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Decision Support System with Uncertain Data: Bayesian Networks Approach

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Abstract: This paper is concerned with decision support system (DSS) development for aid in decision-making with uncertain data. The decision-making problem is formulated in the framework of state-of-the-world decision-making model and the main purpose of the decision support system that will be developed is to aid to estimate the most likely state of the world according to some external or environmental events that are known to have an influence on it. The mathematical tool used to tackle this problem is bayesian networks (BN). The state of the world is considered to be a random variable depending on some random environmental events organized in probabilistic causal network known as bayesian network. Any evidence on environmental events is then propagated by the bayesian network to estimate the state of the world with some degree of confidence that is used to make decision.

Keywords: Decision Support System, Bayesian Networks, Decision-Making, State-of-the-World Decision Model.

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1. Statement of the Problem

In many decision-making problems, the reward of a given decision depends on the state of the world which in return depends on environmental (external) events and/or variables. Classically, such decision-making problems are formulated in terms of state-of-the-world decision-making model [5]. The principle is as follow: there are *n* possible actions $\{a_1, a_2, ..., a_n\}$ and *m* possible states of the world $\{s_1, s_2, ..., s_m\}$; if an action a_i is decided and the observed state of the world is s_j then decision maker receives a reward r_{ij}.

Different algorithms are used in the literature to solve this decision-making problems, that is deciding the best action (or optimal action) knowing that the state of the world will be observed a posteriori; most popular algorithms are: Maximin Reward, Maximax Reward, Minimax Regret and Expected Value approach [5] that are presented below.

- Maximin Reward: here the optimal action a^* is selected as

$$a^* = \arg\max_{a_i}\left(\min_{s_j} r_{ij}\right)$$

- Maximax Reward: in this case, we have

$$a^* = \arg\max_{a_i}\left(\max_{s_j} r_{ij}\right)$$

- **Minimax Regret**: the regret matrix is defined as follow: let call $i^*(j)$ the optimal action index if we know that the state of the world is s_j ,then the regret of taking action a_i when the state of the world is s_j is given by $r_{i^*(j)j} - r_{ij}$ and so the minimax regret algorithm selects as optimal action, a^* the action such that

$$a^* = \arg\min_{a_i}\left(\max_{s_j}\left(r_{i^*(j)j} - r_{ij}\right)\right)$$

- **Maximum Expected Value**: here it is supposed that each state *s* has probability *p(s)* of occurrence and the optimal action is selected as

$$a^* = \arg\max_{a_i}\left(\sum_{j=1}^{m} p(s_j)r_{ij}\right)$$

Previous algorithms are easy to implement and computationally efficient ; their main drawback is that they give an off-line (open loop) type solution. They do not consider a possible evidence on environmental events in order to have an adaptive behavior during decision process. If the state of the world were known in advance, the decision would be trivial ; but we argue that in most domains of decision-making the state of the world depends on some environmental and/or historical events that can be known with some degree of certainty. The approach we consider in this paper is to establish a model of the interaction between environmental events and the state of the world so that at any time one will use the evidence on such events to estimate the state of the world that will determine the action to be taken; this leads to a decision making in closed loop (adaptive) type in the sense that we estimate the state of the world before taking actions. The mathematical tool we will use to establish events/state of the world interaction model is bayesian network (BN). Bayesian networks [2][3] are tools that can put together probabilistic causal relationships between variables, events or parameters. This tool permits one to integrate uncertainty into an expert knowledge system in terms of probability of occurrence of an event knowing that some particular event occurred. Another advantage of BN is that they are able to derive a priori unknown relationship [2] between events by learning process. This approach is preferred here to fuzzy logic theory approach because in the context of real time decision making, causal events (events that have influence on the state of the world) can be known with a great precision (presence of concurrents on the market of a product, reliability of production resources, weather conditions of the day, etc.) instead of being vague.

The remainder of the paper is organized like this: in the second section a rapid presentation of BN is given. The third section is devoted to the main contribution of this paper: the use of BN to establish environmental events/state of the world interaction model that will be used for real time estimation of the state of the world. The last section shows the use of the established approach on an example.

2. Bayesian Networks: Brief Description

2.1 Definition of BN

Bayesian Networks (BN) derive from convergence of statistical methods that permit one to go from information (data) to knowledge (probability laws, relationship between variables, ...) and Artificial Intelligence (AI) that permit computers to deal with knowledge (not only information) (see for example [2]). The terminology BN comes from work by Thomas Bayes [1] in eighteenth century. Its actually development is due to [4]. The main purpose of BN is to integrate uncertainty in expert system. Indeed, an expert, most of the time, has only an approximative knowledge of the system that he or she formulates in terms like: *A* has an influence on *B* ; if *B* is observed, there exists a great chance that *C* occurs and so on. On the other hand, there are data (measurements for example) that contain some information which must be transformed into causality between variables. Thanks to probability, BN will solve this duality problem.

BN consist in a graphical representation of causality relationship between a cause and its effects. The following Figure 1 means that *A* is the cause and *B* its effect.

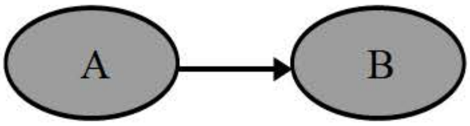

Figure 1: Causality Representation in BN

But as this relationship of causality is not strict, the next step is to quantify it by giving the probability of occurrence of B when A is realized. So a BN consists of an oriented graph where nodes represent variables and oriented arcs represent the causality relationship and a set of some probabilities. The rigorous definition of a BN is given below.

Definition([3]). A bayesian network consists of the following.

- A set X of variables and a set of directed edges between variables.
- Each variable has a finite set of mutual exclusive states.
- The variables together with the directed edge form a directed acyclic graph.
- To each variable X_i with parents (all variables that are causes of this variable) contained in the set $C(X_i)$,there is attached a conditional probability table $P(X_i / C(X_i))$.

($P(A/B)$ = probability of A knowing B).

Following assumptions [2] are added for mathematically convenience.

1. Subjective probabilities (that used by an expert to describe relationship between variables) are considered as mathematical probabilities.
2. Frequencies (table of measures for instance) are considered as mathematical probabilities.
3. The causality graph is a representation of a certain probability law.

2.2 Algorithms

2.2.1 Propagation

The main purpose of BN is to propagate a certain knowledge of the state of one or more particular nodes through the network so that one can get how the beliefs of the expert in the BN will change. So, given a BN it returns to compute the following conditional probability

$$P(X_i / Y) \quad where \quad Y \subset X, \quad X_i \notin Y.$$

Using the properties of chains, trees, networks, d-separation concept (see [3]), and the properties of conditional probabilities one can derive algorithms that can propagate a certain knowledge in the BN. For instance, for a chain of length n, if X_i is downstream of X_j but not a direct relative, then

$$P(X_i / X_j) = \sum_{X_{i-1}} P(X_i / X_{i-1}) P(X_{i-1} / X_j).$$

If X_{i-1} is a direct relative of X_j then stop; if not, decompose $P(X_{i-1} / X_j)$ as previously. For other forms (trees, general networks), it is more complicated but there exists algorithms based on other well known algorithms of networks such as: maximum flow, short path, maximal weight trees, etc. For more information one can consult specialized literature as [2][3] and references therein.

2.2.2 Learning

BN constitutes a mode of knowledge representation funded on the description of relationship between variables in a given domain. If we consider knowledge as relationship between variables and information as the fact of a given situation, then the inference is the way to go from knowledge model to conclusion by computing some probabilities. The problem then is how to compute a knowledge model in order to use it in the future ? This can be done in different ways: by expert knowledge that is known as knowledge acquisition, by doing experiences and measuring some data that will be used to fit some model, that is statistical or Bayesian approach. We will briefly recall in the following the two last cases. These two methods consider the best approximation in some sense of distribution of probabilities (frequencies of some events). The two fundamental components in a BN are relationships between variables represented by arcs that we will refer to as the *structure* of the BN and the conditional probabilities referred to as *parameters*. Learning process is then to compute these two components. If the structure is known a priori, statistics and Bayesian analysis can be used to determine the parameters. In other hand the same techniques can be used to choose the best structure in some sense among a given set of structures.

2.3 Existing BN Software

The computation of parameters and the computation of prediction given that some events occur by hand can be heavy if the BN is large (many variables interconnected). To overcome this and because of great use of BN in many domain: data-mining, diagnostics, planning, banks, finance and defense [2][3] to name few, some software are developed to aid quickly modelling and analysis of BN. The leader in this domain is probably the company Hugin that develop a graphical oriented software Hugin Explorer. A free version Hugin Lite is available for download from the website of the company; Microsoft proposes MSBN, a graphical tool for constructing networks. For more information on the developer related to BN, one can consults [2].

3. Information-Based Decision-Making

We call *information-based decision-making* an on-line type procedure which works as follow: before making a decision, decision-maker looks at some environmental signals or events that are known to have an influence on the state of the world ; from the evidence of these events, the sate of the world is estimated which in return determines decision to be implemented. This means that prior to building the decision systems, experiences or observations will be carried to identify most influential environmental events and construct a causal relationship between these events and the state of the world. Different mathematical tools for learning (statistics, data analysis, neural networks, fuzzy logic, bayesian networks, etc.) can be used to build such relationship. In this paper we use bayesian networks because of their advantages exposed in previous sections. We organize this information based decision-making procedure in three stages. First events/state of the world interaction model is built using bayesian network, that is the structure and parameters of the model are determined (learned) using experimental data or expert knowledge for instance. This model is then used to propagate evidence on environmental events to modify beliefs on the state of the world; for a given evidence of environmental events one estimate the most likely state using some procedure; finally this estimated state is used to decide the action to be implemented. In the following paragraphs these three stages will be considered in details.

3.1 Events/State Interaction Modeling

As stated previously, the prime task to consider is to identify different environmental influential events and the construction of the relationship between them and the state of the world. We consider that a set of N possible events $\mathrm{E} = \{E_1, E_2, ..., E_N\}$ that have influence on the state of the world is identified. Each event E_i can be in one of discrete exclusive state $E_i^j,\ j = 1, 2, ..., n_{E_i}$.

This set of events is organized hierarchically in two levels: the upper level is constituted by events (or node in BN language) whose evidence can be determined easily; let call them *prime events* and their subset E_p.

In terms of BN these nodes do not have parents and have influence directly on the state of the world or on a low level network called intermediate BN that in return has influence on the state of the world (see Figure 2). This defined the structure or causal network. Once the structure is defined, the second task is to define, using an expert knowledge for example or to estimate using experimental data, all parameters of the system. These parameters are all conditional probabilities of the state of the world given different configurations of its parents in the intermediate BN and conditional probabilities of nodes in the intermediate BN given evidence of prime events. These conditional probabilities are in the form $P(S = s_k / C(S)) \quad and \quad P(E_i = E_i^j / C(E_i)), \quad E_i \notin \mathrm{E}_p$ where *C(S)* is the set of all parents of the state and $C(E_i)$ is the set of all parents of node E_i; these parameters give the probability that the state and intermediate nodes are in some state given some evidence on their parents.

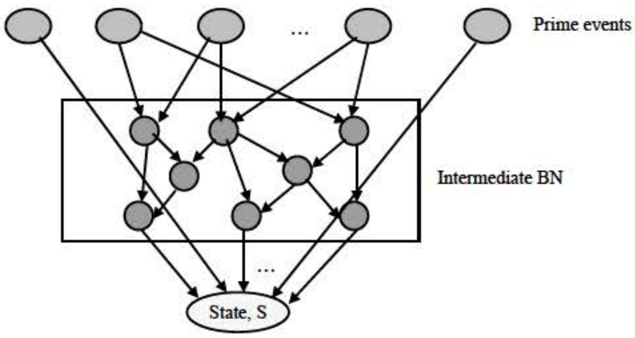

Figure 2: BN Model

3.2 State Estimation

The principal use of the model built in the previous paragraph is to propagate evidence on prime events in order to estimate the actual most likely state of the world. We consider that the state of the world *S* is a numerical value and give below some procedures to estimate it given some evidence on prime events.

3.2.1 Mean Value Procedure

This procedure is probably the first idea one may have for estimating the state. Indeed, it considers that the likely state S^* to be observed is the conditional mean value, namely

$$S^* = \sum_{k=1}^{m} P(S = s_k / \mathrm{E}_p) s_k.$$

A partial mean procedure can be considered in order to ensure that all contributing states have conditional probability greater than a given threshold δ; in this case the estimated state S^* is given by

$$S^* = \frac{\sum_{k,\, P(S=s_k/\mathrm{E}_p)\geq\delta} P(S = s_k / \mathrm{E}_p) s_k}{\sum_{k,\, P(S=s_k/\mathrm{E}_p)\geq\delta} P(S = s_k / \mathrm{E}_p)}.$$

3.2.2 Maximum Probability Procedure

The mean value procedure is not necessarily the appropriate one. According to confidence one has in its experts or data, one can select the state of the world S^* that has a maximum conditional probability of occurrence, that is $S^* = \arg \underset{s_k}{max} \{P(S = s_k / \mathrm{E}_p)\}$.

3.2.3 Decision-Making

The estimated state is used to decide what is the best action to be implemented knowing S^*. If we call a^* this action then a^* is the action that realizes the maximum reward if the state of the world is S^* that is $a^* = \arg \max_{a_i} (r_{ij^*})$ where j^* is such that $s_{j^*} \in \{s_1, s_2, ..., s_m\}$ is the nearest state (in some sense) to S^*.

This action is refreshed periodically according to changes in the environmental events; it makes this approach for decision making suitable for real time adjustments of actions and looks like feedback adaptive control approach.

3.2.4 Post Analysis

It may be interesting to study the benefit of setting up such information-based decision-making system mainly if information acquisition needs an investment. This study may be difficult to carry out because of random nature of the problems ; but by doing initial trials, a statistical study can be done and decide according to some performance measures or ask for performance analysis by experts of the domain. Possible measures of performance can be derived as follow: let call $r_{i^*j^*}$ the predicted reward, that is the reward one get by using previous information-based procedure where $i^* \quad and \quad j^*$ are defined by $a_{i^*} = a^* \quad and \quad s_{j^*} = S^*$;let suppose that the actual observed state is *s* so that the real reward is r_{i^*j} and let define r^{cl} as the reward when using one of the classical procedures given in first section.

Statistical study (mean, standard deviation) of difference $r_{i^*j^*} - r_{i^*j}$ will give an idea of accurate previsions and the benefit gained by using an information-based procedure can be measured by statistics (mean, standard deviation) of the difference $r_{i^*j} - c - r^{cl}$ where *c* is the information acquisition cost.

4. Application

In this section we will use the proposed method to solve a small but instructive application that arise in the domain of marketing.

4.1 Modelling

This application is adapted from an example considered in page 727 of [5]. It concerns a news papers vendor that must determine each day how many papers to order. The vendor is sure to sell between 6 and 10 papers every day. This problem was formulated in terms of state-of-the-world decision-making problem in [5] and different procedures (Maximin Reward, Maximax Reward, Minimax Regret and Expected Value) were used to make decision considering that each possibility (number of papers demanded) is equally likely. Figure 3 gives the rewards matrix (coefficients are given in some money units)

	Papers Demanded				
Papers Ordered					
	6	7	8	9	10
6	30	30	30	30	30
7	10	35	35	35	35
8	-10	15	40	40	40
9	-30	-5	20	45	45
10	-50	-25	0	25	50

Figure 3: Rewards Matrix

and a rapid calculation gives the regret matrix of Figure 4.

	Papers Demanded				
Papers Ordered					
	6	7	8	9	10
6	0	5	10	15	20
7	20	0	5	10	15
8	40	20	0	5	10
9	60	40	20	0	5
10	80	60	40	20	0

Figure 4: Regret Matrix

Solutions of this problem according to different classical procedures (see first section) and under equally likely hypothesis are given in the following Table

Criterion	Decision
Maximin Reward	6
Maximax Reward	10
Minimax Regret	6 or 7
Maximum Expected Value	6 or 7

One can see from the reward matrix and regret matrix that if the state of the world (that is the demand) is known then the action or decision that maximizes the reward is determined. Actually, if the demand $j \in \{6, 7, 8, 9, 10\}$ is observed, the optimal action is to order $i = j$ papers. Now let us modify hypothesis considered in [5] and suppose that the vendor has identified some events that have a great influence on the demand, namely the day of the week (Sunday excluded) and the weather. The information concerning weather is given by the weather bureau and is not very accurate. In terms of BN, we have Figure 5 where we consider that the variable demand has two parents, Day that consists in six states (Monday, Tuesday, Wednesday, Thursday, Friday, Saturday) and Forecast Weather that we suppose, has three states (Sunny, Cloudy, Raining). The Weather Forecast is estimated from information given by weather bureau which is then its parent and has the same states. Demand is a variable with 5 states $\{D_1 = 6, D_2 = 7, D_3 = 8, D_4 = 9, D_5 = 10\}$.

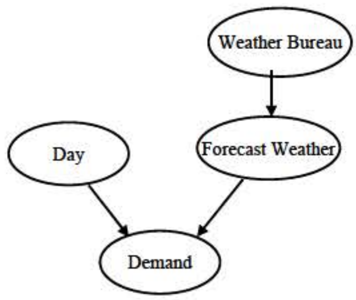

Figure 5: Bayesian Network of News Paper Vendor Problem

Let suppose that from experience the vendor establishes that conditional probabilities for demand according to the day of the week and the forecast weather and those for Forecast Weather according to Weather Bureau are given by Figures 6 & 7[1] respectively. This completes modeling process.

1 Notice that these values do not come from a realistic experience but are chosen by the author with effort to make them consistent and close to reality.

Demand					
Weather			Sunny		
Day	Monday	Tuesday	Wednesday	Friday	Saturday
D1	0,1	0,1	0,1	0,1	0
D2	0,2	0,2	0,2	0,2	0,03
D3	0,4	0,4	0,4	0,4	0,07
D4	0,2	0,2	0,2	0,2	0,2
D5	0,1	0,1	0,1	0,1	0,7
Weather			Cloudy		
Day	Monday	Tuesday	Wednesday	Friday	Saturday
D1	0,4	0,4	0,4	0,4	0,3
D2	0,3	0,3	0,3	0,3	0,3
D3	0,2	0,2	0,2	0,2	0,2
D4	0,1	0,1	0,1	0,1	0,15
D5	0	0	0	0	0,05
Weather			Raining		
Day	Monday	Tuesday	Wednesday	Friday	Saturday
D1	0,6	0,6	0,6	0,6	0,5
D2	0,3	0,3	0,3	0,3	0,3
D3	0,1	0,1	0,1	0,1	0,15
D4	0	0	0	0	0,05
D5	0	0	0	0	0

Figure 6: Conditional Probabilities for Demand

Forecast Weather		Weather Bureau	
	Sunny	Cloudy	Raining
Sunny	0,8	0,05	0,05
Cloudy	0,1	0,8	0,15
Raining	0,1	0,15	0,8

Figure 7: Conditional Probabilities for Forecast Weather

4.2 Simulation Results

The model shown by Figure 5 was introduced in a BN software, Hugin Lite for simulation. For given evidences on the day and weather bureau, Hugin Lite propagate the information through the BN and determine probability of each state of demand. Then demand is selected by mean procedure or maximum probability procedure. Following items give results of few simulations.

Results for a sunny (predicted by weather bureau) Saturday

Dem	D1(6)	D2(7)	D3(8)	D4(9)	D5(10)
Prob	0,08	0,084	0,091	0,18	0,565

- Estimated demand by maximum probability procedure: 10; *decision*: order 10 papers.
- Estimated demand by mean procedure: 9.0660; *decision*: order 9 papers.

From Monday to Friday with a sunny weather (prediction of weather bureau)

Dem	D1(6)	D2(7)	D3(8)	D4(9)	D5(10)
Prob	0,18	0,22	0,35	0,17	0,08

- Estimated demand by maximum probability procedure: 8; *decision*: order 8 papers.
- Estimated demand by mean procedure: 7.7500; *decision*: order 7 (or 8) papers.
- Estimated demand by partial mean procedure (only probabilities greater than or equal to 0.1 are considered): 7.5543; *decision*: order 7 papers.

Results for a raining (predicted by weather bureau) Saturday

Dem	D1(6)	D2(7)	D3(8)	D4(9)	D5(10)
Prob	0,445	0,2865	0,1535	0,0725	0,0425

- Estimated demand by maximum probability procedure: 6 ; *decision*: order 6 papers.
- Estimated demand by mean procedure: 6.9810 ; *decision*: order 7 papers.

From Monday to Friday with a cloudy weather (prediction of weather bureau)

Dem	D1(6)	D2(7)	D3(8)	D4(9)	D5(10)
Prob	0,415	0,295	0,195	0,09	0,005

- Estimated demand by maximum probability procedure: 6; d*ecision*: order 6 papers.
- Estimated demand by mean procedure: 6.9750 ; *decision*: order 7 papers.

5. Conclusion

A methodology for developing a decision support system for state-of-the-world decision-making model where the state of the world can be related to some causal random environmental events has been considered through bayesian network in this paper. This method leads to an on-line decision making that permits a real time adjustments instead of off-line decision making that is usually derived by classical approach presented in the first section of the paper. Furthermore, bayesian networks are suitable here because of their learning capabilities that can be used to modify conditional probabilities periodically to take into account environmental changing. The simple example considered shows the applicability and efficiency of this methodology in adaptive decision making manner and proves a friendly usability. It can then be with great help to decision maker with no prior knowledge of BN. We think that the drawbacks here is the lack of software with bayesian network and optimization capabilities and we are working towards developing such tools.

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