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## FURTHER EVALUATION OF THE MODIFIED AFFINITY LAWS FOR THE PREDICTION OF VISCOSITY EFFECT ON THE PUMP HEAD PERFORMANCE

### Abhay Patil

Research Engineer  
Texas A&M University  
College Station, Texas, USA

### Gerald Morrison

Professor Emeritus  
Texas A&M University  
College Station, TX, USA

### Adolfo Delgado

Associate Professor  
Texas A&M University  
College Station



*Abhay R. Patil is a Research Engineer working at the Turbomachinery Laboratory, TAMU. His research includes numerical and experimental investigations of performance and reliability of artificial lifting methods for multiphase flow. He is leading and working on various projects involving current and new technology initiatives including ESPs, Turbopump, multiphase flow meters and process lubricated bearings. Dr. Abhay Patil holds a PhD degree in Mechanical Engineering from Texas A&M University, College Station.*



*Gerald L. Morrison is a Professor Emeritus of Mechanical Engineering at Texas A&M University. He has been a member of the Turbomachinery Laboratory for over 35 years investigating aircraft jet noise, space shuttle turbopump and standard pump seals, and single and multiphase flow meters and pumps.*



*Dr. Adolfo Delgado is an associate professor of mechanical engineering at Texas A&M University. His research focuses on rotordynamics, structural vibration, energy dissipation mechanisms, thin film lubrication and fluid-structure interaction applied to the design, modeling and improvement of rotating machinery systems and components. Prior to joining Texas A&M, Delgado was a research engineer at the General Electric Global Research Center where he led and worked on multiple initiatives involving improvement of existing rotating equipment and development of new rotor-bearing system architectures and turbomachinery components, such as variable geometry bearings, annular seals, dampers and oil-free bearings. Dr. Delgado received his B.S. in mechanical engineering from Simón Bolívar University and his M.S. and Ph.D. in mechanical engineering from Texas A&M University.*

**Abstract:** A method is presented that predicts a pump's performance for different viscosity fluids. It is a unique combination of the affinity law blended with energy loss phenomenon. Presented correlations are based on the limited data available. Although the method has a scientific base, it requires further validation and refinement to cover different pump types over a wide range of operating conditions. The current study has focused on validating the applicability of the modified affinity Laws on newly acquired data and understanding the effect of impeller size on the predictions using the CFD simulations. The simulations were performed for 4" and 8" sizes of geometrically similar pumps for varying viscosities and rotational speeds. The modified affinity law provides consistent results using the Morrison number which varies with the flow regime and pump specific speed.

## Introduction:

Performance of the pump is affected by various factors such as fluid properties, operating conditions and pump type. As with the flow in a pipe, energy losses in pumps depend on Reynolds number  $Re$  and relative surface roughness [1]. Hence while sizing the pump or evaluating the performance it is important to account for these effects. To predict the effect of viscosity, usually, correction factors are developed by pump manufacturers for specific pump under consideration, [2], [3]. Correction factors are usually developed based on interpolation methods for specific pump type and application which may not be applicable to other pump types, besides those methods are not disclosed due to commercial interests. The Hydraulic Institute method [4] is the most commonly followed procedure to predict the effect of viscosity on pump performance. It provides graphical representation for correction factors at different operating points for different viscosities. However the HI predictions do not include the variation in the performance due to the specific pump types.

Dimensional analysis proposed by Buckingham [5] has been used for more than 100 years to establish the correlations between co-dependent factors. Dimensional analysis was performed on pumps many decades ago which included the effects of density, pump speed, impeller size, and flow rate upon the pressure generated and power required to operate the pump. From this analysis, the Affinity Laws were obtained that reduce the complex pump performance map into three distinct curves for head coefficient ( $\Psi$ ), power coefficient for the power supplied to the pump shaft ( $\Pi$ ), and pump efficiency ( $\eta$ ) as a function of flow coefficient ( $\phi$ ). Two curves ( $\Psi$  vs  $\phi$  and  $\eta$  vs  $\phi$ ) define the entire performance map of the pump for a single viscosity since the fourth non-dimensional group can be calculated from the other three, i.e.  $\Pi = f(\phi, \Psi, \eta)$ . If the viscosity of the pumped fluid remains close to the value of the fluid (usually water) used in the experimental test to obtain the data for the flow map, it is still a good representation of the pump performance.

Pump design is usually characterized by specific speed. The pump type varies from radial to axial flow as the specific speed increases. Viscous loss inside the pump may vary based on variation in the flow field which is a direct function of specific pump geometry. However, so far no universal method exist which can include the effect of viscosity to predict the performance degradation for all pump types. Morrison et al [6], [7] and Patil et al [8] proposed a theory to predict the pump head for different viscosity fluids which uses a unique combination of the current affinity law blended with energy loss phenomenon such as friction factor modeling to form the Modified Affinity Law. Initial evaluation was focused on forming a generalized law to predict the pump performance based on available literature and CFD data of mixed flow pump. The purpose of the CFD simulations was mainly to obtain useful information from hydraulic path modeling to investigate the effects of viscosity and the development of a proposed model. Losses associated with secondary flow path were ignored. The initial study utilized viscosity up to 400 mPa-s cp only and mainly focused on the turbulent flow regime. All the curves for head for all viscosities collapsed on a single line for the independent variable  $\phi * Re_w^{-a}$  with variable  $a$  having different values for different pump types. The exponent “ $a$ ” now called as the Morrison number ( $Mo$ ) was further investigated in [8]. It was found that the Morrison number is a function of rotational Reynolds number. The flow regime inside the pump was characterized based on the Rotation Reynolds number. Further study was conducted to evaluate the change in value of the Morrison number as a function of specific speed. Further relations are established as a function of specific speed. Initial evaluation was based on limited data from published literature and CFD data. CFD data doesn't include losses due to secondary leakage flow and the stage wise performance change. Moreover, the effect of pump size has not been evaluated in previous study. This is the work in progress and current study will focus on the effect of impeller size and utilize more data from literature to further build the confidence in the model.

## Background:

This study utilizes dimensional parameters to characterize the pump performance. The flow coefficient  $\Phi$  represents the flow through the pump. Head Coefficient,  $\Psi$  represents the head developed by the pump. The additional nondimensional term called rotational Reynolds number is utilized to include the effect of viscosity and rotational speed.

$$\text{Flow Coefficient, } \Phi = \frac{Q}{\omega D_s^3} \quad (1)$$

$$\text{Head Coefficient, } \Psi = \frac{\Delta P}{\rho D_s^2 \omega^2} \quad (2)$$

$$\text{Rotational Reynolds Number, } Re_w = \frac{\rho \omega D_s^2}{\mu} \quad (3)$$

It was opted to model the pump as a lumped parameter system much like an elbow or tee, which is represented by an equivalent length that includes the effects of all the internal flow paths, including roughness. The rotational Reynolds number is based upon the pump rotational speed, not the flow inside the pump. Considering the velocity triangle of a pump, the flow speed through the pump depends upon both the rotational speed and the flow rate. Thus in this analysis, the fluid friction effects are considered a function of the flow through the pump which is represented by the flow coefficient and the rotational speed is which represented by the Reynolds number. These two nondimensional groups are combined to represent the effects of viscosity [7].

By utilizing the Affinity Law coefficients, the effects of pump speed and density are included in the nondimensional groups resulting in a single curve for each viscosity for all the data. The effects of viscosity are clearly evident with the head produced decreasing with increasing viscosity. Using the Buckingham Pi analysis and assuming the viscosity effects can be normalized using a Reynolds

number, Figure 4 a) was produced. This figure illustrates that there is a well behaved relationship of the head coefficient as a function of the flow coefficient and the Reynolds number. The goal was to collapse of all the head coefficient data on a single line as a function of  $\Phi$  &  $Re_w$  to provide the basis for the affinity laws to include the effect of viscosity. In order to achieve this, head coefficient was plotted against the factor which is the multiplication of the flow coefficient by the Reynolds number raised to a power similar to the definition of friction factor in turbulent pipe flow regime. The CFD simulation data collapse to a single 2D curve which includes the effects of pump speed, flow rate, fluid density, and fluid viscosity over a range of viscosities from 1 to 400 cp. Analysis also indicated that the exponent may vary based on the specific pump types. Further analysis was performed to evaluate the relationship between the exponents now called as the Morrison number and pump specific speed. No specific relationship was established during the initial evaluation.

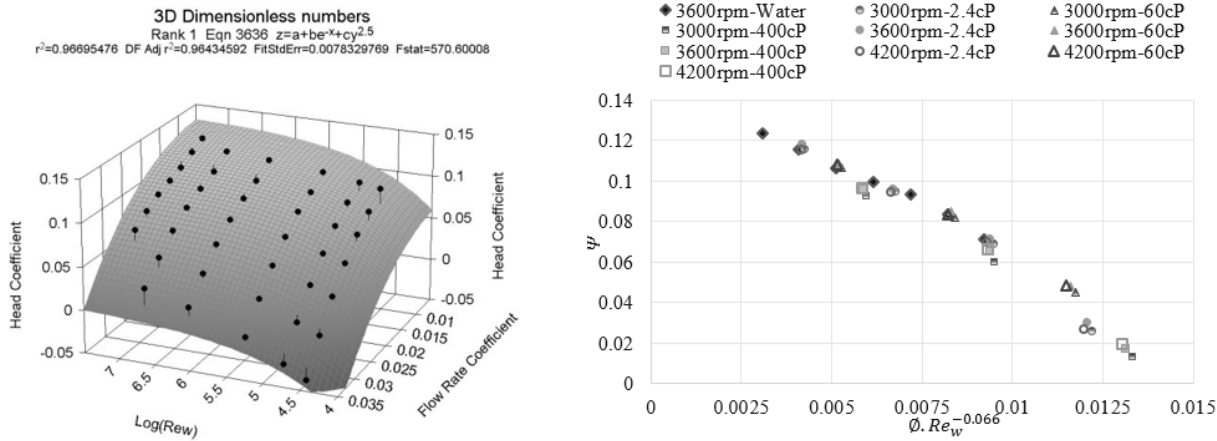


Figure 1: a) Head coefficient versus common logarithm of rotating Reynolds number and flow rate coefficient using the CFD data b) Modified Affinity Law relationship for a pump’s head-flow rate data expressed in terms of head coefficient, flow coefficient, and rotational Reynolds number [13]

The initial evaluation was based on the viscosity range from 1 to 400 cp. Further evaluation was carried out in [8] to understand the effect of flow regime change from turbulent to laminar due to various factors ranging from pump specific speed, fluid properties and operational parameters. Correlations were established based on flow regime and the pump specific speed. This study will consider

### Validation using the data from published Literature

The pump performance data from published literature [2] was utilized to further evaluate the modified affinity laws. The pump under consideration delivers 580 gpm with pressure head of 80 feet of water at the best efficiency point. It has specific speed of 3200. Three types of fluids utilized for the testing consisting of low, medium, and high viscosity fluid. The desired viscosity was achieved by reaching the required temperature based on viscosity temperature plot. The table below shows the operating condition and fluid viscosity.

Fig 2 shows typical performance plot of the pump. Fig 3 and 4 show the normalized head for different fluid viscosities for 2333 rpm and 3617 rpm respectively. As expected, the pump head performance degrades as the fluid viscosity increases. Rate of head degradation increases at higher flow rate indicating a shift in best efficiency point to a lower flowrates. Fig 5 presents the normalized head as a function of  $\Phi * Re_w^{-Mo}$  with constant value of Morrison number set as -0.059 based on the initial evaluation.

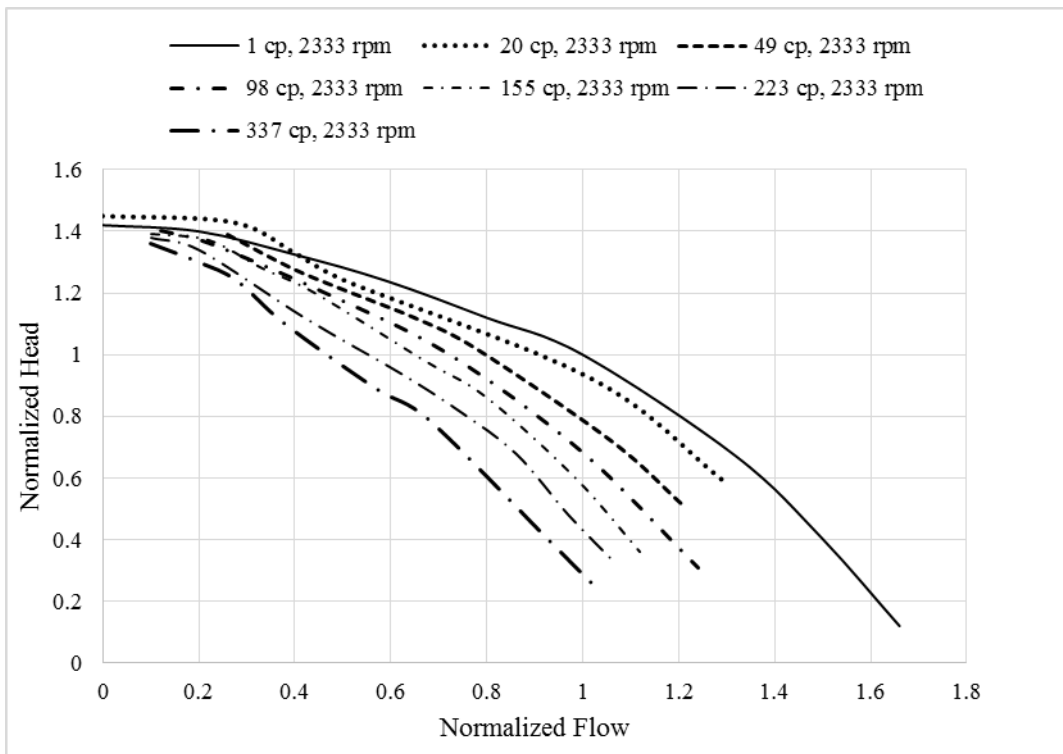


Figure 2: Normalized performance data for 2333 rpm from []

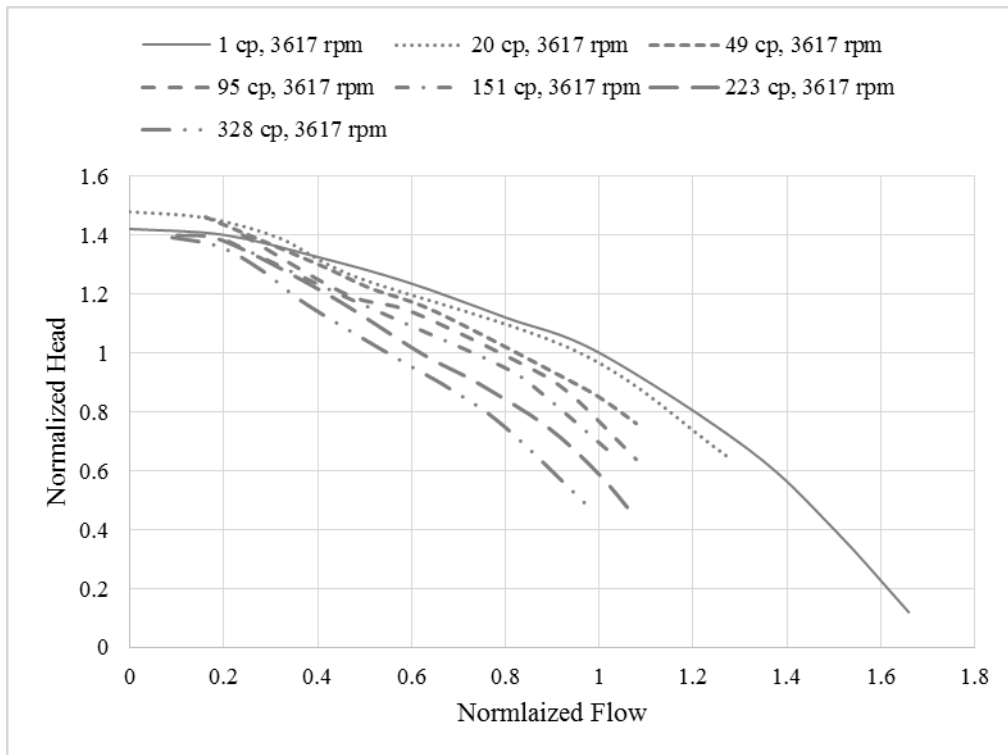


Figure 3: Normalized performance data for 3617 rpm from []

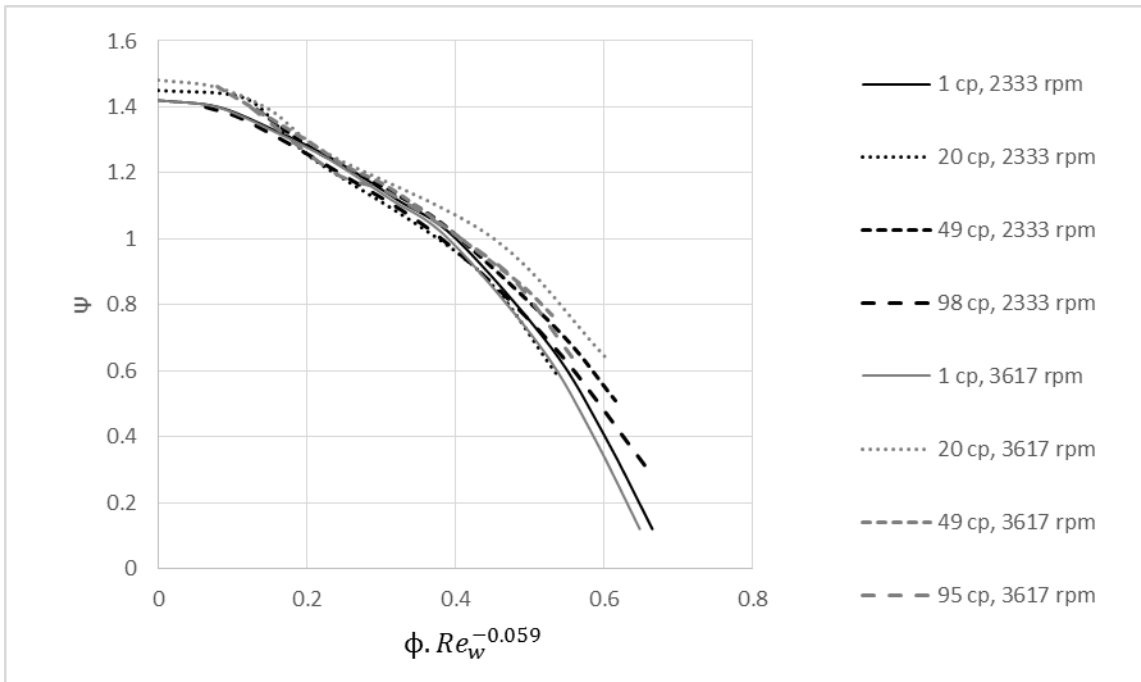


Figure 4: Performance correction using the constant value of Morrison number

The data points for varying viscosities start to diverge from the curve at higher values of  $\phi * Re_w^{-Mo}$  indicating that the value of the Morrison number might change based on flow velocity and viscous losses. This is similar to the variation in Darcy Weisbach friction factor with change in hydraulic Reynolds number. This prompted further investigation to allow for the change in the Morrison number with change in viscosity in order for all the curves to fall on the single line. This modification led to two distinct curves as shown in the fig 6. All the performance curves for rotational Reynolds number greater than 45000 collapse onto a single line. While the remaining performance curves for rotational Reynolds numbers less than 40000 collapse onto another separate curve. This means the performance of the pump under different fluid viscosities can be characterized in terms of laminar and turbulent flow regimes as a function of rotational Reynolds number. To understand the variation in the Morrison number, the Morrison number was plotted as a function of  $\log_{10} Re_w$  similar to a pipe friction factor. Starting from the highest  $Re_w$  ( $\log_{10} Re_w \cong 7$ ), the Morrison number increases with decreasing  $Re_w$ . The maximum value of Mo is achieved around  $\log_{10} Re_w \cong 5$  then decreases slightly. A discontinuity is then observed which clearly hints at a potential change in the flow regime. The Mo then decreases linearly with further reduction in the  $\log_{10} Re_w$  providing a basis for a rational relationship between Mo and  $Re_w$ . This analysis shows agreement with previous study [8]. This method provides the way to determine the flow regime inside the pump which can be utilized to correct the performance for the different fluid viscosities. At this point the method to determine the pump performance is established for the given water performance curve data. The next target is to establish the relationship between Mo and different pump types, achieving this target will provide a universal plot of the Mo for all pump types.

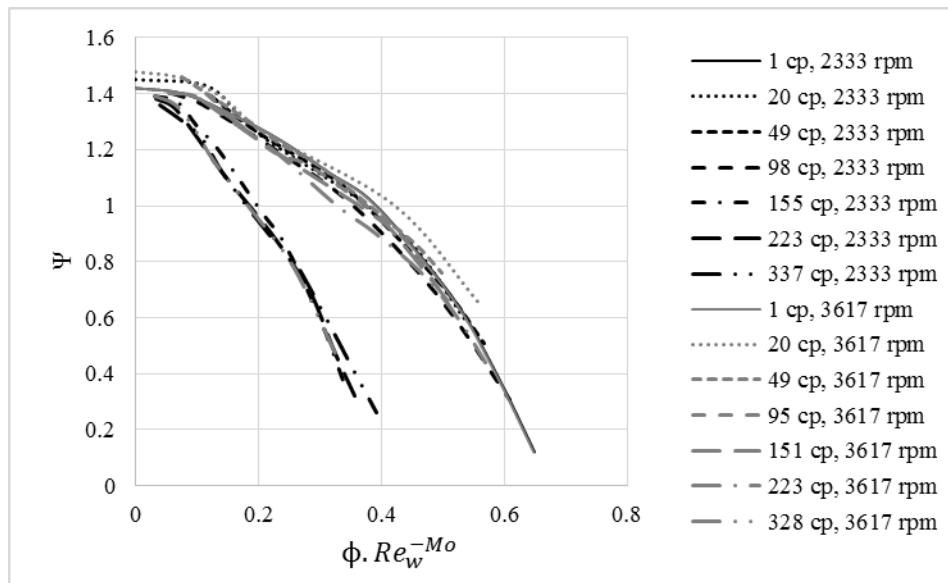


Figure 5: Performance correction using varying values of the Morrison number

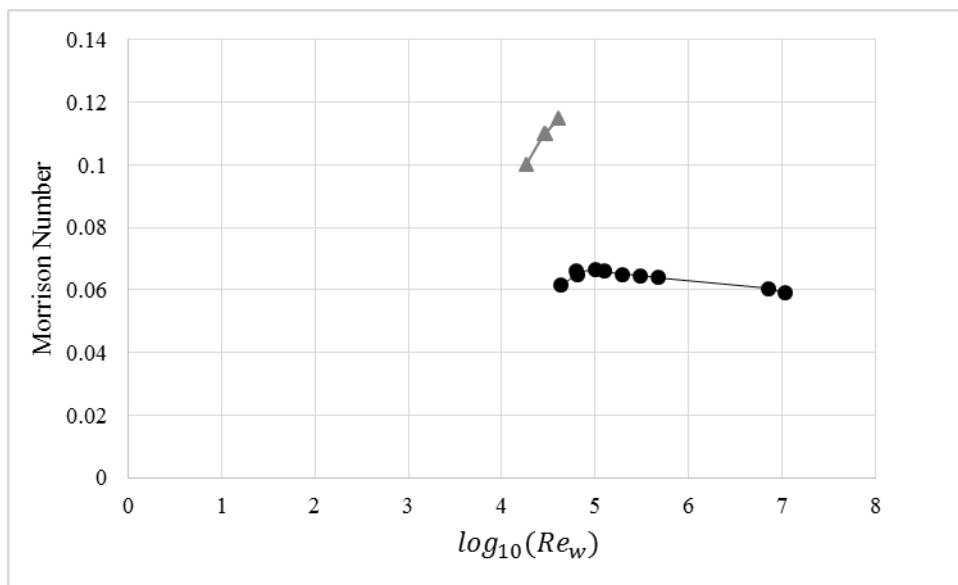


Figure 6: Variation in Morrison number with Rotational Reynolds number

#### Application of Modified Affinity Laws to the data by Dr. Le Fur

Data by Dr. Le Fur [9] were utilized in a previous study utilizing a constant value of the Morrison number. The head coefficient for all the high viscosity fluids collapsed onto a single curve while the head curve for water was separate as shown in the figure 7. Further evaluation was carried out and the Morrison number was allowed to vary to collapse all the curves on a single line. Figure 8 shows the head curve using the modified affinity law. By varying the Morrison number, the accuracy is significantly improved and results in the water curve conforming to the same common line.

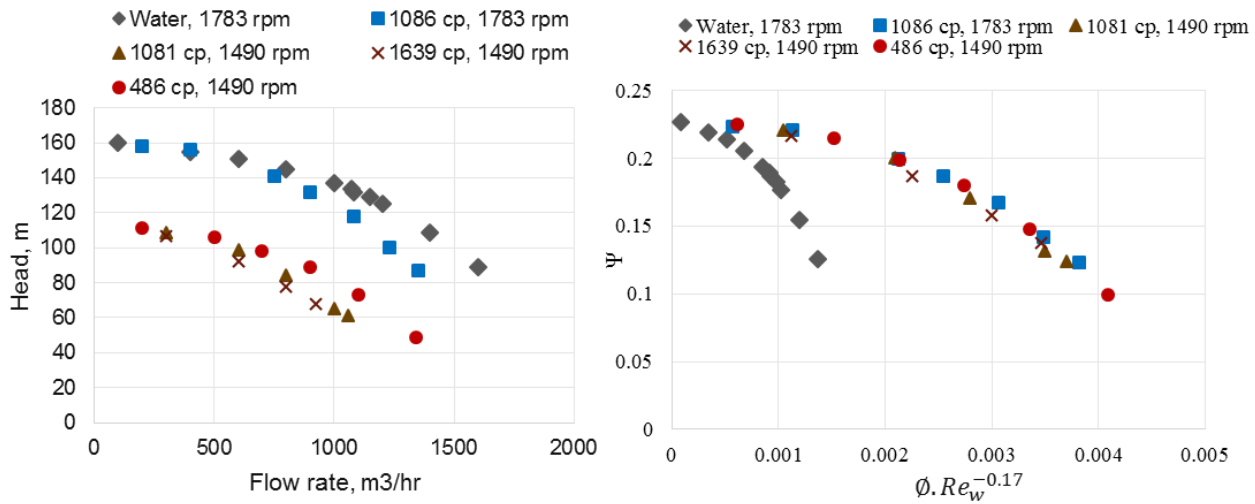


Figure 7: Modified affinity laws applied to the Pump data from Le Fur [9] using constant Morrison number

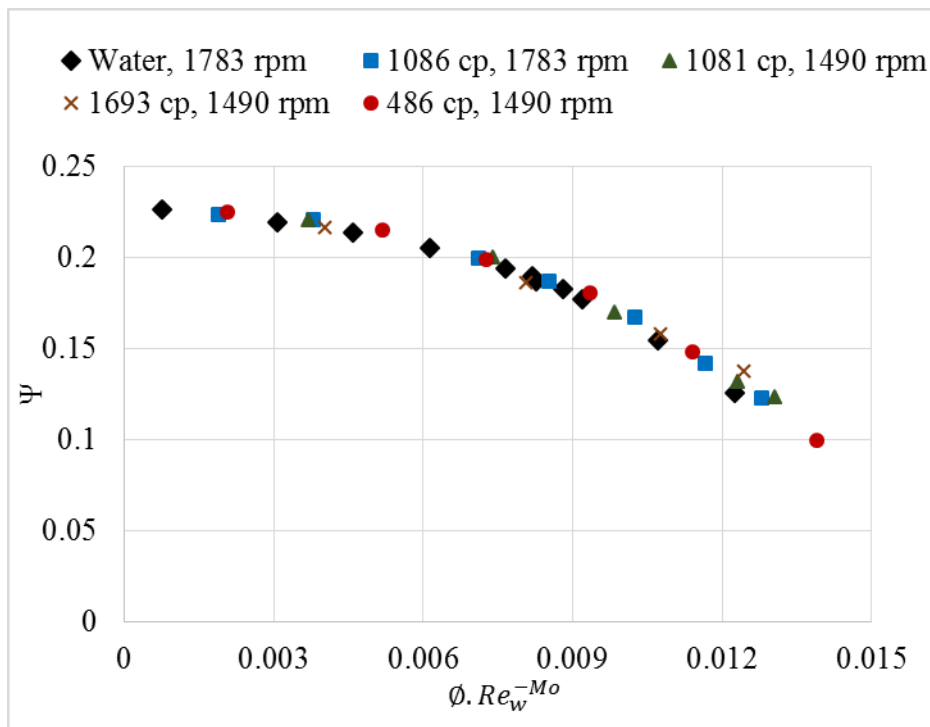


Figure 8: Head coefficient correction using varying values of the Morrison number

Figure 9 shows the variation in the Morrison number with the rotational Reynolds number. The water (lowest viscosity) data point is located at the highest Reynolds number. As viscosity is increased  $Re_w$  decreases. Initially  $Mo$  increases from 0.058 to 0.082 but then decrease to a low value of 0.056 at the lowest Reynolds number. The flow in the pump is laminar below  $Re_w \cong 1.5 \cdot 10^4$ . The flow is transitional between  $1.5 \cdot 10^4 < Re_w < 7 \cdot 10^4$ . Above this range the flow is fully turbulent and  $Mo$  decreases with increasing  $Re_w$  as shown in figure 7 as well.

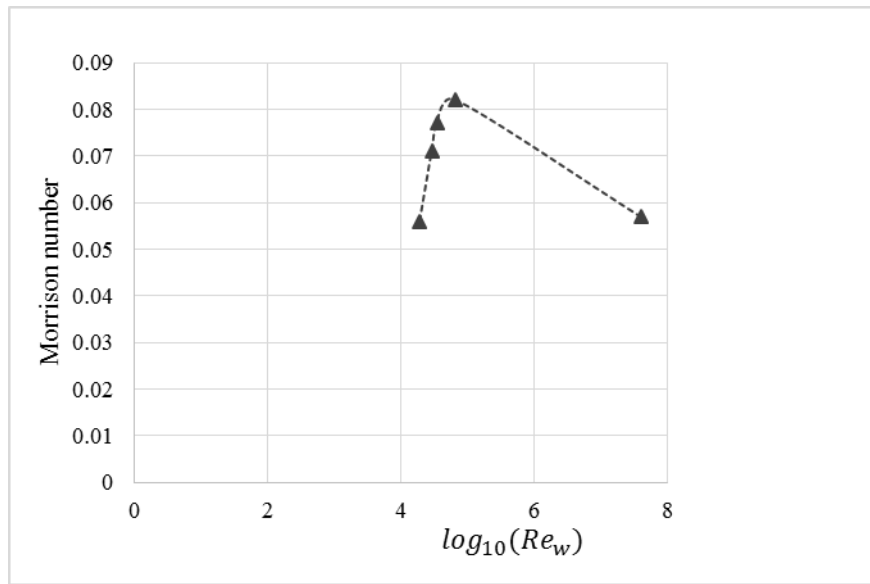


Figure 9: Variation in Morrison number with Rotational Reynolds number for the data by Le Fur [18]

**Variation in the Morrison Number with Specific speed of the pump:** So far the pump performance viscosity correction has been established using Morrison number. Establishing the relationship between the Morrison number and the specific speed will help decipher a universal plot to determine the performance of any pump. The specific speed represents the pump design varying from radial to axial type. The viscous losses which depend on the velocity field can be correlated based on the specific pump type due to change in velocity field. The Morrison number generated from the data by [2], [9], [10] is plotted against the rotational Reynolds number for transitional and turbulent flow in Figure 10. There is a systematic change in the Morrison number as a function of specific speed and  $Re_w$ . Further evaluation is underway to fully establish the relationship.

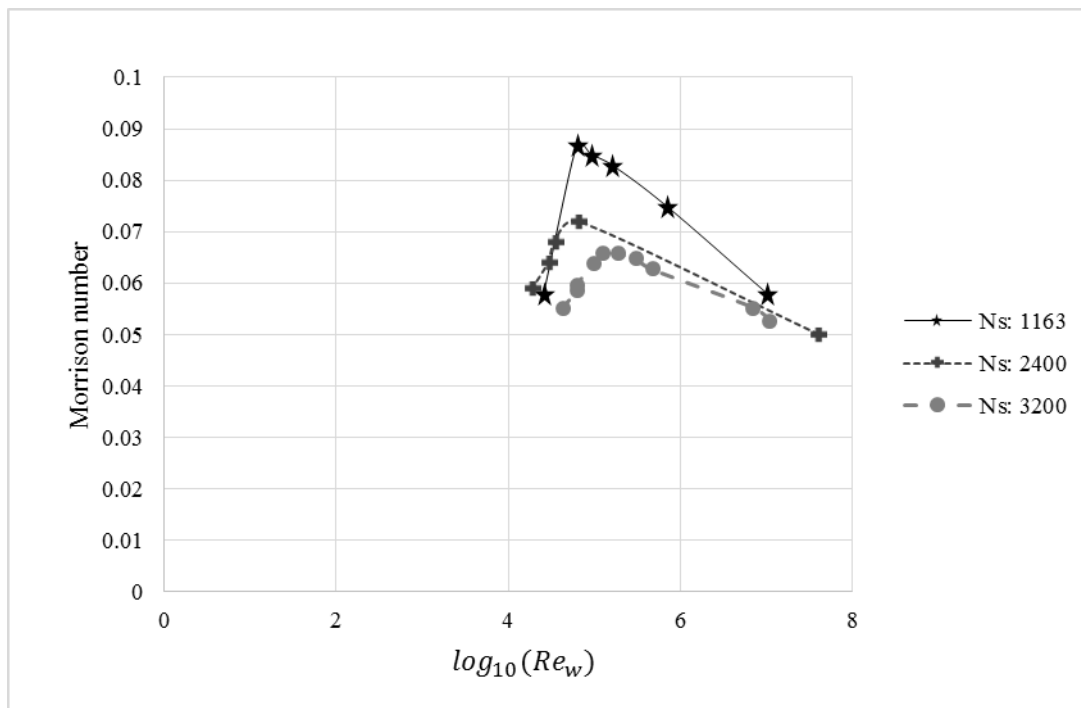


Figure 10: Variation in Morrison number with Rotational Reynolds number for the different specific speed pump data for transitional and turbulent flow [8]



**The Morrison number in the laminar regime:**

In the laminar regime for the pipe flow, the friction factor is independent of surface roughness so should the Morrison number, however based on the available data, the Morrison number appears to be changing with the specific speed. In the laminar regime the Morrison number curve is steeper for lower specific speeds. Further analysis is ongoing to characterize the Morrison number in laminar regime. This analysis represents the foundation to analyze the energy loss in a pump to determine the pressure loss as a function of fluid viscosity.

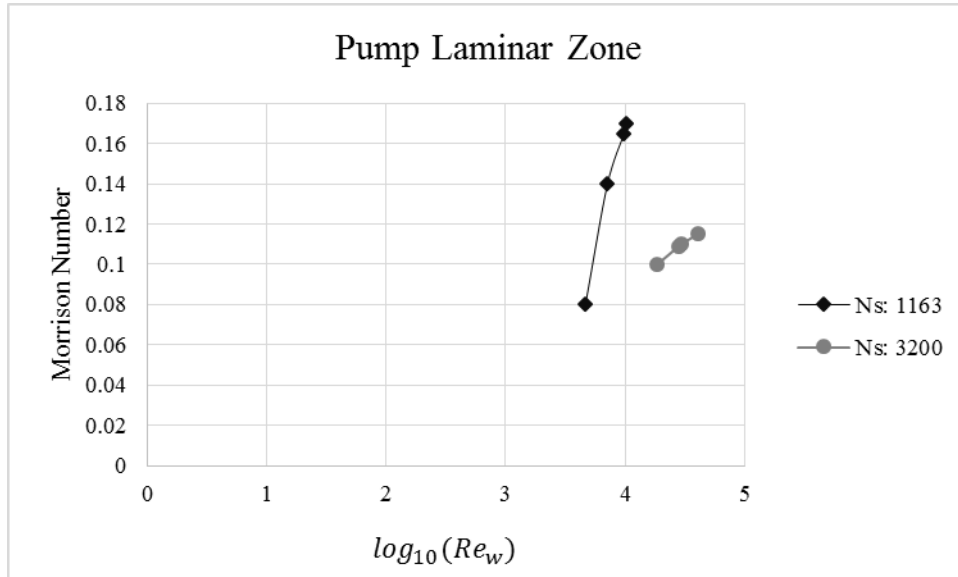


Figure 11: Variation in Morrison number with Rotational Reynolds number for the different specific speed pump data [8]

**Effect of Pump Size:** Pump size is an important factor affecting the flow regime based on variation in the velocity field for same rotational speed. This study utilizes geometrically similar pump with two different sizes to generate the data using CFD simulations. Complete flow path including secondary leakage flow was included. Hybrid mesh was used including mapped hexahedral elements and unstructured polyhedral elements. Figure 12 a) shows the flow path and Figure 12 b) shows the mesh model for the pump stage.

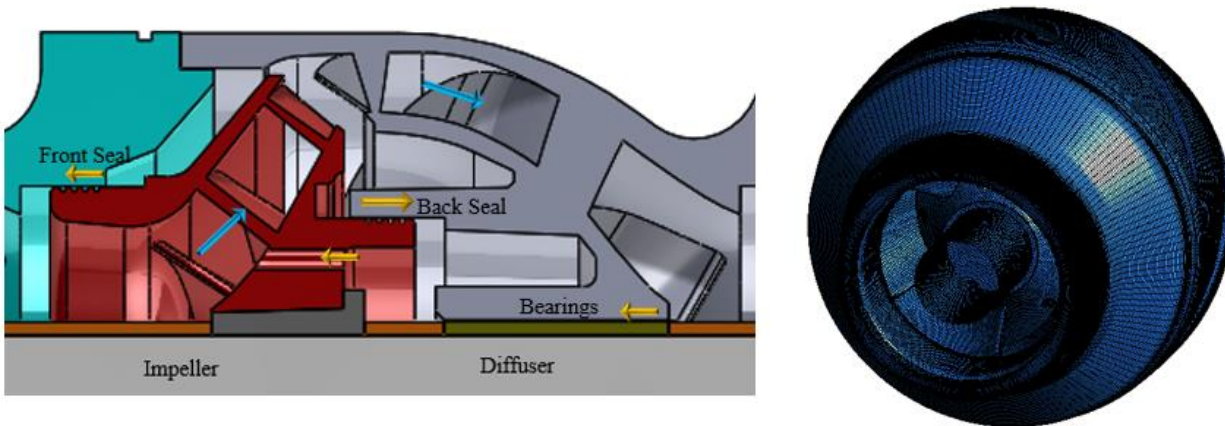


Figure 12: a) Schematic of flow path in a typical ESP stage b) Mesh model

Figure 13 shows the comparison of 8” pump with experimental data. Head and efficiency data using CFD are slightly higher compared to the experimental data.

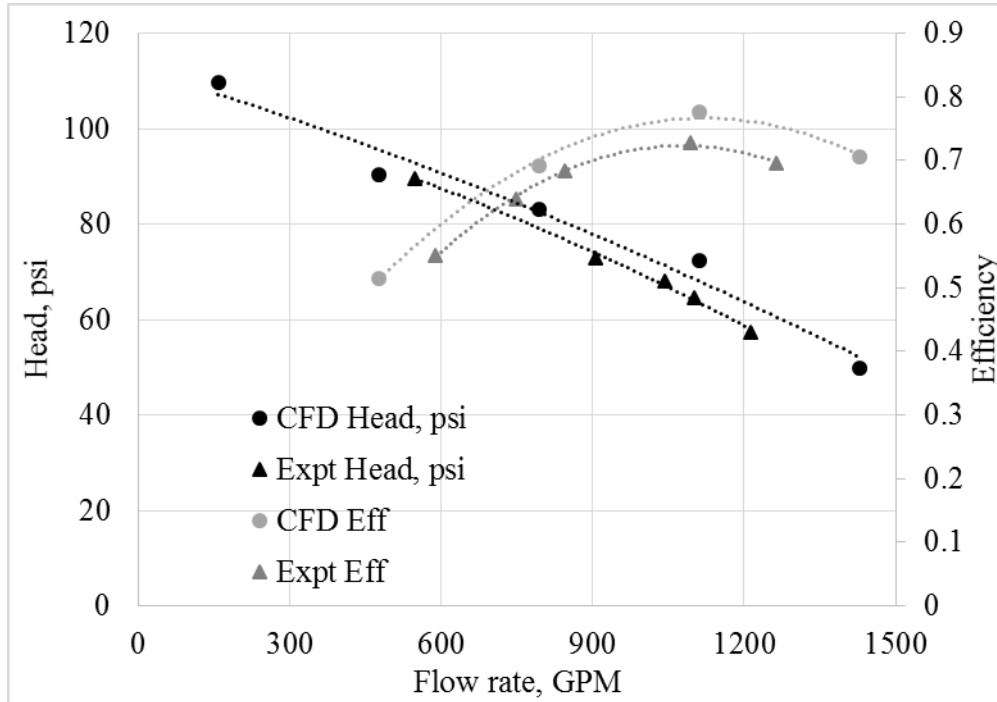


Figure 13: Comparison with experimental data

Numerous simulations under different operational conditions were conducted to evaluate the effect of pump size on the variation in the Morrison number. Various fluid viscosities from 1 to 1500 cp were simulated at 3600 rpm. Figure 14 and 15 shows the plot of head coefficient as a function of  $\phi Re_w^{-Mo}$  for 4” and 8” pump size respectively. As expected, all the head curves collapsed onto two separate lines each representing the separate flow regime. Figure 16 shows the Morrison number for both pump sizes. As expected, the Morrison number plot as a function of rotational Reynolds number does not change with the pump size for given specific speed which also means that the change in flow regime for specific viscosity may deviate the affinity law predictions for geometrically similar pumps with different sizes.

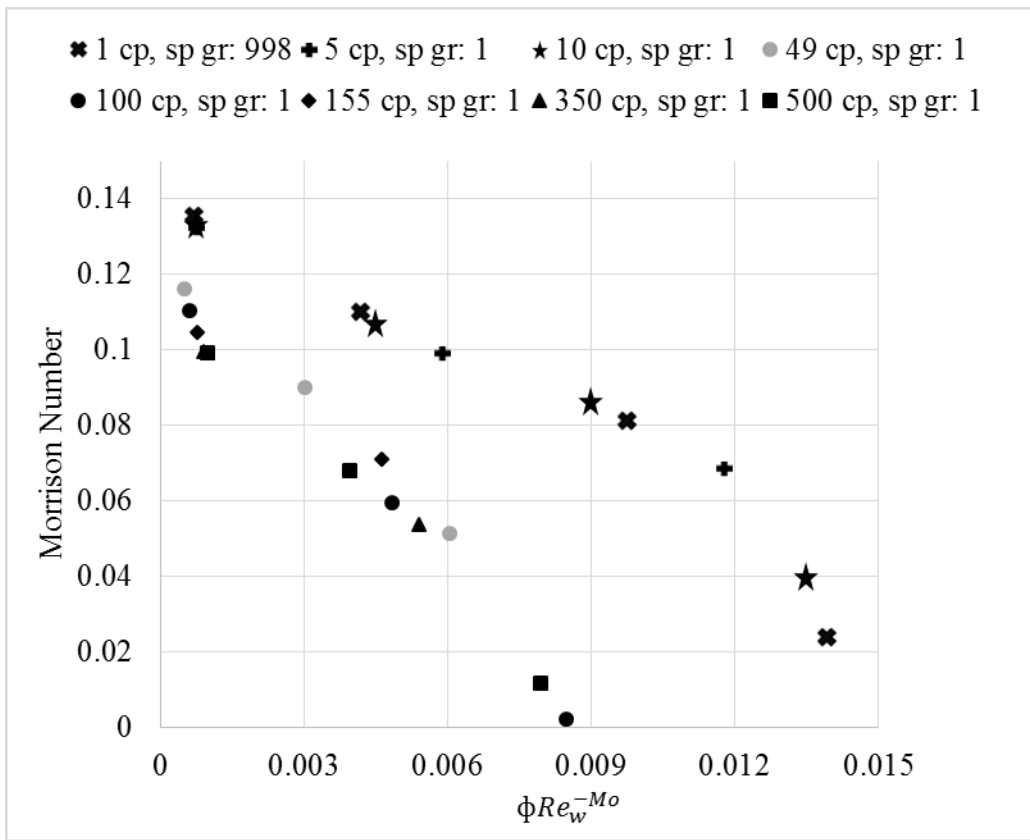


Figure 14: Application of modified affinity laws to 4" pump

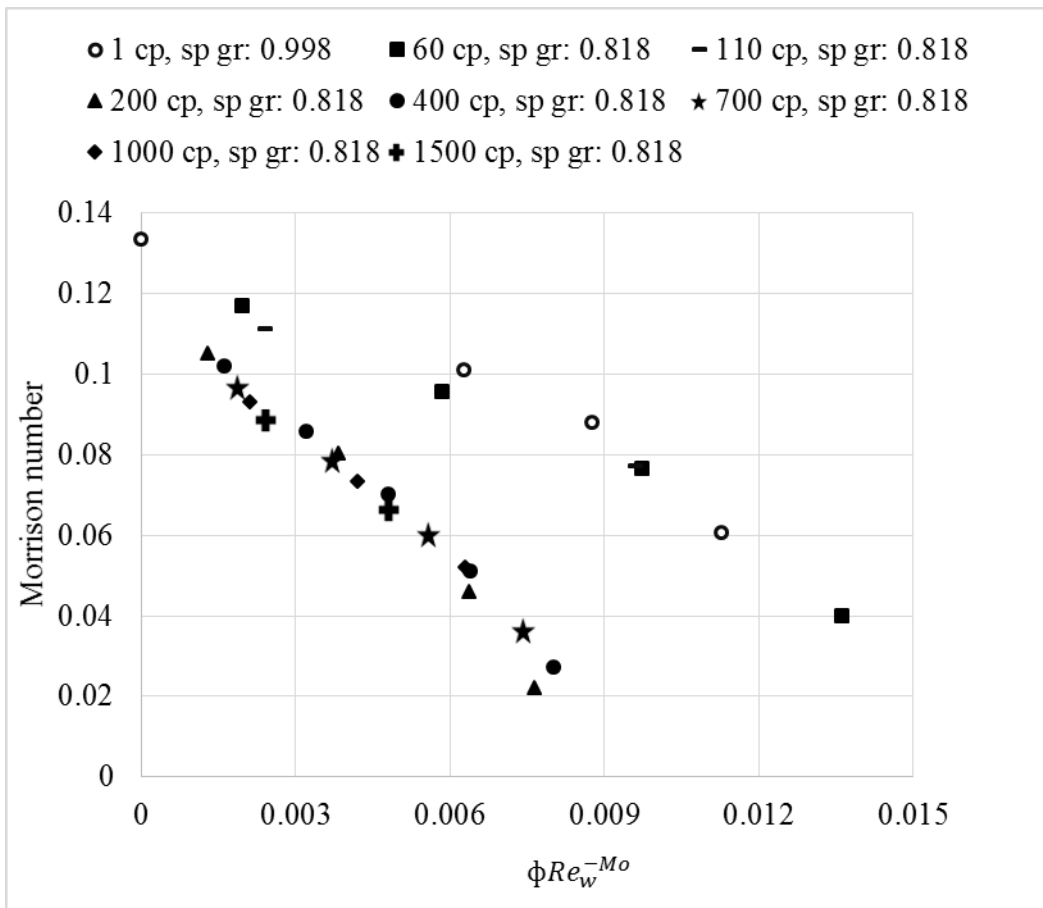


Figure 15: Application of modified affinity law to 8" Pump

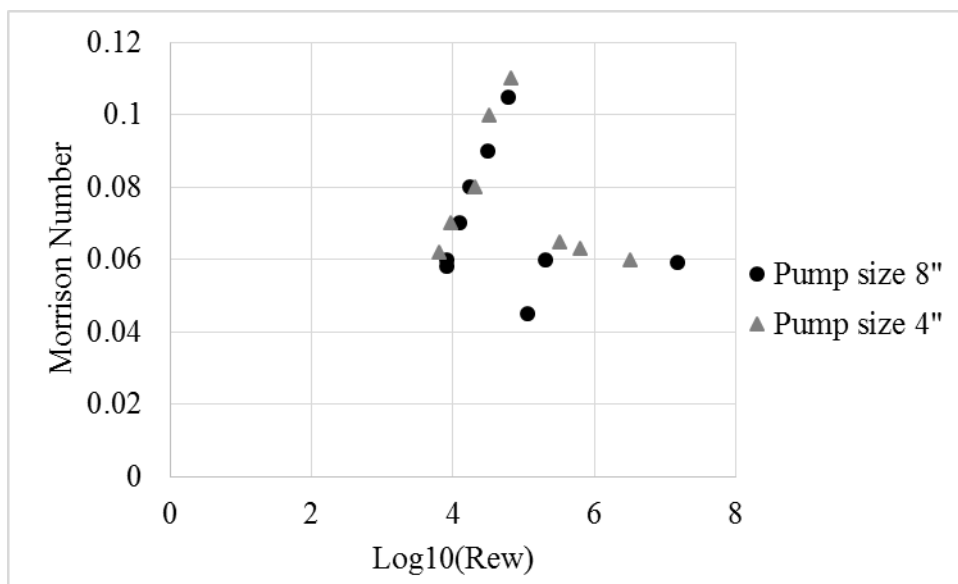


Figure 16: Morrison number for 4" and 8" Pump sizes

**Conclusion:**

The modified affinity laws were further refined to include the effect of viscosity. The pump performance data from published literature was utilized to further evaluate the applicability of the modified affinity laws. The method utilizes the combination of non-dimensional parameters such as rotational Reynolds number and flow coefficient to correct the head coefficient for different fluid viscosities. The dimensionless quantity Morrison number is characterized as a function of flow regime and specific speed. It has been demonstrated that all the head coefficient data is represented by two separate curves with one for turbulent and one for laminar flow regimes of the pump respectively. Effect of pump size on the variation in the Morrison number is investigated. The Morrison number plot do not change with the rotational Reynolds number for different sizes of geometrically similar pump.

The proposed methodology is a unique blend of the affinity laws and friction factor modeling which provides the scientific basis to characterize the energy loss as a function of fluid viscosity, operating conditions and the pump geometry. Using this method, only water performance curve is required to set the shape of the curve. Morrison number chart can then be utilized to evaluate the energy loss due to change in fluid viscosity. The current correlations are based on limited data available from literature. More data is required to characterize the Mo and improve the confidence in the modified affinity laws.

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