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Dynamic epistemic logics: promises, problems, shortcomings, and perspectives

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ABSTRACT

Dynamic epistemic logics provide an account of the evolution of agents' belief and knowledge when they learn the occurrence of an event. These logics started to become popular about 20 years ago and by now there exists a huge number of publications about them. The present paper briefly summarises the existing body of literature, discusses some problems and shortcomings, and proposes some avenues for future research.

KEYWORDS

Dynamic epistemic logic; public announcement logic; logic of action; logic of knowledge; logic of belief; belief revision

1. Introduction

Dynamic epistemic logics (DELs) are an important recent development in the field of non-classical logics. Research on this family of logics blossomed during the last 15 years. The success story began with an early paper by Plaza (1989) that remained basically unnoticed for the next 10 years. It was by the end of the 1990s that it was taken up. Early foundational papers are due to Baltag, Moss, and Solecki (1998), Baltag and Moss (2004) and Gerbrandy and Groeneveld (1997), Gerbrandy (1999), that were followed by many others. This notably led to the republication of Plaza's original paper almost twenty years after the first publication (Plaza, 2007). A rather early textbook was published in 2007 (van Ditmarsch, van Der Hoek, & Kooi, 2007).

The present contribution is an attempt to briefly sum up the state of the art and to provide a critical analysis of the roads the field took in the last years. My main message is that progress was perhaps not as overwhelming as one might think when one sees the impressive number of papers that were published under the DEL label. I will argue that several difficult and involved problems were overlooked or just left aside, including foundational issues such as the integration of belief revision operations and the suitability of DELs as logics of communication, as well as more technical issues such as the closure of a class of epistemic models under updates.

In the next sections, I will start by recalling what DELs are (Section 2). The rest of the paper provides a critical examination of the received views about DELs. It is organised according to the three keywords making up the acronym, in reverse order: I will question the status of DELs as logics (Section 3, point out some weaknesses of the commonly assumed epistemic component (Section 4), and finally discuss some issues with the dynamic component (Section 5). Each time I will point out several mismatches between the discourse about

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DELs and the state of the art; as I will emphasise, several important issues that are difficult to settle received too little attention up to now. I will conclude by pointing out some perspectives and research avenues for future work (Section 6).

2. DELs in a nutshell

I start by a brief overview of DELs. The presentation is standard: language, semantics and axiomatics.

2.1. Language

The DEL language is an interesting and powerful combinations of two kinds of modal operators: epistemic operators of the form K_i where i is an agent and dynamic operators $[\mathcal{E}]$ where $\mathcal{E}$ is an event. The formula $\mathrm{K}_i\varphi$ reads 'i knows that φ' and the formula $\langle\mathcal{E}\rangle\varphi$ reads '$\mathcal{E}$ may occur and φ is true afterwards'. (Many papers also consider the common knowledge operator, which however will play no role in the present exposition.) Such combinations were first considered in AI more than 30 years ago (Moore, 1985). In its simplest form, the event $\mathcal{E}$ is the *public announcement* of (the truth of) a formula χ, written $\chi!$. In its most general form, $\mathcal{E}$ is made up of a set of possible events having pre- and postconditions, an accessibility relation on that set, and an actual event. When $\langle e, e'\rangle \in S_i$ then agent i cannot distinguish the occurrence of e from that of e'. The precondition of an event describes the conditions under which the event may occur; the postcondition describes its effects on the world in terms of assignments of formulas to propositional variables: when p gets assigned φ then after the event the truth value of p equals the truth value of φ before the event (van Ditmarsch, van der Hoek, & Kooi, 2005). For instance, when $\neg p$ is assigned to p then p's truth value gets flipped.

Formally, an event model is a tuple $\langle \mathbb{E}, \{S_i\}_{i\in\mathbb{I}}, \mathrm{pre}, \mathrm{post}, e_0\rangle$ where

- $\mathbb{E}$ is a non-empty set of possible events;
- $S_i \subseteq \mathbb{E} \times \mathbb{E}$ is a binary relation on $\mathbb{E}$, for every element i of the set of agents $\mathbb{I}$;
- $\mathrm{pre} : \mathbb{E} \longrightarrow \mathcal{L}$ is a total function;
- $\mathrm{post} : \mathbb{E} \longrightarrow (\mathbb{P} \longrightarrow \mathcal{L})$ is a partial function with finite domain;
- $e_0 \in \mathbb{E}$.

The set $\mathbb{P}$ is the set of propositional variables; the set $\mathcal{L}$ is either the set of formulas of epistemic logic or the set of formulas of DEL.[1]

Most of the approaches consider events without postconditions, or rather, with the identity postcondition function post_{id} such that $\mathrm{post}_{id}(e)(p) = p$: all variables keep their truth value. Such events have no effect on the world: they are purely epistemic and only change the agents' epistemic state. Figure 1 is a typical example of a purely epistemic event model where p is privately announced to agent 1: there are two possible events $e = \langle p!, \mathrm{post}_{id}\rangle$ and $f = \langle \top!, \mathrm{post}_{id}\rangle$; agent 2 believes that $\top!$ happens, i.e. that nothing is learned, and that this is moreover common knowledge; agent 1 believes that $p!$ happens while 2 does not know this. The actual event is e; the underlining in Figure 1 signals this.

As usual in modal logic, $[\mathcal{E}!]\varphi$ abbreviates $\neg\langle\mathcal{E}!\rangle\neg\varphi$.

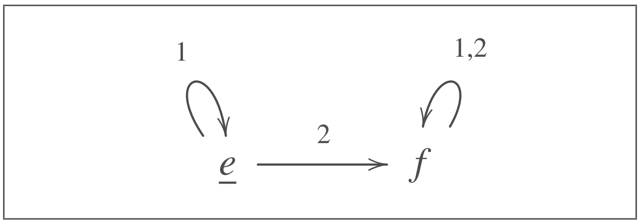

Figure 1. Event model $\mathcal{E}_{1:p!}$ of the private announcement of p to agent 1, where $e = \langle p!, \text{post}_{id}\rangle$ and $f = \langle \top!, \text{post}_{id}\rangle$.

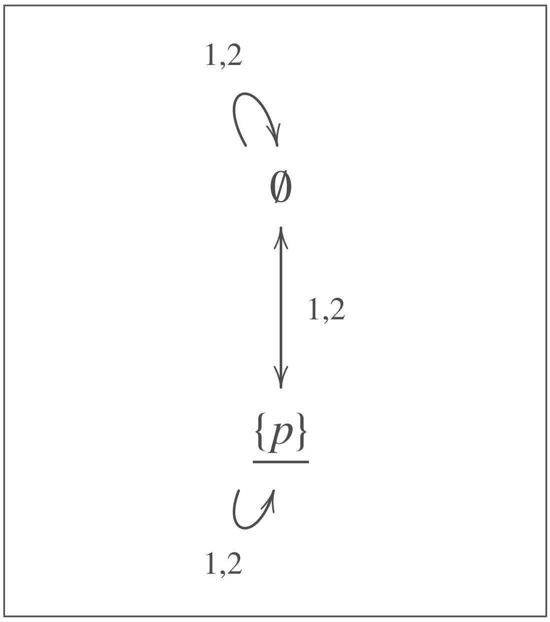

Figure 2. Static epistemic model M_{ign} where both agent 1 and agent 2 do not know whether p (and this is common knowledge).

2.2. Semantics

DEL models have accessibility relations for the epistemic operators, one per agent i. Each relation R_i relates worlds that i cannot distinguish based on her knowledge, and $\mathsf{K}_i\varphi$ is true at a possible world w in a model M if φ is true at every world that i cannot distinguish from w in M. While the interpretation of the epistemic operators is thus standard, the interpretation of the dynamic operators deviates from the traditional modal logic setting: there is no accessibility relation for them. So DEL models are nothing but models of the underlying 'static' epistemic logic. Formally they are tuples of the form $M = \langle W, \{R\}_{i\in\mathbb{I}}, V, w_0\rangle$ where W is a non-empty set of possible worlds, $R_i \subseteq W \times W$ is a binary relation on W for every agent $i \in \mathbb{I}$, $V : \mathbb{P} \longrightarrow 2^W$ is a valuation function associating to every $p \in \mathbb{P}$ the set of worlds where p is true, and $w_0 \in W$ is the actual world. Figure 2 contains an example of a DEL model where both agent 1 and agent 2 do not know whether p is true or not, and where this is common knowledge. The actual world is the lower world $\{p\}$ (underlined in Figure 2).

The interpretation of the epistemic operators is standard: for $M = \langle W, \{R\}_{i\in\mathbb{I}}, V, w_0\rangle$, the truth condition is

$$M \Vdash \mathsf{K}_i\varphi \ \text{ if } \ \langle W, \{R\}_{i\in\mathbb{I}}, V, w\rangle \Vdash \varphi \text{ for every } w \text{ such that } \langle w_0, w\rangle \in R_i.$$

Dynamic operators are interpreted by means of so-called *model updates*. The simplest case is the model update that is associated to the public announcement $\chi!$. It is interpreted by a partial function on the set of pointed Kripke models which restricts the set of possible worlds W of a model M to $||\chi||_M$.[2] The accessibility relation and the valuation are restricted in consequence. So when $M \Vdash \text{pre}(e)$ then[3] the update of the epistemic model $M = \langle W, \{R\}_{i\in\mathbb{I}}, V, w_0\rangle$ by χ is the epistemic model

$$M^\chi = \langle ||\chi||_M, \{R_i^\chi\}_{i\in\mathbb{I}}, V^\chi, w_0\rangle$$

where $R_i^\chi = R_i \cap (||\chi||_M \times ||\chi||_M)$ and $V^\chi(p) = V(p) \cap ||\chi||_M$.[4] The truth condition for the public announcement operator is:

$$M \Vdash \langle \chi!\rangle\varphi \text{ if } M \Vdash \chi \text{ and } M^\chi \Vdash \varphi.$$

In its most general form, an update is a restricted product between an epistemic model and an event model: the formula $\langle\mathcal{E}\rangle\varphi$ is true at possible world w in M if the actual event of $\mathcal{E}$ may occur at w in M and φ is true at w in the update $M^\mathcal{E}$ of M by $\mathcal{E}$. Formally, for $\mathcal{E} = \langle\mathbb{E}, \{S_i\}_{i\in\mathbb{I}}, \text{pre}, \text{post}, e_0\rangle$ and $M = \langle W, \{R\}_{i\in\mathbb{I}}, V, w_0\rangle$:

$$M \Vdash \langle\mathcal{E}\rangle\varphi \text{ if } M \Vdash \text{pre}(e_0) \text{ and } M^\mathcal{E} \Vdash \varphi$$

where $M^\mathcal{E} = \langle W^\mathcal{E}, \{R^\mathcal{E}\}_{i\in\mathbb{I}}, V^\mathcal{E}, (w_0, e_0)\rangle$ with

$$\begin{aligned} W^\mathcal{E} &= \big\{(w,e)\ :\ w \in W,\ e \in \mathbb{E}, \text{ and } \langle W, \{R\}_{i\in\mathbb{I}}, V, w\rangle \Vdash \text{pre}(e)\big\},\\ R_i^\mathcal{E} &= \big\{\langle (w,e),(w',e')\rangle\ :\ \langle w,w'\rangle \in R_i \text{ and } \langle e,e'\rangle \in S_i\big\},\\ V^\mathcal{E}(p) &= \big\{(w,e) \in W^\mathcal{E}\ :\ \langle W, \{R\}_{i\in\mathbb{I}}, V, w\rangle \Vdash \text{post}(e)(p)\big\}. \end{aligned}$$

For example, the update of the static epistemic model M_{ign} of Figure 2 by the event model $\mathcal{E}_{1:p!}$ of Figure 1 is the static epistemic model $(M_{ign})^{\mathcal{E}_{1:p!}}$ depicted in Figure 3. The actual world is $(\{p\}, e)$, underlined in the figure. Observe that the updated model does not contain the world $(\emptyset, e)$ because $M_{ign} \nVdash \text{pre}(e)$. We have $(M_{ign})^{\mathcal{E}_{1:p!}} \Vdash \mathrm{K}_1 p \wedge \neg\mathrm{K}_2 p \wedge \neg\mathrm{K}_2\mathrm{K}_1 p$, from which it follows that $M_{ign} \Vdash \langle\mathcal{E}_{1:p!}\rangle(\mathrm{K}_1 p \wedge \neg\mathrm{K}_2 p \wedge \neg\mathrm{K}_2\mathrm{K}_1 p)$.

2.3. *Axiomatics*

An axiomatisation of DELs that is parametrised by the axiomatisation of the underlying epistemic logic is contained in Table 1.[5] It consists in a complete collection of *reduction axioms*: equivalences whose successive application allows to eliminate all dynamic operators. We therefore end up with a 'static' epistemic formula.[6]

3. Logic?

According to the classical definition, a modal logic is a set of modal formulas that contains all classical propositional theorems and that is closed under uniform substitution, modus ponens, and necessitation (Gabbay, Kurucz, Wolter, & Zakharyaschev, 2003). This is a rather restrictive definition which excludes DELs: indeed, they fail to be closed under uniform substitution.[7]

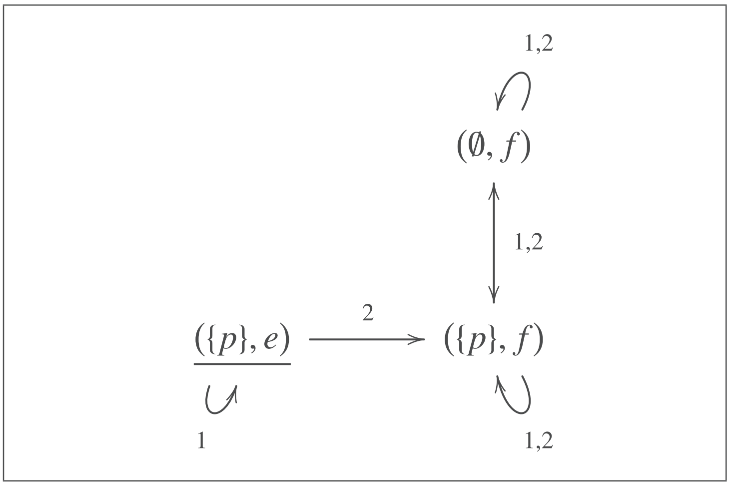

Figure 3. The static epistemic model $(M_{ign})^{\mathcal{E}_{1:p!}}$ after agent 1 has privately learned that p (resulting from the update of the static epistemic model M_{ign} of Figure 2 by the event model $\mathcal{E}_{1:p!}$ of Figure 1).

Table 1. Axiomatisation of DELs, where we suppose that $\mathcal{E} = \langle \mathbb{E}, \{S_i\}_{i\in\mathbb{I}}, \mathsf{pre}, \mathsf{post}, e\rangle$ and $\mathcal{E}' = \langle \mathbb{E}, \{S_i\}_{i\in\mathbb{I}}, \mathsf{pre}, \mathsf{post}, e'\rangle$.

EL	Some axiomatics of epistemic logic (e.g. that of S5)
RE ($[\mathcal{E}]$)	$\dfrac{\psi \leftrightarrow \psi'}{[\mathcal{E}]\psi \leftrightarrow [\mathcal{E}]\psi'}$
Red (p)	$[\mathcal{E}]p \leftrightarrow (\mathsf{pre}(e) \to \mathsf{post}(e)(p))$, for p atomic
Red ($\neg$)	$[\mathcal{E}]\neg\psi \leftrightarrow (\mathsf{pre}(e) \to \neg[\mathcal{E}]\psi)$
Red ($\wedge$)	$[\mathcal{E}](\psi_1 \wedge \psi_2) \leftrightarrow ([\mathcal{E}]\psi_1 \wedge [\mathcal{E}]\psi_2)$
Red ($\mathrm{K_i}$)	$[\mathcal{E}]\mathsf{K}_i\psi \leftrightarrow (\mathsf{pre}(e) \to \bigwedge_{\langle e,e'\rangle\in S_i} \mathsf{K}_i[\mathcal{E}']\psi)$

Let us adopt a more liberal position and accept as logics sets of formulas that are not closed under uniform substitution. DELs fail to satisfy a further, fundamental requirement for logics, viz. that language and semantics should be distinct that are only linked via the interpretation function. DELs with event models violate this principle: the event model $\mathcal{E}$ in the formula $[\mathcal{E}]\varphi$ is a semantical object. This was felt to be a problem right from the start, and several proposals for a language allowing to talk about event models were put forward (Aucher, 2008b, 2009′; Kooi, 2003; van Eijck, Sietsma, & Wang, 2011). Several other approaches allow to at least partly capture event models (Bolander et al., 2016; Gerbrandy, 1999; Gerbrandy & Groeneveld, 1997; van Ditmarsch, 2000, 2002). However, it is only recently that a solution was proposed that I find satisfactory (Hales, Tay, & French, 2014). It combines Gerbrandy's private announcements to groups (Gerbrandy, 1999; Gerbrandy & Groeneveld, 1997) with the PDL program operators.

The perspective of DEL research was mainly aimed at modelling phenomena involving knowledge and action, and only few papers went beyond axiomatisations and investigated mathematical properties. Public Announcement Logic PAL is best studied: it has the same expressivity as epistemic logic due to the reduction axioms; somewhat surprisingly, the complexity of the satisfiability problem is also the same as that of the underlying epistemic logic (Lutz, 2006); however, it is more succinct (French, van der Hoek, Iliev, & Kooi, 2011; Lutz, 2006). For DELs with event models it was proven that the satisfiability problem is

NEXPTIME complete when the underlying epistemic logic is K, contrasting with the PSPACE complexity of the underlying epistemic logic (Aucher & Schwarzentruber, 2013).

Finally, there is a further requirement that is natural for logics extending epistemic logic by dynamic operators: the dynamic extension should be a *conservative extension* of the underlying epistemic logic. We will see in Section 4 that this unfortunately fails to hold for the most relevant underlying epistemic logics.

4. Epistemic?

In the literature it is – sometimes explicitly and sometimes tacitly – supposed that DELs provide a satisfactory formalisation of an agent's representation of the world and its evolution. In this section, I undertake a critical examination of this claim.

In DELs, the term 'epistemic' refers to an agent's representation of the world. It is understood in a broad sense, not only covering knowledge, but also belief. Let us have a closer look at the various logics of knowledge and logics of belief underlying the DELs that one can find in the literature. I first discuss logics of knowledge and then logics of belief.

4.1. Knowledge

Following Halpern et al., many authors chose S5 as their 'official' logic of knowledge. This is however at odds with the philosophical logic literature, where Lenzen and Voorbraak put forward strong arguments against the negative introspection principle of S5 and argued for the weaker modal logic S4.2 (Lenzen, 1978; Voorbraak, 1993). It was shown in Balbiani, van Ditmarsch, Herzig, and de Lima (2012) that this leads to serious technical difficulties: if we choose S4.2 instead of S5 as the logic of knowledge underlying DELs then the restricted product operation of Section 2.2 fails to preserve the class of models. This is due to the fact that the semantical condition that S4.2 models have to satisfy is confluence:[8] suppose the confluence point *u* is the only world where the precondition of the public announcement of φ is false; then *u* is the only world that is eliminated when the restricted product is constructed; therefore the latter lacks the confluence property.

Beyond S4.2, this problem basically plagues every modal logic whose accessibility relation constraints involve an existential formula: the elimination of that point may result in a model violating that constraint.

The situation gets worse in an alternative semantic definition of DELs where the product operation does not restrict the set of possible worlds but only the accessibility relation, as studied e.g. by (Gerbrandy and Groeneveld 1997), (Gerbrandy 1999), Kooi (2007): beyond existential constraints, such a recipe also makes us lose constraints such as reflexivity.

There currently is no satisfactory solution to this issue. Our proposal in (Balbiani et al., 2012) solves the problem from a technical point of view, by means of a truth condition that makes all those updates violating constraints inexecutable. This requires a master modality: we have to be able to express that a constraint is true in every possible world of a model. Moreover, even when the language has such a modality, the solution is somewhat unsatisfactory from the point of view of intuitions.

4.2. Belief

Let us turn to logics of belief. Here, the 'static' logic KD45 is seen by many authors as *the* logic of belief. However, dynamic extensions of KD45 face the same technical problems as dynamic extensions of S4.2: the class of models is not preserved under restricted products. This time it is the seriality constraint[9] that may get violated by the update.

The problem is avoided if one opts for the basic modal logic K as the underlying epistemic logic: K models do not have any constraint to satisfy, and every relativisation of a K model is trivially a K model. However, K is a very weak logic of belief, allowing an agent to simultaneously hold contradictory beliefs.[10] As already highlighted by Hintikka (1962), beyond closure under logical truth and modus ponens, a fundamental property of rational belief is consistency. Consistency of agent *i*'s beliefs is expressed by the formula schema $\neg(\mathsf{B}_i\varphi \wedge \mathsf{B}_i\neg\varphi)$, which corresponds to seriality of the accessibility relation R_i. I would like to add that even if the beliefs of a human or other 'real' agent can sometimes become inconsistent, such an agent nevertheless strives to maintain consistency by revising her beliefs. This aspect is neglected if one chooses the modal logic K. The same criticism applies to all basic modal logics where beliefs can be inconsistent.

My last observation also leads us to the next problem that DEL extensions of logics of belief face. It originates from the observation that while knowledge is always true, beliefs may be false. For example, in PAL an agent may wrongly believe that some announcement ψ cannot be made because he wrongly believes ψ to be false; formally, $\mathsf{B}_i\neg\psi$ is the case, which implies $\mathsf{B}_i[\psi!]\bot$.[11] In such a situation, the right-to-left direction $(\psi \rightarrow \mathsf{B}_i[\psi!]\varphi) \rightarrow [\psi!]\mathsf{B}_i\varphi$ of the belief version of the reduction axiom Red (K_i) of Table 1 should intuitively be invalid. Semantically, a satisfactory doxastic version of PAL should integrate some notion of belief revision. It however is basically still an open problem how to do this: I believe that there are currently no suitable accounts of multiagent belief revision that could be integrated into DELs. Indeed, while there is an abundant literature on belief revision since the seminal 1985 paper by the 'AGM trio' (Alchourrón, Gärdenfors, and Makinson, 1985), – see (Fermé & Hansson 2011) for an overview– it is fair to say that all of the existing formalisms are complex and that they cannot be easily adapted to modal logics. In DELs the situation is worse than in propositional logic because DEL belief revision operations have to be designed in a modal logic setting.[12] This explains that most of the DEL extensions of logics of belief fail to account for multiagent belief revision. There exist a few proposals for an integration (Aucher, 2004, 2008a, 2008b; Baltag & Smets, 2009; van Benthem, 2007; van Ditmarsch, 2005). However, I would like to argue that this is not the final word, for foundational and for implementability reasons. As far as foundations are concerned, I disagree with many in the field who seem to consider that Baltag and Smets's proposal (Baltag & Smets, 2009) is currently the best solution. Their account is grounded on the notion of *safe belief*, which is 'almost knowledge': belief that will never be revised. It is not clear how to use such a logic to design an autonomous agent: how could such she ever distinguish safe beliefs from other, non-safe beliefs? Furthermore, successive application of their belief revision operation result in an ever more complex model, which is a problem plaguing almost all iterated belief revision operations, as pointed out by Konieczny and Pérez (2008). As far as implementability is concerned, I believe that what has not been addressed up to now in a satisfactory manner is that AGM belief revision operations presuppose some kind of preference information, be it in terms of epistemic entrenchment relations between formulas or in terms of comparative possibility relations between worlds. However, in many situations to be modelled it is not so clear where this information comes from. In my opinion, the challenge is to find simple, less demanding theories of revision that can be easily and smoothly integrated into logics of belief and action. A good starting point could be approaches based on language splitting and interpolation (Kourousias and Makinson, 2007; Parikh, 1999). This can be related to uniform interpolation as recently studied in the description logics field in order to account for modularity of ontologies (Lutz, Seylan, & Wolter, 2012; Lutz & Wolter, 2011).

5. Dynamic?

The evolution of an agent's epistemic state may have two different causes: first, the agent may get a new piece of information about a static world; second, the world may evolve and our agent observes this event, possibly only in an imperfect way.

5.1. Evolution of the world

The simplest form of the second kind of change, evolution of the world, is when it is the truth value of some propositional variable that changes. This is captured in DELs by event postconditions, alias the assignment of propositional variables. Previous accounts of such kinds of change include STRIPS (Fikes & Nilsson, 1971) and Reiter's basic action theories (Reiter, 2001). Somewhat surprisingly and as also pointed out by van Benthem (2011), the DEL literature largely ignores the existing literature on reasoning about what is sometimes called ontic actions. In particular, it is well-known in that field that formalisms are plagued by three tenacious problems: the frame problem, the qualification problem and the ramification problem. The frame problem is how to specify the non-effects of an event. The qualification problem is how to specify the preconditions of an event. The ramification problem is to take into account domain constraints (alias integrity constraints).

As to the frame problem, I will just say here that DEL assignments elegantly solve it, as was shown by van Ditmarsch, Herzig, and de Lima (2011), van Benthem (2011).

As to the qualification problem and the ramification problem, it seems that no DEL paper has tried to address them. While some of the approaches could probably be imported (such as (Thielscher 1995, 1996, 1997a, 1997b)), one should nevertheless note that – contrarily to the frame problem, where Reiter's solution was largely adopted due to its simplicity – no consensual solution to these two problems exists in the reasoning about actions field.

5.2. Evolution of the agents' epistemic state

The first kind of change is that of a a private announcement to a group of agents J: some formula φ is announced to J in a way that is public for J. In DELs such an announcement is identified with the precondition of an event that takes place; more precisely, for which it is *possible for the agent* that it takes place. This is captured in DEL event models by an event with precondition φ and an empty postcondition.

This is related to a claim that is often made, viz. that DELs are logics of communication. The claim occurs most prominently and explicitly in the title of Plaza's foundational paper and in those by van Benthem (2006), van Benthem, van Eijck, and Kooi (2006). However, it is clear that DELs lack several key ingredients of speech act theory. To start with, it is not easy to come up with a meaningful notion of a speaker: the agent who makes a public announcement is not part of the situation that is modelled. Furthermore, DELs currently do not provide a good account of communicative intention. Unless such concepts can be integrated into DELs, the field will have to wait for a good logic of communication, and DELs have to be thought of as logics of observation and not of communication or agency.

6. Perspectives

Despite the various criticisms that I have put forward in the preceding sections, I believe that DELs are one of the most fruitful recent developments in the domain of nonclassical logics.

One of its most striking assets is that their models are very compact. This contrasts with standard temporal and dynamic models whose Kripke models typically contain a huge number of possible worlds even for rather simple applications. This makes that model checking procedures working with Kripke models as they stand are not practically feasible.

It, however, remains that DEL model checking has to deal with Kripke models for epistemic logics. Such Kripke models can become pretty big, too, in particular when there are multiple agents. In the literature on model checking for multiagent systems one can find more compact representations where the epistemic accessibility relation is built from information about observability of propositional variables by agents (Lomuscio, van der Meyden, & Ryan, 2000; Su, Sattar, & Luo, 2007). This perspective was recently imported into the DEL setting in a series of papers (Balbiani, Gasquet, & Schwarzentruber, 2013; Herzig & Lorini, 2014; van Benthem, van Eijck, Gattinger, & Su, 2015; van der Hoek, Iliev, & Wooldridge, 2012). All these approaches however suffered from the restriction that the agents' observational abilities are common knowledge. This was overcome by Herzig, Lorini, and Maffre (2015), Herzig and Maffre (2016), Herzig, Lorini, Maffre, and Schwarzentruber (2016), Maffre (2016). Furthermore, an extension with public announcements was introduced by Charrier and Schwarzentruber (2015), Charrier, Herzig, Lorini, Maffre, and Schwarzentruber (2016). I believe these approaches to be promising because they make model checking techniques applicable to non-trivial scenarios.

As to the belief revision problem plaguing DELs, it is not clear to me whether and how it will be addressed in the future. A solution that might be achievable without too much effort is to combine the embedding of Dalal's revision operation of Herzig (2014) with the embedding of the above observation-based DELs.

Notes

1. Precisely, the postcondition is either restricted to epistemic formulas or defined by mutual recursion with the truth conditions.
2. As usual, for a given $M = \langle W, \{R\}_{i\in\mathbb{I}}, V, w_0\rangle$, the notation $||\chi||_M$ stands for the set of possible worlds $w \in W$ such that $\langle W, \{R\}_{i\in\mathbb{I}}, V, w\rangle \Vdash \varphi$.
3. Otherwise the operation $(.)^{\mathcal{E}}$ is undefined.
4. Precisely, updates are defined by mutual recursion with the truth conditions.
5. It is easy to see that it is equivalent to the axiomatisations one usually finds in papers about DELs, such as in Wang and Cao (2013), Wang and Aucher (2013).

6. Observe that there is no reduction axiom for the case of two successive dynamic operators. While it part of the standard axiomatisations of the literature, it is actually not necessary in the presence of the rule of equivalence $\texttt{RE}$ ($[\mathcal{E}]$).
7. To see this consider the formula $[p!]p$: it is a theorem of $\texttt{PAL}$ because its rewriting with Axiom $\texttt{Red(p)}$ results in the classical logic theorem $p \to p$. However, the result of substituting p by the Moore sentence $q \wedge \neg \mathrm{K}_1 q$ fails to be a theorem. Indeed, the formula $[q \wedge \neg \mathrm{K}_1 q!](q \wedge \neg \mathrm{K}_1 q)$ can be rewritten by means of the reduction axioms first to $[q \wedge \neg \mathrm{K}_1 q!]q \wedge [q \wedge \neg \mathrm{K}_1 q!]\neg \mathrm{K}_1 q$; then to $((q \wedge \neg \mathrm{K}_1 q) \to q) \wedge ((q \wedge \neg \mathrm{K}_1 q) \to \neg [q \wedge \neg \mathrm{K}_1 q!]\mathrm{K}_1 q$, which simplifies to $((q \wedge \neg \mathrm{K}_1 q) \to \neg [q \wedge \neg \mathrm{K}_1 q!]\mathrm{K}_1 q$; then to $((q \wedge \neg \mathrm{K}_1 q) \to \neg ((q \wedge \neg \mathrm{K}_1 q) \to \mathrm{K}_1 [q \wedge \neg \mathrm{K}_1 q!]q)$, which simplifies to $((q \wedge \neg \mathrm{K}_1 q) \to \neg \mathrm{K}_1 [q \wedge \neg \mathrm{K}_1 q!]q$; and finally to $((q \wedge \neg \mathrm{K}_1 q) \to \neg \mathrm{K}_1 ((q \wedge \neg \mathrm{K}_1 q) \to q)$, which simplifies to $\neg q \vee \mathrm{K}_1 q$.
8. For every w, if $\langle w, u_1 \rangle \in R_i$ and $\langle w, u_2 \rangle \in R_i$ then there is a u such that $\langle u_1, u \rangle \in R_i$ and $\langle u_2, u \rangle \in R_i$.
9. For every w there is a u such that $\langle w, u \rangle \in R_i$.
10. Some may still object that this is too strong a logic of belief because in $\texttt{K}$, an agent's beliefs are closed under logical truth and modus ponens. This is the so-called omniscience problem. We leave it aside here and suppose that omniscience can be assumed when we are interested in rational agents or in artificial agents.
11. The equivalence $[\psi!]\bot \to \neg\psi$ is valid in any DEL where the truth condition for $[\psi!]$ and the update are defined as in Section 2.2.
12. I note in passing that if we disregard the epistemic operators then DEL public announcements can be identified with AGM expansions, which are considered in the AGM literature to be a very simple case of belief change.
13. The problem however kept on haunting me and I finally succeeded some years later in solving it (Herzig, 1996, 1998).

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This is the right place to say how much I owe to Luis and to express my gratitude towards him. Part of the contents of the present paper can actually be traced back to the first year of my PhD under his supervision that I took up 30 years ago. Indeed, while the subject of my 1989 PhD thesis was automated theorem proving methods for modal logics, we started out in 1986 on a quite different subject: a logical account of database updates. We had come up with a semantics that was close to Winslett's so-called possible models approach and we wanted to find an axiomatisation and a syntactical theorem proving method for it. Despite several months of efforts we failed, and by the end of the first year Luis wisely decided that it was too much a risk to continue our quest (The problem however kept on haunting me and I finally succeeded some years later in solving it (Herzig, 1996, 1998)). The only thing we had been able to deal with was the special case of updates by literals. Such updates can be identified with assignments of truth values to propositional variables, which are a particular case of assignments of formulas to propositional variables as later studied in the DEL . The rise of DELs coincided with Luis's involvement in the administration of science, becoming director of IRIT and serving in several positions and committees on the national level. He consequently lacked time to follow the evolution of the by now huge DEL literature. He however started to get interested in the topic, as witnessed by a paper that we wrote in 2009 together with colleagues where we proposed a generalisation in terms of world- and arrow-updating operations that was taken upby Barteld Kooi and Brian Renne in two papers. Some of the unsolved issues that I have discussed in the present paper were among those Luis and me were struggling with during the first year of my PhD. This is also the place to thank my colleagues of the LILaC group from IRIT with whom I had many discussions on the criticisms of DELs that I put forward here and with many of whom I wrote on the subject, first and foremost Philippe Balbiani and Emiliano Lorini. Thanks are also due to my former or present PhD students with whom I worked and published on the subject, in particular Guillaume Aucher, Tiago de Lima, Faustine Maffre, François Schwarzentruber, Nicolas Troquard, and Zhanhao Xiao. My criticisms have benefitted from several discussions with other colleagues and coauthors, in particular Hans van Ditmarsch. Thanks are also due to Jan van Eijck for provided positive feedback on a previous draft during his stay in Toulouse in June 2016.

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