BUDAPEST TECH
HUNGARIAN FUZZY ASSOCIATION

PROCEEDINGS

OF THE

6th INTERNATIONAL SYMPOSIUM OF HUNGARIAN RESEARCHERS

on

COMPUTATIONAL INTELLIGENCE

MAGYAR KUTATÓK 6. NEMZETKÖZI SZIMPÓZIUMA

2005
NOVEMBER 18-19
BUDAPEST

BUDAPESTI MŰSZAKI FŐISKOLA
MAGYAR FUZZY TÁRSASÁG

Simple Geometric Approach of Identification and Control Using Floating Basis Vectors for Representation

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Abstract: As a plausible alternative of certain sophisticated soft computing approaches trying to identify complete and static system models, a simple adaptive controller is outlined that creates only a temporal model. This model can be built up and maintained step-by-step on the basis of slowly fading information by the use of simple updating rules consisting of finite algebraic steps of lucid geometric interpretation. The method may be used for filling in the lookup tables or rule bases of the above representations experimentally. The method is tested by the use of a simple dynamic system as a typical paradigm via simulation.

1 Introduction

Though strictly stable controller designs already have been proposed on the basis of infinite order models [1, 2], too, the necessary mathematical deductions are very complicated and their complexity strongly increases with the increase in dimensionality. Apart from certain research efforts as [3, 4], the control of infinite-order physical systems are commonly based on finite order approximations in which the infinite modes are neglected for ease of design as e.g. in [5, 6]. These finite order models lead to handling discrete time-series only.

Another interesting class of physical systems are the fractional order ones described by expressions containing non integer $d^{\beta}u(t)/dt^{\beta}$ derivatives that can also be approximated and represented by finite time-series. Fractional order derivatives are defined in various different manners, e.g. [7, 8]. A practically useful definition was given by Caputo [9] in which the full 1st order derivative is causally reintegrated by the use of a kernel function as in (1). It has slowly forgetting nature while its singularity in τ=t enhances the relative weight of the contributions of the $\tau \cong < t$ instants.

$$\frac{d^{\beta}}{dt^{\beta}}u(t) := \frac{1}{\Gamma(1-\beta)}\int_0^t \left[\frac{du(\tau)}{d\tau}\right](t-\tau)^{-\beta}d\tau\,,\ \beta \in (0,1) \tag{1}$$

Its reasonable numerical approximation can be obtained by dividing the region of integration into small disjoint boxes of length δ over which $u'(t)$ is supposed to be approximately constant. In this case the singular integrand in the 1st box can be calculated analytically, too. Furthermore, by limiting the horizon of the retrospective integration it is obtained that

$$\frac{d^{\beta}u(t)}{dt^{\beta}} \cong \frac{u'(t)\delta^{-\beta+1}}{\Gamma(2-\beta)} + \sum_{s1=}^{T/\delta}\frac{\left[(s+1)^{-\beta+1} - s^{-\beta+1}\right]}{\delta^{\beta-1}\Gamma(2-\beta)}u'(t-s\delta)\,. \tag{2}$$

Expression (2) just corresponds to the expected time series representation.

Another important control class is the set of non-stationary stochastic processes in which some deterministic response to an external input and a stationary stochastic process are superimposed. A discrete time model can be formulated in the form of a difference equation with an external input $\{u_k\}$ that is usually considered to be known (AutoRegressive Moving Average model with eXternal input - ARMAX) [10]:

$$y_{k+1} = \sum_{s=0}^{N} a_s y_{k-s} + \sum_{w=0}^{M} b_w u_{k-w} \tag{3}$$

text In the so-called Takagi-Sugeno fuzzy models the consequent parts are expressed by analytical expressions similar to (3). The TS fuzzy controllers use some linear combinations of the (3)-type rules in which the coefficients depend on the antecedents. With the help of such Takagi-Sugeno fuzzy IF-THEN rules sufficient conditions to check the stability of fuzzy control systems are now available. These schemes are based on the stability theory of interval matrices and those of the Lyapunov approach [11].

It was already observed that the fuzzy controller stability conditions can be rewritten in form of Linear Matrix Inequalities (LMIs) [12, 13]. LMIs can be efficiently solved numerically by solving very complex equations for a positive definite solution [14].

Neuro-fuzzy systems provide the fuzzy systems with automatic tuning systems using a Neural Network (NN) as a tool. (The adaptive neuro-fuzzy inference systems are included in this classification.) The Adaptive Neuro-Fuzzy Inference System (ANFIS) is a cross between an artificial neural network and a Fuzzy Inference System (FIS) [11, 15, 16, 17]. The adaptive network can be a multi-layer feed-forward network in which each node (neuron) performs a particular function on incoming signals. Based on the ability of an ANFIS to learn from training data, it is possible to create an ANFIS structure from an extremely limited mathematical representation of the system. The ANFIS system generated by the fuzzy toolbox available in MATLAB allows the generation of a standard Sugeno style fuzzy inference system or a fuzzy inference system based on sub-clustering of the data [18].

Radial Basis Function Networks (RBFNs) provide an attractive alternative to the standard Feedforward Networks using backpropagation learning technique [19]. The linear weights associated with the output layer can be treated separately from the hidden layer neurons. As the hidden layer weights are adjusted through a nonlinear optimization, output layer weights are adjusted through linear optimization [11]. In fact the nodes of a RBFN represent "fuzzified" or "blurred" regions which correspond to the well defined antecedent sets of a fuzzy controller. The neuron's firing achieves its maximum at the centre of the region while its strength decreases with the distance from the center according to some Gaussian function (various distance measures can also be used). Evolutionary methods as e.g. the Particle Swarm Optimization Method that realizes stochastic random search in a multi-dimensional optimization space [20, 21] therefore may also be combined with them. Further interesting possibility is the application of the Self-Organizing Fuzzy Logic Controller (SOLFC) [22]. In the case of certain problem classes similarity relations can also be observed and utilized to simplify the design process [23].

A significant common feature of the above approaches is that they try to develop a "complete" soft computing based model of the system to be controlled. This naturally makes the question arise whether it is always reasonable to try to identify a "complete" model. As a plausible alternative simple adaptive controllers can be imagined that do not wish to create a complete model. Instead of that on the basis of slowly fading recent information a more or less temporal model can be constructed and updated step by step by the use of simple updating rules consisting of finite algebraic steps of lucid geometric interpretation. In the sequel this simple approach is detailed and illustrated via simulation results.

2 Simple Geometric Approach for Dynamic Systems

Consider a simple nonlinear causal Single Input – Single Output (SISO) system described by the equation:

$$y^{(n)}(t) = F\left(y^{(n-1)}(t), y^{(n-2)}(t), \ldots, y^{(0)}(t), f(t)\right) \tag{4}$$

in which $f(t)$ represents the external driving forces to be utilized for controlling purposes. Let us suppose that the time-derivatives can be approached by certain finite element approach using time-resolution δt. To numerically estimate the n^{th} order time-derivatives at least $(n+1)$ discrete values has to be taken into account via considering their linear combination as

$$y^{(n)}(t) \cong \sum_{s=0}^{n} c_s(\delta t) y(t - s\delta t) \tag{5}$$

in which the c_n coefficients depend on δt and can be chosen in various manners. We also note that the number of the coefficients may be somewhat greater that $(n+1)$, e.g. in the case of computing the central first derivatives we may use 3 points, too. Via rearranging (4) and using (5) the following ambiguous representation can be obtained:

$$y(t) \cong \Phi(y(t-\delta t), y(t-2\delta t), \ldots, y(t-n\delta t), f(t-\delta t)) \tag{6}$$

in which the actually used values are concentrated in the vicinity of the values of time t. Supposing that the array of the values $\mathbf{Y}_f$:=$[y(t-\delta t),\ldots, y(t-n\delta t), f(t-\delta t)]^T \neq 0$ (6) can be replaced by a scalar product in ambiguous manner by an array $\mathbf{G}$ as

$$y(t) = \mathbf{G}^T(t)\mathbf{Y}_f(t) \tag{7}$$

in which both the angle between $\mathbf{G}$ and $\mathbf{Y}_f$ and the absolute value of $\mathbf{G}$ are not well defined. If the n^{th} derivative of $y(t)$ is directly measurable then similar ambiguous approximation can be constructed for $y^{(n)}(t)$ as

$$y^{(n)}(t) = \mathbf{g}^T(t)\mathbf{Y}_f(t). \tag{8}$$

Let us suppose that on the basis of some rough initial or preliminary model we can compute the appropriate control action $f(t)$ and can store the $y(t)$ values, too. It is evident that in the case of a time-invariant linear system $\mathbf{g}$ does not depend on t, therefore collecting sufficient information coded in the form of (8) leads to the system of linear equations that belong to the constant array $\mathbf{g}$ as

$$\begin{bmatrix} y^{(n)}(t-\delta t) \\ \dots \\ y^{(n)}(t-M\delta t) \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_f^T(t-\delta t) \\ \dots \\ \mathbf{Y}_f^T(t-M\delta t) \end{bmatrix} \mathbf{g}\,. \tag{9}$$

Equation (9) has very simple and lucid geometric interpretation: the constant vector **g** is represented by time-varying or "floating" system of basis vectors $\mathbf{Y}_f(t-n\delta t)$ (n=1,…,M). If this set is linearly independent **g** can be reproduced as the linear combination of these vectors as

$$\mathbf{g} = \sum_{s=1}^{M} \mu_s(t) \mathbf{Y}_f^T(t-s\delta t)\,. \tag{10}$$

In (10) it is naturally supposed that to a constant **g** for a floating system of basis vectors a floating or time-varying system of the $\mu_s(t)$ coefficients belongs in a special manner: together they have to provide a constant vector. No let us suppose that we have two vectors **a** and **b** having known dot product with **g**. Let us find the component of **b** in the orthogonal subspace of **a** in the form of $\mathbf{b}_\perp=\mathbf{b}+\lambda\mathbf{a}$:

$$0 = \mathbf{a}^T\mathbf{b}_\perp = \mathbf{a}^T\mathbf{b} + \lambda\mathbf{a}^T\mathbf{a} \Rightarrow \lambda = \frac{-\mathbf{a}^T\mathbf{b}}{\mathbf{a}^T\mathbf{a}}\,. \tag{11}$$

Due to the linear property of the dot or scalar product the dot product of $\mathbf{b}_\perp$ with **g** can also be computed as

$$\mathbf{g}^T\mathbf{b}_\perp = \mathbf{g}^T\mathbf{b} + \lambda\mathbf{g}^T\mathbf{a}\,. \tag{12}$$

Now let us apply the following algorithm that is similar to the Gram-Schmidt orthogonalization with the exception of normalizing the vectors: remove the components in the direction of $\mathbf{Y}_f(t-\delta t)$ from $\mathbf{Y}_f(t-2\delta t)$,…, $\mathbf{Y}_f(t-M\delta t)$ with the method given in (11). Then the new set indexed with 2,3,…M-1 will be in the orthogonal subspace of $\mathbf{Y}_f(t-\delta t)$. Then take the 2nd vector of the remaining set and subtract the components of the remaining ones in its direction, etc. while tracing the variation of the dot products according to (12). (To avoid numerical difficulties the components in the direction of very small vectors need no to be subtracted.) Furthermore, since in the case of linear systems it is just enough to obtain sufficient information on the *independent directions* only, the approximately same direction of vectors **a** and **b** can be stated if

$$|\cos\varphi(\mathbf{a},\mathbf{b})| \cong \frac{|\mathbf{a}^T\mathbf{b}|}{|\mathbf{a}|\times|\mathbf{b}| + \varepsilon_1} \geq 1 - \varepsilon_2\,. \tag{13}$$

in which ε_1 and ε_2 are small positive numbers. Otherwise these vectors have *essentially different directions*.

Now let use suppose that we continue the systematic observation and obtain further information on **g** in the form of (8) as

$$y^{(n)}(t+\delta t) = \mathbf{g}^T(t)\mathbf{Y}_f(t+\delta t)\,. \tag{14}$$

Together with the information coded in (9) (14) is *redundant but free of contradiction* if **g** is exactly constant. In this case either (14) or one of the vectors in (9) can be dropped, replaced with the 1st vector in the set in (9), and the orthogonalization algorithm can be repeated. As a result the same constant **g** must be obtained by the use of this new set of basis vectors.

Now let us suppose that our system is *linear* but *not time-invariant*! In this case (9) and (14) are rather controversial than redundant because these vectors do not belong exactly to the same **g** since they were obtained from measurements taken in different time instances. A plausible and lucid method of contradiction resolution may be finding the vector in the closest direction of the last one in the sense of (13) since the remaining vectors convey less relevant information on the system's behavior in this direction. This vector can be omitted in the system in (9) and it can be replaced by the new information conveyed by (14). Then by executing the orthogonalization algorithm on the remaining set the *obsolete information regarding the new direction* can be removed and replaced by the fresh information. [Since the addition in (10) is commutative, in practice the first column of the original set can be put in the place of the dropped vector, and the new one can be placed into the 1st place.]

Finally let us suppose that our system is neither time-invariant nor linear! In this case not only the direction but the absolute values of the vectors also influence the behavior of the system. In this case the old vector closest to the new one in the sense of a norm can be dropped and replaced by the new one because the information mainly conveyed by it is refreshed.

In the possession of some prescribed control strategy formulating the desired trajectory tracking with asymptotic convergence continuous tracking error is expected and the array **g** in (10) can be used for calculating the necessary control action instead of the rough initial model as

$$f(t) = \left\{ y^{(n)Desired}(t) - \begin{bmatrix} g_1 & \dots & g_{M-1} \end{bmatrix} \begin{bmatrix} Y_1 \\ \dots \\ Y_{M-1} \end{bmatrix} \right\} \frac{1}{g_M} \tag{15}$$

To evade numerical problems instead of $1/x$ the approximation

$$\frac{1}{x} \cong \frac{sign(x)}{|x| + \varepsilon_3} \tag{16}$$

can be used with a very small positive ε_3.

To illustrate the above idea consider the forced vibration of a stable, linear, time-invariant, oscillation-free system $y''(t) = -Dy'(t) - P(t) + f(t)$ identified by 3 points, i.e. in which g_1, g_2, g_3 correspond to the y coordinates and g_4 belongs to f of unit amplitude sinusoidal excitation (Fig. 1).

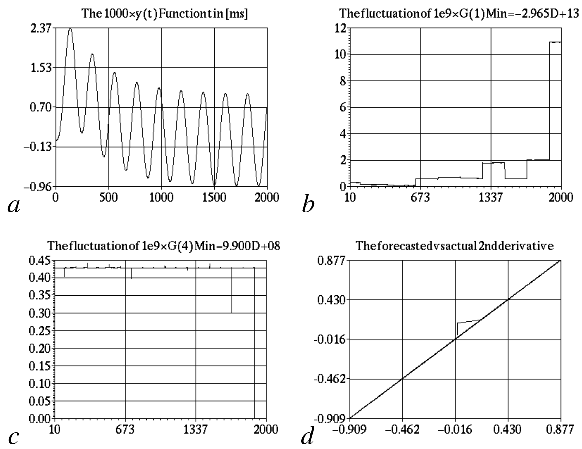

Fig. 1. Typical results for identifying a 2nd order system using three y points for identification: *a*: the forced oscillation, *b*: variation of g_1, *c*: variation of g_4, *d*: the correlation between the actual acceleration and its estimation.

Fig. 1 reveals that the variation of $\mathbf{g}$ is practically negligible. For instance $g_1 \cong -2.665\times10^{13}$ (this value corresponds to the 0 line of graph *b* while the maximum of its variation is about 12×10^{-9}. Graph *c* can be interpreted in similar manner, and similar negligible variations were found for g_2, and g_3. The identified model based acceleration estimation is very accurate according to the expectations.

In the sequel this idea is used for the special control of a cart-pendulum system in which the pendulum's axle is not directly driven: its state can be manipulated due to the nonlinear coupling between the linear and the rotational degrees of freedom by the drive linearly moving the cart.

3 The Model of the Cart and Pendulum System

On the basis of the Euler-Lagrange equations the equation of motion of the pendulum's angle φ is given as the function of the mass of the cart and the pendulum M=1.096 *kg* and m=0.109 *kg*, respectively, the length and the rotational angle of the pendulum with respect to the upper vertical direction (clockwisely) L=0.25 *m* and φ [*rad*]. Variable x [*m*] denotes the horizontal translation of the cart+pendulum system in the right direction, b=0.1 *N/(m/s)* and f=0.00218 $kg\times m^2/s$ are the viscous friction coefficients, and I=0.0034 $kg\times m^2$ denotes the momentum of the arm of the pendulum, and $Q_1 \equiv f(t)$ denotes the linear force moving the cart in the horizontal direction:

$$\ddot{\varphi} = \frac{mL\cos\varphi\left[Q_1 - b\dot{x} - mL\dot{\varphi}^2\sin\varphi\right]}{(mL\cos\varphi)^2 - \left(I + mL^2\right)(M+m)} + \frac{(M+m)(f\dot{\varphi} - mgL\sin\varphi)}{(mL\cos\varphi)^2 - \left(I + mL^2\right)(M+m)}. \tag{17}$$

It is evident that about $\varphi = 0$ $d^2\varphi/dt^2$ is decreasing function of Q_1 which is a kind of specialty of this system. In the forthcoming simulations the x variable was treated as an internal degree of freedom without any interest, and the primitive model $Q_1 = -0.05 \times \ddot{\varphi} + 15$ was used in the beginning of the control.

4 Simulation Results

Fig. 2 describes the trajectory- and the phase trajectory tracking of the rough model based and the 3 Y points based adaptive control. It is evident that the adaptive approach outlined works well in the control of this non-linear and time-variant system (from the point of view of the φ coordinate time-invariance disappears due to the variation of dx/dt that is the velocity of the coupled internal degree of freedom).

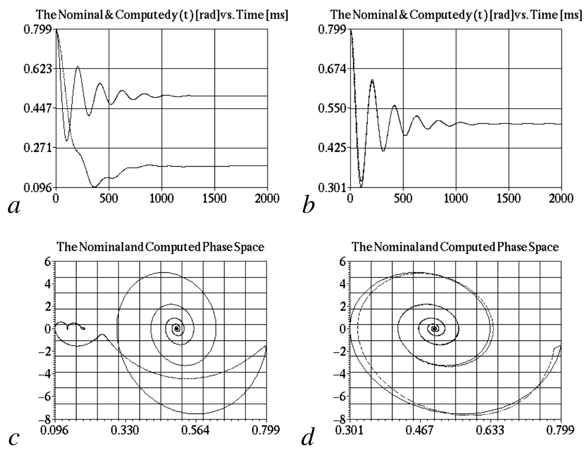

Fig. 2. The trajectory tracking of the rough model based non-adaptive and the 3 Y points based adaptive control (*a* and *b*), and the appropriate phase trajectories (*c* and *d*).

To reveal some details of the operation of the adaptive control Fig. 3 describes the trajectory tracking error, the exerted force, the prediction correlation, and the norm of the basis vectors of the identification in the case of the 3 Y points based adaptive control. It is evident that the prediction with δt=1 *ms* is not so extremely precise as in the case of a time-invariant linear system, however, it seems to lead to acceptable precision and also evades hectic variation of the control signal, i.e. the exerted force. It also reveals that normally the system has a "dominant" basis vector that –due to the operation of the algorithm applied– normally stands in the 1st place. The remaining basis vectors that seem to be responsible for minor corrections in the prediction have small components. As it was expected their little norm does not cause numerical problems in the calculations.

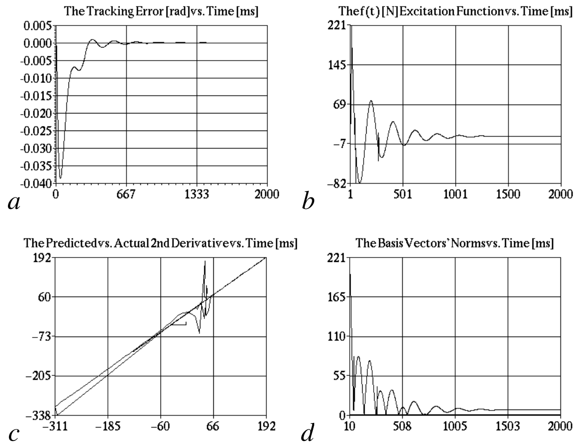

Fig. 3. The trajectory tracking error (*a*), the exerted force (*b*), the prediction correlation (*c*), and the norm of the basis vectors of the identification (*d*) in the case of the 3 Y points based adaptive control

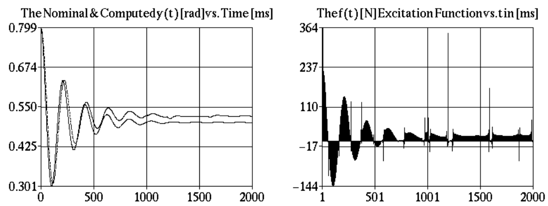

Fig. 4. The trajectory tracking and the excitation function generated by the "alternative" controller

To reveal the significance of maintaining the partly obsolete information an "alternative" adaptive controller was developed in which $[y_k,(y_k-y_{k-1})/\delta t,f(t)]$ were associated with g_1, g_2, g_3, and the "obsolete" information was not taken into account at all, i.e. the vector **g** that was identified in cycle *i* was used for prediction in cycle *i*+1. The lower quality of trajectory tracking as well as the bang-bang type control signal covered by a smooth hull in Fig. 4 reveals the superiority and necessity of gradually letting the “obsolete” information fade.

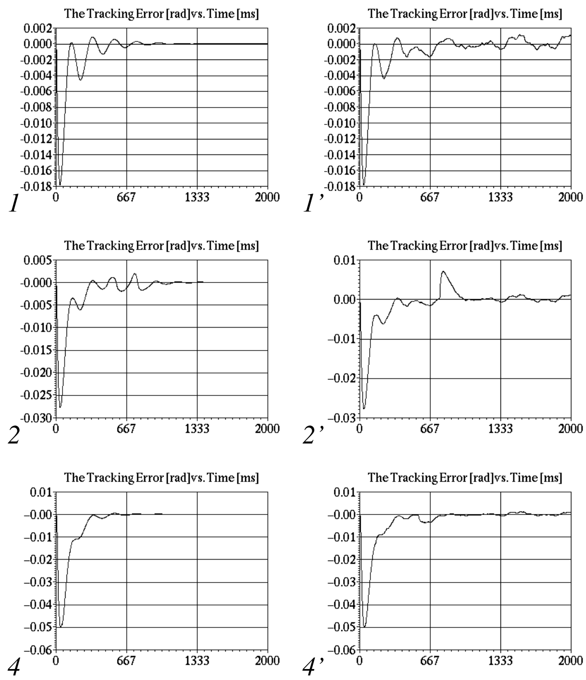

Fig. 5. The trajectory tracking error in the case of the adaptive controller designed for the maximal index of **g** (1, 2, and 4) without noise in the exerted force (the numbers without ‘), and with even distribution noise (the numbers with ‘).

The ambiguity and noise sensitivity of the proposed method is also an interesting question. For this purpose the control calculating with 1, 2, and 4 y_k points were investigated with noiseless and noisy exerted force *f*. For the noise even distribution was chosen in the [-5,+5] *N* interval. The results are displayed in Fig. 5. It can be seen that the bigger the dimensions of the $\mathbf{Y}_f$ vectors are the less precise control can be achieved. Fig. 6 displays the prediction correlation for the appropriate cases of Fig. 5. It does not seem to reveal significant differences.

Conclusions 10

In this paper, as a plausible alternative of certain sophisticated soft computing approaches trying to identify “complete” system models, a simple adaptive controller dealing with continuously updated temporal model was investigated via simulation. This model utilizes the slowly fading information via applying finite algebraic steps of lucid geometric interpretation based on the Gram-Schmidt orthogonalization algorithm. The simulation investigations indicated that this approach can be useful. Its great advantage is simplicity, limited number of algebraic operations and lucid interpretation. In contrast to the mathematically far more intricate solutions based on the Lyapunov technique normally guaranteeing Lyapunov stability without making it possible to prescribe dynamic details of trajectory tracking this simple approach makes it possible to prescribe arbitrary error relaxation by the use of simple kinematic terms. Neither complicated evolutionary computation or LMIs based optimization seem to be necessary for its use. The method may be used for filling in the lookup tables or rule bases of the other representations experimentally, too. Further investigations concerning the operation of this approach in the cases of fractional order linear or nonlinear systems seem to be expedient in the future.

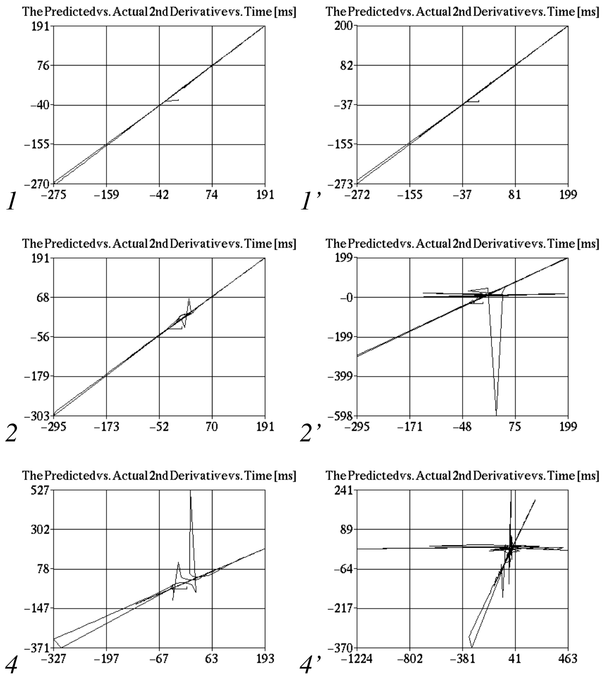

Fig. 6. The prediction correlation in the case of the adaptive controller designed for the maximal index of **g** (1, 2, and 4) without noise in the exerted force (the numbers without '), and with even distribution noise (the numbers with ').

Acknowledgment

The authors gratefully acknowledge the support received within the Hungarian-Portuguese S&T Project No. Port-6/03 financed by the Hungarian Research and

Technology Innovation Fund as well as by the Portuguese Instituto de Cooperacao Cientifica e Tecnológica.

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