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Proof without Words: On Sums of Squares and Triangles

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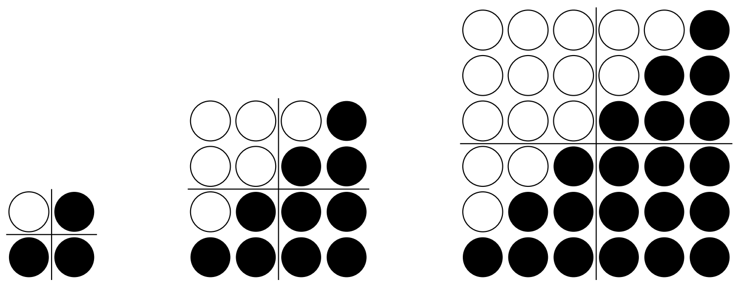

$$T_k = \sum_{j=1}^{k} j \qquad \Longrightarrow \qquad \sum_{k=1}^{2n} T_k = 4 \sum_{k=1}^{n} k^2.$$

Remarks: If the diagram above is extended to contain the odd squares too, then it becomes clear that $\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n} T_k + \sum_{k=1}^{n-1} T_k$, which can also be derived from the results given in either of the PWW [**4**] or [**5**]. Thus,

$$\sum_{k=1}^{2n} T_k = 4 \left(\sum_{k=1}^{n} T_k + \sum_{k=1}^{n-1} T_k \right) = 4T_n + 8 \sum_{k=1}^{n-1} T_k,$$

which can also be derived by summing the squares of the even integers using the result given in the PWW [**2**] and the definition of T_n. Finally, the reader is invited to compare this to other PWW involving sums of squares and triangular numbers, especially [**1**] and [**3**].

REFERENCES

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Summary. We visually display a relationship between sums of squares and the sum of an even number of triangular numbers. Connections to some PWW appearing in the literature are briefly discussed.

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