http://dx.doi.org/10.17654/BO013020223

SURVIVAL ANALYSIS IN LIVING AND ENGINEERING SCIENCES

Abdoulaye Diamoutene[1,4], Diakarya Barro[2,*], M. A. Serge Somda[3], Farid Noureddine[4] and Bernard Kamsu Foguem[4]

[1]IPRFRA de Katibougou
BP: 06 Koulikoro
Republique Du Mali

[2]Université Ouaga 2
BP: 417 Ouagadougou 12
Burkina Faso
e-mail: dbarro2@gmail.com

[3]UPB-Centre Muraz
BP: 1091 Bobo-Dioulasso 01
Burkina Faso

[4]Université de Toulouse
ENIT, LGP, EA 1905
47 Avenue d'Azereix BP: 1629
65016 Tarbes cedex, France

Abstract

Survival or reliability analysis is one of the most significant advancements of statistics in the last quarter of the 20th century.

Keywords and phrases: survival analysis, reliability, hazard function, parametric proportional hazard model, extreme value theory.

[*]Corresponding author

This domain of statistics takes an important place in biomedical and industrial framework. In this paper, we propose a new baseline hazard function from extreme value theory. The newly suggested function is non-monotone and is named as a generalized extreme values baseline hazard function. We prove that this function satisfies hazard properties. A study of the characteristics of the function related to time is made. Conditions for applicability of the model are obtained.

1. Introduction

Survival analysis is generally defined as a set of statistical methods for analyzing data where the outcome variable is the time until the occurrence of an event of interest. The event can be death, occurrence of disease, failure of equipment or complex system. The time to event can be measured in days, weeks, years, etc. For example, if the event of interest is heart attack, then the survival time can be time in years until a person develops attack. In recent years, the field of applied survival analysis has attracted a growing amount of interest due to its applications in many domains such as biomedical, engineering and reliability in industrial equipment safety. This increase is due to the performance of parametric survival model. Benefits are recognized and the availability of some flexible methods are now available in standard software like R and SAS [2, 9, 14]. From the parametric approach, we can obtain biomedical useful measures of absolute risk allowing greater understanding of individual system risk profile, particularly, when we focus on personalized medicine or sensitive equipments [2, 15].

Here, we focus on parametric proportional hazard model. We remember that the problem which frequently arises in failure data analyzing is that not all of the data have been obtained from similar sources. For example, the characteristics of a diabetic patient may be different from another patient. A number of equipments may have been used in different environments depending on ages with modifications [20, 13, 15]. These factors affect survival data quality or failure records of various equipments. Most often survival data are censored [5, 7, 14]. Therefore, it is desirable to isolate the effects of such explanatory variables. Both estimate the size of these effects and enable all the failure data to be analyzed on a common basis [6, 22]. The flexible model which can be used to separate effects of explanatory variables in such contexts is the proportional hazard model introduced by Cox. This model assumes that explanatory variables have multiplicative rather than additive effects on the hazard rate [3, 35]. In Cox model, when the baseline hazard function is specified, we have the parametric proportional hazard model [3, 10]. Weibull distribution is the most used baseline hazard function, but there is initial need to precisely describe the shape parameter [1, 3, 6]. Crowther and Lambert [1] proposed a parametric model through the use of restricted cubic splines and relative risk [3]. Many studies have shown the necessity to modify Weibull distribution for more flexibility [1, 5].

In this study, we introduce a new baseline function through the Generalized Extreme Value (GEV) theory. The GEV distribution is a continuous probability distribution which deals with extreme events such as catastrophe, earthquake, heart attack, aircraft [4, 7, 15]. This distribution can be obtained as limiting distributions of properly normalized maxima of n independent and identically distributed (iid) random variables. Extreme values analysis finds wide applications in many areas including engineering for health engine managing and signal determination [6, 19], biomedical risk management for lung cancer data and high cholesterol modelling [18, 26], biology and bioinformatics modelling [20, 34]. Zio [24] illustrated the flexibility of GEV distribution. Commonly, the distribution which is used widely in survival model is viewed as a parameterization of Weibull distribution.

The paper is structured as follows. Section 2 exposes an overview on survival or reliability analysis. Section 3 presents extreme values statistics and its main components concerning the link between extreme values theory and a new baseline hazard function. In Section 4, we are interested to the analysis of theoretical properties and the applicability conditions. Finally, Section 5 concludes and discusses future challenges.

2. Survival or Reliability Analysis

Let T be a non-negative random variable representing the waiting time until the occurrence of an event. We will assume here that T is a continuous random variable with probability density function (pdf) $f(t)$ and cumulative distribution (cdf) $F(t) = \Pr\{T < t\}$, giving the probability that the event has occurred by duration t. It will be convenient to work with the complement of the cdf, the survival or reliability function

$$S(t) = \Pr\{T \geq t\} = 1 - F(t) = \int_t^{+\infty} f(x)dx$$

which gives the probability of being alive just before duration t, or more generally, the probability that the event of interest has not occurred by duration t [13, 19, 20]. An alternative characterization of the distribution of T is given by the hazard function, or instantaneous rate of occurrence of the event, defined as follows [18, 19]:

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t \leq T < t + \Delta t \,|\, T \geq t)}{\Delta t} = \frac{f(t)}{S(t)}. \tag{2.1}$$

The cumulative hazard function giving amount rates until time t, is defined as

$$H(t) = \int_0^t h(x)dx = -\ln(S(t)). \tag{2.2}$$

The hazard rate goes by several aliases:

- In engineering sciences, it is known as the failure rate.
- In initial statistics and in the life sciences, it is known as the age specific death rate.
- In point process and extreme value theory, it is known as the rate function or the intensity function.

The reliability (survival) function examines the chance that breakdown of organisms, of technical units, etc. occur beyond a given point in time. To monitor the life time of a unit across the support of the life time distribution, the hazard rate $h(t)$ is used.

In survival or reliability data analysis, the hazard function could be monotone or non-monotone. In epidemiology studies, the intensity of contamination must change over time, often the same happens in degradation situation. At the beginning of the epidemic, the intensity is high (increasing) and when the health authorities take some dispositions epidemic intensity becomes weak. In equipment framework also, we have an improvement. In the following section, we focus on the relationship between extreme value analysis and hazard function.

3. Extreme Values Analysis and Survival Reliability

Suppose $Y_1, \ldots, Y_n$ is a sequence of independent and identical copies of a random variable Y and let $M_n = \max(Y_1, \ldots, Y_n)$. If the distribution of Y_i is specified, then the exact distribution of M_n is known. On the other hand, in the absence of such specifications, extreme values consider the existence of $\lim_{n \to +\infty} P[(M_n - b_n)a_n^{-1} \leq y] = F(y)$ for some sequence of real numbers $\{a_n > 0\}$, and $\{b_n \in \mathbb{R}\}$. If the cumulative distribution function $F(y)$ is a non-degenerate distribution, then it follows that

$$F(y) = G_{(\mu, \sigma, \xi)}(y) = \begin{cases} \exp\left[-\left(1 + \xi \dfrac{y - \mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] & \text{if } \xi \neq 0 \\ \exp\left[-\exp\left(-\dfrac{y - \mu}{\sigma}\right)\right] & \text{if } \xi = 0, \end{cases} \tag{3.1}$$

where $\mu \in \mathbb{R}$ is the location parameter, σ is a scale parameter, ξ is the shape parameter and $x_+ = \max(x, 0)$. The Gumbel, Fréchet and the Weibull family of distributions are obtained from (2.1) by considering $\xi = 0$, $\xi > 0$, $\xi < 0$, respectively [4, 7, 19]. The importance of GEV distribution as a link function arises from the fact that the shape parameter ξ purely controls the tail behavior of the distribution [7, 24]. The Gumbel distribution is the least positively skewed distribution in the GEV class when ξ is non-negative. Moreover, Zio [24] provided a plot of the probability distributions of the

GEV family which proves the flexibility of the GEV distribution. This flexibility is an important property in the modelling complex data [7, 23].

3.1. Hazard function modelling with generalized extremal model

The hazard or failure rate function measures the propensity to fail or to die depending on the age reached and it thus plays a key role in characterizing the process of aging and classifying lifetime distributions. We will distinguish between

- monotone hazard rates, either increasing, when the system is wearing out with time, or decreasing, when the system is improving with time;
- non-monotone hazard rates, when the system deteriorates with time, then it improves with time or vice versa.

The hazard function which is non-monotone is sparse in literature. We focus on the baseline hazard function of proportional hazard model to have a monotone function which is very important to modelling some datasets. Roy and Dey [5] proposed flexible hazard function for $\log T$ in accelerated failure time model with GEV errors. Here, we introduce a new baseline hazard function and prove that the proposed function satisfies hazard rate properties and study the characteristics of the new function for the shape parameter $\xi = 0$.

The GEV distribution is a particular case that includes the Gumbel distribution widely used in parametric survival model. However, the commonly used version of the extreme value model can be viewed as a parameterization of the Weibull distribution. If T has a Weibull distribution, then $\log T$ has the Gumbel distribution for the minimum of extremes, Barro et al. [9]. In this case, the hazard function is a Weibull distribution, which is monotone. To obtain a flexible function, we consider that $\log T$ has a GEV distribution. From formula (3.1) and $\log T \sim GEV(0, 1, \xi)$, we have the following probability distribution function (pdf) of T:

$$f_\xi(t) = \begin{cases} \dfrac{\exp\left[-1(1+\xi\log t)^{-\frac{1}{\xi}}\right]}{t(1+\xi\log t)^{\frac{1}{\xi}+1}} & \text{if } t \neq \exp\left(\dfrac{-1}{t}\right); \quad \xi \neq 0 \\ \exp\left(\dfrac{-1}{t}\right)\dfrac{1}{t^2} & \text{if } \xi = 0. \end{cases} \tag{3.2}$$

The corresponding survival function $S_\xi = P(T \geq t)$ is given by

$$S_\xi(t) = \begin{cases} 1 - \exp\left[-(1+\xi\log t)^{-\frac{1}{\xi}}\right] & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\dfrac{1}{t}\right) & \text{if } \xi = 0. \end{cases} \tag{3.3}$$

Proposition 1. *A function* $h(t)$ *is a non-monotone hazard rate if and only if it satisfies the properties*: (1) $h(t) \geq 0, \forall t \geq 0$; (2) $h(t)$ *is increasing and then decreasing or vice versa with time*; (3) $\int_0^{+\infty} h(t)dt = +\infty$.

Proof. (1) Let $f(t)$ be a pdf and $S(t)$ be a survival function. Since *f* and *S* are positive functions, $h(t) = \dfrac{f(t)}{S(t)} \geq 0$.

(2) $h(t) \geq 0$ and $\lim\limits_{t\to 0} f(t) = \lim\limits_{t\to 0} h(t)$; $f(t) \geq h(t), \ \forall t > 0$; thus, there is at least an interval *J* such that *h* is increasing or decreasing, or else vice versa.

(3) $\int_0^{+\infty} h(t) = [H(t)]_0^{+\infty}$, and using formula (2.2), we have $[H(t)]_0^{+\infty}$ = $[-\ln S(t)]_0^{+\infty} = +\infty$. □

By definition, the hazard function $h_\xi(t) = \dfrac{f_\xi(t)}{S_t(t)}$. Using (3.2) and (3.3), we obtain the following expression:

$$h_\xi(t) = \begin{cases} \dfrac{1}{t(1+\xi\log t)^{\frac{1}{\xi}+1}\left\{\exp\left((1+\xi\log t)^{-\frac{1}{\xi}}\right)-1\right\}} & \text{if } \xi \neq 0 \\ \dfrac{1}{t^2\left(\exp\left(\frac{1}{t}\right)-1\right)} & \text{if } \xi = 0. \end{cases} \quad (3.4)$$

3.2. A new baseline hazard function

In this subsection, we focus on the particular case when $\xi = 0$. From the previous subsection, we have $f_0(t) = \exp\left(\frac{-1}{t}\right)\frac{1}{t^2}$, $F_0(t) = \exp\left(-\frac{1}{t}\right)$, $S_0(t) = 1 - \exp\left(-\frac{1}{t}\right)$, $h_0(t) = \frac{f_0(t)}{S_0(t)} = \frac{1}{t^2\left(\exp\left(\frac{1}{t}\right)-1\right)}$.

Proposition 2. *The function* $t \mapsto \frac{1}{t^2\left(\exp\left(\frac{1}{t}\right)-1\right)}$ *is a non-monotone* (*increasing and then decreasing*) *hazard function*.

Proof. The proof of this result uses Proposition 1.

(1) Monotonicity: Let $h_0(t) = \frac{1}{t^2\left(\exp\left(\frac{1}{t}\right)-1\right)}$ and $g(t) = t^2\left(\exp\left(\frac{1}{t}\right)-1\right)$, $t > 0$. Then $g'(t) = (2t-1)\exp(1/t) - 2t$, and the software Geogebra 4 shows that the function $g > 0$, and has extrema at $t = 0, 63$. Moreover, g is decreasing and then increasing. Since $h_0(t) = \frac{1}{g(t)}$, h_0 is increasing and then decreasing.

(2) Hazard rate, f_0 is a pdf function and S_0 a survival function. Both are positive, and hence $h_0(t) = \frac{f_0(t)}{S_0(t)} \geq 0$. Thus,

$$\int_0^{+\infty} h_0(t)\,dt = [-\ln S_0(t)]_0^{+\infty} = \ln S_0(0) - \ln S_0(+\infty) = +\infty. \qquad \square$$

Remark 3. Let f be a function on a set D; f has an upside down bathtub shape if f is increasing and then decreasing. In the rest of this work for $\xi = 0$, the hazard function $h_0(t) = \dfrac{1}{t^2\left(\exp\left(\frac{1}{t}\right) - 1\right)}$ satisfies these conditions.

The last limit predicts an improvement of baseline risk over time, but in practical cases, it is another issue. We will talk about this in Section 4. This function is non-monotone (increasing and next decreasing); this property is very important in complex dataset like clinical trials and engineering system. In some complex data modelling, we have a turning point where the structure of the system takes another aspect. The increasing hazard function is a characteristic of the system that consistently deteriorates with time, whereas decreasing hazard function is a property of the system that consistently improves with time.

3.3. Parametric proportional hazards models

The assumption of the proportional hazard model is that the failure rate of an equipment is the product of a baseline failure rate $h_0(.)$, depending on time and a positive function $\exp(X\beta')$. The classical semi-parametric proportional hazard model, firstly proposed by Barro et al. [9] has the following form where $h_0(t)$ is an arbitrary unspecified baseline hazard function for continuous failure time T, $X = (x_1, x_2, ..., x_p)$ is the vector of covariates, $\beta = (\beta_1, \beta_2, ..., \beta_p)$ is the parameter effect.

The most of survival data is censored. Let Y_i denote the observed time (either censoring time or event time) for the subject i. Let C_i be the indicator that the time corresponds to an event, i.e., if $C_i = 1$, the event occurred and if $C_i = 0$, the time is a censoring time; $X_i = \{X_{i1}, X_{i2}, ..., X_{ip}\}$ is the realized value of the covariates for the subject or system [5, 6].

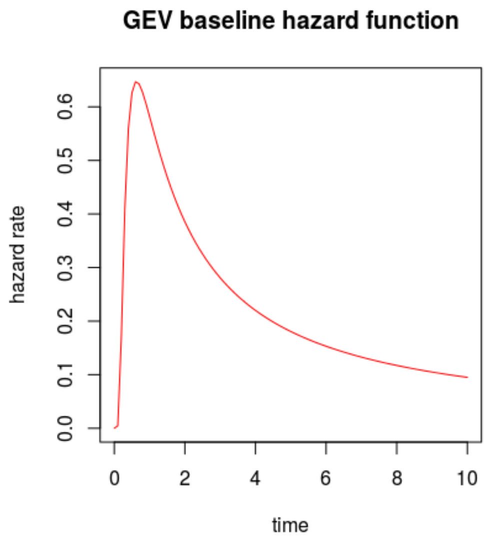

Figure 1. GEV baseline hazard function.

The proportional hazard model has the following form:

$$h(t,\ X_i) = h_0(t)\exp(\beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip}) = h_0(t)\exp(X_i\beta'). \quad (3.5)$$

The last expression gives hazard rate at time t for subject i with covariates vector (explanatory variables) X_i [6, 9].

Commonly the parametric form $h_0(t)$ is specified like Weibull, lognormal, exponential distribution. Parametric distribution type most commonly used like baseline hazard function is the Weibull distribution [3, 9]. The disadvantage of this approach is that the assumption on monotonicity (increasing or decreasing) provides only the failure time distribution of the average system under average conditions of operations and does not correctly take account of extreme behaviors such as harsh conditions [2, 15]. Moreover, it is expected that the system under harsh conditions will fail at earlier times than system operating in mild environments [3, 9, 11]. To take into account the external covariates of such an environment, operating conditions must be brought into the model. This leads to the improvement of degradation models including explanatory covariates that describe the operation environment. For example, the proportional hazard model is a statistical method capable of including information on the environment and operating harsh conditions to take into account extreme events [3, 10, 11]. In the following subsection, we introduce the new hazard proportional hazard model using the baseline hazard function proposed in the previous subsection. This function is extremely flexible, any assumption on monotonicity is taken up.

3.4. Generalized extreme values proportional hazard model

Commonly in parametric proportional hazard model, the baseline risk function is Weibull distribution. With object to take into account the extreme behaviour and operating harsh conditions, using Proposition 2, the newly proportional hazard model is named as GEV proportional hazard model.

In general framework, the analysis of system often sees that on some interval, the system is running smoothly and on another interval the system runs in a degradation phase. Additionally, in the surface roughness, first it has a degradation trend and then does proper operation. The GEV proportional hazard function has the following form:

$$h(t,\ X_i) = \frac{1}{t^2\left(\exp\left(\frac{1}{t}\right) - 1\right)} \exp(X_i \beta') \tag{3.6}$$

and the corresponding cumulative hazard function has the next form

$$H(t,\ X_i) = \left[-\ln\left(\exp\left(\frac{1}{t}\right) - 1\right) + \frac{1}{t}\right] \exp(X_i \beta').$$

4. Theoretical Analysis and Conditions for Applicability of the Model

The purpose of this section is to develop theoretical properties of the proposed model. We also focus on conditions for applicability in life and engineering sciences.

4.1. Theoretical analysis of our proposed model

In this part, we focus on our model, notably the theoretical properties and asymptotic behaviours for some particular values. The hazard rate is the product of two factors, the baseline function and the covariates risk function.

From the second factor, we substitute the term $X_i \beta'$ by the prognostic index (PI) which indicates the weight of the covariates in hazard rate modelling. Thus, the formula (3.6) takes the following form:

$$h(t,\ PI) = \frac{1}{t^2\left(\exp\left(\frac{1}{t}\right) - 1\right)} \exp(PI). \tag{4.1}$$

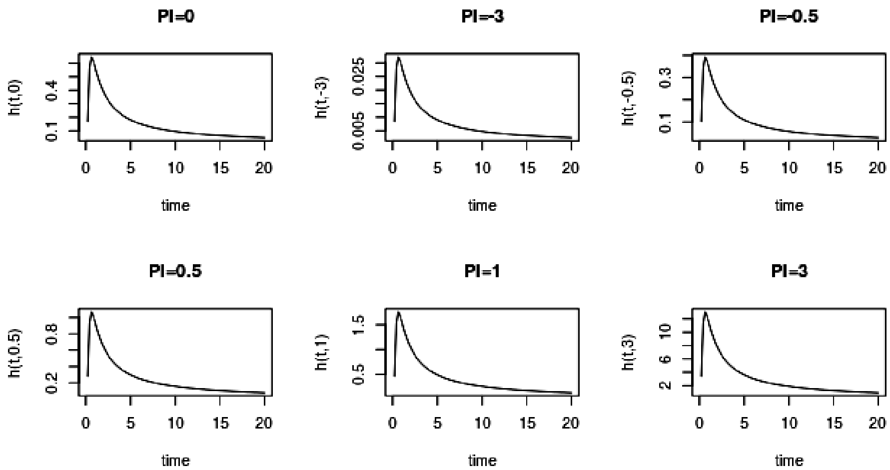

Figure 2. Hazard function simulation depending on PI and time.

The covariates accelerate $(PI > 0)$ or decelerate $(PI < 0)$, the hazard rate which makes a unit move through time with respect to the baseline case. However, as our baseline function is non-monotone (increasing then decreasing), the proposed model recognizes the two aspects $(PI > 0,\ PI < 0)$ and also $(PI = 0)$. To assess the predominant trend of the model, we have to investigate the variation of the function depending on time and PI. We begin our study by a simulation with some values of PI and time, as shown in the following graph The analysis of Figure 3 shows, firstly that hazard rate accelerate, when PI increases with time and the threshold of the hazard rate $(PI > 0)$ is greater than the threshold $(PI < 0)$; secondly, we remark that the decreasing phase is more important than increasing phase because increasing is on short interval. This shows a clear trend of the model towards an improvement situation (decreasing hazard rate).

Furthermore, to support our arguments we propose another simulation study with the following table value degradation phase and then a large improvement phase. In the parametric proportional hazard model, PI is very important in hazard rate assessment. But the PI value is obtained from model parameters estimation; this will involve the selection of relevant covariates.

Time	0.1	0.2	0.3	0.4	0.5	1	2	3	4	5	6	7	8	9
PI	5.1	5	4.9	4.8	4	3	2	1	0	–1	–2	–3	–4	–5

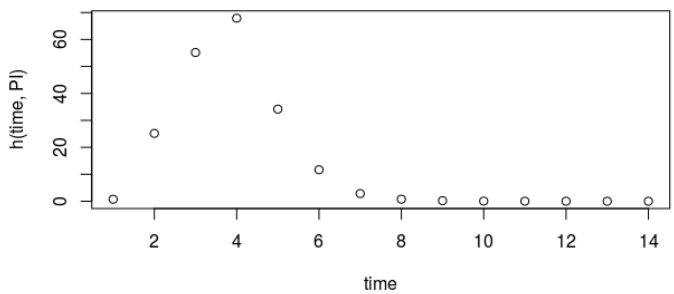

Figure 3. This figure confirms the previous analysis.

4.2. Conditions for applicability on real data

Here, we give the main conditions to apply our proposed model. There are certain constraints on the baseline hazard function and prognostic index.

- In Subsection 3.2, we have $\lim_{t \to +\infty} h_0(t) = 0,$ this means that the system is "dead" or fails completely. Thus, when work is performed on a limited interval, such situation is excluded. For practical reasons, we introduce the baseline relative risk value denoted by ε such as $\varepsilon \ll 1.$ Hence, we have $h_0^*(t) = h_0(t) + \varepsilon$ and $\lim_{t \to +\infty} h_0^*(t) = \varepsilon.$
- We have introduced the prognostic index in previous subsection like the weight of covariates in hazard rate modelling. Figure 2 shows that when PI is increasing, hazard rate is accelerated; for the stability of model, we suggest that $PI \le 1.$

Our model could be used in component roughness and clinical trials studies, because in these we have often increasing phase and then decreasing phase for the hazard function. However, it would be difficult to have available data in industrial and medical framework.

5. Conclusion and Discussion

We presented a new approach to the development of the survival analysis in biomedical and industrial framework. The main contribution is the modelling and characterization of a new baseline hazard function from extreme value theory in proportional hazard modelling. A theoretical study is proposed. This function could be an alternative to Weibull function when the degradation or system behaviour is non-monotone. Many situations of this kind occur in biomedical and surface roughness in industrial equipment. The choice of baseline hazard function is arbitrary in many practical cases; but in biomedical and industrial framework, this function should be flexible, increasing and decreasing. Thus, the task of the parametric baseline hazard function modelling is a challenge for further developments. We have also proposed the practical conditions for the model.

Further development should include the search of available data for model and the implementation of the new approach to experimental step and simulation study on real data.

References

[1] Michael J. Crowther and Paul C. Lambert, A general framework for parametric survival analysis, Stat. Med. 33(30) (2014), 5280-5297.

[2] Helena Penalva, M. Neves and S. Nunes, Topic in data analysis using R in extreme value theory, Metodološki zvezki 10(1) (2013), 17-29.

[3] Kash Barker and Hiba Baroud, Proportional hazards models of infrastructure system recovery, Reliability Engineering and System Safety 124 (2014), 201-206. http://dx.org/10.1016/j.ress.2013.12.004.

[4] Zhiguo Li, S. Zhou, S. Choubey and C. Sievenpiper, Failure event prediction using the Cox proportional hazard model driven by frequent failure signatures, IIE Transaction 39 (2007), 303-315. DOI: 10.1080/07408170600847168.

[5] Vivekananda Roy and Dipak K. Dey, Propriety of posterior distributions arising in categorical and survival model under generalized extreme value distribution, Statist. Sinica 24 (2014), 699-722.

[6] R. W. Platt, K. S. Joseph, C. V. Ananth, J. Grondines, M. Abrahamowicz and M. S. Kramer, A proportional hazards models with time-dependent covariates and time-varying effects for analysis of fetal and infant death, Am. J. Epidemiol. 160(3) (2004), 199-206. DOI: 10.1093/aje/kw/201.

[7] Sameer Al-Dahidi, Francesco Di Maio, Piero Baraldi and Enrico Zio, Remaining useful life estimation in heterogeneous fleets working under variable operating conditions, Reliability Engineering and System Safety 156 (2016), 109-124.

[8] Noureddine Zerhouni, Degradations analysis and aging modeling for health assessment and prognostics of PEMFC, Reliability Engineering and System Safety 148 (2016), 78-95.

[9] Diakarya Barro, Ouédraogo Jean Etienne and Somé Longin, Survival analysis by sampling mixtures of extremal copulas, Int. J. Stat. Econ. 11(2) (2013), 73-82.

[10] Theodoro Dimitiakopoulou et al., A lifetime distribution with an upside-down Bathtub-shaped hazard function, IEEE Transactions on Reliability 56(2) (2007), 308-311.

[11] S. G. Coles, An Introduction to Statistical Modeling of Extreme Values, Springer-Verlag, New York, 2001.

[12] William J. Reed, A flexible parametric survival model which allows a bathtub-shaped hazard rate function, J. Appl. Stat. 38(8) (2011), 1665-1680.

[13] J. B. Broadwater and R. Chellappa, Adaptive threshold estimation via extreme value theory, IEEE Trans. Signal Process. 58(2) (2010), 490-500. DOI: 10.1109/TSP.2009.2031285.

[14] Olivier Bastien and Eric Maréchal, Evolution of biological sequences implies an extreme value distribution of type 1 for both global and local pairwise alignment scores, BMC Bioinformatics 9 (2008), 332. DOI: 10.1186/1471-2105-9-332.

[15] Patrick Royston and Douglas, External validation of a Cox prognostic model: principles and methods, BMC Medical Research Methodology 13 (2013), 33. http://www.biomedcentral.com/1471-2288/13/33.

[16] Sangun Park, Jihwan Park and Youngsik Choi, A new flexible Weibull distribution, Communications for Statistical Application and Methods 23(5) (2016), 399-409. http://dx.doi.org/10.5351/CSAM.2016.23.5.399.

[17] Kevin Granville and Zhaozhi Fan, Accelerated Failure Time Models with auxiliary covariates, J. Biom. Biostat. 3 (2012), 152. http://dx.doi.org/10.4172/2155-6180.1000152/2012.

[18] Ziajia Zhang and Andrew B. Lawson, Bayesian parametric accelerated failure time spatial models and its application to prostate cancer, J. Appl. Stat. 38(2) (2011), 591-603. DOI: 10.1080/02664760903521476.

[19] William R. Swindell, Accelerated failure time models provide a useful statistical framework for aging research, NIH Public Access 44(3) (2006), 190-200. DOI: 10.1016/j.exger.2008.10.005.

[20] E. Ortiz, A. Babbar and V. L. Syrmos, Extreme value theory for engine health monitoring and diagnosis, 2009 IEEE Control Applications, (CCA) and Intelligent Control, (ISIC), St. Petersburg, 2009, pp. 1069-1074. DOI: 10.1109/CCA.2009.5280957.

[21] Malika Chassan, J.-M. Azaïs, G. Buscarlet and N. Suard, A proportional hazard model for the estimation of ionosphere storm occurrence risk, Special Issue on Extreme Theory and Application (Part II) 8 (2015), 71-84. arXiv:1305.5812 [math.ST].

[22] Miguel P. Martinez, E. Vazquez, E. Walter, G. Fleury and R. Kielbasa, Estimation of extreme values, with application to uncertain systems, IFAC Proceedings 39(1) (2006), 1027-1032.

[23] M. Thomas, M. Lemaitre, M. L. Wilson, C. Vibiud, Y. Yordanov, H. Wackernagel and F. Carrat, Applications of extreme values theory in public health, PLOS ONE 11(7) (2016), e0159312. DOI: 10.1371/journal.pone.0159312.

[24] Enrico Zio, Challenges in the vulnerability and risk analysis of critical infrastructures, Reliability Engineering and System Safety 152 (2016), 137-150.