Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: S Y Yoon *et al* 2018 *J. Phys.: Conf. Ser.* **1063** 012021

FE implementation of HAH model using FDM-based stress update algorithm for springback prediction of AHSS sheets

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Abstract. The homogeneous anisotropic hardening (HAH) model was implemented into a finite element (FE) code in order to predict springback for an advanced high strength steel (AHSS) sheet sample after double-stage U-draw bending. The finite difference method (FDM) was utilized as an alternative way to calculate the derivatives of this advanced distortional plasticity model allowing the update of the equivalent plastic strain and stress tensor at each time step in the user-material subroutines (UMAT and VUMAT). The FDM makes it easier to derive the stress gradient of complex yield surfaces. The proposed FDM-based stress update algorithm was verified by comparing the springback profiles after the single- and double-stage U-draw bending tests for a DP980 sheet sample predicted with analytical and numerical approaches. In addition, the springback measurement parameters and computational efficiencies depending on both approaches were also compared. The results indicate that the computational efficiency and accuracy of the FE simulations with the FDM-based stress update algorithm were similar to those of the analytical method.

1. Introduction

In various industrial fields, the demand for advanced high strength steels (AHSS) has increased because of the outstanding properties of these materials: high strength leading to low structural weight, and cost-effectiveness in manufacturing. However, since AHSS are multi-phase materials, they exhibit very complex deformation mechanisms including twinning or phase transformation during deformation. As a result, accurate predictions of plastic deformation in AHSS during forming processes is difficult to achieve and require advanced constitutive descriptions.

The homogeneous anisotropic hardening (HAH) approach is such an advanced model that has been employed to predict springback after U-draw bending of AHSS sheets. In order to implement an advanced constitutive model such as HAH into a FE code, the stress at each time step should be calculated through a stress update (or integration) algorithm in which the equivalent plastic strain and stress components are obtained based on the gradient of the yield surface usually derived analytically. As the constitutive models become more and more intricate to describe the complex mechanical behavior of advanced materials, the analytical derivation of the gradient gets cumbersome. As an alternative to this conventional way, numerical differentiation was used to calculate plastic strain rate in the associated flow rule [1]. Recently, a numerical differentiation method was utilized to calculate the gradient of the yield surface generated by the HAH model, which makes the implementation of this constitutive description much easier than before [2]. However, the proposed numerical differentiation was performed only after the total number of sub-steps in the stress update algorithm was defined.

In this work, the numerical derivatives of the HAH model were calculated using a finite difference method (FDM) through the cutting plane method, which is one of the most successful stress update algorithms in FE simulations. For verification purpose, the springback profiles after single- and double-U-draw bending tests for a dual-phase (DP) steel grade (DP980) sheet by computing the plastic equivalent strain and stress tensors with the introduced FDM method were compared with those computed with the analytical expressions. Then, the springback measurement parameters were also evaluated. Finally, the computational performance of the FDM-based approach was assessed.

2. Theoretical description

2.1. Constitutive model: HAH model

The HAH model [3] is an anisotropic hardening approach in which the distortion of the yield surface after strain path changes can be predicted. The constitutive equation for the effective stress is as follows:

$$\bar{\sigma}(s) = \left\{\phi^q + \phi_h^q\right\}^{\frac{1}{q}} = \left\{\phi^q + f_1^q\left|\hat{h}^s : s - \left|\hat{h}^s : s\right|\right|^q + f_2^q\left|\hat{h}^s : s + \left|\hat{h}^s : s\right|\right|^q\right\}^{\frac{1}{q}} \quad (1)$$

where the stable component, ϕ, is a homogeneous yield function and the fluctuating component, ϕ_h, distorts the yield surface generated by the stable component. The anisotropic Yld2000-2d yield function was used as a stable component in this work. s is the deviatoric part of the Cauchy stress tensor and $\hat{h}^s$ is the normalized microstructure deviator representing the microstructure evolution during the material loading history. Additional details regarding HAH are provided elsewhere [3].

2.2. Stress update algorithm: Cutting plane method

When a constitutive model is implemented in a FE code, the stress update (or integration) algorithm is necessary to get the unique solution for the given problem. The cutting plane method [4] was used to update the stress tensor, the equivalent plastic strain and all the state variables in the HAH model. The equivalent stress and associated flow rule are defined based on the incremental deformation theory [5]:

$$\bar{\sigma}(\sigma_{n+1}) = \sigma_{n+1} : \frac{\partial \bar{\sigma}_{n+1}}{\partial \sigma_{n+1}} \quad (2)$$

and, for the flow rule:

$$\Delta\varepsilon_{n+1}^p = \Delta\lambda \frac{\partial \bar{\sigma}_{n+1}}{\partial \sigma_{n+1}} \quad (3)$$

Using Eqs. (2), (3), the equivalent plastic strain increment can be derived from the plastic work equivalence:

$$\Delta\bar{\varepsilon} = \frac{\sigma : \Delta\lambda \frac{\partial \bar{\sigma}}{\partial \sigma}}{\bar{\sigma}} = \Delta\lambda \frac{\sigma : \frac{\partial \bar{\sigma}}{\partial \sigma}}{\bar{\sigma}} = \Delta\lambda \quad (4)$$

where λ is the plastic multiplier.

In addition, the stress increment is divided into an elastic predictor (ep) and a plastic corrector (pc) as:

$$\Delta\sigma = C : \left[\Delta\varepsilon - \Delta\varepsilon^p\right] = \Delta\sigma^{ep} + \Delta\sigma^{pc} \quad (5)$$

The trial stress is determined by the elastic predictor:

$$\sigma^{trial} = \sigma_n + \Delta\sigma^{ep} \quad (6)$$

In the cutting plane method, the stress at the current step is calculated by reducing the trial stress with the plastic corrector. Details about the plastic corrector and the overall correction algorithm are described in [6].

2.3. *Numerical differentiation: finite difference method (FDM)*

In the stress update algorithm such as the cutting plane method, Euler backward method, and closest point projection, the first or second derivative of a constitutive equation, $\partial\bar{\sigma}/\partial\sigma$, is required to get the equivalent plastic strain and stress increments at a sub-step, k, as follows:

$$\delta\bar{\varepsilon}_{n+1}^{(k)} = \frac{F_{n+1}^{(k)}}{\dfrac{\partial\bar{\sigma}_{n+1}^{(k)}}{\partial\sigma_{n+1}^{(k)}} : C : \dfrac{\partial\bar{\sigma}_{n+1}^{(k)}}{\partial\sigma_{n+1}^{(k)}} + \dfrac{\partial H_{n+1}^{(k)}}{\partial\bar{\varepsilon}_{n+1}^{(k)}}} \tag{7}$$

where $F_{n+1}^{(k)} = F\left(\sigma_{n+1}^{(k)}, \bar{\varepsilon}_{n+1}^{(k)}\right) = \bar{\sigma}\left(\sigma_{n+1}^{(k)}\right) - H\left(\bar{\varepsilon}_{n+1}^{(k)}\right)$,

$$\delta\sigma_{n+1}^{(k)} = -C : \delta\bar{\varepsilon}_{n+1}^{(k)} \frac{\partial\bar{\sigma}_{n+1}^{(k)}}{\partial\sigma_{n+1}^{(k)}} \tag{8}$$

Unfortunately, it is really a laborious work to analytically express the derivatives of an advanced constitutive model like HAH. Indeed, this has been an obstacle for implementation of plasticity models into finite element codes. To solve this problem, the finite difference method was applied to a multi-step Euler backward (MSEB) approach in a previous work [2]. However, here the finite difference method was applied in a different way. The five-point midpoint rule was adopted because of its high order error, $O(h^4)$:

$$f'(x_0) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right] + O(h^4) \tag{9}$$

In this article, the five-point midpoint rule was modified to calculate the partial derivatives of the constitutive equation, Eq. (1), in terms of each stress component:

$$\begin{Bmatrix} \dfrac{\partial\bar{\sigma}}{\partial\sigma_{11}} \\ \dfrac{\partial\bar{\sigma}}{\partial\sigma_{22}} \\ \dfrac{\partial\bar{\sigma}}{\partial\sigma_{12}} \end{Bmatrix} = \begin{Bmatrix} \dfrac{1}{12h_1}\left[\bar{\sigma}(\sigma_{11}-2h_1,\sigma_{22},\sigma_{12}) - 8\bar{\sigma}(\sigma_{11}-h_1,\sigma_{22},\sigma_{12}) + 8\bar{\sigma}(\sigma_{11}+h_1,\sigma_{22},\sigma_{12}) - \bar{\sigma}(\sigma_{11}+2h_1,\sigma_{22},\sigma_{12})\right] \\ \dfrac{1}{12h_2}\left[\bar{\sigma}(\sigma_{11},\sigma_{22}-2h_2,\sigma_{12}) - 8\bar{\sigma}(\sigma_{11},\sigma_{22}-h_2,\sigma_{12}) + 8\bar{\sigma}(\sigma_{11},\sigma_{22}+h_2,\sigma_{12}) - \bar{\sigma}(\sigma_{11},\sigma_{22}+2h_2,\sigma_{12})\right] \\ \dfrac{1}{12h_3}\left[\bar{\sigma}(\sigma_{11},\sigma_{22},\sigma_{12}-2h_3) - 8\bar{\sigma}(\sigma_{11},\sigma_{22},\sigma_{12}-h_3) + 8\bar{\sigma}(\sigma_{11},\sigma_{22},\sigma_{12}+h_3) - \bar{\sigma}(\sigma_{11},\sigma_{22},\sigma_{12}+2h_3)\right] \end{Bmatrix} \tag{10}$$

where the variation, h, is defined as each stress component divided by an arbitrary number, 500.

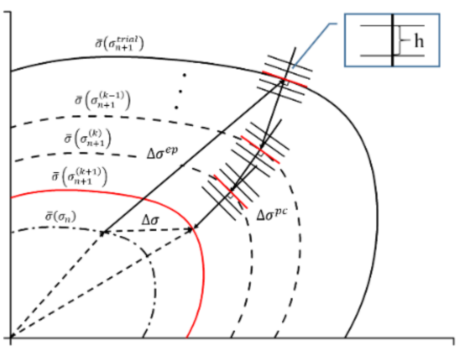

Figure 1 Schematic view of FDM-based cutting plane method

The gradients of the yield surface are defined through four imaginary stress points around the stress at a sub-step (red line) as shown in Figure 1. Therefore, the derivatives can be obtained easily with the simple equation, Eq. (10), regardless of their order. A verification of this FDM approach in the stress update algorithm is discussed in a later section.

3. FE simulation: Double-stage U-draw bending

The FE simulations for springback of the DP980 sheet after double-stage U-draw bending were conducted after implementing the HAH model into the FE analysis software, ABAQUS 6.17 through the user material subroutines (UMAT and VUMAT). The simulations of the bending process were conducted with ABAQUS/Explicit and those of springback with ABAQUS/Standard.

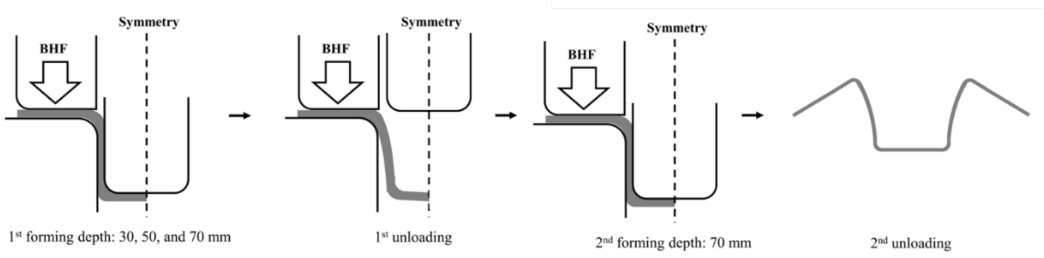

Figure 2 Process of double-stage U-draw bending [7]

The overall process of the double-stage U-draw bending is schematically illustrated in Figure 2. The first forming stage is similar to a single U-draw bending process with a stroke of 70 mm followed by removal of the punch while the blank-holding force (BHF), 50 kN, is maintained. Then, an additional punch stroke of 70 mm is carried out as the second forming stage, allowing a significant reduction of springback compared to that of the standard single stage forming [7].

4. Results and discussion

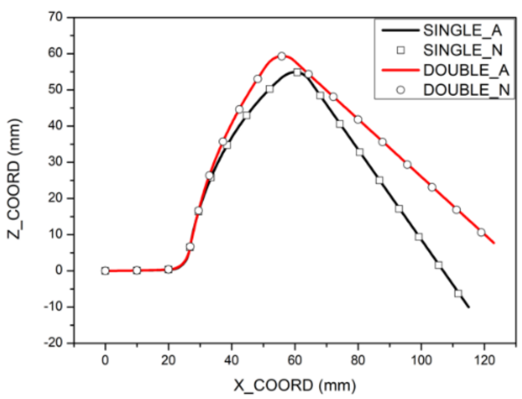

Figure 3 Springback profiles after single-stage and double-stage U-draw bending processes using analytical (_A) and numerical (_N) derivatives

Table 1 Springback measurement parameters

Parameter	Single_A	Single_N	Double_A	Double_N
θ_1 (˚)	120.47	120.48	114.86	114.94
P (mm)	69.27	69.13	76.38	76.32
θ_2 (˚)	81.42	81.44	114.43	113.72

The validation of the proposed FDM-based cutting plane method was done by comparing the springback profiles and the parameters defined in the Numisheet'93 benchmark [8] after single-stage (conventional) and double-stage U-draw bending simulations. The shapes of the part cross-section after both U-draw bending simulations are shown in Figure 3 where _A and _N indicate that stresses were updated based on analytical and numerical approaches, respectively. The springback profile obtained through the proposed numerical approach is in good accordance with those obtained through the analytical approach. The maximum differences in z coordinate in single- and double-stage cases are only 0.1601 mm and 0.2364 mm, respectively. The computed springback parameters are listed in Table 1 as well as the experimental values, which were determined in a previous work [7].

Table 2 Computational performance of FDM-based cutting plane method

Name	CPU time	Improvement
DP_Single_A	05:53:32	-4.95 %
DP_Single_N	06:11:02	
TWIP_Single_A	11:58:34	2.49 %
TWIP_Single_N	11:40:40	
DP_Double_A	38:05:17	2.71 %
DP_Double_N	37:03:19	
DP_Double_N100	37:14:26	-0.5 %
DP_Double_N1000	37:16:02	-0.57 %

The computational performance of the numerical derivative approach was compared to that of the analytical approach in Table 2 where TWIP_Single indicates the single-stage U-draw bending simulation with TWIP980 sheet. The numerical differentiation is repeated but composed of simple calculations while the calculation of analytical derivative is non-repeated but complicated. As the computation time gets longer, the FDM-based cutting plane method becomes faster than the analytical derivative-based cutting plane method. This result implies that the proposed numerical approach shows the same convergence rate as the analytical one. Moreover, because of its simplicity, the computational efficiency of the numerical approach is better than that of analytical approach

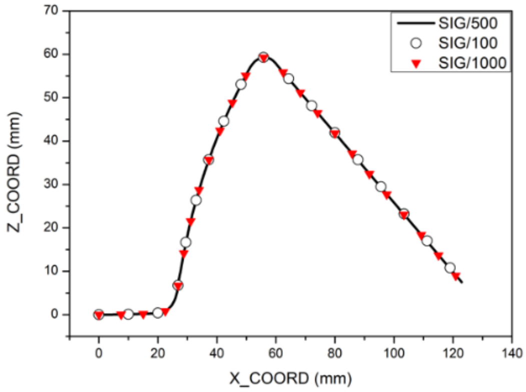

Figure 4 Springback profiles predicted with various step-sizes

The influence of the differentiation step-size, h, in Eq. (9) on the accuracy and computational efficiency of the results was investigated in order to find the optimal value. Regardless of the step-size, h, the predicted springback profiles are in good agreement as shown in Figure 4. The CPU times of the FE analysis are compared in Table 2 where N100 and N1000 correspond to step-sizes of σ/100 and σ/1000, respectively. The computational efficiencies depending on the step-size are similar because of the high order error of the five-point midpoint rule. In addition, the introduced FDM-based cutting plane method is applicable to the various other types of FDM equations such as three-point midpoint rule with showing similar accuracy and computational efficiency.

5. Conclusions

In this article, the use of FDM into the cutting plane method for stress integration was applied to the prediction of springback for a DP980 sheet after double-stage U-draw bending. The main results are summarized as follows:

1. FE simulation results for double-stage U-draw bending and springback prediction with FDM-based cutting plane method was in good accordance with the results using analytical derivatives.
2. No degradation of CPU time in the application of FDM was observed. The simple but repeated calculations in numerical differentiation were faster than complex and non-repeated calculations with analytical derivatives. Therefore, the FE simulation with the FDM-based cutting plane method can be an alternative method to obtain the derivatives of the constitutive models.
3. The step-size, h, in FDM did not have a remarkable influence on the accuracy of the simulations and the computational efficiency because the used five-point midpoint equation has high order error which can make the stress update algorithm converge well. It is necessary to utilize an adequate FDM equation in calculating numerical derivatives as the equations have different errors.

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