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FRACTIONAL CONTROL OF TWO ARMS WORKING IN COOPERATION

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Abstract: This paper analyzes the performance of two cooperative robot manipulators. It is studied the implementation of fractional-order algorithms in the position/force control of two robots holding an object. The experiments reveal that fractional algorithms lead to performances superior to classical integer-order controllers.

Keywords: Cooperative robots, fractional calculus, control

1. INTRODUCTION

Two robots carrying a common object are a logical alternative for the case in which a single robot is not able to handle the load. The choice of a robotic mechanism depends on the task or the type of work to be performed and, consequently, is determined by the position of the robots and by their dimensions and structure. In general, the selection is done through experience and intuition; nevertheless, it is important to measure the manipulation capability of the robotic system (Tsai and Soni, 1981), that can be useful in the robot operation. In this perspective it was proposed the concept of kinematic manipulability (Yoshikawa, 1985) and its generalization by including the dynamics (Asada, 1983) or, alternatively, the statistical evaluation of manipulation (Machado and Galhano, 1997). Other related aspects such as the coordination of two robots handling objects, collision avoidance and free path planning have been also investigated (Nakamura *et al.*, 1989).

With two cooperative robots the resulting interaction forces have to be accommodated and consequently, in addition to position feedback, force control is also required to accomplish adequate performances (Bejczy and Tarn, 2000; Raibert and Craig, 1981). There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first method (Ferreira *et al.*, 2004) separates the task into two orthogonal sub-spaces corresponding to the force and the position controlled variables. Once established the subspace decomposition two

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independent controllers are designed. The second method (Hogan, 1985) requires the definition of the arm mechanical impedance. The impedance accommodates the interaction forces that can be controlled to obtain an adequate response. This paper addresses the control of two arm systems, through the dynamical analysis and a statistical evaluation (Galhano and Machado, 2001) of the joint torques, using fractional-order (FO) control algorithms (Ferreira and Machado, 2003; Oustaloup, 1995; Machado, 1997; Podlubny, 1999).

Bearing these facts in mind this article is organized as follows. Section two presents the controller architecture for the position/force control of two robotic arms. Based on these concepts, section three develops several experiments for the statistical analysis and the performance evaluation of FO and the PID controllers, for robots having several types of dynamic phenomena at the joints. Finally, section four outlines the main conclusions.

2. CONTROL OF TWO ARMS

The dynamics of a robot with n links interacting with the environment is modeled as:

$$\tau = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) - \mathbf{J^T}(\mathbf{q})\mathbf{F} + \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} \qquad (1)$$

where τ is the $n \times 1$ vector of actuator torques, $\mathbf{q}$ is the $n \times 1$ vector of joint coordinates, $\mathbf{H(q)}$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the $n \times 1$ vector of centrifugal / Coriolis terms and $\mathbf{G(q)}$ is the $n \times 1$ vector of gravitational effects. The matrix $\mathbf{J}^T\mathbf{(q)}$ is the transpose of the Jacobian and $\mathbf{F}$ is the force that the load exerts in the robot gripper. For a RR manipulator ($n = 2$) the dynamics yields:

$$\mathbf{C}\left(\mathbf{q},\dot{\mathbf{q}}\right) = \begin{bmatrix} -m_2r_1r_2S_2\dot{q}_2^2 - 2m_2r_1r_2S_2\dot{q}_1\dot{q}_2 \\ m_2r_1r_2S_2\dot{q}_1^2 \end{bmatrix} \qquad (2)$$

$$\mathbf{G}\left(\mathbf{q}\right) = \begin{bmatrix} g\left(m_1r_1C_1 + m_2r_1C_1 + m_2r_2C_{12}\right) \\ gm_2r_2C_{12} \end{bmatrix} \qquad (3)$$

$$\mathbf{J^T}\left(\mathbf{q}\right) = \begin{bmatrix} -r_1S_1 - r_2S_{12} & r_1C_{11} + r_2C_{12} \\ -r_2S_{12} & r_2C_{12} \end{bmatrix} \qquad (4)$$

$$\mathbf{H(q)} = \begin{bmatrix} (m_1+m_2)r_1^2 - m_2r_2^2 & m_2r_2^2 + m_2r_1r_2C_2 \\ +2m_2r_1r_2C_2 + J_{1m} + J_{1g} & \\ -m_2r_2^2 + m_2r_1r_2C_2 & m_2r_2^2 + J_{2m} + J_{2g} \end{bmatrix} \qquad (5)$$

where $C_{ij} = \cos(q_i + q_j)$ and $S_{ij} = \sin(q_i + q_j)$.

We consider two robots with identical dimensions (Fig. 1). The contact of the robot gripper with the load is modeled through a linear system with a mass M, a damping B and a stiffness K. The numerical values adopted for the RR robots and the object are $m_1 = m_2 = 1.0$ kg, $l_1 = l_2 = l_b = l_0 = 1.0$ m, $\alpha_0 = 0$ deg, $B_1 = B_2 = 1$ Ns.m^{-1} and $K_1 = K_2 = 10^4$ Nm^{-1}.

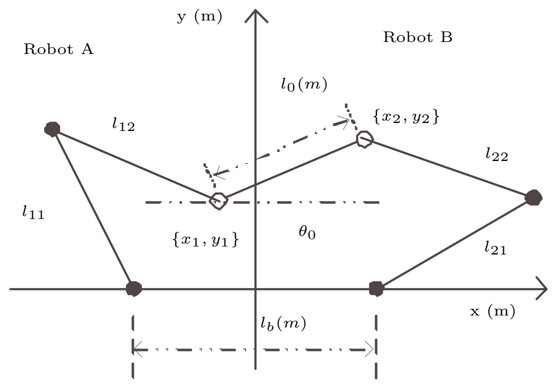


Fig. 1. Two RR robots working in cooperation for the manipulation of an object with length l_0, orientation θ_0 and distance l_b between the shoulders.

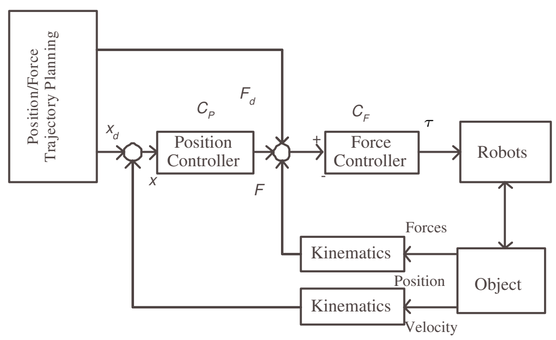


Fig. 2. The position/force cascade controller.

The controller architecture (Fig. 2), namely the cascade controller, is inspired on the impedance and compliance schemes. Therefore, we establish a cascade of force and position algorithms as internal an external feedback loops, respectively, where $\boldsymbol{x}_d$ and $\boldsymbol{F}_d$ are the payload desired position coordinates and contact forces.

In the position and force control loops we consider FO controllers: of the type $C(s) = K_p + K_\alpha\ s^\alpha$, $-1 < \alpha < 1$, that are approximated by $4^{th}-$order discrete$-$time Pade expressions ($a_{i,}$, $b_{i,} \in \mathbb{R}$, $k = 4$):

$$C\left(z\right) \approx K\frac{a_0z^k + a_1z^{k-1} + .. + a_k}{b_0z^k + b_1z^{k-1} + ... + b_k} \qquad (6)$$

To analyze the system performance we consider robots with ideal transmissions, and robots with joint backlash and flexibility. Moreover, we compare the response of FO and classical $PD-PI$

Table 1. The parameters of the position and force FO controllers

(a) Position controller

i	K_p	K_α	α
1	0.1259	$1.55\,10^{-3}$	$\frac{1}{2}$
2	0.1259	$1.55\,10^{-3}$	$\frac{1}{2}$

(b) Force controller

i	K_p	K_α	α
1	10.59	$2\,10^{-3}$	$-\frac{1}{5}$
2	10.59	$2\,10^{-3}$	$-\frac{1}{5}$

Table 2. The parameters of the position and force $PD-PI$ controllers

(a) Position controller

i	K_p	K_d
1	$25\,10^3$	$25\,10^1$
2	$25\,10^3$	$25\,10^1$

(b) Force controller

i	K_p	K_i
1	$5\,10^2$	$10\,10^2$
2	$5\,10^2$	$10\,10^2$

algorithms, in the position and force loops, respectively.

$$C(s) = K_p + K_d s \tag{7}$$

$$C(s) = K_p + K_i \frac{1}{s} \tag{8}$$

Both algorithms were tuned by trial and error, having in mind getting a similar performance in the two cases (Tables 1 and 2). In order to study the system dynamics we apply a small amplitude rectangular pulse δy_d at the position reference and we analyze the system response.

The experiments adopt a controller sampling frequency $f_c = 10$ kHz, contact forces of the grippers $\{Fx_j,Fy_j\}\equiv\{0.5,5\}$ Nm, a operating point of the center of the object A$\equiv\{x,y\}\equiv\{0,1\}$ and $\theta = 0^o$.

In a first phase we consider robots with ideal transmissions at the joints. Figure 3 depicts the time response of robot A under the action of the FO and $PD-PI$ algorithms.

In a second phase (figure 4) we analyze the response of robots with dynamic backlash at the joints. For the i^{th} joint gear (i= 1, 2), with clearance h_i, the backlash reveals impact phenomena between the inertias, which obey the principle of conservation of momentum and the Newton law:

$$\dot{q}_i' = \frac{\dot{q}_i\,(J_{ii} - \varepsilon J_{im}) + \dot{q}_{im} J_{im}\,(1+\varepsilon)}{J_{ii} + J_{im}} \tag{9}$$

$$\dot{q}_{im}' = \frac{\dot{q}_i J_i\,(1+\varepsilon) + \dot{q}_{im}\,(J_{im} - \varepsilon J_{ii})}{J_{ii} + J_{im}} \tag{10}$$

where $0 \le \varepsilon \le 1$ is a constant that defines the type of impact ($\varepsilon = 0$ inelastic impact, $\varepsilon = 1$ elastic impact) and $\dot{q}_i$ and $\dot{q}_{im}$ ($\dot{q}_i'$ and $\dot{q}_{im}'$) are the velocities of the i^{th} joint and motor before (after) the collision, respectively. The parameter J_{ii} (J_{im}) stands for the link (motor) inertias of joint i. The numerical values adopted are $h_i = 1.8\ 10^{-4}$ rad and $\varepsilon_i = 0.8$.

In a third phase (figure 5) we study the RR robot with compliant joints. For this case the dynamic model corresponds to model (1) augmented by the equations:

$$T = J_m \ddot{q}_m + B_m \dot{q}_m + K_m(q_m - q) \tag{11}$$

$$K_m(q_m - q) = J(q)\ddot{q} + C(q,\dot{q}) + G(q) \tag{12}$$

where J_m, B_m and K_m are the $n \times n$ diagonal matrices of the motor and transmission inertias, damping and stiffness, respectively. In the simulations we adopt $K_{mi} = 2\ 10^6$ Nm rad^{-1} and $B_{mi} = 10^4$ Nms rad^{-1} (i = 1,2).

The low-pass characteristics of $|y(j\omega)/yd(j\omega)|$ reveal the existence of some coupling between the position and force loops due to the non-ideal performance of both algorithms. Figure 6 show the frequency responses for robots with ideal joints, having backlash and transmissions flexibility, both under the action of the FO and the $PD-PI$ controllers, for a pulse perturbation, at the robot reference δyd. The charts reveal that the FO algorithms have a superior performance, namely a good robustness and larger bandwidth.

3. STATISTICAL EVALUATION

Usually system descriptions are based on a set of differential equations which, in general, require laborious computations and may be difficult to analyze. These facts motivate the need of alternative models based on different mathematical concepts. The proposed statistical method give clear guidelines towards the robotic system evaluation.

A statistical sample for the variables is obtained by driving the cooperating robots though a large numbers of trajectories, having appropriate time/space evolutions. All variables are calculated, sampled in the time domain, and the resulting numerical values are organized in histograms.

In order to illustrate the method, we specify different desired motions and planed $N = 10000$ distinct trajectories with different types of accelerations. The performance of the controller, using fractional order and classical integer order control algorithms, is characterized by the torque variations of the two robots. We can observe that the $PD-PI$ controller requires higher actuators torques in the cases of backlash and flexible joints.

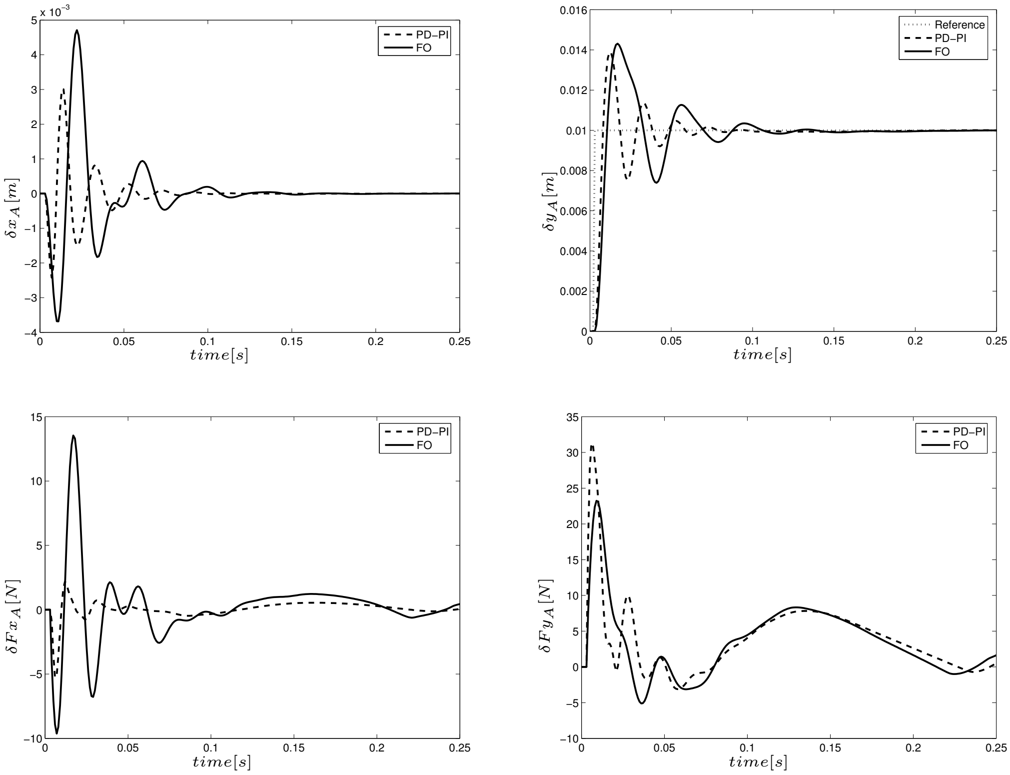


Fig. 3. Time response for robots with ideal joints under the action of the FO and the $PD{-}PI$ algorithms for a reference position perturbation $\delta y_d = 0.1$ m and a payload with $M = 1$ kg, $B_i = 10$ Ns/m and $K_i = 10^3$ N/m.

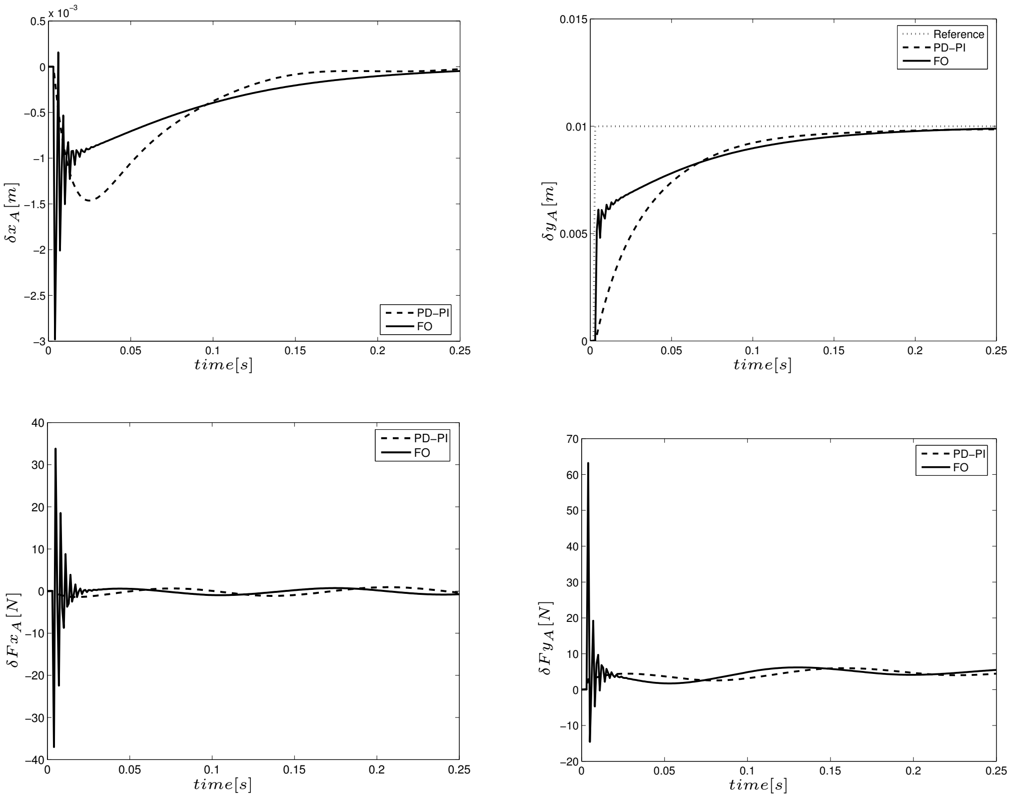


Fig. 4. Time response for robots with joints having backlash under the action of the FO and the $PD{-}PI$ algorithms, for a pulse perturbation at the robot A position reference $\delta y_d = 10^{-3}$ m and a payload $M = 1$ kg, $B_i = 1$ Ns/m and $K_i = 10^3$ N/m.

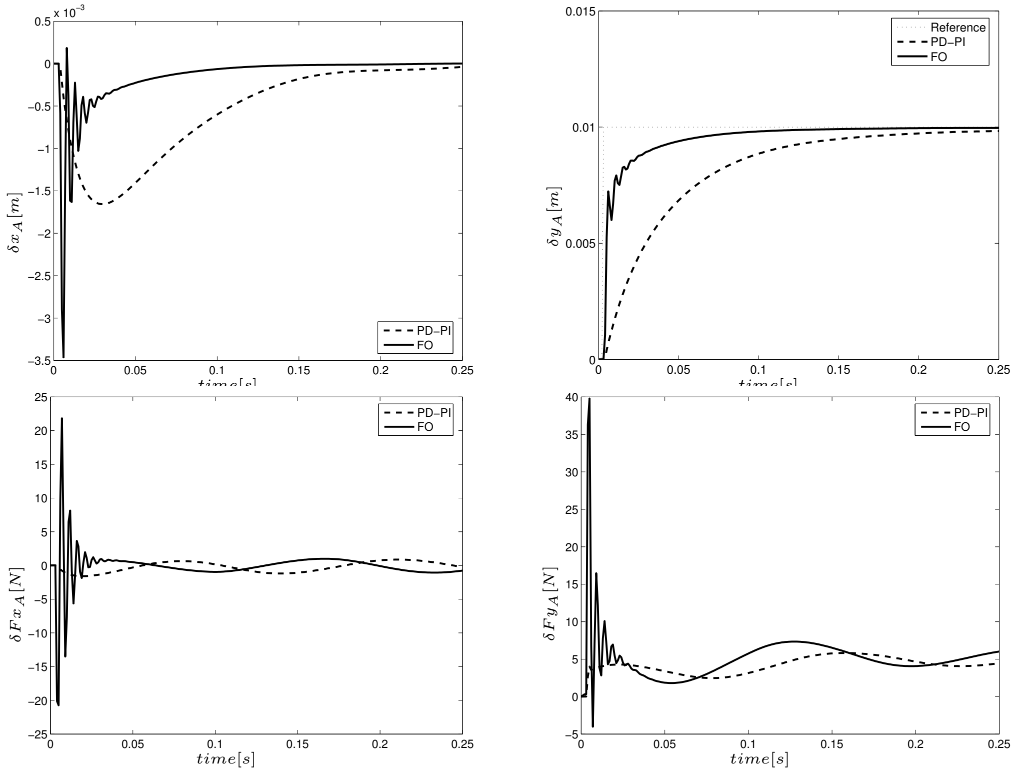
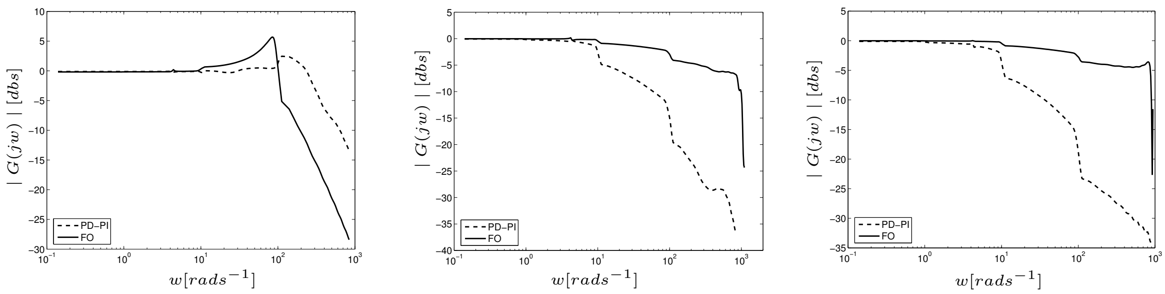


Fig. 5. Time response for robots with joints having flexibility under the action of the FO and the $PD-PI$ algorithms, for a pulse perturbation at the robot A position reference $\delta y_d = 10^{-3}$ m and a payload $M = 1$ kg, $B_i = 1$ Ns/m and $K_i = 10^3$ N/m.



(a) Ideal joints. (b) Joints having backlash. (c) Joints having flexibility.

Fig. 6. The Bode diagram of the closed-loop transfer function $G(jw) = \frac{F\{\delta y(t)\}}{F\{\delta yd(t)\}}$ for two cooperating RR robots A with: a) ideal joints, b) joints having backlash and c) joints having flexibility.

The figure 7 shows the relative frequency of the dynamics and required actuators torques for the performance of the classical and fractional controllers.

4. CONCLUSIONS

This paper studied the position/force control of two robots working in cooperation using fractional order and classical integer order control algorithms. The system dynamics was analyzed for manipulators having several types of dynamical phenomena at the joints. The results demonstrated that the fractional-order algorithm reveals a good performance and a high robustness. The transient response of the system, shows the quality of the performance of the fractional order controllers.

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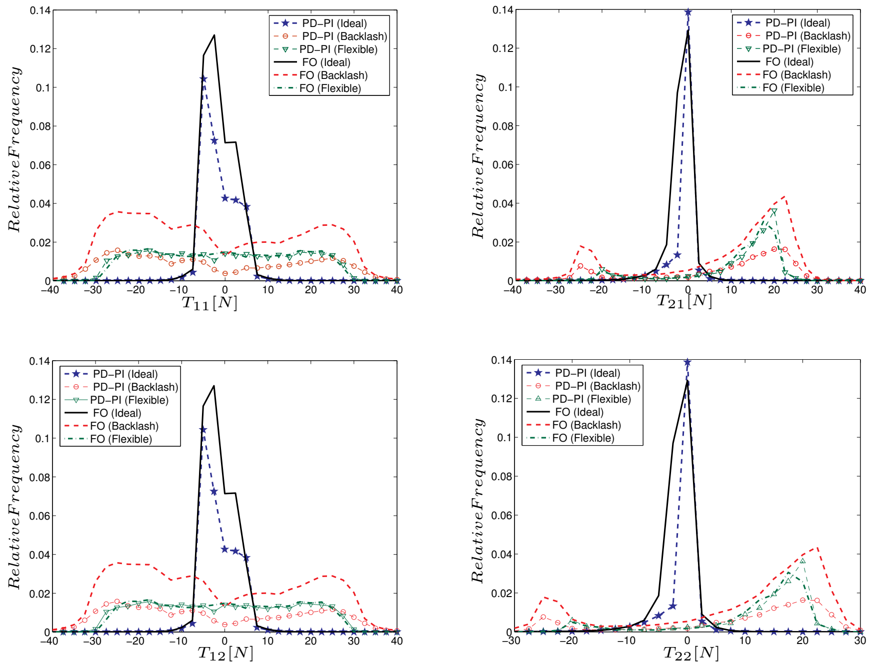

Fig. 7. Comparison of the torque distribution for two cooperating RR robots with ideal joints, joints with backlash, joints with flexibility, for a payload with $M = 1$ kg, $B_i = 1$ Ns/m and $K_i = 10^3$ N/m, under the action of the $PD - PI$ and FO algorithms, for a constant sinusoidal acceleration.

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