EVALUATION MODEL FOR MULTIATTRIBUTES–MULTIAGENTS DECISION MAKING: SATISFICING GAME APPROACH

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This paper considers the *evaluation step* in a decision-making process that follows decision-making *goals* setting, feasible *alternatives* and *attributes* or *criteria* that characterize them determination steps. Evaluation step must establish a model or algorithm to evaluate alternatives taking into account their performances with regard to criteria as well as decision makers or stakeholders preferences. Though this problem is rather a classic one, researches related to evaluation model construction continue to be active to find models that cope with more realities or that fit well how human beings behave in group and proceed when facing the problem of choosing, ranking or sorting alternatives or options. The purpose of this paper is to construct an evaluation model that integrate the performances of alternatives with regard to attributes or criteria and decision makers or agents opinions with regard to the importance to assign to each criterion in order to obtain a value function. As any decision problem is almost always a matter of trade-off, among attributes characterizing alternatives there will be those acting toward the achievement of decision makers goal (benefit) and those that decision makers would like to reduce as much as possible (cost); we will designate the first ones as *positive attributes* and the later ones as *negative attributes*. The process of dividing attributes into positive attributes and negative attributes is beyond the scope of this paper and this partition will be considered as a part of the problem specification. The model is constructed in two steps: firstly, *satisfiability* (*selectability* and *rejectability*) measures or functions are obtained for each alternative using attributes values (positive attributes will contribute to selectability measure whereas negative ones are used in the derivation of rejectability measure) and agents opinions in the framework of satisficing game theory and secondly a value function is built on that measures. Agents opinions with regard to attributes will be expressed locally by weighting them by category (positive/negative).

Keywords: Evaluation model; multiattributes–multiagents decision making; satisficing game theory; value function.

1. Introduction and Statement of the Problem

Decision making is a daily activity of humans, individually or collectively, that can be more or less complex. Decision making is, in general, a process with many steps such as formulating the decision *goal or objective*, identifying *attributes* through which to measure the performance of potential *alternatives* that can respond to the decision goal and making *recommendation* regarding the alternatives to be implemented so that the results will be closed as much as possible to the decision goal. The final recommendation scheme of a decision process can be reduced to three main processes: *choosing* (this is a relative evaluation that finds a subset of alternatives that satisfy the decision goal), *ranking* (relative evaluation that ranks alternatives from the best to the worst with regard to the decision goal) or *sorting* (an absolute evaluation that assigns alternatives to some predefined categories according to a prescribed norm regarding the decision goal). The construction of an evaluation model is then an important step in the decision process that is often carried by an expert known in the literature as the *analyst* (see Ref. 4); this step is the main purpose of this paper: we suppose that the upstream processes have been considered and we are in possession of the set of alternatives, the set of attributes and their values for each alternative as well as the decision goal; our duty is to construct an evaluation model for the final recommendation purpose. The remainder aspect of the problem is that the decision process involves most of the time many decision makers or agents (stakeholders) with possible conflicting points of view that must be taken into account by the analyst who is in charge of constructing the evaluation model. The context in which an evaluation model must be constructed in this paper is given by the following definition.

Definition 1.1. The multiattributes multiagents decision-making problem considered here consists in:

- a discrete set U of n alternatives (actions or decisions),

$$U = \{u_1, u_2, \ldots, u_n\}; \tag{1}$$

- a discrete set A of m attributes or criteria that characterize each alternative,

$$A = \{a_1, a_2, \ldots, a_m\}; \tag{2}$$

- a performance function (or matrix), $\rho : U \times A \to D$ where $D = \times_{a_i \in A} D_i$ and D_i is the *domain* of attribute a_i (the set of possible values that attribute a_i is allowed to take), $\rho(u, a_i) \in D_i$, $\forall\ u \in U$, $a_i \in A$;
- a set of d decision makers or agents that will express their preferences;
- the purpose of the evaluation model is to be used to choose the best (in some sense) alternatives, to rank the set of alternatives from the best to the worst or to sort them according to the decision goal.

Remark 1.1. $\rho(u, a)$ can be a numerical value or an ordinal evaluation (qualitative value) such as good, bad, medium, etc. Here we consider only numerical value; ordinal values (if they exist) are translated to numerical values by some consistent procedure and the domain of the attribute i is defined by $D_i = [\rho^i_{\min}, \rho^i_{\max}]$ where lower and upper values $\rho^i_{\min}$ and $\rho^i_{\max}$ are supplied by decision maker(s). If decision makers are not able to supply these values, we consider that $\rho^i_{\min} = \min_{u \in U} \rho(u, a_i)$ and $\rho^i_{\max} = \max_{u \in U} \rho(u, a_i)$.

Such a decision-making problem can represent practical situations in many domains such as management, engineering, economics, social, etc., see for instance Refs. 4, 10, 12, 15, 20 and references therein. In the rapidly expanding domain of e-commerce, researchers are designing negotiation algorithms using agent technology that allow consumers to negotiate over multiple attributes of a product besides the price (see for instance Ref. 8). The different approaches that deal with evaluation process modeling for decision problems as specified by the previous definition in the literature can be regrouped into two main categories briefly recalled below.

- Evaluation models based on *value function*(*s*): roughly speaking, these techniques consider a numerical function π defined on the set U such that

$$\pi(u) \geq \pi(v) \Leftrightarrow u \succsim v, \tag{3}$$

  where "$u \succsim v$" stands for "u is at least as good, with regard to decision goal, as v" leading to a weak order. The evaluation modeling process then consists in building such a function based on the performance function and decision makers preference (obtained in general by answering some particular questions of the analyst); there are many techniques employed in the literature for constructing such a value function where a number of them suppose a particular form for π such as expected utility or additive value function (see Refs. 4, 10, 20 and references therein).
- Evaluation models allowing incomparability and/or intransitivity known in the literature as outranking methods such as the family of ELECTRE procedures and PROMETHEE techniques (see Refs. 4, 10, 20).

The majority of techniques to construct an evaluation model previously evoked do not make difference between attributes that act in the sense of decision goal achievement (positive attributes) and those that act against the goal achievement (negative attributes); we argue that this situation is common in a decision-making problem (decision-making is a matter of tradeoff). We do think that humans, to judge a complex situation or an alternative, will first balance its positive attributes vs its negative attributes in order to see if this situation or alternative is feasible before possibly comparing it to other situations or alternatives; the approach considered in this paper falls in this way of thinking. In this paper, we consider a point of view similar to evaluation models based on value function defined previously where the value function will be constructed using a pair of measures known as satisfiability measures or functions derived, for each alternative, from its performance function and decision makers preferences expressed over the attributes instead of alternatives in the framework of satisficing game theory. Notice that concepts from classical game theory have been also considered to solve multiattributes multiagents decision-making problems in domains like e-commerce (see for instance Refs. 3, 5, 21). The remainder of this paper is organized as follows: in Sec. 2, the satisficing game theory is briefly presented (only the elements that are relevant to our objective will be presented); Sec. 3 is the core of this paper and is concerned by formulating a multiattributes multiagents decision-making problem as a satisficing game. Finally in Sec. 4, we consider an example to show potential applicability of this approach.

2. Satisficing Game Theory

This section is devoted to the presentation of the basis of satisficing game theory; the materials (satisficing set, equilibrium set, etc.) presented in this section are adapted or come from Ref. 16. The underlying philosophy of most of the techniques used in the literature to construct the evaluation model is the superlative rationality, looking for the best or optimizing; all the alternatives must be compared against each other. But the superlative rationality paradigm is not necessarily the way humans evaluate alternatives (and maybe not the best one). Most of the time humans content themselves with alternatives that are just "*good enough or satisficing.*" The concept of satisficing, instead of the idea behind the H. Simon's bounded rationality (see Ref. 14) where the acceptation of a suboptimal solution is only due to the limited cognitive capabilities of decision makers and the imperfection of information (if one can optimize it will), is considered here as a decision-making paradigm. This concept of being good enough allows to have always a nonempty solutions set because one can adjust its aspiration level to obtain at least an alternative that is satisficing. On the other hand, decision makers more probably tend to classify alternatives as good enough or not good enough in terms of their positive attributes (benefit) and their negative attributes (cost) with regard to the decision goal instead of ranking them with regard to each other. For instance, to evaluate a number of cars from which we wish to select one (decision objective), we often make a list of positive attributes (driving comfort, speed, robustness, etc.) and a list of negative attributes (price, gas consumption per kilometer, maintainability, etc.) of each car and then make a list of cars for which positive attributes "exceed" negative attributes in some sense. This way of evaluation falls into the framework of praxeology or the study of theory of practical activity (the science of efficient action). Here decision maker(s), instead of looking for the best options or alternatives, look for the satisficing alternatives. Satisficing is a term that refers to a decision-making strategy where options, units, or alternatives are selected which are "good enough" instead of being the best.[16] Let us consider a universe U of alternatives; then for each alternative $u \in U$, a *selectability measure or function* $p_S(u)$ and a *rejectability measure function* $p_R(u)$ are defined to measure the degree to which u works towards success in achieving the decision maker goal and costs associated with this alternative, respectively. This pair of measures called *satisfiability functions* must have the mathematical structure of probability[16]: they are nonnegative and sum to one on U. In fact, the probability structure requirement for satisfiability functions in this paper is not rigorously necessary as we do not consider that decision makers influence each other (see later). When decision makers can interact, this requirement become necessary in order to compute easily the joint satisfiability functions using praxeic networks (see Refs. 1, 2), a graphical tool that works exactly as Bayesian networks.[7,9] The following definition then gives the set of alternatives arguable to be "good enough" because for these alternatives, the "benefit" expressed by the function p_S exceeds the cost expressed by the function p_R with regard to an index of caution q.

Definition 2.1. (See Ref. 16). The satisficing set $S_q \subseteq U$ is the set of alternatives defined by the following equation

$$S_q = \{u \in U : p_s(u) \geq qp_r(u)\}. \tag{4}$$

The caution index q can be used to adjust the aspiration level: small values of this index will lead to lot of alternatives being declared satisficing whereas large values of q will reduce the number of satisficing alternatives. A sensitivity analysis can be carried up to determine the value $q_{\min}$ below which all the alternatives of U will be declared satisficing and a value $q_{\max}$ above which no alternative will be satisficing. For all alternatives of U to be declared satisficing the following inequality (5)

$$p_s(u) \geq qp_r(u)\ \forall\ u \in U \Leftrightarrow q \leq q_{\min} = \min_{u\in U}\left(\frac{p_s(u)}{p_r(u)}\right), \tag{5}$$

must be verified so that for such an indices of caution q we have Eq. (6)

$$\Sigma_q = U. \tag{6}$$

On the contrary, there is no satisficing alternative, that is

$$\Sigma_q = \varnothing, \tag{7}$$

if and only if the following inequality (8)

$$p_s(u) < qp_r(u)\ \forall\ u \in U \Leftrightarrow q > q_{\max} = \max_{u\in U}\left(\frac{p_s(u)}{p_r(u)}\right), \tag{8}$$

is verified. Finally, if the index of caution verifies $q \in [q_{\min}, q_{\max}]$ then we have Eq. (9)

$$\Sigma_q \subseteq U. \tag{9}$$

But for a satisficing alternative, there can exist other satisficing alternatives that are better (having more selectability and at most the same rejectability or having less rejectability and at least the same selectability) than the previous one; it is obvious that in this case any rational decision maker will prefer the later alternatives. So the interesting set is that containing satisficing alternatives for which there are no better alternatives: this is the *satisficing equilibrium* set E_q^S. To define this set, let us define first, for any alternative $u \in U$, the set $B(u)$ of alternatives that are strictly better than u[16]

$$B(u) = B_S(u) \cup B_R(u), \tag{10}$$

where $B_S(u)$ and $B_R(u)$ are defined by Eqs. (11)

$$\begin{aligned} B_S(u) &= \{v \in U : p_R(v) < p_R(u) \text{ and } p_S(v) \geq p_S(u)\}, \\ B_R(u) &= \{v \in U : p_R(v) \leq p_R(u) \text{ and } p_S(v) > p_S(u)\}. \end{aligned} \tag{11}$$

The equilibrium set E (alternatives for which there are no strictly better alternatives) is then defined by Eq. (12)

$$E = \{u \in U : B(u) = \varnothing\}, \tag{12}$$

and the satisficing equilibrium set, E_q^S is finally given by Eq. (13)

$$E_q^S = E \cap S_q. \tag{13}$$

Figure 1 shows the satisficing set Σ_q (Fig. 1(a)) and for a given alternative u the space where lay the set $B(u)$ of alternatives that dominate it (Fig. 1(b)) in the plane (p_R, p_S).

Remark 2.1. Notice that the equilibrium set E cannot be empty otherwise there would be at least an alternative laying in the colored space of Fig. 1(b) for any alternative u and this is impossible; as the satisficing set Σ_q can always be rendered nonempty by managing the index of caution q, one can always find satificing equilibria for any problem respecting then the spirit of satisficing game theory that is any problem can have a solution as soon as decision makers are disposed to revise their aspiration level.

In Sec. 3, we will establish a method that puts the problem of multiattributes–multiagents decision-making as defined in the introduction section into the satisficing game theory framework by setting up a systematic method to compute satisfiability functions $p_S(u)$ and $p_R(u)$ for each alternative u from its attributes performance and decision makers preferences.

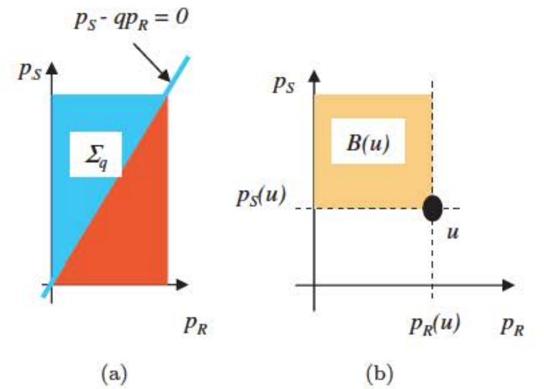

Fig. 1. Satisficing set Σ_q and dominating alternatives set $B(u)$ for alternative u in the plan (p_R, p_S).

3. Satisficing Evaluation Model

To establish the model of evaluation of a multiattributes–multiagents decision-making problem in the framework of satisficing game theory, let us make a parallel with the evaluation of production units. A production unit is a system that uses inputs (or resources) to produce outputs (goods or services). Defining an evaluation model of production units problem as a satisficing game is straightforward because the *rejectability function* will be related to inputs or resource consumption (*negative attributes*) and the selectability function will be a function of output performances (*positive attributes*); this approach has been used by the author in Refs. 17 and 18 to establish a performance evaluation procedure of a group of homogeneous production units. The negative attributes (resources) can be interpreted as the price to pay to obtain the positive attributes (goods or deliveries); we do think that this interpretation hold for a multicriteria decision problem because there will be always some attributes that work toward the achievement of the decision goal (positive attributes) and those that work against this achievement (negative attributes). We can then interpret each alternative as a production system that consume negative attributes to produce positive ones. The distinction between negative attributes and positive attributes will probably depend on the psychology of decision makers instead of a formal procedure. This consideration is beyond the scope of this paper; we suppose that this distinction has been done before tackling the construction of the evaluation model. The approach of decision-making that distinguishes positive and negative attributes is widely used in decision-making literature; for instance, in Ref. 11 the benefits, opportunities, costs and risks approach clearly lay in positive/negative attributes approach where benefits and opportunities are related with positive attributes and costs and risks to negative ones. We suppose then that the attributes set A is partitioned as $A = A_p \cup A_n$ and $A_p \cap A_n = \varnothing$ where A_p contains the positive attributes and A_n is the subset of negative attributes. The following definition gives a formal characterization of this distinction.

Definition 3.1. Let us define by $\mu_u(a)$ a numerical function that measures the degree to which the attribute a works toward the realization of the decision goal by the alternative u; then for positive attributes a and b we have

$$\rho(u,a) \geq \rho(u,b) \Rightarrow \mu_u(a) \geq \mu_u(b), \quad \forall\ u, \tag{14}$$

and for negative attributes a and b the following inequality

$$\rho(u,a) \geq \rho(u,b) \Rightarrow \mu_u(a) \leq \mu_u(b), \quad \forall\ u, \tag{15}$$

holds.

For instance if the decision goal is to buy a car, we will pay a price (the cost of the car) and the gas consumption to obtain driving comfort, speed, acceleration, etc.; so that, for us price and gas consumption per kilometer will constitute the negative attributes whereas the acceleration, the driving comfort and the speed will represent positive attributes; another decision maker may consider another distinction. In the following paragraphs, we will establish a systematic procedure to compute satisfiability functions using attributes performances and decision makers or agents preferences expressed by weighting attributes.

3.1. *Evaluation model*

3.1.1. *Agents preferences*

The fundamental characteristic of multiagents decision-making is the possible conflicting interests among agents in terms of importance to assign to each negative attribute as well as to each positive attribute (we suppose that decision makers agree on the partition of the attributes set into positive/negative attributes). Our purpose in this paper is to derive a method that integrates different point of view of the agents expressed through weights assigned to attributes. We assume that d agents express their preferences (concerns) with regard to negative attributes and positive attributes through following weights (α_{kj} and β_{kj}) defined on the same scale for each class of attributes; but the scale does not need to be the same for the negative attributes and positive attributes:

- α_{kj} ($k = 1, 2, \ldots, d$; $a_j \in A_p$) is the weight assigned by the agent k to the positive attribute $a_j \in A_p$; the more, the attribute a_j works toward achieving the decision goal (selectability), in the view of the agent k, the more important is the weight α_{kj};
- β_{kj} ($k = 1, 2, \ldots, d$; $a_j \in A_n$) is the weight assigned by the agent k to the negative attribute $a_j \in A_n$; the more, the attribute a_j works against the achievement of the decision goal (rejectability), in the view of the agent k, the more important is the weight β_{kj}.

We do think that it is easier to ask the agents to compare attributes in order to express their preferences rather than to pair-wise compare alternatives as it is often done in multicriteria decision literature. These weights are then aggregated to define the selectability weight ω_j^S and the rejectability weight ω_j^R for each attribute $a_j \in A$ by taking the mean value over agents' preference as given by Eq. (16)

$$\omega_j^S = \frac{\sum_{k=1}^{d} \alpha_{kj}}{\sum_{u_j \in A_p} \sum_{k=1}^{d} \alpha_{kj}} \quad \text{and} \quad \omega_j^R = \frac{\sum_{k=1}^{d} \beta_{kj}}{\sum_{u_j \in A_n} \sum_{k=1}^{d} \beta_{kj}}. \tag{16}$$

The weights ω_j^S and ω_j^R measure the aggregate strength that agents attach to the positive attribute $a_j \in A_p$ and the negative attribute $a_j \in A_n$, respectively, with regard to other items of the same category; let us denote by ω^S and ω^R the following row vectors (see Eq. (17))

$$\begin{aligned} \omega^S &= [\omega_1^S, \omega_2^S, \ldots, \omega_{|A_p|}^S], \\ \omega^R &= [\omega_1^R, \omega_2^R, \ldots, \omega_{|A_n|}^R], \end{aligned} \tag{17}$$

where $|M|$ is the cardinal, the number of elements, of the set M.

Derivation of weights α_{kj} and β_{kj} by decision makers may be not easy in practice mainly for those decision makers that are not familiar with decision analysis subjects. To overcome such possible difficulty, the analyst may set up a procedure based, for instance, on analytic hierarchy process (AHP) approach (see Ref. 11) where each decision maker will be asked to choose, for each category (positive/negative) of attributes, a pivot attribute and compare other attributes to it using AHP scale and then the analyst will deduce the concerned weight using this comparison (see Refs. 18 and 19 for detail formulation of this idea). Furthermore, by using web technology, this way of doing permit to possibly have decision makers be remotely and geographically distributed; a decision maker that is connected to gives his/her opinion will be presented only with attributes and their description without knowing neither their values nor the alternatives.

The multiagents decision-making approach considered in this paper differs from the multiagent component of the original theory of satisficing game.[16] Indeed in this paper, the agents or decision makers are implicitly supposed to be independent and the model just aggregate their opinions whereas in the original theory dependence is widely considered to express some social behavior of agents such altruism or aggressiveness, see for instance Refs. 1, 2 and 16. In this case, the decision problem are formulated in terms of praxeic networks that work like Bayesian networks to compute the joint satisfiability functions; this explain in part the need for satisfiability mass functions to satisfy a mathematical structure of probability. Rigorously speaking, for our approach here, there is no need for satisfiability functions to satisfy probabilistic structure but as this approach can be thought as a particular case of that of dependence, we maintain this constraint that do not influence the result. As a Bayesian Network is a direct acyclic graph that represents some probabilistic dependence (correlation, causality, ...) between variables of a given knowledge domain, a praxeic network is a direct acyclic graph that represents influence between personas of a group of decision makers. Each decision makers has two persona: *selectability* that works towards achieving goals without caring about resources consumption and *rejectability* that tends to preserve resources without caring about goals achievement. But the persona of a given decision maker may depend on the behavior of other decision makers and this interaction can be represented by a direct acyclic graph (a praxeic network), see for instance Refs. 1, 2 and 16, with a joint interdependence function p_{SR}; the joint selectability and rejectability measures are then obtained by marginalization of the interdependence function. Concerning our evaluation problem here, one can imagine possible dependency; in this case, we can simplify the situation by constructing two praxeic networks to derive separately the selectability measure $p_S(u)$ and the rejectability measure $p_R(u)$ given the particular features of this problem (negative/positive attributes distinction). This can be done in the framework of Bayesian networks by specifying a conditional selectability table and a conditional rejectability table to measure the strength of interaction represented by praxeic networks, as well as the satisfiability measures for decision makers that are not influenced; this later process can be carried up by the approach presented in this paper; then using inference algorithms of Bayesian networks, see for instance Ref. 7, ultimate satisfiability measures $p_S(u)$ and $p_R(u)$ will be determined.

3.1.2. *Normalized performance function*

The normalization of the original performance function is necessary before weighting because attributes performances are not, in general, expressed in the same units (money, memory capacity, human resources, surface, machines, qualitative, etc.). Let us then define the normalized column vectors (utilities) $\rho_p^S(u)$ of length $|A_p|$ of the performance function corresponding to the positive attributes and $\rho_n^R(u)$ of length $|A_n|$ corresponding to the negative attributes as shown by Eq. (18)

$$\begin{aligned} \rho_n^S(u) &= [\rho_n^S(u,a_1), \rho_n^S(u,a_2), \ldots, \rho_n^S(u,a_{|A_p|})]^{\mathrm{T}}, \\ \rho_n^R(u) &= [\rho_n^R(u,a_1), \rho_n^R(u,a_2), \ldots, \rho_n^R(u,a_{|A_n|})]^{\mathrm{T}}, \end{aligned} \tag{18}$$

where x^{T} stands for the transpose of the vector x. There is not a unique way to define $\rho_n^{\times}(u,a_i)$, $\times = S$ or R; but as the utilities are unique only up to a positive affine transformation (see for instance Ref. 13), to ensure comparability of utilities, both the scale and the zero point need to be chosen, so we consider the following normalization scheme, Eq. (19)

$$\rho_n^{\times}(u,a_i) = \frac{\rho(u,a_i) - \rho_{\min}^i}{\rho_{\max}^i - \rho_{\min}^i}. \tag{19}$$

The next paragraph gives necessary materials that define the evaluation model of a multiattributes multiagents decision-making problem as a satisficing game.

3.1.3. *Satisfiability functions*

The following definition gives the data that define a multiattributes–multiagents decision-making problem as a satisficing game and by which evaluation of alternatives as well as sensitivity analysis can be considered.

Definition 3.2. The satisfiability functions p_S and p_R are defined by

$$\begin{aligned} p_S(u) &= \frac{\omega^S \rho_n^S(u)}{\sum_{x \in U} \omega^S \rho_n^S(x)} \\ p_R(u) &= \frac{\omega^R \rho_n^R(u)}{\sum_{x \in U} \omega^R \rho_n^R(x)}, \quad \forall\, u \in U; \end{aligned} \tag{20}$$

the set of satisficing alternatives S_q at the caution index q is defined by

$$S_q = \{u \in U : p_S(u) \geq q p_R(u)\}, \tag{21}$$

and the satisficing equilibrium alternatives set E_q^S is defined by

$$E_q^S = S_q \cap E \quad \text{with } E = \{u \in U : B(u) = \emptyset\}, \tag{22}$$

where $B(u)$ is defined by Eq. (10).

It is worth noticing that p_S and p_R have probability structure on U and thus fall into satisficing game theoretic framework. One may wonder if this approach is consistent: that is, if there is an alternative which negative attributes performances are more important and which positive attributes performances are less important than the corresponding performances for another alternative, is there a chance that the former alternative be declared as a satisficing equilibrium one? Let us consider the following definition that formalizes this idea.

Definition 3.3. An alternative $u \in U$ dominates (is preferred to or is better than) an alternative $v \in U$, noted $u \succ v$, if and only if the following inequalities

$$\begin{aligned} &\rho(u, a_i) \geq \rho(v, a_i), \quad \forall\ a_i \in A_p \quad \text{and} \\ &\rho(u, a_j) \leq \rho(v, a_j), \quad \forall\ a_j \in A_n, \end{aligned} \tag{23}$$

hold with at least one strict inequality.

The following theorem establishes the consistency of the approach established in this paper: a dominated alternative cannot be declared as a satisficing equilibrium alternative.

Theorem 3.1. *Let u and v belong to U. Then $u \succ v \Rightarrow u \in B(v)$ and so $v \notin E$.*

Proof. $u \succ v \Rightarrow \rho_n^S(u, a_i) \geq \rho_n^S(v, a_i) \geq 0$, $\forall a_i \in A_p$ and $0 \leq \rho_n^R(u, a_j) \leq \rho_n^R(v, a_j)$, $\forall a_j \in A_n$ with at least one strict inequality and as $\omega_l^S \geq 0$, $\omega_l^R \geq 0$, $\forall l$ we have $\omega^S \rho_n^S(u) \geq \omega^S \rho_n^S(v)$ and $\omega^R \rho_n^R(u) \leq \omega^R \rho_n^R(v)$ and finally $p_S(u) \geq p_S(v)$ and $p_R(u) \leq p_R(v)$ with at least one strict inequality so $u \in B(v)$ that is $B(v) \neq \varnothing$ and v is not an equilibrium. □

3.2. *Recommendation procedure and sensitivity analysis*

As stated in the introduction section, we consider the recommendation process to be reduced to choosing, ranking or sorting alternatives in order to achieve decision makers goals.

3.2.1. *Choosing and ranking*

Choosing and ranking are relative evaluation operations[4] over the alternatives set U. Based on the previously derived materials, a value function $\pi(u)$, $\forall\ u \in U$ can be defined as a function of the satisfiability measures $p_S(u)$ and $p_r(u)$, see Eq. (24)

$$\pi(u) = \pi\left(p_S(u), p_R(u)\right) \tag{24}$$

which can take particular form depending on the decision goal. Here is some of these possible forms (see Ref. 16):

- maximally discriminant, $\pi(u) = p_S(u) - qp_r(u)$, gives the priority to alternatives with large difference between the selectability measure and the rejectability measure given the index of caution;

- maximum caution index, $\pi(u) = \frac{p_S(u)}{p_r(u)}$, considers alternatives with the largest index of caution to have priority;
- most selectable, $\pi(u) = p_S(u)$ or least rejectable, $\pi(u) = \frac{1}{p_r(u)}$, give priority to alternatives with the largest selectability or lowest rejectability.

The selected subset U_s that responds to the decision goal is then given by

$$U_s = \left\{ u : u = \arg\max_{v \in E_q^S} \{\pi(v)\} \right\}, \tag{25}$$

and the ranking process is carried by the following relationship, Eq. (26)

$$u \succeq v \Leftrightarrow \pi(u) \geq \pi(v). \tag{26}$$

3.2.2. *Sorting*

Sorting is an absolute operation[4] that requires defining norms and categories; different norms can be derived by using the value function $\pi(u)$ defined in the previous paragraph by setting a threshold on it for instance and defining categories based on these thresholds. For instance, in the case $\pi(u) = p_s(u) - qp_r(u)$, two natural partitions of U is given by

$$C_1 = S_q = \{u \in U : \pi(u) \geq 0\} \quad \text{and} \quad C_2 = U - C_1. \tag{27}$$

Besides this possibility of sorting, the satisficing game approach leads to a natural categorization of the alternatives set U into four subsets, namely E_q^S, $E - E_q^S$, $S_q - E_q^S$, and $U - S_q \cup E$. In terms of preference, the subset E_q^S is obviously preferred to the rest; it contains alternatives arguable to be "good enough" (their selectability exceeds their rejectability and there are no alternatives that are better than them) and the subset $U - S_q \cup E$ contains completely irrelevant alternatives (they are not satisficing alternatives nor equilibrium); there is no obvious conclusion for the subsets $E - E_q^S$ and $S_q - E_q^S$ and a sensitivity analysis can be done for these alternatives. Notice that the set-theoretic breakdown considered here is different from that considered by Ref. 16 in terms of gratification (alternatives for which $p_S > \frac{1}{n}$ and $p_R < \frac{1}{n}$), ambivalence (alternatives for which $p_S > \frac{1}{n}$ and $p_R > \frac{1}{n}$), dubiety (alternatives for which $p_S < \frac{1}{n}$ and $p_R < \frac{1}{n}$), and relief (alternatives for which $p_S < \frac{1}{n}$ and $p_R > \frac{1}{n}$) that is related to the uniform distribution, $p_s(u) = p_r(u) = \frac{1}{n}$ where n is the number of alternatives, of the satisfiability mass functions.

3.2.3. *Sensitivity analysis*

Before considering the sensitivity analysis, let us represent all the alternatives in a plane (p_R, p_S) using their selectability and rejectability measures as their coordinates as shown in Fig. 2; then the satisficing alternatives, that is the subset S_q, lay above the line (OB) characterized by the equation $p_S - qp_R = 0$ and the equilibrium

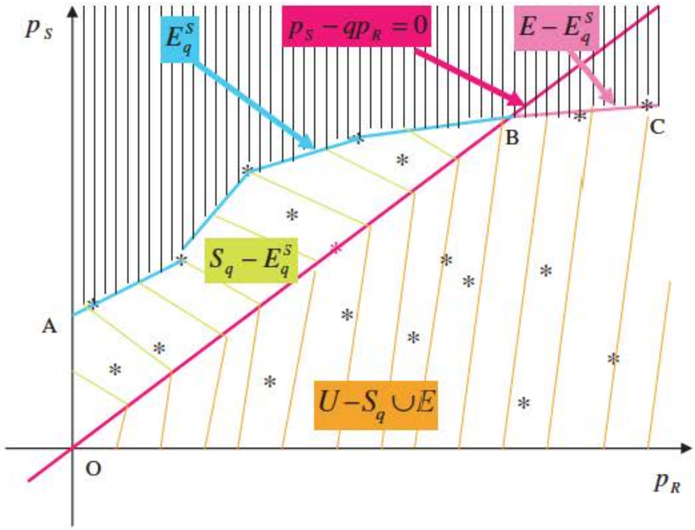

Fig. 2. Different subsets obtained by satisficing evaluation model.

set E constitutes a border of the universe U in the plane; indeed, if we joint all the points that represent the equilibrium alternatives in the plane (p_R, p_S) to form a curve (ABC) where B is the intersection of the satisficing limit line (OB) and the equilibrium border, then all other alternatives will lay below that curve (see Fig. 2). The border E is fix, but the position of B can vary by varying the caution index q. The portion (AB) of the equilibrium curve contains the satisficing equilibrium set E_q^S. The alternatives laying in the region situated between (AB) and (OB) are satisficing, but not equilibrium (the subset $S_q - E_q^S$) and those of the portion (BC) are equilibrium, but not satisficing (the subset $E - E_q^S$).

A sensitivity analysis for negotiation purpose for instance can be engaged for the alternatives of the subsets $E - E_q^S$ and $S_q - E_q^S$.

- The alternatives of the set $E - E_q^S$ are equilibrium alternatives but not satisficing; for an alternative u of this subset, one can do a sensitivity analysis to determine the way to render it satisficing by computing the amount by which its positive attributes performances must be increased and/or the amount by which its negative attributes performances must be reduced in order to be satisficing if other alternatives attributes performances remain unchanged. To do so, one can compute sensitivity parameters $\delta(u, a_i) \geq 0$, $\forall a_i \in A_p$, and $\gamma(u, a_i) \geq 0$, $\forall a_i \in A_n$, such that, if one replaces $\rho_n^S(u, a_i)$ and $\rho_n^R(u, a_i)$ by $\rho_n^S(u, a_i) + \delta(u, a_i)$ and $\rho_n^R(u, a_i) - \gamma(u, a_i)$, respectively, with conditions (28)

$$\begin{aligned} &0 < \rho_n^S(u, a_i) + \delta(u, a_i) \leq 1, \\ &0 < \rho_n^R(u, a_i) - \gamma(u, a_i) \leq 1, \quad \forall a_i, \end{aligned} \tag{28}$$

then one obtains condition (29)

$$p_S(u) \geq q p_R(u). \tag{29}$$

One can find these parameters by solving the following nonlinear program (30)–(33):

$$\min_{\delta(u),\ \gamma(u)} \{0\} \tag{30}$$

$$0 < \rho_n^S(u, a_i) + \delta(u, a_i) \leq 1, \quad \forall a_i \in A_p \tag{31}$$

$$0 < \rho_n^R(u, a_i) - \gamma(u, a_i) \leq 1, \quad \forall a_i \in A_n \tag{32}$$

$$C_S(\delta(u)) \geq q C_R(\gamma(u)), \tag{33}$$

where

$$\begin{aligned} C_S(\delta(u)) &= \frac{\omega^S(\rho_n^S(u) + \delta(u))}{(\sum_{v \in U, v \neq u} \omega^S \rho_n^S(v)) + \omega^S(\rho_n^S(u) + \delta(u))}, \\ C_R(\gamma(u)) &= \frac{\omega^R(\rho_n^R(u) - \gamma(u))}{(\sum_{v \in U, v \neq u} \omega^R \rho_n^R(v)) + \omega^R(\rho_n^R(u) - \gamma(u))}; \end{aligned} \tag{34}$$

$\delta(u)$ and $\gamma(u)$ are column vectors of dimensions $|A_p|$ and $|A_n|$ with entries equal to $\delta(u, a_i)$, $a_i \in A_p$ and $\gamma(u, a_i)$, $a_i \in A_n$, respectively. Notice that, other constraints can be added to take into account practical requirements such as uniform distribution of effort for a class of attributes for instance or on the contrary concentrating the effort on some particular attributes.

- The alternatives of the subset $S_q - E_q^S$ are satisficing but not equilibrium; that is there exists alternatives that are better than them ($\exists\ u^* \in B(u)$) and this information can be used to identify weak points of u. A procedure similar to that presented in the previous point can be used to look for how u can be rendered as good as u^*, that is, to determine parameters $\delta(u, u^*)$ and $\gamma(u, u^*)$ defined as $\delta(u)$ and $\gamma(u)$ of the previous point to have conditions of Eq. (35)

$$p_S(u) = p_S(u^*) \quad \text{and} \quad p_R(u) = p_R(u^*), \tag{35}$$

that leads to linear constraints (36)

$$\begin{aligned} \omega^S \delta(u, u^*) &= \frac{p_S(u^*)(\sum_{v \in U,\ v \neq u} \omega^S \rho_n^S(v)) - \omega^S \rho_n^S(u)}{1 - p_S(u^*)}, \\ \omega^R \gamma(u, u^*) &= -\frac{p_R(u^*)(\sum_{v \in U,\ v \neq u} \omega^R \rho_n^R(v)) - \omega^R \rho_n^R(u)}{1 - p_R(u^*)}; \end{aligned} \tag{36}$$

to be added to conditions similar to (28) in the linear programming problem to be solved to obtain $\delta(u, u^*)$ and $\gamma(u, u^*)$.

In Sec. 4, we will apply the approach established so fare to a real world problem to aid selecting a car that must have some characteristics.

4. Example of Application

This application is taken from Ref. 4; the decision goal is to choose a car with sportive characteristics that will be used in everyday life by somebody (a student) who cannot afford buying an expensive car. A list of 14 possible cars with attributes considered to be relevant to the decision goal by the decision maker has been established (see the following Table 1); the attributes are: Cost (the price of the vehicle measured in euros); Accel (the acceleration performance measured by the time in seconds needed to cover a distance of 1 km starting from rest); Pick up (the suppleness performance of the engine in urban traffic measured by the time in seconds needed for covering 1 km when starting in fifth gear at 40 km/h); Brakes and Road-h (road holding) are related to the safety and are measured by ordinal evaluations defined as: serious deficiency (0), below average (1), average (2), above average (3), exceptional (4); numbers in brackets are numerical conversion done by the decision maker. The values of these attributes in Table 1 are obtained by taking a mean from different sources.

To use the method established in this paper, we interpret the Cost as the negative attribute and other characteristics as positive attributes (we pay the price to obtain acceleration, pick up and safety performances) so that the data used for "Accel" and "Pick up" in the calculation of the selectability function p_S are the inverse of those given in Table 1. The following Table 2 shows the normalized performance function and the satisfiability functions (the two last columns) where we suppose that the decision maker considers all the positive attributes to have the same importance.

Table 1. Performance function of 14 cars to be analyzed.

No.	Cost	Accel	Pick up	Brakes	Road-h
01	18342	30.7	37.2	2.33	3
02	15335	30.2	41.6	2	2.5
03	16973	29	34.9	2.66	2.5
04	15460	30.4	35.8	1.66	1.5
05	15131	29.7	35.6	1.66	1.75
06	13841	30.8	36.5	1.33	2
07	18971	28	35.6	2.33	2
08	18319	28.9	35.3	1.66	2
09	19800	29.4	34.7	2	1.75
10	16966	30	37.7	2.33	3.25
11	17537	28.3	34.8	2.33	2.75
12	15980	29.6	35.3	2.33	2.75
13	17219	30.2	36.9	1.66	1.25
14	21334	28.9	36.7	2	2.25

Table 2. Normalized performance function and satisfiability functions.

No.	Cost	Accel	Pick up	Brakes	Road-h	p_S	p_R
01	0.6007	0.0326	0.5948	0.7519	0.8750	0.0741	0.0949
02	0.1994	0.1987	0.0000	0.5038	0.6250	0.0436	0.0315
03	0.4180	0.6207	0.9654	1.0000	0.6250	0.1056	0.0660
04	0.2161	0.1316	0.8148	0.2481	0.1250	0.0434	0.0341
05	0.1722	0.3704	0.8476	0.2481	0.2500	0.0564	0.0272
06	0.0000	0.0000	0.7027	0.0000	0.3750	0.0354	0.0000
07	0.6846	1.0000	0.8476	0.7519	0.3750	0.0978	0.1082
08	0.5976	0.6574	0.8975	0.2481	0.3750	0.0716	0.0944
09	0.7953	0.4762	1.0000	0.5038	0.2500	0.0733	0.1256
10	0.4171	0.2667	0.5202	0.7519	1.0000	0.0835	0.0659
11	0.4933	0.8834	0.9827	0.7519	0.7500	0.1107	0.0779
12	0.2855	0.4054	0.8975	0.7519	0.7500	0.0922	0.0451
13	0.4508	0.1987	0.6405	0.2481	0.0000	0.0357	0.0712
14	1.0000	0.6574	0.6714	0.5038	0.5000	0.0767	0.1580

If we set the index of caution to $q = 1$, we obtain the following results:

$$
\begin{aligned}
S_1 &= \{02, 03, 04, 05, 06, 10, 11, 12\},\\
E &= \{03, 05, 06, 11, 12\},\\
E_1^S &= E,\\
S_1 - E_1^S &= \{02, 04, 10\},\\
B(02) &= \{05\},\\
B(04) &= \{02, 05\},\\
B(10) &= \{12\},\\
U - S_1 \cup E &= \{01, 07, 08, 09, 13, 14\}.
\end{aligned}
\tag{37}
$$

Figure 3 shows the set breakdown considered in this paper compared to that of Ref. 16 (gratification, dubiety, ambivalence, relief).

The final recommendation using previously defined value functions is given by the following Table 3.

We notice that our satisficing equilibrium subset is almost equivalent (except the alternative 03) to the subset qualified as "can be chosen" by UTA techniques (see Ref. 6 for UTA techniques) in Ref. 4 that is conditioned to the fact that the decision maker stated the following preferences in advance

$$11 \succ 03 \succ 13 \succ 09 \succ 14, \tag{38}$$

based on a feeling; the alternative 12 (gratification alternative see Fig. 3) considered as close contender to alternative 11 (ambivalence alternative) as good alternative in Ref. 4 is recommended as selected by one of our value functions and alternatives declared as clearly poor (except 04 that is dubiety according to Ref. 16), namely $\{09, 13, 14\}$ by the approach considered in Ref. 4 belong to the completely inefficient subset $U - S_1 \cup E$ in our study. The recommendation for selection of the alternative 06 (dubiety alternative) is due to its lower rejectability. We can say that our

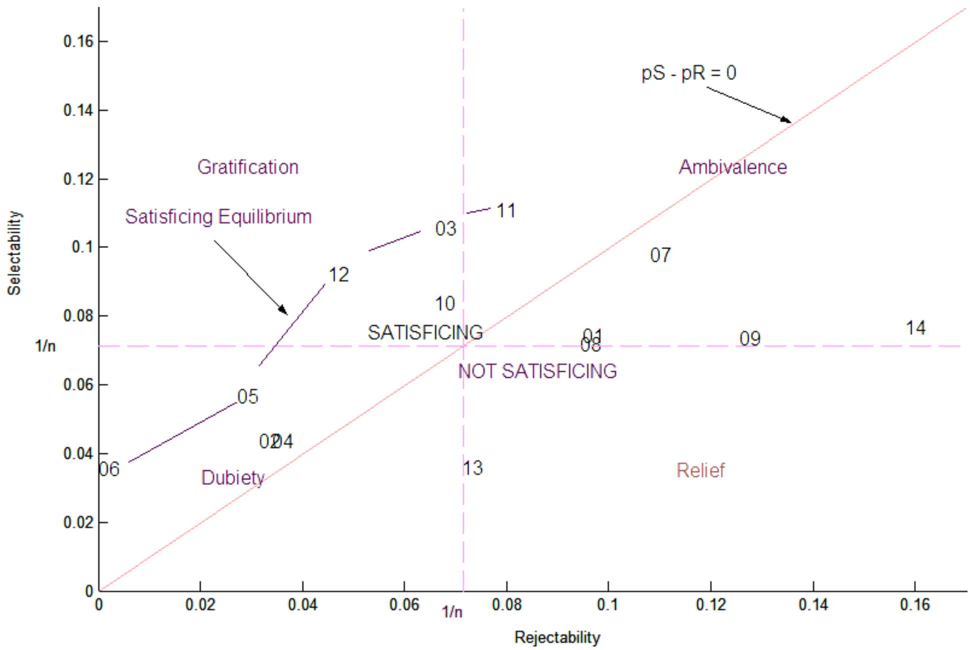

Fig. 3. The representation of alternatives considered in the application in the plane (p_R, p_S).

Table 3. Recommendation results.

Value Function $\pi(u)$	Selected Alternative(s)
$\pi(u) = p_S(u) - p_r(u)$	$\{12\}$
$\pi(u) = \frac{p_S(u)}{p_r(u)}$	$\{06\}$
$\pi(u) = p_S(u)$	$\{11\}$
$\pi(u) = \frac{1}{p_r(u)}$	$\{06\}$

approach, without complicated hypotheses, complicated model parameters setting and/or decision makers required to answer complicated questions, perform as well as existing approaches.

5. Conclusions

The problem of modeling the evaluation step in a decision-making process has been considered in this paper; this step is the stepping stone of the decision-making process as it must establish a model that must reflect as faithfully as possible how human beings proceed in practice like when there are many decision makers or stakeholders with possible antagonist opinions with regard to the importance of different attributes that characterize alternatives. Furthermore, it is worth noticing that for any real world decision problem there are always attributes that contribute to achieving decision goals whereas other attributes will act against obtaining satisfaction as resources consumption for instance. By utilizing this duality of attributes, this paper has formulated the evaluation model in the framework of satisficing game which principal data are selectability and rejectability measures for each alternative. Attributes that act as benefit, known as positive attributes, enter into the derivation of the selectability measure whereas those acting as resources consumption are used to obtain the rejectability measure along with decisions makers preference expressed by assigning weights to attributes in each category (positive/negative). These measures are then used to derive a value function on the set of alternatives that will be used to compare any pair of alternatives for final recommendation purpose. The contribution of this paper is the formulation of the evaluation model in the framework of satisficing games with an integration of stakeholders opinions expressed locally by judging attributes instead of comparing alternatives. Furthermore, a sensitivity analysis scheme has been established that allows a rapid analysis in any change of the performance of an attribute and this can be used to benchmark alternatives. This model is easy to understand and to apply with potentially many domains of applications; a possible drawback could be the fact that the satisfiability measures do not represent a meaningful characterization of alternatives and the necessity to divide attributes into positive/negative attributes. The application considered shows that this method performs as well as existing approaches with all other advantages such as flexibility and simplicity.

References

1. J. K. Archibald, R. L. Frost and W. C. Stirling, Situational altruism: Making conditional commitments, *Proc. Fifth Workshop on Game Theoretic and Decision Theoretic Agents* (Melbourne, Australia, 2003).
2. J. K. Archibald, J. C. Hill, F. R. Johnson and W. C. Stirling, Satisficing negotiations, *IEEE Trans. Syst., Man and Cybern., Part C* **36**(1) (2006) 4–18.
3. W. M. Conklin and S. Lipovetsky, Marketing decision analysis by TURF and shapley value, *Int. J. Inf. Technol. Decision Making* **4**(1) (2005) 5–20.
4. D. Bouyssou, T. Marchant, P. Perny, M. Pirlot, A. Tsoukiàs and P. Vincke, *Evaluation and Decision Models: A Critical Perspective* (Kluwer Academic Publishers, Dordrecht, 2001).
5. P. Dasgupta, P. M. Melliard-Smith and L. E. Moser, Maximizing welfare through cooperative negotiation in a multi-agent internet economy, *Int. J. Inf. Technol. Decision Making* **5**(2) (2006) 331–352.
6. E. Jacquet-Lagrèze and J. Sisko, Assessing a set of additive utility functions for multicriteria decision-making: The UTA method, *Eur. J. Oper. Res.* **10** 151–164.
7. F. V. Jensen, *Lecture Notes on Bayesian Networks and Influence Diagrams*, Department of Computer Science (Aalborg University, 1999).
8. W. Kim, J. S. Hong and Y. U. Song, Multi-attributes-based agent negotiation framework under incremental information disclosing strategy, *Int. J. Inf. Technol. Decision Making* **6**(1) (2007) 61–84.
9. J. Pearl, *Probabilistic Reasoning in Intelligent Systems* (Morgan Kaufmann, 1988).
10. B. Roy and D. Bouyssou, Aide multicritere a la decision: Methodes et cas, *Economica* (1993).

11. T. L. Saaty, *Theory and Applications of the Analytic Networks Process: Decision Making with Benefits, Opportunities, Costs, and Risks* (RWS Publications, Pittsburgh, 2005).
12. P. Salminen, J. Hokkanen and R. Lahdelma, Multicriteria decision analysis project on environmental problems, *Report 5/1996, Department of Mathematics, Laboratory of Scientific Computing* (University of Jyväskylä, 1996).
13. A. A. Salo and R. P. Hämäläinen, On the measurement of preferences in the analytic hierarchy process, *J. Multicriteria Decis. Anal.* **6** (1997) 309–319.
14. H. A. Simon, Administrative Behavior, *A Study of Decision-Making Processes in Administrative Organizations*, 4th edn. (The Free Press, 1997).
15. R. E. Steuer, *Multicriteria Optimization: Theory, Computation, and Application* (Wiley, New York, 1986).
16. W. C. Stirling, *Satisficing Games and Decision Making: With Applications to Engineering and Computer Science* (Cambridge University Press, 2003).
17. A. P. Tchangani, A satisficing game approach for group evaluation of production units, *Decis. Supp. Syst.* **42**(2) (2006) 778–788, Elsevier.
18. A. P. Tchangani, SANPEV: A satisficing analytic network process framework for efficiency evaluation of alternatives, *Found. Comput. Decis. Sci. J.* **31**(3–4) (2006a) 291–319, PUT.
19. A. P. Tchangani, Multiple objectives and multiple actors load/resource dispatching or priority setting: Satisficing game approach, *AMO – Adv. Model. Opt.: An Electronic Int. J.* **8**(2) (2006b) 111–134.
20. Ph. Vincke, L'aide multicritere à la decision, *Editions de l'Université Libre de Bruxelles* (1989).
21. H. Yu, C. Dang and S.-Y. Wang, Game theoretical analysis of buy-it-now price auctions, *Int. J. Inf. Technol. Decision Making* **5**(3) (2006) 557–581.