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Scicos Based Investigation of an Adaptive Vibration Damping Technique Using Fractional Order Derivatives

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Abstract – **Detailed investigation of a simple nonlinear, active, adaptive approach of controlling the oscillation of a car proceeding on a bumpy road is presented. Its key idea is a frequency dependent control of the strictness of a traditional PID controller by applying fractional order derivatives in a simple kinematic design without any respect to the dynamic model of the system. The adaptive part of the controller relieves the designer of dealing with the system's dynamics within the frames of some linear control, and guarantees the implementation of this design. The operation of the approach is illustrated by the use of INRIA's scientific co-simulator Scicos for a rough model of a car. Well interpretable trends were revealed regarding the effect of the variation of the order of derivation, and that of the sampling time of the adaptive loop. These results seem to be promising for actively damping the vibration of systems having unmodeled, uncontrolled internal degrees of freedom.**

I. INTRODUCTION

Externally excited oscillation normally is undesired phenomenon that occurs in various physical systems therefore its efficient damping is of great practical significance. In the practice the task has the „delicate" nature that the controller cannot be provided with the "exact", and/or with complete information on the actual physical state of the system to be controlled. The immediate antecedent of this paper [1] reported on the preliminary application of a novel branch of soft computing for solving such problems by the use of ideas and methods reported e.g. in [2] and [3].

As an input, this control method requires the desired trajectory of the generalized coordinates of the system that has to directly be controlled. In our case some more or less "free" variation of the distance between the chassis and the wheel has to be allowed within certain limited range. Whenever the motion of the wheel is of limited amplitude and high frequency it passes through small bumps or dips, and relieving the chassis of such oscillation is desirable. However, when the elevation of the wheel is relatively big and slow, the car has to climb or climb down a bigger hill and the height of the position of the chassis has to follow this motion. The most plausible means of control would be the application of a simple PID terms to keep a finite error at bay. A small integrating term of this controller does not „forget" the past, and for an even small but constant error it generates infinite signal for feedback, so it can be used for eliminating small, constant trajectory tracking errors. However, for the compensation of "abrupt" changes in the tracking error the proportional and the derivative terms are responsible. Due to these terms the vibration of the wheel would be transmitted to the chassis.

As generalization of the concept of the traditional derivative the concept of fractional order derivative found more and more physical applications. The problem of designing fractional order control systems within the frames of linear control obtained considerable attention recently, e.g. [4]. The French expression invented by Oustaloup „*CRONE: Commande Robuste d'Ordre Non Entier*" [5] hallmarks a well-elaborated design methodology that obtained application in vibration control [6]. Understanding and application of this method requires deep engineering knowledge in the realm of linear systems, frequency spectrum analysis, the use of Laplace transforms and complex integrals, various typical diagrams, etc.

The aim of the present paper is to demonstrate an alternative approach not strictly restricted to the traditional "linear" way of thinking. Tackling the problem from a more general nonlinear basis requires less amount of profound and specific engineering knowledge, the application of which can be evaded by the controller's adaptive nature or learning abilities. The operation of the approach is illustrated by the use of INRIA's scientific co-simulator Scicos for a rough model of a car. Well interpretable trends are revealed regarding the effect of the variation of the order of derivation, and that of the sampling time of the adaptive loop. These results seem to be promising for actively damping the vibration of systems having unmodeled, uncontrolled internal degrees of freedom.

II. APPLICATION OF FRACTIONAL ORDER DERIVATIVES

In the case of a normal PID-type controller the desired trajectory reproduction can be prescribed in a purely kinematics based manner. For the second time-derivative of the actual coordinate errors the desired relation can be formulated as:

$$\ddot{\mathbf{e}}^d = -P\mathbf{e} - D\dot{\mathbf{e}} - I\int_{-\infty}^{t}\mathbf{e}(t')dt' \tag{1}$$

The use of a quite small integrating term in (1) is expedient whenever very accurate and slow tracking of the nominal trajectory is needed, because even for very small permanent error sooner or later it yields quite considerable feedback. Therefore, in contrast to the approach presented in [1] a small integrating term also is applied. For the desired acceleration of the controlled generalized coordinates (1) then yields

$$\ddot{\mathbf{q}}^{d} = \frac{d}{dt}\dot{\mathbf{q}}^{N} - \left[P + D\frac{d}{dt}\right]\left(\mathbf{q}^{r} - \mathbf{q}^{N}\right) - I\int\left(\mathbf{q}^{r} - \mathbf{q}^{N}\right)d\tau \qquad (2)$$

in which the superscript "N" refers to the nominal coordinates prescribed. If the dynamics of the system to be controlled is not very well known the feedback force calculated on the basis of (2) cannot be accurate enough, therefore very big gains (i.e. P, and D coefficients) has to be applied. Normally such a choice results in a noisy control. This situation can be improved by using fractional order derivative in (2) as

$$\ddot{\mathbf{q}}^{d} = \frac{d^{\beta}}{dt^{\beta}}\dot{\mathbf{q}}^{N} - \left[P + D\frac{d^{\beta}}{dt^{\beta}}\right]\left(\mathbf{q}^{r} - \mathbf{q}^{N}\right) - I\int\left(\mathbf{q}^{r} - \mathbf{q}^{N}\right)d\tau \qquad (3)$$

in which the symbol d^{β}/dt^{β} denotes some fractional order derivative. As in [1], the numerical approximation of the definition by Caputo

$$\frac{d^{\beta}}{dt^{\beta}}u(t) := \frac{1}{\Gamma(1-\beta)}\int_{0}^{t}\left[\frac{du(\tau)}{d\tau}\right](t-\tau)^{-\beta}d\tau,\ \beta\in(0,1) \qquad (4)$$

as

$$\frac{d^{\beta}}{dt^{\beta}}u(t) \cong \frac{u'(t)\delta^{-\beta+1}}{\Gamma(2-\beta)} + \sum_{0<s \text{ while } s\delta<T} \frac{\delta^{-\beta+1}\left[(s+1)^{-\beta+1} - s^{-\beta+1}\right]}{\Gamma(2-\beta)}u'(t-s\delta) \qquad (5)$$

is applied. In [4] the full 1st order derivative is "causally reintegrated" by the use of a kernel function having slowly forgetting nature (the contribution of the far past becomes more and more negligible in it), while its singularity in τ=t enhances the relative weight of the contribution of the τ=<t instants. The slowly decreasing "tail" of this function also acts as a frequency filter that rejects the high-frequency components of the traditional 1st derivative. Furthermore, to introduce symmetry against the translation of the signal in time in [5] we can go back in time only to some time t-T instead of 0. In the numerical simulations in this paper δ=4 *ms*, T=30×4 *ms* were chosen, and the appropriate value for β was as well as the sampling time of the adaptive loop were varied.

III. THE MODEL OF THE CAR AND ITS CONTROL

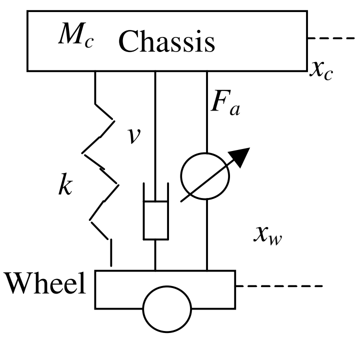

Fig. 1. The rough model of the suspension system

The model of the system considered is described in Fig. 1. The mass of the wheel is supposed to be negligible with respect to that of the chassis of mass M_c=100 *kg* the model value of which was supposed to be 150 *kg*. (This value cannot exactly be known *a priori* since one or more than one travelers of even 100 *kg* weight each can sit in the car.) The passive suspension system consisted of a spring of stiffness k=2×10^4 *N/m* and viscosity of v=1×10^4 *N/(m/s)*. The force of the active suspension F^a was supposed to be generated according to the control law. The coordinates x_c and x_w in *m* units describe the height of the chassis and the wheel, respectively, with respect to an inertial frame, i.e. with respect to the sea level, so they are not available as direct data for the controller. The nominal height of the chassis while rigidly following the wheel was prescribed to be $x_{cnom}=x_w+L_0$ with L_0=0.5 *m.* In the case of loose trajectory tracking little humps and dips need not be traced by the chassis, but climbing a higher hill or deeper valley must be traced. However, the error of this trajectory tracking is available via local measurements within the car as $e=x_c-x_{cnom}=(x_c-x_w)-L_0$. The 1st order time-derivative of the error can be numerically estimated by finite element methods. Because x_w and x_c are measured with respect to an inertial frame their second traditional time-derivatives also are measurable even by the use of micro-sensors developed on a chip.

Eq. (3) represents a tracing requirement expressed by the use of purely kinematic terms. The main expectation behind it is the supposition that for small proportional coefficient P some loose tracking can be achieved the accuracy of which is increased by the "filtered" integrals at low frequency (that is for hill climbing), while for the higher frequency components occurring when small dips are passed it remains loose. By the use of the approximate dynamic model of the system the appropriate active force can be estimated. Due to the approximate nature of the dynamic model exertion of this force will not result in the desired acceleration of the chassis. For the realization of (3) adaptive control is needed. Its main principles are given in the following part.

IV. THE ADAPTIVE CONTROL

For the adaptive control there is given an imperfect system model as a starting point. On the basis of that some excitation is calculated to obtain a desired system response $\mathbf{i}^d$ as $\mathbf{e}=\boldsymbol{\varphi}(\mathbf{i}^d)$. This model is step by step refined in the following manner. If we apply the above approximate excitation, according to the actual system's inverse dynamics described by the unknown function a realized response $\mathbf{i}^r=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d))=\boldsymbol{f}(\mathbf{i}^d)$ is obtained instead of the desired one, $\mathbf{i}^d$. Normally one can obtain information via observation only on the function $\boldsymbol{f}()$ considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only $\mathbf{i}^d$ as the input of $\boldsymbol{f}()$ can be "deformed" to $\mathbf{i}^{d*}$ to achieve and maintain the $\mathbf{i}^d=\boldsymbol{f}(\mathbf{i}^{d*})$ state. The following "scaling iteration" was suggested for finding the proper deformation:

$$\mathbf{i}_0; \mathbf{S}_1\mathbf{f}(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1\mathbf{i}_0; \ldots; \mathbf{S}_n\mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_0; \quad \mathbf{i}_{n+1} = \mathbf{S}_{n+1}\mathbf{i}_n; \mathbf{S}_n \xrightarrow[n\to\infty]{} \mathbf{I} \qquad (6)$$

in which the $\mathbf{S}_n$ matrices denote some linear transformations that map the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, and the controller „learns” the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (6) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. In this paper the Special Symplectic Transformations were chosen as algebraic means. Due to their special structure these matrices automatically approach the unit matrix as $\mathbf{f}\rightarrow\mathbf{i}_0$. In the lack of enough free space, regarding the details we refer to [3].

V. SIMULATION RESULTS

The new version of INRIA’s SCILAB 3.0 was issued about the end of the summer of 2004. This software package is freely usable for scientific research. In its basic form it corresponds to a programming language and a development system that makes it possible to develop and use user-defined functions in similar way as its own built-in functions. Scicos an application developed in SCILAB to support programming via defining block diagrams and symbolic “wires”. Besides this graphical possibility its main virtue is the use of sophisticated program packages for solving Ordinary Differential Equations (ODEs) either in explicit $\dot{\mathbf{y}}=\mathbf{f}(t,\mathbf{x}), \mathbf{y}(t_0)=\mathbf{y}_0$ or in implicit form as $\mathbf{g}(t,\mathbf{y},\dot{\mathbf{y}})=\mathbf{0}, \mathbf{y}(0)=\mathbf{y}_0, \dot{\mathbf{y}}(0)=\dot{\mathbf{y}}_0$. The program is defined “graphically” at first, then it is compiled to bring about an ODE system that is solved by the use of one of these packages automatically. The user-developed functions can be given as common SCILAB instructions that are “interpreted” by Scicos. To speed up the operation of the simulator an alternative method is loading and compiling the user-defined functions instead of directly writing them into the user blocks. (In this case the user block contains only a simple call for the compiled function.) The compilation of the necessary user functions at the beginning can be prescribed in the so-called “*Context*” box of the simulator. The here defined variables behave as “global” ones. They can be referred to as “global” variables in the heading (beginning lines) of the user’s functions.

At the time being Scicos has not very extended documentation. Regarding the reproduction of the simulation results it has been found “experimentally” that the option of loading a “New Scilab” before running the simulation program always leads to the same results. Most probably the ODE solver is loaded according to its default settings when it is called at the first occasion. These settings can vary in time as the program runs, and remain in the memory after finishing it. (Since the package has to adapt itself to solve various problems it probably adapts itself to the last task solved within this SCILAB session, this supposition is reasonable. Even quitting Scicos within the SCILAB window seems to leave the last settings valid.)

The absolute and relative error of the ODE solver (0.0001 and 0.00001 respectively, in the here presented

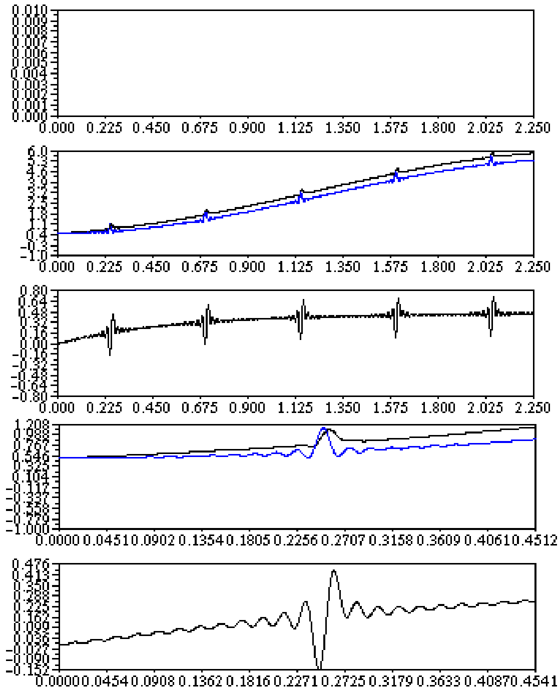

Fig. 2: Passive vibration control only: 1st box: the norm of the **S-I** matrices (now identical to 0); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line); 3rd box: tracking error (x_c-L_0-x_w) [m]; 4th and 5th boxes: zoomed details of the trajectory of the wheel and chassis and that of the tracking error vs. time [s]

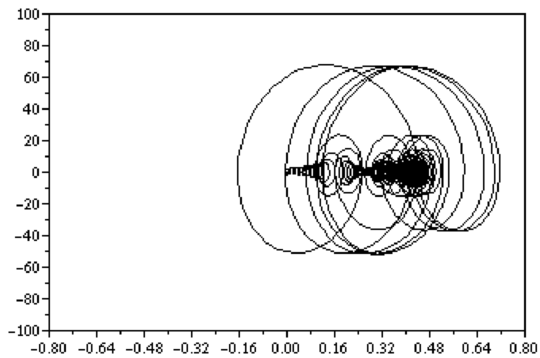

Fig. 3. Passive vibration control: the phase space of the (x_c-L_0-x_w) distance [m, m/s]

simulations), the accuracy in computing time (“Tolerance on time” 1.000D-06 in our case), and the maximal allowable step size in the integration (“maximum step size” 0.0001 s in our case) can be prescribed before running the program. In the simulation examples the car was supposed to move with a velocity of 10 m/s (36 km/h) while climbing a hill covered by a bumpy road as given in the figures containing the results. The bumps/dips were modeled by a Fourier series containing ω=1,2,…,314 circular frequencies with equal weights. For the highest frequency component this corresponds to T_{min}=2×10^{-2} s period (frequency of 50 Hz), which, at 10 m/s velocity means a combination of pairs of 10 cm wide dips and bumps that corresponds to a road built up of granite blocks of this size.

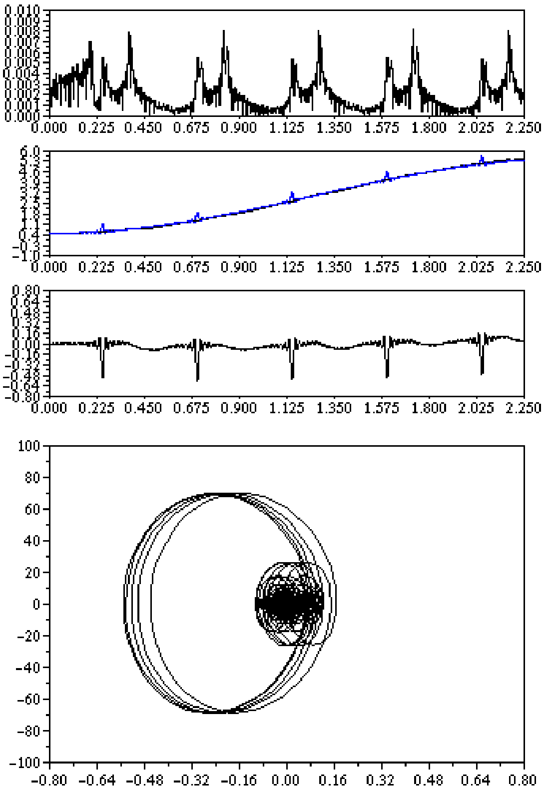

Fig. 4. Active, adaptive vibration control: 1st box: the norm of the **S-I** matrices; 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line); 3rd box: tracking error (x_c-L_0-x_w) [m] vs. time [s]; 4th box: Phase space of (x_c-L_0-x_w) [m, m/s], order of derivation: β=0.01

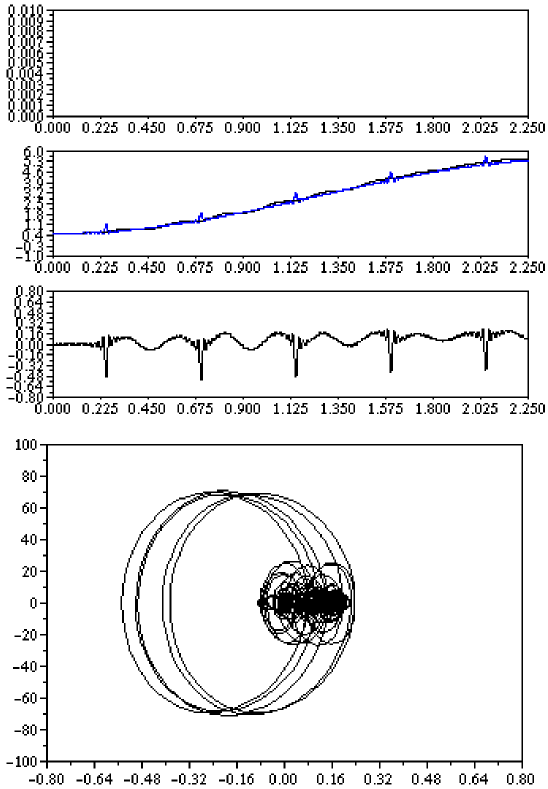

Fig. 5. Active, non adaptive vibration control: 1st box: the norm of the **S-I** matrices (now equals to 0); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line); 3rd box: tracking error (x_c-L_0-x_w) [m] vs. time [s]; 4th box: Phase space of (x_c-L_0-x_w) [m, m/s], order of derivation: β=0.01

In Fig. 2 typical results are given for the passive suspension when no active force is applied. It is evident that in this case it results in a poor oscillation damping. The vibration of the wheel is transmitted to the chassis. Besides this the imprecise estimation of the load leads to the appearance of a "static error" in the distance between the wheel and the chassis .Fig. 3: reveals the phase space of the (x_c-L_0-x_w) distance.

The adaptive version of this control with δ=4 *ms* sampling time for the internal and the adaptive loops and β=0.01 order of derivation is given in Fig. 4. The improvement in decreasing the vibration of the chassis is evident from the zoomed part of the graph. Furthermore, the mean value of the (x_c-L_0-x_w) distance has been moved to around 0, too.The results belonging to the non-adaptive active counterpart of the same controller are given in Fig. 5. While the high-frequency terms are well suppressed due to the filtered goal of the control, the lack of adaptivity constrains the controller to compensate the modeling error within the frames of the linear controller. This leads to a typical oscillation in the tracking error.

It interesting question to what extent the sampling time of the "outer" adaptive loop can be increased, in order to reduce the computational burden of the controller. According to the simulations 3δ=12 *ms* for this value does not reveal significant differences in comparison with the results given in Fig. 4. However, further increase in the sampling time of the adaptive loop seems to approach the results of the non adaptive control (Figs. 6-7).

It is also interesting to see the effect of increasing the order of derivation in the adaptive control. Figs. 8 and 9 belong to the case of δ=4 *ms*, of sampling time 3δ=12 *ms*, and β=0.14. The adaptive control still is stable but shows considerable oscillations. The non-adaptive control seems to lose its stability.

VI. CONCLUSIONS

In this paper the combination of the concept of fractional order derivatives and a novel branch of soft computing was applied for vibration suppression purposes in the case of a simple car model.

The main contribution of the non-integer order derivatives lies in providing the controller with appropriate causal goal functions of significantly filtered high-frequency components.

The adaptive law gives help in implementing the result of this essentially purely kinematic design without requiring *a priori* accurate information on the dynamics of the system. It takes away the burden of dealing with dynamic effects from the linear fractional order controller. Furthermore, in this case no particular suppositions are needed for the nature of the vibration that assumptions used to be typical in the traditional control literature, e.g. that vibration can be treated with low order Taylor series expansion around some equilibrium position, or that the suspension system and the external excitation have characteristic eigenfrequencies or peaks in their Fourier spectra for the absorption of which the eigenfrequency of

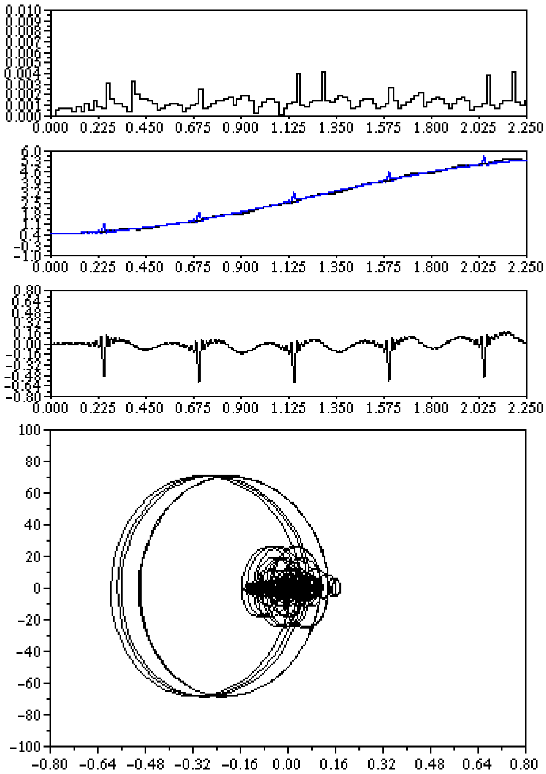

Fig. 6. Active, adaptive vibration control with 5δ=20 ms sampling in the "outer" adaptive loop: 1st box: the norm of the **S-I** matrices (now equals to 0); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line); 3rd box: tracking error (x_c-L_0-x_w) [m] vs. time [s]; 4th box: Phase space of (x_c-L_0-x_w) [m, m/s], order of derivation: β=0.01

the damping medium has to be properly tuned.

It needs the possibility only for fast feedback signals the characteristic time of which was found equal to be equal to with δ=4 *ms*. It was also found that for the external adaptive loop this sampling time can be tripled.

Regarding the order of derivation it cropped up that choosing a relatively small value as β=0.01 seems to be expedient.

For further research the active adaptive vibration control of physical systems having unmodeled internal degrees of freedom can be considered for which the adaptive approach was also found to be applicable.

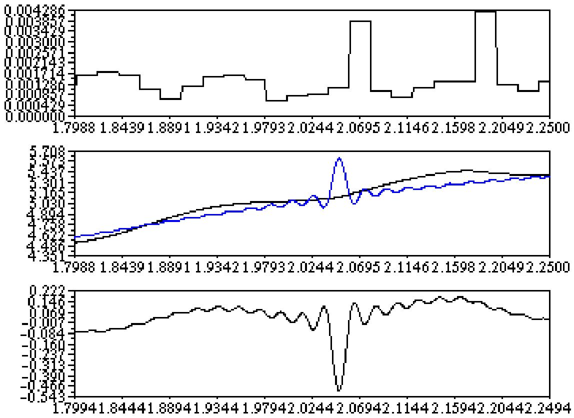

Fig. 7. Zoomed excerpts of Fig. 6: Active, adaptive vibration control with 5δ=20 ms sampling in the "outer" adaptive loop: 1st box: the norm of the **S-I** matrices (now equals to 0); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line); 3rd box: tracking error (x_c-L_0-x_w) [m] vs. time [s]; order of derivation: β=0.01

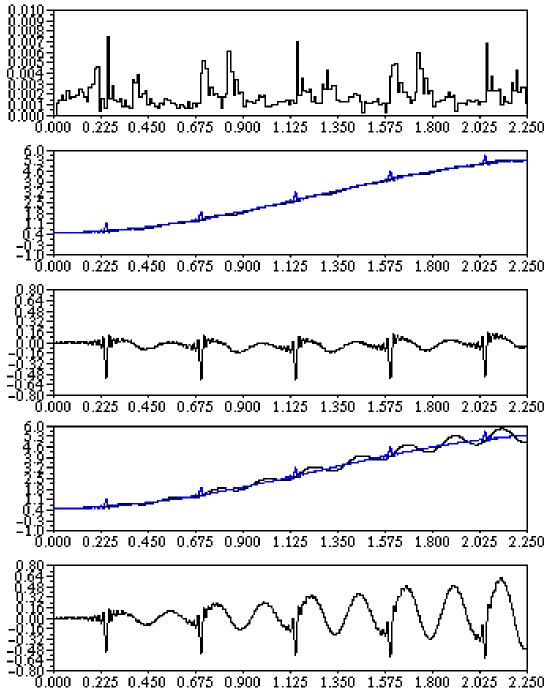

Fig. 8. Active, adaptive and non adaptive vibration control with 3δ=12 ms sampling time in the "outer" adaptive loop and order of derivation: β=0.14: 1st box: the norm of the **S-I** matrices (adaptive); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [m] (upper line) (adaptive); 3rd box: tracking error (x_c-L_0-x_w) [m] vs. time [s] (adaptive); 4th box: wheel and chassis position (non adaptive), 5th box: tracking error (non-adaptive)

VII. ACKNOWLEDGMENT

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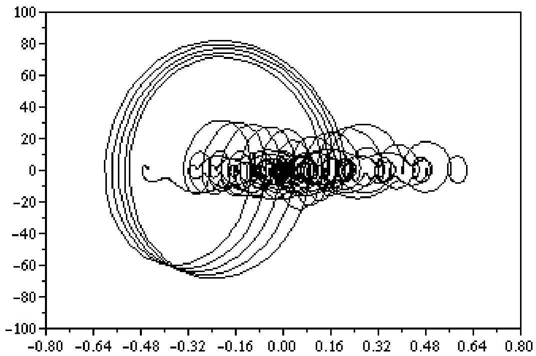

Fig. 9. Non adaptive active vibration control: the phase space of the (x_c-L_0-x_w) distance [*m, m/s*] with 3δ=12 ms sampling time in the "outer" adaptive loop and order of derivation: β=0.14

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