = 0.20$	$\hat{\rho} = 0.50$
$\rho = 0.01$	R 45.651 45.651	–	–	–
$\rho = 0.07$	N1 45.651 16.128	S1 19.684 19.684	–	–
$\rho = 0.20$	N2 45.651 11.152	P1 19.684 13.204	S2 13.676 13.676	–
$\rho = 0.50$	N3 45.651 9.744	P2 19.684 10.497	P3 13.676 10.870	S3 10.992 10.992

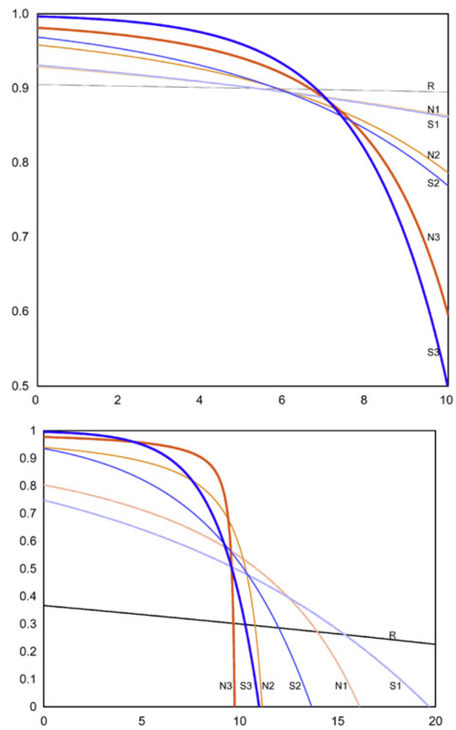

Fig. 2. Graph of $x(\tau)$ for selected types of agents (a) when T is fixed and (top) (b) when T is free (bottom).

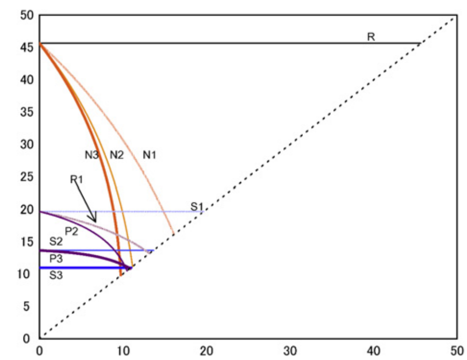

Fig. 3. The graph of $\hat{T}(\tau, S_\tau, \hat{\rho})$ against the time τ for all types of agents.

Fig. 4(b) shows that when T is free, $\tilde{\gamma}$ changes dramatically over time and may far exceed ρ. In fact, $\tilde{\gamma}$ may become so high that an implausible rate of time preference could be inferred from observations under the assumption that the agent is rational. Hence, we cannot find a rational agent that appropriately approximates the behavior of the generalized agent when T is free. This is true even when the difference between the myopic discount rate and the rational discount rate is small.

This finding is strikingly different from Fujii and Karp (2008), in which the behavior of non-constant discounter can be well approximated by a constant discounter, even when the agent is highly present-biased. This is because Fujii and Karp (2008) use a model with an infinite time horizon and thus the agent does not suffer from the wrong projection of the terminal time. In contrast, our naïve agent consistently overestimates $\hat{T}$ and heavily discounts the distant future utility at the time of decision-making. This leads to implausibly high observationally-equivalent discount rates.

Our numerical results offer some policy implications. Imagine that a government wants to sell resource extraction permits to agents who may be myopic. The government has an option to sell the permit with or without expiration. The decision of which option is better would depend on the degree of the agents' myopia. By comparing the total lifetime utility reported in Table 1, we can see which options are more desirable for which agents. It should be obvious that the free T is always desirable for the rational agent. Indeed, according to our model, this is the case for all the sophisticated agents as well but not others. This suggests that, when it is likely that the agents are more or less ignorant of their own myopia, the government should sell the extraction permits with expiry.

Why, then, are naïve agents better off with the expiry? The basic logic is similar to the theory of second best; the government may be able to improve social welfare by imposing a regulation when there is a pre-existing distortion, where the distortion in this model is myopia. The interpretation of the situation, however, depends on how the nature of myopia is understood. If we interpret the naïve agents as ultra-optimistic, the logic is as follows. When there is no expiry, current action is not so important because they think there is a long time to go, during which time they believe will behave like their ideal self. When there is an expiry, they know they do not have a lot of time to make up for the current excessive extraction.

If the naïve agents are considered paranoiac, on the other hand, the period of sub-optimal extraction from the perspective of the *decision* utility is long without an expiry. This in turn means that the marginal decision utility from current consumption is higher when there is no expiry. This leads to the rapid depletion of resource.

If we adopt this second interpretation, the results are akin to the Green Paradox, where the owner of the resource accelerates her extraction activity when a decline in the value of the resource is expected. The government may want to sell permits without expiry in the expectation that the buyer maintain the resource for a long period of time. However, this could have an effect opposite to the expectation because the naïve (and paranoiac) agents extract more today to reduce the influence of restriction.

5. Discussion and conclusions

In this study, we developed a model of myopia with an applications to non-renewable extraction. Our model is related

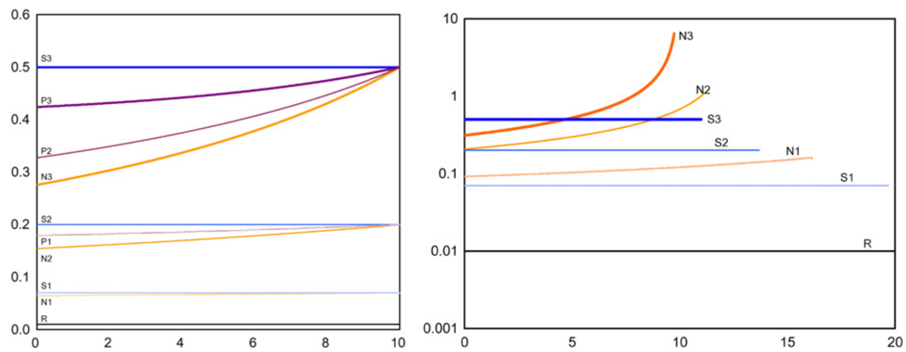

Fig. 4. Graph of the observationally equivalent rational discount rate $\tilde{\gamma}(\tau)$ against the time τ for all types of agents (a) when T is fixed (left) and (b) when T is free (right).

to the standard quasi-hyperbolic discounting model. However, in contrast to the standard model, the agent can commit to her control only for an infinitesimal period of time in our model. Unlike the standard model, the time-inconsistent behavior arises not from ignorance of the present bias but from ignorance of the decision discount rate. Our model is parsimonious, transparent, and both analytically and numerically convenient. It allows us to describe a variety of degrees of myopia. Although we applied our model to a resource management problem because the commitment period is typically very short relative to the time horizon of the problem, the model is general and potentially applicable to many other economic issues, including household saving and fiscal deficits.

Our results exhibit two features that do not appear in other models. First, by explicitly modeling the process of planned future controls, our model shows how the time-inconsistent agent adjusts her planning time horizon. For example, the model allows us to present both the predicted and actual terminal time of extraction in Fig. 3. Such a feature does not appear in a typical standard model, because T is typically fixed or infinite. Although the standard model can also incorporate such a feature, our model is more convenient for application because we can simulate the trajectory of the state and control in a forward-looking manner.

Second, our model does not suffer from the kind of multiplicity of equilibria noted in Karp (2007), where there are multiple solutions associated with a range of steady states. On the contrary, our model can guarantee a unique solution with sufficient concavity in f and g. This is an attractive feature for practical purposes because the choice of solution is usually not obvious.

We applied our model to a simple non-renewable resource extraction problem. This example offers some intriguing lessons. First, *knowing thyself* – that is, the agent knows the decision discount rate and use it for planning (i.e., $\hat{\rho} = \rho$) – does not always constitute an advantage. We saw that when T is fixed, naïve agents do better than the sophisticated ones. Naïve agents optimistically think that they will behave like a rational agent, and this optimism realizes itself. That is, optimism about the future leads the naïve agents to conserve the resource and to smooth the extraction over time. Hence, *knowing thyself* is harmful when T is fixed because a bad future prospect will realize itself.

In contrast, *knowing thyself* can be helpful when T is free. The agent may still benefit from optimism, but this is not always the case. When the agent is naïve and thinks that she behaves like a rational agent, her evaluation of the future stream of payoffs under the myopic discount rate ρ is low because it involves the payoffs in the distant future. Thus, the marginal gains from increasing the current control relative to the marginal gains in the decision utility from conserving the resource become higher when T is free. As a result, naïve agents tend to extract resources faster than sophisticated agents.

Appendix A. Conditions and assumptions for the model of myopia

C1 $\dot{\lambda}(t) = -L_S(t, x^*(t), S^*(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{I}})$.

C2 $\lambda(T) = \sum_{i=1}^{I} \mu_i(T) h_S^i(x^*(T), S^*(T)) + \phi_S(S^*(T))$ for $\forall i \in \mathcal{I}$.

C3 $L_x(t, x^*(t), S^*(t), \lambda(t), \{\mu_i(t)\}_{i\in\mathcal{I}}) = 0$ for $\forall t \in [\tau, T]$.

C4 $H(t, x^*(t), S^*(t), \lambda(t)) \geq H(t, x(t), S^*(t), \lambda(t))$ at each $t \in [\tau, T]$ satisfying $h^i(x(t), S^*(t)) \geq 0$ for $\forall i \in \{1, \ldots, I\}$.

C5 $S^*(\tau) = S_\tau$.

C6 $\dot{S}^*(t) = g(x^*(t), S^*(t))$ for $\forall t \in [\tau, T)$.

C7 $h(x^*(t), S^*(t)) \geq 0$ for $\forall i \in \mathcal{I}$ and $t \in [\tau, T]$.

C8 $S(t), \dot{S}(t)\lambda(t)$ and $\dot{\lambda}(t)$ are continuous on $[\tau, T]$ and continuously differentiable on $[\tau, T] \setminus \mathcal{J}$.

A1 There is a set of $(0 \leq)K(< \infty)$ distinct points $\mathcal{J} \equiv \{j_1, \ldots, j_K\} \subset [\tau, \mathcal{T}]$ in time such that for $\forall l \in \{1, \ldots, K\}$ and $\forall \epsilon > 0$, there exists $\delta_{l,\epsilon} \in \mathbb{R}$ such that $\mathcal{B}(j_l) \neq \mathcal{B}(j_l + \delta_{l,\epsilon})$ and $|\delta_{l,\epsilon}| < \epsilon$, where $\mathcal{B}(t) \equiv \{i \in \mathcal{I} : h^i(x(t), S(t)) = 0\}$ is the set of binding constraints. The sets $\mathcal{J}$ and $\mathcal{B}(\cdot)$ may be empty.

A2 There exists a unique quadruple $\{x^*(t), S^*(t), \lambda(t), (\mu_i)_{i=1}^{I}\}$ that maximizes Eq. (1) and satisfy C1–C7.

C1 and C2 above are the costate and transversality equations, respectively. C3 is the complementary-slackness condition for the inequality constraints. C4 is the optimality condition. C5–C7 simply require that the initial condition, the transition equation $\mathcal{G}_\tau^T$ and the constraint $\mathcal{H}_\tau^T$ are satisfied, respectively.

We assume that assumptions A1 and A2 are satisfied. A1 states that there are at most a finite number of junction times at which the set of binding constraints just changes. Under A2, the rational agent's control rule x^R and the set of junction times $\mathcal{J}$ in A1 can be defined uniquely. Oniki (1973) showed that the differentiability condition C8 is satisfied under A2.

Appendix B. Stock and control for fixed T in Section 4

Here, we consider the time-evolution of the stock and control for the general agent using Eq. (10) when T is fixed. We denote the time remaining for extraction at time τ by $r \equiv T - \tau$. Further, let us define $M \equiv ar - S(\tau)$, which is a measure of the scarcity of the stock. It is the difference between the amount of stock when the resource is not scarce and the actual remaining stock at time τ. Using the transition equation, we have $\mathrm{d}S(\tau)/\mathrm{d}r = -\mathrm{d}S(\tau)/\mathrm{d}\tau = x^G(\tau, S_\tau, \hat{\rho}, T, \rho)$. Then, subtracting a from both sides and dividing by M, Eq. (10) can be written as follows:

$$\frac{\mathrm{d}M}{\mathrm{d}r}\cdot\frac{1}{M}=\frac{\hat{\rho}^2(e^{(2\hat{\rho}-\rho)r}-1)}{(2\hat{\rho}-\rho)(e^{\hat{\rho}r}-1)^2}.$$

Integrating over r and noting that $e^{-\hat{\rho}r}<1$ for $r>0$, we have

$$\begin{aligned}\ln M &= \frac{\hat{\rho}^2}{(2\hat{\rho}-\rho)}\int\frac{(e^{(2\hat{\rho}-\rho)r}-1)}{(e^{\hat{\rho}r}-1)^2}\mathrm{d}r\\ &= -\frac{e^{(2\hat{\rho}-\rho)r}-1}{\hat{\rho}(e^{\hat{\rho}r}-1)}-r-\ln(e^{\hat{\rho}r}-1)\\ &\quad+\frac{(\hat{\rho}-\rho)}{\hat{\rho}}\int\frac{e^{(\hat{\rho}-\rho)r}}{1-e^{-\hat{\rho}r}}\mathrm{d}r\\ &= -\frac{e^{(2\hat{\rho}-\rho)r}-1}{\hat{\rho}(e^{\hat{\rho}r}-1)}-r-\ln(e^{\hat{\rho}r}-1)\\ &\quad+\frac{(\hat{\rho}-\rho)}{\hat{\rho}}\sum_{k=0}^{\infty}\frac{e^{((\hat{\rho}-\rho)-k\hat{\rho})r}}{(\hat{\rho}-\rho)-k\hat{\rho}}+A\\ &= -\frac{e^{(\hat{\rho}-\rho)r}-1}{\hat{\rho}(e^{\hat{\rho}r}-1)}-r-\ln(e^{\hat{\rho}r}-1)\\ &\quad+\frac{(\hat{\rho}-\rho)e^{(2\hat{\rho}-\rho)}}{\hat{\rho}}F\left(1,\frac{\rho}{\hat{\rho}}-2,\frac{\rho}{\hat{\rho}}-1;e^{-\hat{\rho}r}\right)+A,\qquad(*)\end{aligned}$$

where $A\in\mathbb{R}$ is a constant and $F(a_1,a_2,a_3;z)$ is a hypergeometric function defined as follows:

$$F(a_1,a_2,a_3;z)\equiv\sum_{k=0}^{\infty}\frac{\Pi_{l=0}^{k-1}(a_1+l)\cdot\Pi_{l=1}^{k-1}(a_2+l)}{\Pi_{l=1}^{k-1}(a_3+l)}\cdot\frac{z^k}{k!}.$$

Hence, letting $r=T$ (i.e., $t=0$), we have:

$$\begin{aligned}A &= \ln(aT-S_0)+\frac{e^{(\hat{\rho}-\rho)T}-1}{\hat{\rho}(e^{\hat{\rho}r}-1)}+T+\ln(e^{\hat{\rho}T}-1)\\ &\quad-\frac{(\hat{\rho}-\rho)e^{(2\hat{\rho}-\rho)}}{\hat{\rho}}F\left(1,\frac{\rho}{\hat{\rho}}-2,\frac{\rho}{\hat{\rho}}-1;e^{-\hat{\rho}T}\right).\end{aligned}$$

Plugging this back in Eq. (*) and solving for $S(\tau)$, we can write the stock as a function of S_0, τ, ρ and $\hat{\rho}$, which, in turn, allows us to write the control as a function of S_0, τ, ρ and $\hat{\rho}$ using Eq. (10). □

Appendix C. Proof of $\partial x_G/\partial\rho > 0$

Define $B\equiv 1-e^{(2\hat{\rho}-\rho)T}(<1)$ and $C\equiv e^{\hat{\rho}T}-1(>0)$. Taking the derivative of x_G with respect to $\hat{\rho}$, we have:

$$\frac{\partial x_G}{\partial\hat{\rho}}=\frac{2\ln(1+C)Z}{(\ln(1-B))^2C^3}(aT-S_0),$$

where $Z\equiv B\ln(1-B)(C-\ln(1+C))-C\ln(1+C)(B+\ln(1-B))$. We prove $\partial x_G/\partial\hat{\rho}>0$ by showing $Z>0$. Note here that $C-\ln(1+C)>0$ for all $C>0$. Therefore, if $B<0$, we have $B\ln(1-B)>0$ and $B+\ln(1-B)<0$ and thus we have $Z>0$.

Now, suppose that we have $0<B(<1)$. To prove $Z>0$ in this case, it is useful to Taylor-expand Z with respect to B around $B=0$ in the following manner:

$$\begin{aligned}Z &= \frac{1}{T}\sum_{k=0}^{\infty}\frac{B^k}{k!}\cdot\left.\frac{\partial^kZ}{\partial B^k}\right|_{B=0}\\ &= \frac{1}{T}\sum_{k=2}^{\infty}\frac{B^k}{k}\cdot\left[C\ln(1+C)+\frac{k}{k-1}(\ln(1+C)-C)\right].\end{aligned}$$

We prove $Z>0$ by showing $D_0(C)\equiv C\ln(1+C)+k/(k-1)\cdot(\ln(1+C)-C)\geq 0$ for $C>0$ and $k\geq 2$. To this end, first note $D_0(0)=0$ and $D_0'(C)=\ln(1+C)-C/((k-1)(1+C))\geq\ln(1+C)-C/(1+C)\equiv D_1(C)$. Because $D_1(0)=0$ and $D_1'(C)=C/(1+C)^2>0$ for $C>0$, we have $D_0(C)>0$ and $D_1(C)>0$ for $C>0$ and $k\geq 2$. □

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