Weakly (I, J)-continuous multifunctions and contra (I, J)-continuous multifunctions

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August 24, 2018

Abstract

The purpose of the present paper is to introduce, study and characterize upper and lower weakly (I, J)-continuous multifunctions and contra (I, J)-continuous multifunctions. Also, we investigate its relation with another class of continuous multifunctions.

AMS Subject Classification: 54C10, 54C08, 54C05, 54C60

Key Words and Phrases:weakly (I, J)-continuous multifunctions, I-open set, I-closed set, contra (I, J)-continuous multifunctions, (I, J)-continuous multifunctions

1 Introduction

It is well known today, that the notion of multifunction is playing a very important role in general topology, upper and lower continuity have been extensively studied on multifunctions $F : (X, \tau) \to (Y, \sigma)$. Currently using the notion of topological ideal, different types of upper and lower continuity in multifunction $F : (X, \tau, I) \to (Y, \sigma)$ have been studied and characterized [2], [8], [9], [15], [18]. The concept of ideal topological spaces has been introduced and studied by Kuratowski[12] and the local function of a subset A of a topological space (X, τ) was introduced by Vaidyanathaswamy [17] as follows: given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called the local function of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(\tau, I) = \{x \in X \ / \ U \cap A \notin I$ for every $U \in \tau_x\}$, where $\tau_x = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(,)$ for a topology $\tau^*(\tau, I)$ called the *-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\tau, I)$. We will denote $A^*(\tau, I)$ by A^*. In 1990, Jankovic and Hamlett[10], introduced the notion of I-open set in a topological space (X, τ) with an ideal I on X. In 1992, Abd El-Monsef et al.[1] further investigated I-open sets and I-continuous functions. In 2007, Akdag [2], introduce the concept of I-continuous multifunctions in a topological space with and ideal on it. In 2007, A. Al-Omari and M. S. M. Noorani [3] introduce the notions of Contra-I-continuous and almost I-continuous functions. Given a multifunction $F : (X, \tau) \to (Y, \sigma)$, and two ideals I, J associate, now with the topological spaces (X, τ, I) and (Y, σ, J), consider the multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$. We want to study some type of upper and lower continuity of F as doing Rosas et al. [14]. In this paper, we introduce and study a two new classes of multifunction called a weakly (I, J)-continuous multifunctions and contra (I, J)-continuous multifunctions in topological spaces. Investigate its relation with another classes of continuous multifunctions. Also its relation when the ideal $J = \{\emptyset\}$.

2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed, unless explicitly stated and if I is and ideal on X, (X, τ, I) mean an ideal topological space. For a subset A of (X, τ), $Cl(A)$ and $\text{int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset A is said to be regular open [16] (resp. semiopen [11], preopen[13] , semi preopen [4]) if $A = \text{int}(Cl(A))$ (resp.$A \subseteq Cl(\text{int}(A)), A \subseteq \text{int}(Cl(A)), A \subseteq Cl(\text{int}(Cl(A))))$. The complement of regular open (resp. semiopen, semi-preopen) set is called regular closed (resp. semiclosed, semi-preclosed) set. A subset S of (X, τ, I) is an I-open[10], if $S \subseteq \text{int}(S^*)$. The complement of an I-open set is called I-closed set. The I-closure and the I-interior, can be defined in the same way as $Cl(A)$ and $\text{int}(A)$, respectively, will be denoted by $I\,Cl(A)$ and $I\text{int}(A)$, respectively. The family of all I-open (resp. I-closed, regular open, regular closed, semiopen, semi closed, preopen, semi-preclosed) subsets of a (X, τ, I), denoted by $IO(X)$(resp. $IC(X)$, $RO(X), RC(X), SO(X), SC(X), PO(X)$, $SPO(X), SPC(X)$). We set $IO(X, x) = \{A : A \in IO(X)$ and $x \in A\}$. It is well known that in a topological space (X, τ, I), $X^* \subseteq X$ but if the ideal is codense, that is $\tau \cap I = \emptyset$, then $X^* = X$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, also we always assume that $F(x) \neq \varnothing$ for all $x \in X$. For a multifunction $F : X \to Y$, the upper and lower inverse of any subset A of Y denoted by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A) = \{x \in X : F(x) \subseteq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. In particular, $F^+(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$.

Definition 2.1. *[7] A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be*

1. *upper semi continuous at a point $x \in X$ if for each open set V of Y with $x \in F^+(V)$, there exists an open set U containing x such that $F(U) \subseteq V$.*

2. *lower semi continuous at a point $x \in X$ if for each open set V of Y with $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(a) \cap V \neq \emptyset$ for all $a \in U$.*

Definition 2.2. *[15] A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be*

1. *upper weakly continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an open set U containing x such that $U \subseteq F^+(Cl(V))$.*

2. *lower weakly continuous if for each $x \in X$ and each open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(u) \cap Cl(V) \neq \emptyset$ for every $u \in U$.*

3. *weakly continuous if it is both upper weakly continuous and lower weakly continuous.*

Definition 2.3. *[2] A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be*

1. *upper I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(V)$.*

2. *lower I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I-open set U containing x such that $U \subseteq F^-(V)$.*

3. *I-continuous if it is both upper and lower I-continuous.*

Definition 2.4. *[5] A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be*

1. *upper weakly I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(Cl(V))$.*

2. *lower weakly I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^{-}(V)$, there exists an I-open set U containing x such that $U \subseteq F^{-}(Cl(V))$*

3. *weakly I-continuous if it is both upper weakly I-continuous and lower I-weakly continuous.*

3 Weakly (I, J)-continuous multifunctions

Definition 3.1. *A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:*

1. *upper weakly (I, J)-continuous at a point $x \in X$ if for each J-open set V such that $x \in F^{+}(V)$, there exists an I-open set U containing x such that $U \subseteq F^{+}(J\,Cl(V))$*

2. *lower weakly (I, J)-continuous at a point $x \in X$ if for each J-open set V of Y such that $x \in F^{-}(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^{-}(J\,Cl(V))$.*

3. *upper (resp. lower) (I, J)-continuous on X if it has this property at every point of X.*

Example 3.2. *Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(a) = \{a\}$, $F(b) = \{c\}$ and $F(c) = \{b\}$. It is easy to see that:*
The set of all I-open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.
In consequence, F is upper(resp. lower) weakly (I, J)-continuous on X.

Example 3.3. *Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and two ideals $I = J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(a) = \{a\}$, $F(b) = \{c\}$ and $F(c) = \{b\}$. It is easy to see that:*
The set of all I-open is $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. In consequence, F is not upper (resp. lower) weakly (I, J)-continuous.

Recall that if (X, τ, I) is an ideal topological space and I is the empty ideal, then for each $A \subseteq X$, $A^* = cl(A)$, that is to said, every I-open set is a preopen set, in consequence, if $F : (X, \tau, I) \to (Y, \sigma, \{\emptyset\})$ is upper weakly $(I, \{\emptyset\})$-continuous, then F is upper weakly I-continuous.

Example 3.4. *Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a, c\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(a) = \{b\}$, $F(b) = \{c\}$ and $F(c) = \{a\}$. It is easy to see that:*
The set of all I-open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.
$F : (X, \tau, I) \to (Y, \sigma)$ is upper weakly I-continuous but $F : (X, \tau, I) \to (Y, \sigma, \{\emptyset\})$ is not upper weakly $(I, \{\emptyset\})$-continuous.

Now consider (X, τ, I) and (Y, σ, J) two ideals topological spaces. If $J \neq \{\emptyset\}$, then the concepts of upper weakly (I, J)-continuous and upper weakly I-continuous are independent, as we can see in the following examples.

Example 3.5. *In the Example 3.4, the multifunction F is upper weakly (I, J)-continuous on X but is not upper weakly I-continuous on X.*

Example 3.6. *In the Example 3.3, the multifunction F is upper weakly I-continuous on X but is not upper weakly (I, J)-continuous on X.*

Remark 3.7. *It is easy to see that if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \subset \sigma$ and F is upper (lower) weakly I-continuous, then F is upper (lower) weakly (I, J)-continuous. Even more, if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \nsubseteq \sigma$, we can find upper (resp. lower) weakly (I, J)-continuous on X that are not upper (lower) weakly I-continuous.*

The following theorem characterize the upper weakly (I, J) continuous multifunctions.

Theorem 3.8. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is upper weakly (I, J)-continuous.*

2. $F^+(V) \subseteq I\,int(F^+(J\,Cl(V)))$ *for any* J*-open set* V *of* Y*.*

3. $I\,Cl(F^-(J\,int(B))) \subset F^-(B)$ *for any every* J*-closed subset* B *of* Y*.*

Proof. (1)⇒(2): Let $x \in F^+(V)$ and V be any J- open set of Y. From (1), there exists an I-open set U_x containing x such that $U_x \subset F^+(J\,Cl(V))$. It follows that $x \in I\mathrm{int}(F^+(J\,Cl(V)))$, in consequence, $F^+(V) \subseteq I\mathrm{int}(F^+(J\,Cl(V)))$ for any J-open set V of Y.
(2)⇒(1): Let V any J-open subset of Y such that $x \in F^+(V)$. By (2), $x \in F^+(V) \subseteq I\mathrm{int}(F^+(J\,Cl(V))) \subseteq F^+(J\,Cl(V))$. Choose $U = I\mathrm{int}(F^+(J\,Cl(V)))$. U is an I-open subset of X, containing x. It follows that F is upper weakly (I, J)-continuous.
(2)⇒(3): Let B be any J- closed set of Y. Then by (2), $F^+(Y\backslash B) = X\backslash F^-(B) \subseteq I\mathrm{int}(F^+(J\,Cl(Y\backslash B))) = I\mathrm{int}(F^+(J\,Cl(Y\backslash I\mathrm{int}(B)))) = X\backslash I\,Cl(F^-(J\mathrm{int}(B)))$. Thus, $I\,Cl(F^-(J\mathrm{int}(B))) \subset F^-(B)$.
(3)⇒(2): Let V be any J- open set of Y. Then by (3), $I\,Cl(F^-(J\mathrm{int}(Y\backslash V))) \subset F^-(Y\backslash V) = X\backslash F^+(V)$. It follows that $I\,Cl(X\backslash F^+(I\,Cl(V)) = I\,Cl(F^-(Y\backslash I\,Cl(V))) = I\,Cl(F^-(J\mathrm{int}(Y\backslash V))) \subset X\backslash F^+(V)$, and then $X\backslash I\mathrm{int}(F^+(I\,Cl(V))) \subseteq X\backslash F^+(V)$. And the result follows. □

Theorem 3.9. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$*, the following statements are equivalent:*

1. F *is lower weakly* (I, J)*-continuous.*

2. $F^-(V) \subseteq I\,int(F^-(J\,Cl(V)))$ *for any* J*-open set* V *of* Y*.*

3. $I\,Cl(F^+(J\,int(B))) \subset F^+(B)$ *for any every* J*-closed subset* B *of* Y*.*

Proof. The proof is similar to that of Theorem 3.8. □

Definition 3.10. *[14] A multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$ *is said to be:*

1. *upper* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *containing* $F(x)$*, there exists an* I*-open set* U *containing* x *such that* $F(U) \subset V$*.*

2. *lower* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *of* Y *meeting* $F(x)$*, there exists an* I*-open set* U *of* X *containing* x *such that* $F(u) \cap V \neq \emptyset$ *for each* $u \in U$*.*

3. *upper (resp. lower) (I, J)-continuous on X if it has this property at every point of X.*

Example 3.11. *The multifunction defined in Example 3.2 is upper weakly (I, J)-continuous on X but is not upper (I, J)-continuous on X.*

Remark 3.12. *Every upper (resp. lower) (I, J)-continuous multifunction on X is upper (resp. lower) weakly (I, J)-continuous multifunction on X, but the converse is not necessarily true, as we can see in the following example.*

Example 3.13. *Let $X = \mathbb{R}$ the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\}$=J. Define $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. Recall that in this case the I-open sets are the preopen sets. f is upper (resp. lower) weakly (I, J)-continuous on X, but is not upper(resp. lower) (I, J)-continuous on X..*

Theorem 3.14. *[14] For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is upper (I, J)-continuous.*
2. *$F^+(V)$ is I-open for each J-open set V of Y.*
3. *$F^-(K)$ is I-closed for every J-closed subset K of Y.*
4. *$I\,Cl(F^-(B)) \subset F^-(J\,Cl(B))$ for every subset B of Y.*
5. *For each point $x \in X$ and each J-open set V containing $F(x)$, $F^+(V)$ is an I-open containing x.*

There exist any additional condition in order to say that every upper (resp. lower) (I, J)-continuous if upper (resp. lower) weakly (I, J)-continuous.

Theorem 3.15. *Let $F : (X, \tau, I) \to (Y, \sigma, J)$ be a multifunction such that $F(x)$ is a J-open subset of Y for each $x \in X$. Then F is lower (I, J)-continuous if and only if lower weakly (I, J)-continuous.*

Proof. Let $x \in X$ and V any J-open subset of Y such that $x \in F^-(V)$. Then there exists an I-open subset U of X containing x such that $U \subset F^-(J\,Cl(V)$. It follows that $F(u) \cap J\,Cl(V) \neq \emptyset$ for each $u \in U$. Since $F(u)$ is a J-open subset of Y for each $u \in U$, It follows that $F(u) \cap V \neq \emptyset$ and then F is lower (I, J)-continuous. The converse is clear because every (I, J)-continuous multifunction is weakly (I, J)-continuous. □

Theorem 3.16. *Let $F : (X, \tau, I) \to (Y, \sigma, J)$ be a multifunction such that $F(x)$ is a J-open subset of Y for each $x \in X$. Then F is upper (I, J)-continuous if and only if upper weakly (I, J)-continuous.*

Proof. The proof is similar to the above Theorem. □

4 Contra (I, J)-continuous multifunctions

Definition 4.1. *A multifunction $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:*

1. *upper contra (I, J)-continuous if for each $x \in X$ and each J-closed set V such that $x \in F^+(V)$, there exists an I-open set U containing x such that $F(U) \subset V$.*

2. *lower contra (I, J)-continuous if for each $x \in X$ and each J-closed set V of Y such that $x \in F^-(V)$, there exists an I-open set U of X containing x such that $U \subseteq F^-(V)$.*

3. *Contra (I, J)-continuous if it is upper contra (I, J)-continuous and lower contra (I, J)-continuous.*

Example 4.2. *Let $X = \mathbb{R}$ the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\}$=J. Define $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. Recall that in this case the I-open sets are the preopen sets. It is easy to see that F is upper (resp. lower) contra (I, J)-continuous.*

Example 4.3. *Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J =$*

$\{\emptyset, \{b\}\}$. *Define a multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$ *as follows:* $F(a) = \{b\}$, $F(b) = \{a\}$ *and* $F(c) = \{c\}$. *It is easy to see that:*
The set of all I*-open is* $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J*-open is* $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.
The set of all J*-closed is* $\{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$.
In consequence, f *is upper(resp. lower)* (I, J)*-continuous on* X *but is not upper (resp. lower) contra* (I, J)*-continuous.*

Example 4.4. *The multifunction* F *defined in Example 4.2 is upper (resp. lower) contra* (I, J)*-continuous but is not upper (resp. lower)* (I, J)*-continuous on* X *and the multifunction* F *defined in Example 4.3 is upper (resp. lower)* (I, J)*-continuous but is not upper (resp. lower) contra* (I, J)*-continuous. In consequence both concepts are independent of each other.*

Theorem 4.5. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$*, the following statements are equivalent:*

1. F *is upper contra* (I, J)*-continuous.*
2. $F^+(V)$ *is* I*-open for each* J*-closed set* V *of* Y.
3. $F^-(K)$ *is* I*-closed for every* J*-open subset* K *of* Y.

Proof. (1)$\Leftrightarrow$(2): Let $x \in F^+(V)$ and V be any J-closed set of Y. From (1), there exists an I-open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x \in F^+(V)} U_x$. Since any union of I-open sets is I-open, $F^+(V)$ is I-open in (X, τ). The converse is similar.

(2)$\Leftrightarrow$(3): Let K be any J- open set of Y. Then $Y \backslash K$ is a J-closed set of Y by (2), $F^+(Y \backslash K) = X \backslash F^-(K)$ is an I-open set. Then it is obtained that $F^-(K)$ is an I-closed set. The converse is similar. $\square$

Theorem 4.6. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$*, the following statements are equivalent:*

1. F *is lower contra* (I, J)*-continuous.*
2. $F^-(V)$ *is* I*-open for each* J*-closed set* V *of* Y.

3. *$F^+(K)$ is I-closed for every J-open subset K of Y.*

4. *For each $x \in X$ and each J-closed set K of Y such that $F(x) \cap K \neq \emptyset$, there exists an I-open set U containing x such that $F(y) \cap K \neq \emptyset$ for each $y \in U$.*

Proof. The proof is similar to the proof of Theorem 4.5. □

Remark 4.7. *It is easy to see that if $J = \{\emptyset\}$ and $F : (X, \tau, I) \to (Y, \sigma, J)$ is upper (resp. lower) contra (I, J)-continuous then F is upper (resp. lower) contra I-continuous.*

The following example shows the existence of upper (resp. lower) contra I-continuous that is not upper (resp. lower) contra $(I, \{\emptyset\})$-continuous.

Example 4.8. *Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a, c\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(a) = \{c\}$, $F(b) = \{b\}$ and $F(c) = \{a\}$. It is easy to see that:*
The set of all I-open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
The set of all J-open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, Y\}$.
The set of all J-closed is $\{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$.
Observe that $F : (X, \tau, I) \to (Y, \sigma)$ is upper contra I-continuous but $F : (X, \tau, I) \to (Y, \sigma, \{\emptyset\})$ is not upper contra $(I, \{\emptyset\})$-continuous.

Remark 4.9. *It is easy to see that if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \subset \sigma$. If F is upper (lower) contra I-continuous, then F is upper (lower) (I, J)-continuous. Even more, if $F : (X, \tau, I) \to (Y, \sigma, J)$ is a multifunction and $JO(Y) \nsubseteq \sigma$, we can find upper (resp. lower) contra (I, J)-continuous on X that are not upper (lower) contra I-continuous.*

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