Quantitative modelling of benchmarking process

Ayeley Philippe Tchangani

Université de Toulouse, UPS,
IUT de Tarbes,
1, rue Lautréamont,
Tarbes Cedex 65016, France
Fax: +33 (0)5 62 44 42 19
E-mail: ayeley.tchangani@iut-tarbes.fr
and
Université de Toulouse,
Laboratoire Génie de Production (LGP),
47 Avenue d'Azereix, BP 1629,
Tarbes Cedex 65016, France
E-mail: Ayeley.Tchangani@enit fr

Abstract: In this paper, we consider the problem of establishing a quantitative model allowing, given a set of production units (enterprises, plants, banks, university departments, etc.), to determine those units that can be considered as benchmarks in terms of production efficiency and to evaluate for a unit, that is not a benchmark the gap that separates it from the benchmarks. A production unit is considered here as a transformation centre that consumes resources (input items) of different nature (information, human resources, energy, money, etc.) to deliver some products (output items) of different nature as well (manufactured products, services, information, energy, etc.). This benchmarking problem is, therefore, a multicriteria ranking problem that necessitates sensitivity analysis process to determine which items a given unit must improve in order to become as efficient as benchmark unit(s). We propose in this paper to formulate this problem using satisficing games, an evaluation method, that is, based on two measures namely *selectability* measure (that measures production level) and *rejectability* measure (that is, related to resources consumption) for each unit or alternative. Units for which the selectability measure exceeds the rejectability one will be considered as satisficing units and the benchmark units are those satisficing units that are not dominated.

Keywords: benchmarking; production units; efficiency; group decision; multicriteria ranking; satisficing games, analytic hierarchy process.

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Biographical notes: Ayeley Philippe Tchangani received his Ingénieur Degree (1995) from Ecole Centrale de Lille, France, and an MSc (1995) and a PhD (1999) from Université des Sciences et Technologies de Lille, France in Control and Automation. Since 2001, he has been with Université Toulouse (IUT de Tarbes) and Laboratoire Génie de Production (LGP), since 2003. Currently, he is an Associate Professor. His current research interests are in decision analysis and risk management. His work has been published in international journals including *DSS, IJITDM, JUS,* etc. He is a Member of IEEE, SIAM and MCDM society.

1 Introduction and statement of the problem

The benchmarking is a technique that consists in comparing a production unit, generally an enterprise, to other units considered as benchmarks (Roux, 2007). The goal of benchmarking depends on the pursued objectives but in general, its purpose is to reveal the weak points of the considered unit compared to benchmark units in order to propose possible improvement actions. There are different types of benchmarking such as:

- *competitive benchmarking*: one compares units of the same sector; units doing the same business
- *generic benchmarking*: here, the comparison is made between units of different sectors in order to improve some practices or processes
- *internal benchmarking*: the purpose here is to put different departments of a given enterprise for instance in competition.

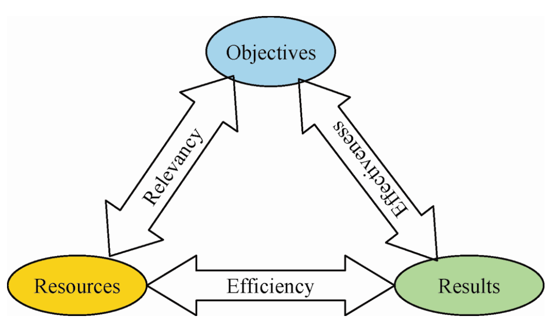
Benchmarking is becoming in nowadays highly competitive world, a management tool largely used for performance improvement purpose. As results, many studies as well as researches are being carried up all over the world by numerous researchers and practitioners to derive new approaches or methodologies or to apply existing ones to real-world applications. For instance, in Moffett et al. (2008), authors explore the theoretical understanding and practical application of lead benchmarking and performance measurement as a way to achieve organisational changes. Pursglove and Simpson (2007) examined the effectiveness of teaching and widening participation as measures to assess, compare and benchmark the performance of English universities. Two case studies were conducted in Åhrén and Parida (2009) using maintenance indicators in order to improve best practices in railway infrastructures. The assessment of the comparative strengths and weaknesses of leading third-party logistics providers (3PLs) in the USA with respect to their financial efficiencies during the period of 2005–2007 have been considered in Min and Joo (2009) using Data Envelopment Analysis (DEA) approach. In Deros et al. (2006), a conceptual framework for benchmarking implementation in small medium-sized enterprises (SMEs) taking into consideration their characteristics has been proposed. To identify the overall best-in-class (BIC) performer for performance metrics involving inventory record accuracy within a public sector warehouse, Collins et al. (2006) used multiattribute utility theory (MAUT) to aid in the decision making in benchmarking gap analysis. Using a simulation approach, the study considered in Rehman and Babu (2008) help quantifying how a manufacturing system that accepts customer orders for any combination of products should perform under different scenarios characterised by the combinations of various operational features where the performance of the system is measured using machine utilisation, throughput time, product earliness, product lateness and product block time. This paper (Karuppusami et al., 2006) used the Quality Function Deployment (QFD) approach to derive a methodology to benchmark the quality-related action programmes and critical success factors of strategic quality management (SQM) for manufacturing industries. Kounis and Panagopoulos (2007) addressed the difficulties associated with benchmarking techniques and the implication of total quality management (TQM) tools in companies of public and private sectors.

In this paper, a production unit is understood in a broad sense; it represents a generic term to designate any entity that consumes resources (input items) to deliver products (output items). The considered benchmarking is that of competitive benchmarking and consists in considering *n* production units that consume the same kind of input items to produce the same kind of output items, to determine those units that can be considered as benchmarks and given a unit, that is, not a benchmark to evaluate how much it must improve its input and/or output items in order to become a benchmark. The benchmarking process is undertaken with regards to efficiency index of the performance diagram shown in Figure 1, that is, the adequacy between the delivered results and the engaged resources (are results enough given the resources consumed?). The two other indices of this diagram: the relevancy (adequacy between the engaged resources and the pursued objectives, are engaged resources reasonable given the pursued objectives?) and the effectiveness (relation between the obtained results and the pursued objectives, at which level the obtained results realise the pursued objectives?) are not easy to use as they are related to the pursued objectives, a component of performance diagram, that is, neither easy to define nor to measure in general.

A benchmark unit must not only produce at least as much results as it consumes resources but also it must not be dominated in the sense that there must not exist units that could produce the same results with less resources or produce more results with the same resources.

Given how a production unit is defined (transformation of input items into output items), data envelopment analysis (Charnes et al., 1978) or DEA for short is a candidate approach that can be used to determine a relative efficiency index when input and output items are numerically measured. This relative index may be determined with regards to a virtual benchmark. But this approach does have some drawbacks such as the possibility to render arbitrary the relative efficiency index of a unit equal or near to one by choosing adequately the weights to assign to items, see for instance Tchangani (2006a) and references therein. Furthermore, this approach does not allow to integrate multiple opinions of decision makers regarding the importance to assign to each item for instance, whereas; we consider that this situation is frequently encountered in benchmarking process; it does not permit a sensitivity analysis to identify the items to improve for a non-efficient unit too.

Figure 1 Performance diagram (see online version for colours)



It is also possible to use all the approaches developed in multicriteria decision analysis literature to rank production units; some of main references of this literature are Brans et al. (1984), Brans et al. (1986), Pomerol and Barba-Romero (1993), Roy and Bouyssou (1993), Saaty (1980), Steuer (1986) and Zahir et al. (2009) to name few. These methods can be roughly classified into two classes or categories that are briefly presented below.

- Transformation of multicriteria problem into single criterion problem: here, criteria or items are aggregated into one criterion or some criteria are transformed into constraints and then single criterion optimisation algorithms are used to solve the obtained problem. The main advantage of this approach is that many algorithms and software developed by operational research community do exist for this purpose (see for instance Teghem, 1996). The drawback of this approach is that the aggregation procedure and the transformation of criteria into constraints are not easy to do mainly when some actors involved in the benchmarking process do not have scientific skills. This approach is mainly developed by the North America researchers such as Saaty (1980), Steuer (1986), Ignizio (1976), etc.
- Outranking methods: mainly developed by European operational research community (Pomerol and Barba-Romero, 1993; Roy and Bouyssou, 1993; Vincke, 1989) with methods such as ELimination Et Choix Traduisant la REalité (ELECTRE) and Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE); this approach has the advantage to allow the impossibility to compare two alternatives rendering possible for a decision maker to be indifferent between two alternatives; but the procedure to define preferences by decision makers is not always easy.

The approach we propose to develop in this paper will lead to a benchmarking model that integrates following features:

- evaluation of adequacy between production level and resources consumption of each unit and its relative efficiency compared to other units
- integration of experts or decision makers judgement because we do think that a benchmarking process can necessitate intervention of many actors that do not necessarily have the same point of view regarding the relative importance of items
- sensitivity analysis with regards to the variation of items values in order for managers to be able to recommend to a non-efficient unit which of its items it must improve.

To achieve this goal, we will use satisficing games theory (Stirling, 2003) as the underlying mathematical tool. This tool has been developed firstly in the context of group decision making and agents' coordination in artificial intelligence domain as an alternative to classical (Von Neumann and Morgenstern, 1944) games theory to deal with situational social behaviour of actors engaged in a decision process in some circumstances. It is now showing successful application in other domains such as performance evaluation (Tchangani, 2006a,c), loads or resources dispatching between agents (Tchangani, 2006b), retrieval of objects that must satisfy some requirements from a database (Tchangani, 2006d) or multiattributes and multiobjectives decision making (Tchangani, 2009a,b). Satisficing games theory is based on the notion of good enough units, alternatives, decisions or options (as opposed to optimal notion) that is, units for which the 'benefit' exceeds the 'cost'; benefit and cost being represented by selectability and rejectability measures, respectively, that are defined locally for each unit. The main motivations that guide us to use this theory for benchmarking process is on one hand; the dual character of items (input items will be associated with the cost and will be used to compute the rejectability measure, whereas, the output ones will be considered as benefit and will serve to define the selectability measure) and on the other hand; the notion of benchmark: a benchmark unit must not only produce at least as much as it consumes resources but also at least as good as other units. Furthermore, the author proved recently in Tchangani (2009b), the possibility to extent this approach to the case where units are no necessarily characterised by the same items but are pursuing the same objectives.

The remainder of this paper is organised as the following: Section 2 presents the basics of satisficing games theory that are relevant to our goal, more details of this theory can be found in Stirling (2003); Section 3 shows how the benchmarking problem we are considering can be formulated using satisficing games; in Section 4, an example will be considered to show the potentiality of this approach and finally, conclusion is presented in Section 5.

2 Satisficing games: presentation

The main objective of this section is to present relevant features of satisficing games theory that we need in order to develop our quantitative benchmarking model. In the simplest version, (see Stirling, 2003 for complex cases with many decision makers in interaction where each decision maker has its own options set), a satisficing game is given by Definition 1.

Definition 1: *A satisficing game consists in the triplet* $\langle U, p_S, p_R \rangle$ *where*

U is the set (discrete) of units, alternatives, decisions or options, etc.

p_S *and* p_R *represent mass functions or measures defined from U onto the interval [0, 1] where* p_S *measures the selectability degree and* p_R *that of rejectability; a function p is said to be a mass function over a discrete set U if it possesses a probability structure, that is, it verifies* $p(u) \geq 0$ *for any element u of U and the sum of p(u) over U is one.*

The interesting units that can be qualified as satisficing units are those units for which the selectability measure does exceed the rejectability one as given by Definition 2.

Definition 2: *Let* Σ_q *be the set of satisficing units at the boldness or caution index q (according to the value of q, more or less units, options, alternatives or decisions will be declared satisficing);* Σ_q *is then given by Equation (1)*

$$\Sigma_q = \{u \in U : p_S(u) \geq q p_R(u)\} \tag{1}$$

But a satisficing unit may be dominated in the sense that there may exist another unit for which the selectability is greater or equal and the rejectability less than or the rejectability less or equal and the selectability greater than the corresponding measures of former one. Finally, units that can be qualified as 'good enough or benchmarks' are those satisficing units that are not dominated. To characterise them, let us denote by *D*(*u*) the set of units that dominate the unit *u*; *D*(*u*) is then given by Equation (2)

$$D(u) = D_S(u) \cup D_R(u) \tag{2}$$

where

$$D_S(u) = \{v \in U : p_R(v) < p_R(u) \text{ and } p_S(v) \geq p_S(u)\}$$
$$D_R(u) = \{v \in U : p_R(v) \leq p_R(u) \text{ and } p_S(v) > p_S(u)\}$$

Non-dominated units known as equilibriums and the benchmark units are then given by Definition 3.

Definition 3: *The set E of non-dominated or equilibrium units is given by Equation (3)*

$$E = \{u \in U : D(u) = \varnothing\} \tag{3}$$

The set B_q of benchmark or good enough units at the boldness or caution index q is then given by Equation (4)

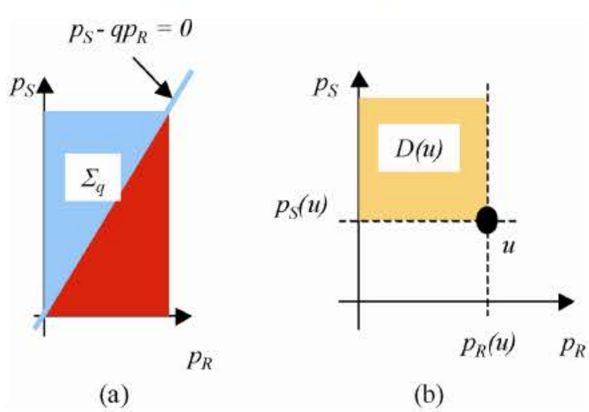
$$B_q = \Sigma_q \cap E \tag{4}$$

Figure 2 shows in the plan (p_R, p_S), the position of the satisficing set Σ_q (Figure 2(a)), and for a given unit *u*; the position of the set *D*(*u*) of units that dominates it (Figure 2(b)). A non-dominated unit is that for which the coloured space of Figure 2(b) does not contain a unit (is empty).

One can notice that the equilibrium set *E* of non-dominated units cannot be empty; indeed, it would means that for any unit *u*, the coloured space of Figure 2(b) should contain at least one unit and, that is, impossible over the set *U*. If one joins all the equilibrium units by a line in the plan (p_R, p_S), we will obtain a curve above which there is no unit.

The boldness index *q* permits to adjust the number of satisficing units: if the satisficing units set is empty, by reducing the boldness index new units will be declared satisficing; in the contrary if too much units are satisficing, increasing this index will reduce the number of satisficing units. The range on which this index can vary can be determined as follows: the lower bound $q_{\min}$ is the value below which all the units of *U* are considered as satisficing and is given by Equation (5)

Figure 2 Representation of sets Σ_q and *D*(*u*) in the plan (p_R, p_S) (see online version for colours)



$$p_S(u) \geq qp_R(u), \quad \forall\, u \in U \Leftrightarrow q \leq q_{\min} = \min_{u \in U}\left(\frac{p_S(u)}{p_R(u)}\right) \tag{5}$$

whereas, the upper bound $q_{\max}$ is the value above which no unit is satisficing and it is determined by of Equation (6)

$$p_S(u) < qp_R(u), \quad \forall\, u \in U \Leftrightarrow q > q_{\max} = \max_{u \in U}\left(\frac{p_S(u)}{p_R(u)}\right) \tag{6}$$

Finally, for any value of index q in the interval $[q_{\min}, q_{\max}]$, we have $\Sigma_q \subseteq U$. As the equilibrium set E of non-dominated units cannot be empty, one can always adjust the value of boldness or caution index q to obtain at least one benchmark unit respecting by the way the spirit of satisficing games theory that gives preference to the notion of 'good enough' instead of 'optimal' which allow one to have a solution to any problem provided one revises of one's aspiration level. With regards to benchmarking problem, we are considering in this paper, the main task will be to define selectability and rejectability measures; this is the purpose of Section 3.

3 Proposed benchmarking model

In this section (main contribution of this paper), we will first present a method to compute the two main parameters of a satisficing game namely selectability and rejectability measures using items values as well as actors preferences, and in a second step; we show by a sensitivity analysis how the benchmarking process can be carried up.

3.1 *Defining the satisficing game parameters*

Applying satisficing games theory as defined previously to solve the benchmarking problem return in a first step to establish a method that permit to compute the selectability and the rejectability measures for each unit using the problem data namely the items values and actors preferences. Here, the role of actors will consist in supplying the analyst with their opinions in terms of relative importance to assign to each item. We consider that units consume *I* items of resources as inputs to produce *O* products as outputs; the value (numerical or rendered numerical by any procedure) of the resource item *i* and the product item *j* for the unit *u* are given by $I(u, i)$ and $O(u, j)$ respectively. The analyst will then ask each actor *k* to supply a weight α_{kj} (in some scale) for *j* belonging to output items; α_{kj} is as high as actor *k* considers the contribution of output item *j* to the selectability measure to be highly important (important product). In the same way, each actor *k* will supply a weight β_{kj} for *j* belonging to resources items; β_{kj} is as high as actor *k* considers the contribution of item *j* to the rejectability (resources consumption) to be highly important. Determination of weights α_{kj} and β_{kj} by actors may not be easy mainly for those actors that do not have scientific culture. To assist them in this process, some formal methods do exist. One of such method is the analytic hierarchy process (Saaty, 1980) or AHP for short; it is widely used in multicriteria decision analysis and particular for those decision problems with intangible or non-numeric items. One easy version of this method for determining the weights α_{kj} and β_{kj} works like this: one will ask each actor k to choose a pivot item p (for each category, input or output items) and to complete a table similar to that of Table 1 below.

by giving the weight v_{ip}^{k} (the relative importance of item i with regard to pivot item p according to actor k) on a scale going from 1 to 9 with verbal signification shown on Table 2. The weights 2, 4, 6 and 8 are used to express intermediary opinions. From these weights v_{ip}^{k}, the analyst will construct, for each actor k, a consistent[1] comparison matrix A^{k} for output items as the following: $A_{ip}^{k} = v_{ip}^{k}$, $A_{pi}^{k} = 1/v_{ip}^{k}$; other coefficients of this matrix are deduced using the rule $A_{il}^{k} = A_{ij}^{k} A_{jl}^{k}$. A similar consistent comparison matrix B^{k} is constructed for input items. The weights α_{kj} and β_{kj} are then given by Equation (7) (see Saaty, 1980).

$$\alpha_{kj} = \frac{1}{O}\sum_{l=1}^{O}\left(\frac{A_{jl}^{k}}{\sum_{t=1}^{O} A_{tl}^{k}}\right) \quad \text{and} \quad \beta_{kj} = \frac{1}{I}\sum_{l=1}^{I}\left(\frac{B_{jl}^{k}}{\sum_{t=1}^{I} B_{tl}^{k}}\right) \tag{7}$$

These weights are finally aggregated to obtain weights ω_j^{S} for the output items and ω_j^{R} for the input items as given by Equation (8)

$$\omega_j^{\mathrm{S}} = \frac{\sum_{k=1}^{d} \alpha_{kj}}{\sum_j \sum_{k=1}^{d} \alpha_{kj}} \quad \text{and} \quad \omega_j^{\mathrm{R}} = \frac{\sum_{k=1}^{d} \beta_{kj}}{\sum_j \sum_{k=1}^{d} \beta_{kj}} \tag{8}$$

Table 1 inter-category items comparison scheme

Items of a category	*Pivot item (p)*
…	…
i	v_{ip}^{k}
…	…

Table 2 Inter-same items comparison scale

Relative importance	v_{ip}^{k}
Equally important	1
Moderately more important	3
Strongly more important	5
Very strongly more important	7
Extremely more important	9

The weights ω_j^{S} and ω_j^{R} measure the global importance actors assign to the corresponding items; these global weights are ultimately organised in row vectors as shown by Equation (9)

$$\omega^{\mathrm{S}} = \begin{bmatrix} \omega_1^{\mathrm{S}} & \omega_2^{\mathrm{S}} & \ldots & \omega_O^{\mathrm{S}} \end{bmatrix} \quad \text{and} \quad \omega^{\mathrm{R}} = \begin{bmatrix} \omega_1^{\mathrm{R}} & \omega_2^{\mathrm{R}} & \ldots & \omega_I^{\mathrm{R}} \end{bmatrix} \tag{9}$$

The second step is to normalise items values as shown by Equation (10)[2]

$$O_n(u,i) = \frac{O(u,i)}{\max_{v \in U} O(v,i)} \quad \textit{and} \quad I_n(u,i) = \frac{I(u,i)}{\max_{v \in U} I(v,i)} \tag{10}$$

that are also organised as column vectors given by Equation (11)

$$\begin{aligned} O_n(u) &= \begin{bmatrix} O_n(u,1) & O_n(u,2) & \ldots & O_n(u,O) \end{bmatrix}^{\mathrm{T}} \\ I_n(u) &= \begin{bmatrix} I_n(u,1) & I_n(u,2) & \ldots & I_n(u,I) \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{11}$$

where x^{T} stands for the transpose of the vector x. A normalisation scheme of items is needed because they are in general evaluated in different units and/or scales. The global aggregated functions $g_{\mathrm{S}}(u)$ and $g_{\mathrm{R}}(u)$ that act in the sense of selectability and rejectability of the unit u are then given by Equation (12)

$$g_{\mathrm{S}}(u) = \omega^{\mathrm{S}} O_n(u) \quad \text{and} \quad g_{\mathrm{R}}(u) = \omega^{\mathrm{R}} I_n(u) \tag{12}$$

Definition 4 establishes the stepping stones of the satisficing games theory that are the selectability and the rejectability measures of each unit.

Definition 4: *The selectability and the rejectability measures for the benchmarking problem are given by Equation (13)*

$$p_S(u) = \frac{g_S(u)}{\sum_{x \in U} g_S(x)} \quad \textit{and} \quad p_R(u) = \frac{g_R(u)}{\sum_{x \in U} g_R(x)} \tag{13}$$

An issue that may be raised at this stage is that of coherency: Is the presented approach coherent in the sense that if there is a unit that consumes more resources to produce less products than another unit? Is it possible that the approach presented declares it as satisficing equilibrium? To formalise this idea, let us consider Definition 5.

Definition 5: *A unit $u \in U$ dominates another unit $v \in U$, denoted $u \succ v$, if and only if the inequalities of Equation (14) hold with at least one strict inequality,*

$$O(u,i) \geq O(v,i), \quad i = 1,2,\ldots,O \quad \textit{and} \quad I(u,j) \leq I(v,j), \quad j = 1,2,\ldots,I \tag{14}$$

The proposition then proves the coherency of the selectability and the rejectability determination procedure considered so far.

Proposition: *Let u and v be two units, then $u \succ v \Rightarrow u \in B(v)$ and so $v \notin E$, that is, v cannot be a satisficing equilibrium unit.*

Proof: $u \succ v$ means that $O_n(u,i) \geq O_n(v,i) \geq 0, \quad i = 1,2,\ldots,O$ and $0 \leq I_n(u,j) \leq I_n(v,j)$, $j = 1,2,\ldots,I$ with at least one strict inequality and as $\omega_l^{\mathrm{S}} > 0,\ \omega_l^{\mathrm{R}} > 0$, we have

$g_S(u) \ge g_S(v)$ and $g_R(u) \le g_R(v)$ and finally, $p_S(u) \ge p_S(v)$ and $p_R(u) \le p_R(v)$ with at least one strict inequality so that $u \in B(v)$, that is, $B(v) \ne \varnothing$ and v is not an equilibrium unit.

One can now consider undertaking the analysis of benchmarking problem.

3.2 *Analysis of benchmarking problem*

Necessary information for analysing the benchmarking problem is summarised in the sets Σ_q, E and B_q as well as set $D(u)$ for each unit u.

- The units of the set B_q are those that can be qualified as benchmark units because they are using adequately their resources to produce their products and there are no other units that are doing better.
- Σ_q contains units that are using correctly their resources but not necessarily in the best way as there can exist other units that can do better with the same resources. If a unit u does not belong to this set ($u \notin \Sigma_q$), it may be interesting for it to know its weak points and to compute the improvement that must be made for each items in order to become satisficing. To this end, we propose to do a sensitivity analysis to determine parameters δ_u^i and γ_u^i that verify $\delta_u^i \ge 0, i = 1, 2, \ldots, O$ and $\gamma_u^i \ge 0, i = 1, 2, \ldots, I$, so that if one replace $O_n(u, i)$ and $I_n(u, i)$ by $O_n(u,i) + \delta_u^i$ and $I_n(u,i) \quad \gamma_u^i$, respectively, when respecting conditions of Equation (15)

$$0 < O_n(u,i) + \delta_u^i \le 1 \quad \text{and} \quad 0 < I_n(u,i) - \gamma_u^i \le 1 \tag{15}$$

then one can have the inequality Equation (16)

$$p_S(u) \ge q p_R(u) \tag{16}$$

It is easy to show that these parameters can be determined by solving the non-linear programming problem given by Equation (17)

$$\min_{\delta_u, \gamma_u} 0 \quad \text{s.t.} \quad \begin{cases} C_O(\delta_u) \ge q C_I(\delta_u) \\ \varepsilon_O \le O_n(u) + \delta_u \le 1, \delta_u \ge 0 \\ \varepsilon_I \le I_n(u) \quad \gamma_u \le 1, \gamma_u \ge 0 \\ \text{where} \\ \delta_u = \begin{bmatrix} \delta_u^1 & \delta_u^2 & \ldots & \delta_u^O \end{bmatrix}^T \\ \gamma_u = \begin{bmatrix} \gamma_u^1 & \gamma_u^2 & \ldots & \gamma_u^I \end{bmatrix}^T \end{cases} \tag{17}$$

where 1 and 0 are in fact vectors of dimension O and I with all entries equal to 1, respectively, to 0; in the same way ε_O and ε_I are vectors with compatible dimensions; s.t. stands for 'subjected to' and finally, the non-linear functions $C_O(\delta_u)$ and $C_I(\gamma_u)$ are given by Equation (18).

$$C_O(\delta_u) \quad \frac{\omega^S(O_n(u) + \delta_u)}{\sum_{v \in U, v \ne u} \omega^S O_n(v) + \omega^S(O_n(u) + \delta_u)} \quad \text{and} \quad C_I(\gamma_u) \quad \frac{\omega^R(I_n(u) \quad \gamma_u)}{\sum_{v \in U, v \ne u} \omega^R I_n(v) + \omega^R(I_n(u) \quad \gamma_u)} \tag{18}$$

Notice that the mathematical programming problem given by Equation (17) is expressed in a very general way so that other constraints may be added to take into account practical considerations such as the uniform distribution of effort between items or in contrary putting the effort on some particular items. This analysis is suitable for units in the set $E - B_q$ (units that are equilibrium but with a poor adequacy between their resources consumption and their delivered outputs); $\delta_u^i / (O_n(u,i))$ and $\gamma_u^i / I_n(u,i)$ represent the proportions by which production must increase and by which resources consumption must decrease, respectively, in order for a non-satisficing unit ($u \in E - B_q$) to become satisficing.

- The sets $D(u)$ are sources of important information that may be used for performance improvement purpose; indeed if $u^* \in D(u)$, by observing the environment in which the unit u^* is operating and/or its organisation, one may find the reasons of weakness of the unit u mainly for those units of the set $\Sigma_q - B_q$, the units that are satisficing but not equilibrium. An analysis procedure similar to that of the former point can be setup to determine how much the unit u must improve its items in order to become as good as the unit u^*. To do so, one may compute the parameters $\delta_u^{u^*}$ and $\gamma_u^{u^*}$ (defined as parameters δ_u and γ_u of the previous point) such that Equation (19) are verified

$$\begin{aligned} p_{\mathrm{S}}(u) &= \frac{\omega^{\mathrm{S}}\left(O_n(u)+\delta_u^{u^*}\right)}{\sum_{v\in U, v\neq u} \omega^{\mathrm{S}} O_n(v) + \omega^{\mathrm{S}}\left(O_n(u)+\delta_u^{u^*}\right)} = p_{\mathrm{S}}(u^*) \\ p_{\mathrm{R}}(u) &= \frac{\omega^{\mathrm{R}}\left(I_n(u) \quad \gamma_u^{u^*}\right)}{\sum_{v\in U, v\neq u} \omega^{\mathrm{R}} I_n(v) + \omega^{\mathrm{R}}\left(I_n(u) \quad \gamma_u^{u^*}\right)} = p_{\mathrm{R}}(u^*) \end{aligned} \tag{19}$$

by solving a linear mathematical programming problem of the form given by Equation (20)

$$\begin{aligned} &\min_{\delta_u^{u^*}, \gamma_u^{u^*}} 0 \\ &\text{s.t.} \begin{cases} \omega^{\mathrm{S}} \delta_u^{u^*} = \dfrac{p_{\mathrm{S}}\left(u^*\right)\left(\sum_{v\in U} \omega^{\mathrm{S}} O_n(v)\right) \quad \omega^{\mathrm{S}} O_n(u)}{1 \quad p_{\mathrm{S}}(u^*)} \\ \omega^{\mathrm{R}} \gamma_u^{u^*} = \dfrac{p_{\mathrm{R}}\left(u^*\right)\left(\sum_{v\in U} \omega^{\mathrm{R}} I_n(v)\right) \quad \omega^{\mathrm{R}} I_n(u)}{1 \quad p_{\mathrm{R}}\left(u^*\right)} \\ \varepsilon_O \le O_n(u) + \delta_u^{u^*} \le 1, \ \delta_u^{u^*} \ge 0 \\ \varepsilon_I \le I_n(u) \quad \gamma_u^{u^*} \le 1, \ \gamma_u^{u^*} \ge 0 \end{cases} \end{aligned} \tag{20}$$

- The set $U - \Sigma_q \cup E$ contains units that are completely inefficient because they use inefficiently their resources; for these units, one can consider one or both of the former analysis so that their productions are in adequacy with their resources consumptions and/or so to be as good as a unit that dominates them.

Remark: *One can notice that mathematical programming problems (17) and (20) are mathematically ill-posed problems (possibility of many solutions); by adding some other constraints such as uniform distribution for parameters* δ_u *and* γ_u *or* $\delta_u^{u^*}$ *and* $\gamma_u^{u^*}$*, or by adding lower and upper bounds for these parameters one can render these problems mathematically well-posed. To facilitate the presentation of the results of the analysis for the communication purpose (for instance), one can represent them in the plan (*p_R*,* p_S*).*

3.3 Comments

The boldness or caution index q permits actors to modulate their aspiration level; if few units are declared satisficing for an index of boldness q, actors may decide to reduce this index; in the contrary, if too much units are declared satisficing, actors may increase their aspiration level by increasing the boldness or caution index q so to reduce the number of satisficing units. This procedure can be programmed in order to make a variation of the value of boldness index q from its minimum value $q_{\min}$ till the number of satisficing equilibrium units (that is, less or equal to the number of equilibrium units) that one wish to include in the set B_q is obtained. This number may depend on the problem at hand and the preferences of actor, a preliminary task that must be carried up before utilising the model. Sensitivity analysis may furnish important information for other activities like negotiation: let us suppose that the set U is constituted by suppliers of a certain enterprise, this enterprise may use this sensitivity analysis to put them in competition and in reverse, they can use this analysis to see which of their weak positions can be improved. In terms of management, solving mathematical programming problems (17) and/or (20) to find improvable items for inefficient units correspond to tactical or operational decisions processes; this analysis can also contribute to strategic decisions such as stopping the activities of some inefficient units or emerging some of them to create synergy that contribute to improve the performance of the obtained unit.

One can notice that the approach presented in this paper does have the following strong advantages:

- it is easy to understand and to integrate in decision aid software
- actors preferences are expressed locally with regards to items regardless of their value and the process is transparent with regards to units; only their pursued objectives that may differ from one actor to another guide them
- knowing the units that dominate a given unit gives necessary information to determine the weak points of the last one
- sensitivity analysis furnishes important information for strategic and operational decision aid purpose
- it does not necessitate important computing power even for important real-world problems.

Nevertheless, this approach does present the following weak points:

- it is necessary to normalise data and there can be a problem of how to choose the normalisation scheme
- selectability and rejectability mass functions do not represent a meaningful parameters to decision makers as they are obtained by a complex aggregation process
- how to express their preferences may be a problem for actors mainly for those of less scientific culture even if the AHP approach may contribute to reduce this problem.

In Section 4, we will apply this approach to a real-world problem that aims to evaluate depots of a large organisation that supplies goods to supermarkets.

4 Application

4.1 Problem

A large retailing organisation which distributes goods to supermarkets consists of 20 depots that must be evaluated; this problem is taken from Emrouznejad (1995). The input items are taken to be the value of the stock (St) measured in *millions* of monetary units and the recurrent costs in the form of wages (Wa) measured in *hundreds of thousands* of monetary units. The output items, corresponding to the activity levels of the depots, are measured by the number of issues (Is) representing deliveries to supermarkets measured in *hundreds* of deliveries, the number of receipts (Rc) in bulk from suppliers measured in *thousands*, and the number of requisitions (Rq) on suppliers where they are out of stock or approaching stock out measured in *thousands* also. Data for this application are presented in Table 3.

Table 3 Data for depots problem

Depots	*St*	*Wa*	*Is*	*Rc*	*Rq*
#01	3	5	40	55	30
#02	2.5	4.5	45	50	40
#03	4	6	55	45	30
#04	6	7	48	20	60
#05	2.3	3.5	28	50	25
#06	4	6.5	48	20	65
#07	7	10	80	65	57
#08	4.4	6.4	25	48	30
#09	3	5	45	64	42
#10	5	7	70	65	48
#11	5	7	45	65	40
#12	2	4	45	40	44
#13	5	7	65	25	35
#14	4	4	38	18	64

Table 3 Data for depots problem (continued)

Depots	*St*	*Wa*	*Is*	*Rc*	*Rq*
#15	2	3	20	50	15
#16	3	6	38	20	60
#17	7	11	68	64	54
#18	4	6	25	38	20
#19	3	4	45	67	32
#20	5	6	57	60	40

4.2 Results

Application of DEA approach leads to the results of the fourth column of Table 4 obtained by Emrouznejad (1995), which shows that the relatively efficient depots are depots 12, 14, 15 and 19; for these depots, there is no (possible) virtual depot that does better. If we look closely, we can see that depot 14 is declared relative efficient because of its performance in requests item, that is, very high compared to other output items; this will be revealed when applying the method established in this paper. Applying the approach established in this paper, with assumption that all items of the same category are equally important leads to results of columns 2 and 3 of Table 4 in terms of selectability and rejectability measures, respectively.

We deduce from Table 4 that the satisficing set Σ_1, the equilibriums set E and benchmark depots set B_1 for an index of boldness or caution of 1 ($q = 1$) are the following (see also Figure 3):

- Σ_1 = {01, 02, 05, 09, 10, 12, 14, 15, 16, 19, 20}
- E = {02, 05, 07, 09, 10, 12, 15, 19, 20}
- B_1 = {02, 05, 09, 10, 12, 15, 19, 20}.

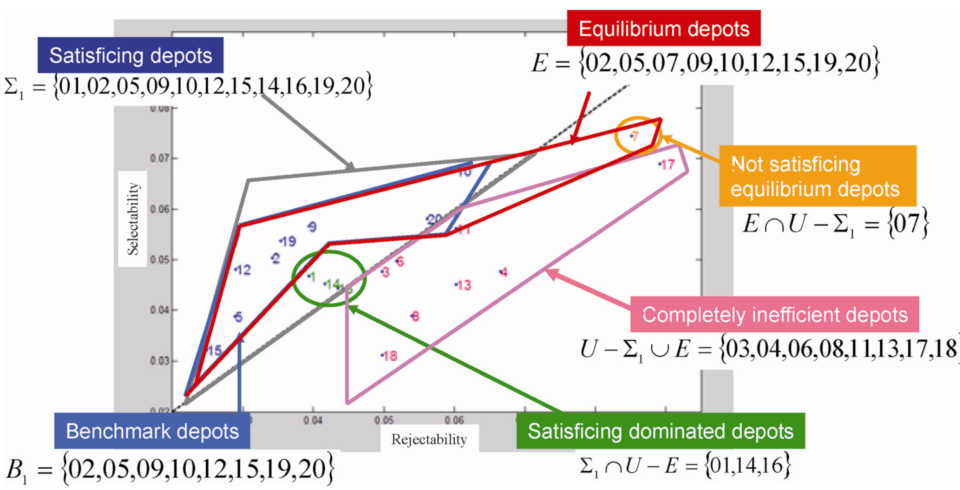
Table 4 Results of depots problem

Depots	p_S	p_R	*DEA*
#01	0.0466	0.0394	0.82
#02	0.0503	0.0342	0.94
#03	0.0476	0.0498	0.82
#04	0.0476	0.0666	0.65
#05	0.0387	0.0289	0.95
#06	0.0496	0.0519	0.83
#07	0.0744	0.0852	0.71
#08	0.0389	0.0540	0.52
#09	0.0565	0.0394	0.96
#10	0.0675	0.0603	0.89
#11	0.0561	0.0603	0.63

Table 4 Results of depots problem (continued)

Depots	p_S	p_R	*DEA*
#12	0.0480	0.0290	1.00
#s13	0.0450	0.0603	0.83
#14	0.0452	0.0417	1.00
#15	0.0321	0.0249	1.00
#16	0.0443	0.0435	0.91
#17	0.0689	0.0892	0.55
#18	0.0310	0.0498	0.42
#19	0.0537	0.0354	1.00
#20	0.0581	0.0562	0.84

Figure 3 Representation of results of depots problem in the plan (p_R, p_S) (see online version for colours)



4.3 *Analysis of results and managerial implications*

The set $E - B_1$ is reduced to depot #07 (see Figure 3); this means that even if this depot is not dominated it can improve its items. By solving a non-linear programming problem similar to that of Equation (17) for this depot, we obtain $\delta_{07} \; \begin{bmatrix} 0.00 & 0.03 & 0.06 \end{bmatrix}^{\mathrm{T}}$ and $\gamma_{07} \; \begin{bmatrix} 0.11 & 0.11 \end{bmatrix}^{\mathrm{T}}$.

Given that $O_n(07) \; \begin{bmatrix} 1.00 & 0.97 & 0.88 \end{bmatrix}^{\mathrm{T}}$ and $I_n(07) \; \begin{bmatrix} 1.00 & 0.91 \end{bmatrix}^{\mathrm{T}}$, a tactical or operational managerial implication of this result could be to demand depot #07 to increase its output items by 0%, 3.08% and 7.10%, respectively, and to reduce its resources consumption by 10.51% and 11.56%, respectively, in order to become satisficing if the performances of other units remain unchanged (see Figure 4). This improvement can be done by observing the organisation, the environment and local management procedures of benchmark depots namely those of the set B_1 = {02, 05, 09, 10, 12, 15, 19, 20}.

In the same way, we have $\Sigma_1 - B_1$ = {01, 14, 16} as satisficing dominated depots with the following corresponding sets of depots that dominate them (see Figure 3).

$D(01)$ = {02, 09, 12, 19}

$D(14)$ = {01, 02, 09, 12, 19}

$D(16)$ = {01, 02, 09, 12, 14, 19}

This means that even if depots #01, #14 and #16 are performing well, individually, they can improve their performance as there are some depots that are performing better than them. For these depots, by solving linear programming problems as that presented in Equation (20), we obtain results present in Tables 5–7. Quantitatively, the effort to be done by these depots in order to become benchmarks is given by Tables 5–7, respectively, and in Figure 4 that also gives important information on how to achieve this effort as we can see in this figure that these depots should just do as better as depot 02 in order to become benchmarks. Mangers can compare operating and organisational methods of these units to that of depot 02 in order to find adequate recommendation for improvement procedure.

Table 5 Parameters for improving performance of depot #01

U^*	*#02*	*#09*	*#12*
$\delta_{01}^{u^*}$	$\begin{bmatrix}0.0663\\0.0073\\0.0767\end{bmatrix}$	$\begin{bmatrix}0.1785\\0.0273\\0.1972\end{bmatrix}$	$\begin{bmatrix}0.0202\\0.0132\\0.0241\end{bmatrix}$
$\gamma_{01}^{u^*}$	$\begin{bmatrix}0.0568\\0.0641\end{bmatrix}$	$\begin{bmatrix}0.0002\\0.0000\end{bmatrix}$	$\begin{bmatrix}0.1152\\0.1250\end{bmatrix}$

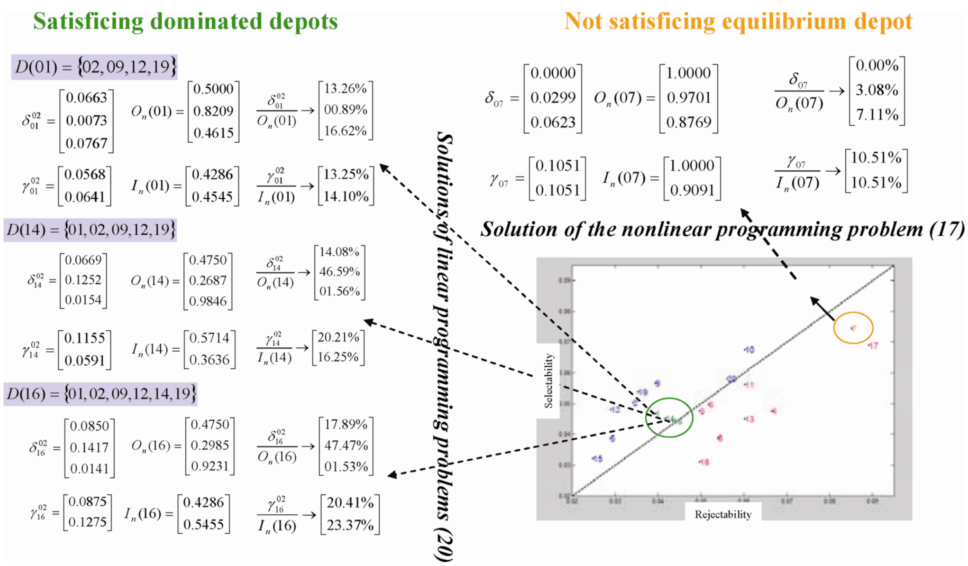
Table 6 Parameters for improving performance of depot #14

U^*	*#01*	*#02*	*#12*	*#19*
$\delta_{14}^{u^*}$	$\begin{bmatrix}0.0084\\0.0344\\0.0152\end{bmatrix}$	$\begin{bmatrix}0.0669\\0.1252\\0.0154\end{bmatrix}$	$\begin{bmatrix}0.0296\\0.0695\\0.0153\end{bmatrix}$	$\begin{bmatrix}0.1270\\0.2068\\0.0119\end{bmatrix}$
$\gamma_{14}^{u^*}$	$\begin{bmatrix}0.0349\\0.0194\end{bmatrix}$	$\begin{bmatrix}0.1155\\0.0591\end{bmatrix}$	$\begin{bmatrix}0.1853\\0.1085\end{bmatrix}$	$\begin{bmatrix}0.1017\\0.0453\end{bmatrix}$

Table 7 Parameters for improving performance of depot #16

U^*	*#01*	*#02*	*#09*	*#12*	*#14*	*#19*
$\delta_{16}^{u^*}$	$\begin{bmatrix}0.02\\0.05\\0.03\end{bmatrix}$	$\begin{bmatrix}0.09\\0.14\\0.01\end{bmatrix}$	$\begin{bmatrix}0.20\\0.29\\0.003\end{bmatrix}$	$\begin{bmatrix}0.04\\0.08\\0.02\end{bmatrix}$	$\begin{bmatrix}0.00\\0.01\\0.02\end{bmatrix}$	$\begin{bmatrix}0.15\\0.22\\0.01\end{bmatrix}$
$\gamma_{16}^{u^*}$	$\begin{bmatrix}0.04\\0.06\end{bmatrix}$	$\begin{bmatrix}0.09\\0.13\end{bmatrix}$	$\begin{bmatrix}0.02\\0.07\end{bmatrix}$	$\begin{bmatrix}0.14\\0.19\end{bmatrix}$	$\begin{bmatrix}0.02\\0.02\end{bmatrix}$	$\begin{bmatrix}0.07\\0.12\end{bmatrix}$

Figure 4 Improvements to be done by depots #01, #07, #14 and #16 to become benchmark depots (results from non-linear programming problem (17) and linear programming problem (20)) (see online version for colours)



The rest of the results are given by (see also the Figure 3):

$U \quad \Sigma_1 \cup E \quad \{03, 04, 06, 08, 11, 13, 17, 18\}$ with the following dominating units for each of these units:

$D(03) = \{02, 09, 12, 19\}$

$D(04) = \{02, 06, 9, 10, 11, 12, 19, 20\}$

$D(06) = \{02, 09, 19\}$

$D(08) = \{01, 02, 03, 06, 09, 12, 14, 16, 19\}$

$D(11) = \{09, 10, 20\}$

$D(13) = \{01, 02, 03, 06, 09, 10, 11, 12, 14, 19, 20\}$

$D(17) = \{07\}$

$D(18) = \{01, 02, 03, 05, 09, 12, 14, 15, 16, 19\}$

In terms of management and depending on strategic objectives of the organisation, one may ask some of these depots to make an effort to improve their performance and/or reorganise them by closing or emerging some of them.

4.4 Comments

When comparing with DEA method, we see that all depots declared efficient by DEA method are efficient benchmark depots according to our approach except depot 14 (satisficing but dominated) that is, dominated by depots #01, #02, #12 and #19; this is due to the fact that in the DEA approach, by putting maximum weight on the third output item, depot 14 can be efficient since the ratios between its third output item and input items are very high compared to other ratios.

Figure 3 shows the graphical representation in the plan (p_R, p_S) of previous analysis. This visualisation is interesting for communication purpose for instance and/or for determining proximity between non-benchmark and benchmark units so that mathematical programming problems (17) and (20) can be formulated more effectively. For instance, Figure 4 shows that it is sufficient for depots #01, #14 and #16 to do as better as depot #02 to become benchmark depots.

5 Conclusion

Competitive benchmarking of production units that consume different resources to deliver different products has been considered in this paper. The presented approach that can integrate possibly antagonist opinion of different actors for the evaluation of the adequacy between resources consumption and production level of a given set of production units is based on satisficing games as the underlying mathematical tool. This tool consists in assigning to each unit two measures: a selectability measure that roughly measures the degree of production and a rejectability measure that evaluates resources consumption leading to the notion of good enough units instead of optimal units. The principal contribution of this paper is the established procedure to compute the selectability and the rejectability measures for each unit using its items performance and actors preferences that are expressed as relative importance (through a weighting process) of each item within its category (input or output). Another point of the approach proposed in this paper to highlight is that of offering a framework for sensitivity analysis that allows to detect the weak points of a given unit and to compute the improvement percentage of each item of this unit in order to render it efficient. The benchmarking problem is then carried up in two steps:

- one determines units that can be considered as benchmark units
- one determines the improvements to be undertaken by each non-benchmark unit in order to become a benchmark.

The application of this approach to a real-world problem proves some advantages over other methods such as DEA and the easiness for its utilisation as a decision support tool.

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References

Åhrén, T. and Parida, A. (2009) 'Maintenance performance indicators (MPIs) for benchmarking the railway infrastructure: a case study', *Benchmarking: An International Journal*, Vol. 16, No. 2, pp.247–258.

Brans, J.P., Mareschal, B. and Vincke, P. (1984) 'PROMETHEE: a new family of outranking methods in multi-criteria analysis', In J.P. Brans (Ed.), *Operational Research*, IFORS; Amsterdam: North Holland, pp.477–490.

Brans, J.P., Mareschal, B. and Vincke, P. (1986) 'How to select and how to rank projects: the PROMETHEE method', *European Journal of Operational Research*, Vol. 24, pp.228–238.

Charnes, A., Cooper, W.W. and Rhodes, E. (1978) 'Measuring the efficiency of decision making units', *European Journal of Operational Research*, Vol. 2, pp.429–444.

Collins, T.R., Rossetti, M.D., Nachtmann, H.L. and Oldham, J.R. (2006) 'The use of multi-attribute utility theory to determine the overall best-in-class performer in a benchmarking study', *Benchmarking: An International Journal*, Vol. 13, No. 4, pp.431–446.

Deros, B.Md, Yusof, S.M. and Salleh, A.Md. (2006) 'A benchmarking implementation framework for automotive manufacturing SMEs', *Benchmarking: An International Journal*, Vol. 13, No. 4, pp.396–430.

Emrouznejad, A. (1995) *DEA Homepage*. Available at: http://www.deazone.com/.

Ignizio, J.P. (1976) *Goal Programming and Extensions*. Lexington: Lexington Books.

Karuppusami, G., Gandhinathan, R. and Rao, R.V. (2006) 'A quality function deployment approach to internal benchmarking of critical success factors of strategic quality management', *Int. J. Services and Operations Management*, Vol. 2, No. 2, pp.178–201.

Kounis, L.D. and Panagopoulos, N. (2007) 'Total quality management and benchmarking: bridging the gap in the public sector', *Int. J. Services and Operations Management*, Vol. 3, No. 2, pp.245–259.

Min, H. and Joo, S-J. (2009) 'Benchmarking third-party logistics providers using data envelopment analysis: an update', *Benchmarking: An International Journal*, Vol. 16, No. 5, pp.572–587.

Moffett, S., Anderson-Gillespie, K. and McAdam, R. (2008) 'Benchmarking and performance measurement: a statistical analysis', *Benchmarking: An International Journal*, Vol. 15, No. 4, pp.368–381.

Pomerol, J-C. and Barba-Romero, S. (1993) *Choix Multicritère dans l'entreprise, principe et pratique*. Paris: Hermes.

Pursglove, J. and Simpson, M. (2007) 'Benchmarking the performance of English universities', *Benchmarking: An International Journal*, Vol. 14, No. 1, pp.102–122.

Rehman, A-U. and Babu, A.S. (2008) 'Manufacturing competitiveness assessment for business excellence', *Int. J. Business Excellence*, Vol. 1, No. 3, pp.231–261.

Roux, D. (2007) *Les 100 Mots de la Gestion*. Paris: PUF.

Roy, B. and Bouyssou, D. (1993) Aide *Multicritere a la Decision: Methodes et Cas*. Paris: Economica.

Saaty, T. (1980) *The Analytic Hierarchy Process*. New York: John Wiley.

Steuer, R.E. (1986) *Muticriteria Optimization: Theory, Computation, and Application*, New York: Wiley.

Stirling, W.C. (2003) *Satisficing Games and Decision Making: With Applications to Engineering and Computer Science*. New York: Cambridge University Press.

Tchangani, A.P. (2006a) 'SANPEV: a satisficing analytic network process framework for efficiency evaluation of alternatives', *Foundations of Computing and Decision Sciences Journal*, Vol. 31, Nos. 3–4, pp.291–319.

Tchangani, A.P. (2006b) 'Multiple objectives and multiple actors load/resource dispatching or priority setting: satisficing game approach', *Advanced Modelling and Optimization: An Electronic International Journal*, Vol. 8, No. 2, pp.111–134.

Tchangani, A.P. (2006c) 'A satisficing game theory approach for group evaluation of production units', *Decision Support Systems*, Vol. 42, No. 2, pp.778–788.

Tchangani, A.P. (2006d) 'A satisficing game theoretic framework for retrieving relevant objects from a database', *Int. J. Computers, Systems and Signals*, Vol. 7, No. 2, pp.18–29.

Tchangani, A.P. (2009a) 'Evaluation model for multi attributes – multi agents decision making: satisficing game approach', *Int. J. Information Technology and Decision Making*, Vol. 8, No. 1. pp.73–91.

Tchangani, A.P. (2009b) 'Modeling selecting and ranking alternatives characterized by multiple attributes to satisfy multiple objectives', *Journal of Information and Computing Science*, Vol. 4, No. 1, pp.3–16.

Teghem, J. (1996) *Programmation Linéaire*. Bruxelles: Université de Bruxelle.

Vincke, Ph. (1989) *L'aide Multicritere a la Décision*. Bruxelles: Université Libre de Bruxelles.

Von Neumann, J. and Morgenstern, O. (1944) *The Theory of Games and Economic Behavior*. Preston: Preston University Press.

Zahir, S., Sarker, R. and Al-Mahmud, Z. (2009) 'An interactive decision support system for implementing sustainable relocation strategies for adaptation to climate change: a multi-objective optimisation approach', *Int. J. Mathematics in Operational Research*, Vol. 1, No. 3, pp.326–350.

Notes

[1] A matrix A is consistent if it verifies:

$$A_{ii} = 1,\ A_{ji} = \frac{1}{A_{ij}} \text{ and } A_{ik} = A_{ij}A_{jk}\ .$$

[2] Other possible normalisation schemes are given by: $O_n(u,i) = \dfrac{O(u,i) - \min_{v\in U} O(v,i)}{\max_{v\in U} O(v,i) - \min_{v\in U} O(v,i)}$

and $I_n(u,i) = \dfrac{I(u,i) - \min_{v\in U} I(v,i)}{\max_{v\in U} I(v,i) - \min_{v\in U} I(v,i)}$ or $O_n(u,i) = \dfrac{O(u,i)}{\sum_{v\in U} O(v,i)}$ and $I_n(u,i) = \dfrac{I(u,i)}{\sum_{v\in U} I(v,i)}$