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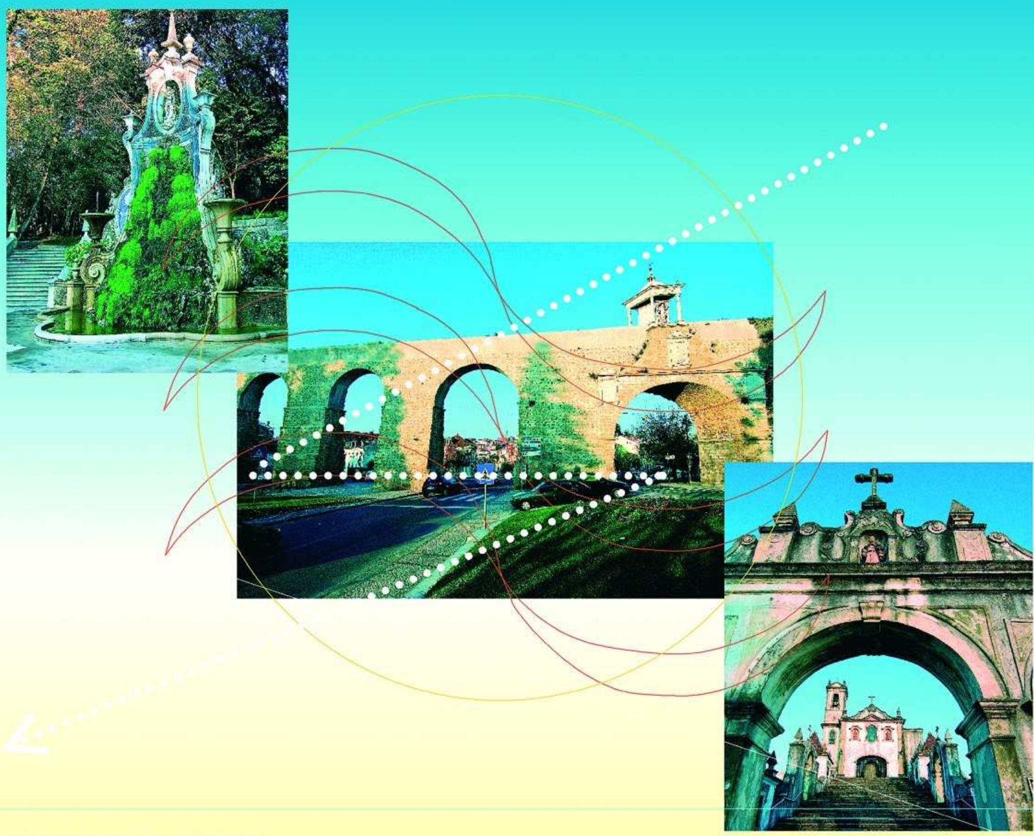

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STATISTICAL MODELLING OF DUAL – ARM ROBOTIC SYSTEMS

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The article presents a new approach to the analysis and design of two-arm robotic systems. Usually, system descriptions are based on a set of differential equations which require laborious computations. This motivates the need of alternative models based on other mathematical concepts. The proposed statistical method gives clear guidelines towards the robotic systems analysis and development.

1. INTRODUCTION

Mechanical manipulators are developed according to engineering and scientific principles that are based on fundamental concepts such as those arising from mathematics and physics. Based on these formulations, the first step on the study of a physical phenomenon is the development of an adequate model [1]. Manipulators are a system where we have for fundamental concepts the differential and matrix calculus and the classical Newtonian physics, while the system model corresponds to the standard kinematic and dynamic descriptions [2-6]. Nevertheless, other classes of phenomena such as quantum physics and thermodynamics are studied using different concepts. Quantum physics requires the use of statistical methods while thermodynamics can be studied both through classical and statistical methods.

These facts suggest that, for a given problem, we may develop different models each with its own merits and drawbacks. This paper presents a framework where these problems are addressed for robotic manipulators. We develop a new modelling approach based on a statistical formalism [8-9]. These concepts are then illustrated on a simple mechanical joint-actuated arm and for two robots carrying an object. In order to develop the method we organise this paper as follows. Section two formulates the new fundamental modelling concepts. Section three illustrates the application of the statistical method to the kinematics of one and two mechanical manipulators working in cooperation. Finally, section four presents the main conclusions.

2. ON THE STATISTICAL MODELLING

The classical modelling of mechanical manipulators is well known. For a n degrees of freedom (*dof*) robot the kinematics is described by a set of non-linear equations:

$$\mathbf{q} = \psi(\mathbf{p}) \tag{1a}$$

$$\dot{\mathbf{q}} = \left[\frac{\partial \psi(\mathbf{q})}{\partial \mathbf{p}}\right]\dot{\mathbf{p}} \tag{1b}$$

$$\ddot{\mathbf{q}} = \left[\left[\frac{\partial \psi(\mathbf{q})}{\partial \mathbf{p}}\right]\right]\ddot{\mathbf{p}} + \left[\frac{\partial^2 \psi(\mathbf{q})}{\partial \mathbf{p}^2}\right]\dot{\mathbf{p}}^2 \tag{1c}$$

where $(\mathbf{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}})$ and $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ are $n \times 1$ vectors of positions, velocities and accelerations in the operational and joint spaces, respectively.

These observations motivate the re-evaluation of the concepts in use because expressions (1) involve a plethora of variables and parameters that give rise to a gigantic number of possible combinations of values in a design stage.

In order to overcome implementation problems, alternative concepts are required. Statistics is a mathematical tool well adapted to this type of problem. If with this method, we lose the 'certainty' of the deterministic model, we gain a simpler and more intuitive viewpoint. This approach has already been used by other researchers [11] in some restricted classes of problems. In the sequel we refer to the new approach, as the statistical model [12-15] to stress the contrast with the standard method. Our modelling procedure comprises: The statistical description of a set of input variables (*IV*s), which is variables that are free to change independently.

- The statistical description of a set of output variables (*OV*s), that is, variables that are functions of the previous ones.
- The above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:
- A set of parameters that are to be optimised in the design stage.

Bearing these ideas in mind, we are stating that, in the kinematics, the independent random variables have probability density functions (*pdf*) similar to the histograms of a long run sampling therefore the statistical description of the variables does not consider the (implicit) time variable.

3. A STATISTICAL MODEL FOR ROBOTICS

In this section we adopt the $2R$ joint-actuated manipulator as the support for the development and implementation of the new modelling concepts.

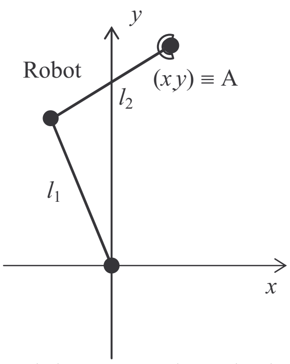


Figure 1: One $2R$ joint-actuated manipulator.

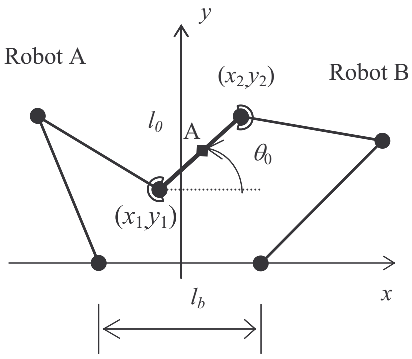


Figure 2: Two $2R$ cooperating robots for the manipulation of an object with length l_0, orientation θ_0 and center point A.

In the first sub-section we begin by introducing our approach in the kinematics of the $2R$ robot (Figure 1). In the second sub-section we analyse the kinematics of two robots working in cooperation (Figure 2).

3.1. ONE ARM KINEMATICS

We begin our study with a numerical-based approach and, in a second stage, we adopt a complementary analytical perspective.

In order to have a statistical description we have to characterise the random variables through appropriate *pdf*. At this point there is no *a priori* knowledge about the statistical properties of the system. Therefore, we start our experiments with some preliminary assumptions and, in the sequel; we demonstrate the conditions that optimise the kinematic performances. For $\mathbf{p} = [p_1, p_2]^T$ and $\mathbf{q} = [q_1, q_2]^T$ we start by considering bidimensional uniform *pdf* for the *IV*s in $f_P(\mathbf{p})$ in $\mathbf{p}\rightarrow\mathbf{q}$ and $f_Q(\mathbf{q})$ in $\mathbf{p}\leftarrow\mathbf{q}$.

The inverse and direct kinematics have *pdf* related by the expressions:

$$f_Q(\mathbf{q}) = |\Im_P| f_P(\mathbf{p}) \tag{2a}$$

$$f_P(\mathbf{p}) = |\Im_P|^{-1} f_Q(\mathbf{q}) \tag{2b}$$

In order to test this idea we perform several numerical experiments. We consider identical links $l_1 = l_2$, that is, the case of maximum manipulability. To test this method, we 'excite' the $\mathbf{p}\leftarrow\mathbf{q}$ kinematics with a numerical random sample of variables with a uniform *pdf* and we compare the relationship between the operational and joints space. Afterwards, we repeat this procedure in the reverse order, that is, we 'excite' the $\mathbf{p}\rightarrow\mathbf{q}$ kinematics with a numerical random sample in the inverse order.

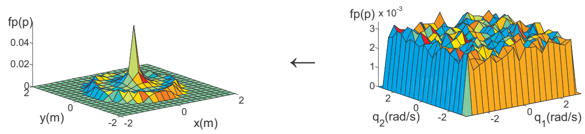


Figure 3: Uniform 'excitation' of the $\mathbf{p}\leftarrow\mathbf{q}$ kinematics of the $2R$ robot ($l_1 = l_2$) with a numerical sample.

From the chart of the $\mathbf{p}\leftarrow\mathbf{q}$ we conclude, that for a uniform *pdf* in the joint space the direct kinematics 'prefers' the singular robot configuration, namely $[p_1, p_2]^T = [0,0]^T$.

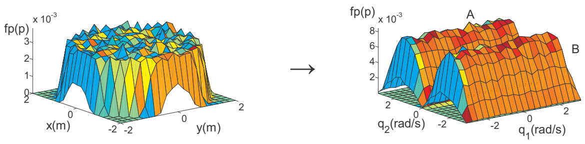


Figure 4: Uniform 'excitation' of the $\mathbf{p}\rightarrow\mathbf{q}$ kinematics of the $2R$ robot ($l_1 = l_2$) with a numerical sample, A – robot with upper elbow and B – robot with lower elbow.

From the chart of the $\mathbf{p}\rightarrow\mathbf{q}$ we conclude, we conclude, that for a uniform *pdf* in the operational space the inverse kinematics 'avoids' the singular robot configurations. Moreover, $f_{Q_2}(q_2)$ reveals maxima at $q_2 = \pm\pi/2$ and minima at $q_2 = \{0, \pm\pi\}$. As *pdf* of the *IV*s is not responsible for this situation the result is an intrinsic property of the kinematics.
The symbolic derivation of the Jacobians requires the classical kinematic model. This indicates that the classical and the statistical models are *not exclusive* but are, in fact, *complementary*. Knowing that for the $2R$ manipulator the transformation $\mathbf{p}\rightarrow\mathbf{q}$ is given by the expression:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} l_1 Cos(q_1) + l_2 Cos(q_{12}) \\ l_1 Sin(q_1) + l_2 Sin(q_{12}) \end{bmatrix} \tag{3}$$

yielding:

$$\Im_P = l_1 l_1 Sin(q_2) \tag{4}$$

For $L = l_1 + l_2$ and $\mu = l_1/l_2$ the maximum of $\Im_P$ occurs when:

$$\mu = 1 \tag{5a}$$

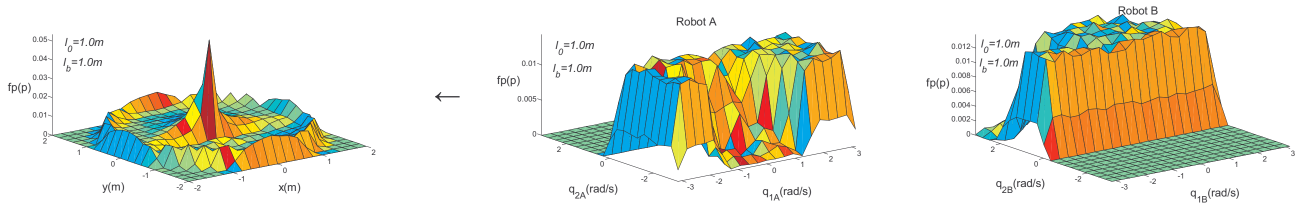
$$q_2 = \pm \pi/2 \qquad (5b)$$

These expressions coincide with the numerical results and have distinct meanings. Condition (6a) defines the optimal kinematic structure of the 2*R* manipulator that must be optimised in a design stage. Expression (5b) points out the best position configuration that must be satisfied through appropriate trajectory planning algorithms. Such conclusions are similar to those obtained in previous studies [7-8] using the classical approach which proves the feasibility of the statistical analysis.

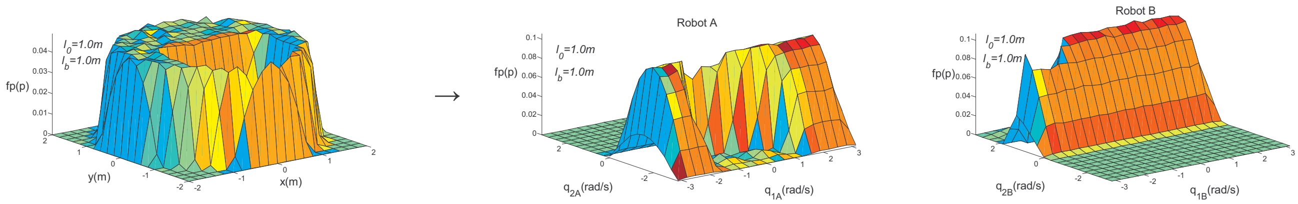
3.2. TWO ARMS KINEMATICS

We consider two manipulators with a configuration similar to the human being, namely robot A has an upper elbow and robot B has a lower elbow. Moreover, points leading to link-link or link-object crossovers are not considered in the system workspace.

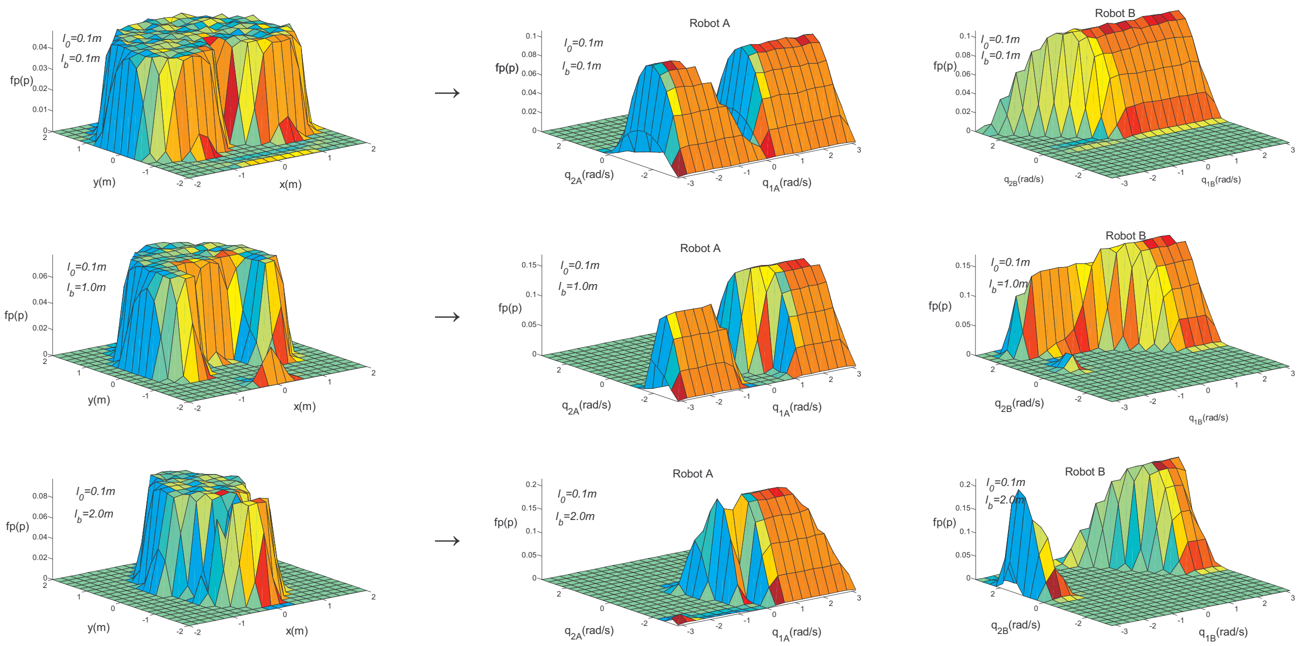
Figures 5 and 6 depict the *pdf* for the kinematic transformations **p←q** and **p→q,** respectively. The charts in Fig. 5 show that a uniform *pdf* of the *IV*s in the joint space leads to the 'preference' of the singular configuration. On the other hand, the charts of Fig. 6 reveal that an 'excitation' through a uniform *pdf* of the *IV*s in the operational space, leads to a ‘preference’ of the configurations $q_{2A} = -\pi/2$ and $q_{2B} = +\pi/2$.



<u>Figure 5:</u> Uniform 'excitation' of the **p←q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = l_b = 1$ m) handling an object with length $l_0 = 1$ m and orientation $\theta_0 = 0º$



<u>Figure 6:</u> Uniform 'excitation' of the **p→q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = l_b = 1$ m) handling an object with length $l_0 = 1$ m and orientation $\theta_0 = 0º$



<u>Figure 7:</u> Uniform 'excitation' of the **p→q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = 1$ m) handling an object with length $l_0 = 0.1$ m and orientation $\theta_0 = 0º$ for different shoulder distances $l_b = \{0.1, 1.0, 2.0\}$ m.

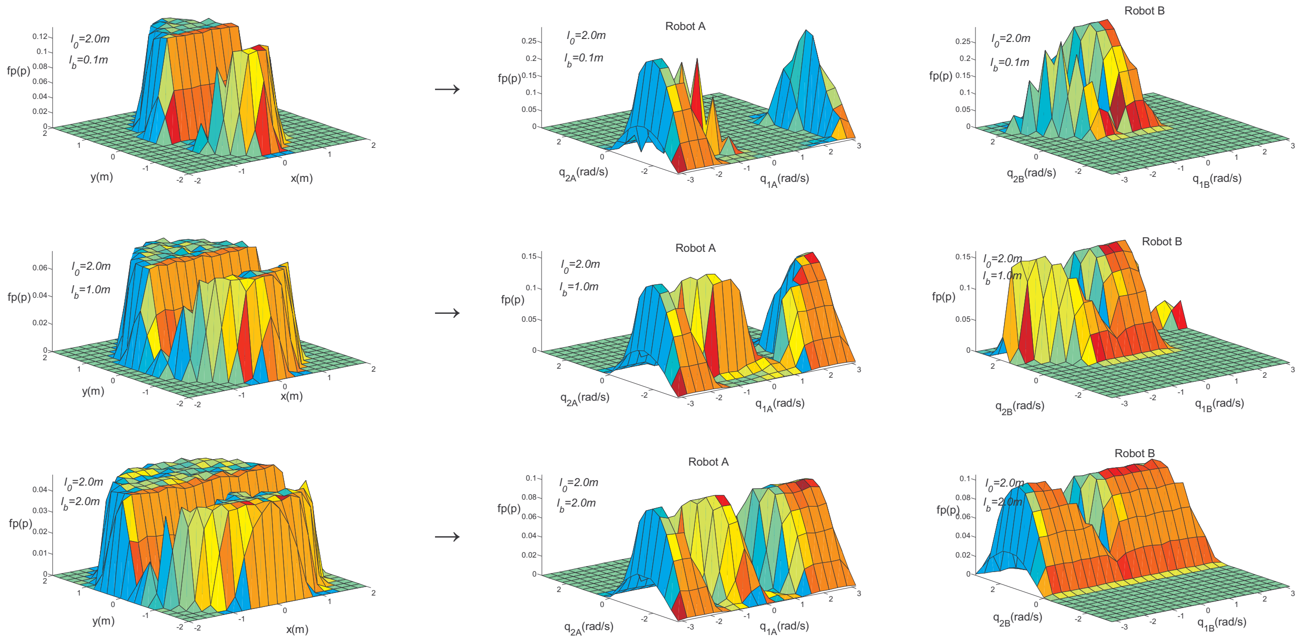


Figure 8: Uniform 'excitation' of the **p**→**q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = 1$ m) handling an object with length $l_0 = 2.0$ m and orientation $\theta_0 = 0^{\rm o}$ for different shoulder distances $l_{\rm b} = \{0.1, 1.0, 2.0\}$ m.

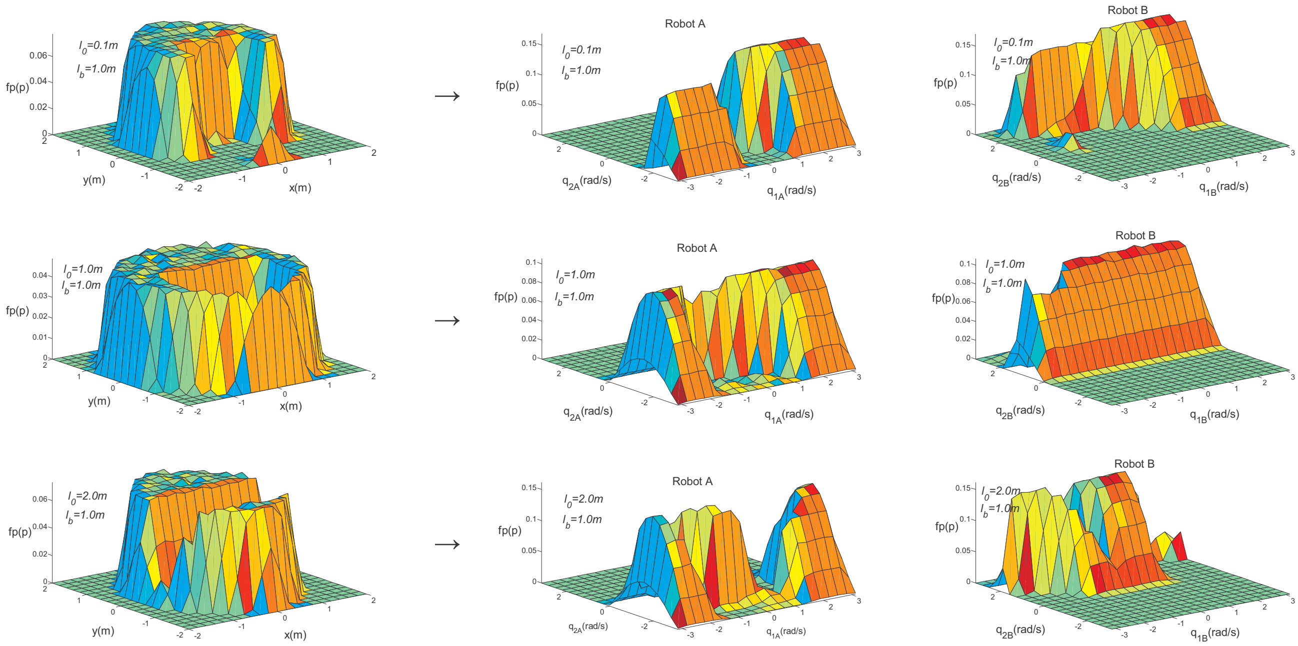


Figure 9: Uniform 'excitation' of the **p**→**q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = l_{\rm b} = 1$ m) handling an object with orientation $\theta_0 = 0^{\rm o}$ for different lengths $l_0 = \{0.1, 1.0, 2.0\}$ m.

Figures 7 and 8 show the uniform 'excitation' of the **p**→**q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = 1$ m) for different shoulder distances $l_{\rm b} = \{0.1, 1.0, 2.0\}$ m when handling an object with orientation $\theta_0 = 0^{\rm o}$ and two distinct lengths $l_0 = 0.1$ m and $l_0 = 2.0$, respectively. We verify that the maximum workspace occurs for $l_{\rm b} = l_0$. Furthermore, we observe a 'preference' of the configurations $q_{2A} = -\pi/2$ and $q_{2B} = +\pi/2$, similarly to the result of Fig. 6.

Alternatively, Fig. 9 depicts the uniform 'excitation' of the **p**→**q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = l_{\rm b} = 1$ m) handling an object with orientation $\theta_0 = 0^{\rm o}$ for different lengths $l_0 = \{0.1, 1.0, 2.0\}$ m. Once again, we get similar conclusions for the robot link and object lengths, and the arm configurations.

Figure 10 shows the results for several object orientations demonstrating that the best case results for $\theta_0 = 0^{\rm o}$.

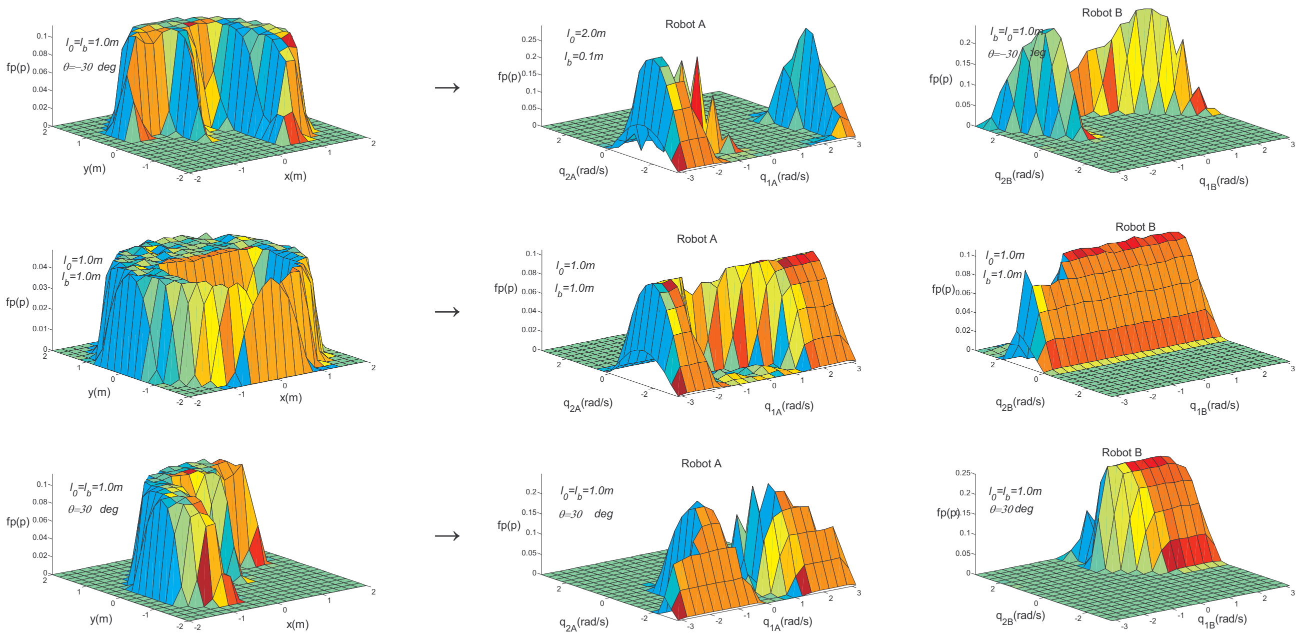


Figure 10: Uniform 'excitation' of the **p**→**q** kinematics for two 2*R* cooperating robots ($l_1 = l_2 = l_b = 1$ m) handling an object with length $l_0 = 1.0$ m for different orientations $\theta_0 = \{-30, 0, +30\}$ degree.

4. CONCLUSIONS

A new method to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modeling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results but can be very complex and hard to tackle. These difficulties motivate the development of models having distinct characteristics. The statistical formalism is a step in that direction which has been shown to give clear guidelines towards the robot structure. Moreover, the experiments demonstrated that the statistical modeling is well suited to a numerical evaluation. This characteristic is of utmost practical importance because it allows the direct treatment of data from sensor measurements.

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