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EXISTENCE AND UNIQUENESS RESULTS FOR SOLUTIONS TO INITIAL VALUE PROBLEMS IN SCALES OF BANACH SPACES

A THESIS PRESENTED IN PARTIAL FULFILMENT FOR
THE DEGREE OF MASTER OF SCIENCE IN
MATHEMATICS AT
MASSEY UNIVERSITY

David Warren Bulger

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Abstract

This thesis addresses existence and uniqueness of solutions to certain classes of initial-value problems with functional differential equations. The technique of scales of Banach spaces is used. A scale of Banach spaces is a collection of Banach spaces varying on a real parameter. A scale consisting of function spaces can be used to suppress one variable in an initial-value problem in a partial differential equation of two independent variables, therefore enabling local existence and uniqueness of a solution to the problem to be shown with the classical method of successive approximations from the Picard-Lindelöf Theorem of ordinary differential equations. Tuschke's presentation (c.f. [7]) of this technique and a related theorem has been adapted in Chapters 1 and 2. Chapters 3 and 4 present original theorems, stating existence and uniqueness of solutions to more general initial-value problems, having a retarded character.

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