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Testing constant absolute and relative ambiguity aversion [☆]

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Abstract

Recent applications have demonstrated the crucial role of decreasing absolute ambiguity aversion in financial and saving decisions. Yet, most ambiguity models predict that ambiguity aversion remains constant when individuals become better off overall. We propose the first tests of constant absolute and relative ambiguity aversion, using simple variations of the Ellsberg paradoxes. Our tests are axiomatically founded and grounded in the theoretical literature. We implemented these tests in an experiment. Our results call for the use of ambiguity models that can accommodate decreasing aversion toward ambiguity.

JEL classification: C91; D81

Keywords: Ambiguity aversion; Ellsberg; CARA; CRRA; Ambiguity models

Does ambiguity attitude change when individuals become better off overall? Addressing risk attitude, seminal papers in finance from the 1960s and 1970s explained portfolio allocations by hypothesizing that absolute risk aversion is decreasing but relative risk aversion is increasing (see in particular Arrow, 1971). Recently, several papers developing applications of

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Klibanoff et al.'s (2005) smooth ambiguity model demonstrated the crucial role of decreasing absolute ambiguity aversion (DAAA) on saving behavior (Berger, 2014; Osaki and Schlesinger, 2013; Gierlinger and Gollier, 2014) and prevention behavior (Berger, 2016) and in the survival of ambiguity-averse agents in a market with expected utility agents (Guerdjikova and Sciubba, 2015). At odds with these applications, most ambiguity models assume constant absolute ambiguity aversion (CAAA), and some predict constant relative ambiguity aversion (CRAA). In particular, CAAA is implied by Gilboa and Schmeidler's (1989) maxmin expected utility, Schmeidler's (1989) Choquet expected utility, Ghirardato et al.'s (2004) invariant biseparable preferences and alpha-maxmin expected utility, Maccheroni et al.'s (2006) variational preferences, Siniscalchi's (2009) vector expected utility and Grant and Polak's (2013) mean dispersion preferences. CRAA is implied by Gilboa and Schmeidler's (1989) maxmin expected utility and Chateauneuf and Faro's (2009) confidence preferences. Klibanoff et al.'s (2005) can accommodate either CAAA (with an exponential function) or CRAA (with a power function).

We propose the first tests of CAAA and CRAA using simple variations of the Ellsberg paradoxes (Ellsberg, 1961). Our tests are axiomatically founded and grounded in the theoretical literature. Consider an urn containing ten red balls and ten balls that are yellow or green in unknown proportion. A decision maker is indifferent between winning a prize if he draws a yellow ball from the urn and winning the same prize with probability p. He is now told that he can win the prize not only if the ball drawn is yellow but also if it is red. Hence, irrespective of his prior(s) about the probability of winning, his chances increase by $\frac{1}{2}$. CAAA predicts that the decision maker should be indifferent between betting on yellow or red and winning with probability $p + \frac{1}{2}$. This prediction provides a simple test of CAAA.[1]

Consider again the initial bet on yellow and imagine that the red balls are removed from the urn. Irrespective of the number of yellow balls, the chance of drawing one of them is now multiplied by 2 with respect to the initial bet on yellow. In other words, regardless of what the decision maker's prior(s) was (were) about the probability of drawing a yellow ball, this probability has doubled. The decision maker exhibits CRAA if he is indifferent between betting on yellow in the urn without red balls and winning with probability $p \times 2$.[2]

We conducted an experiment implementing these and similar tests of absolute and relative risk aversion. At the aggregate level, our results support decreasing absolute and relative ambiguity aversion. At the individual level, although CAAA is a reasonable assumption for about 40% of the subjects, we find that a very similar proportion of subjects exhibit DAAA. Almost half of the subjects also satisfied DRAA. Studying the magnitude of the deviations from constant absolute and relative ambiguity aversion, our results suggest that CAAA would not make accurate predictions for most subjects unless we accept errors of up to 10%. Our findings encourage theoretical and empirical applications of ambiguity to rely on models accounting for decreasing aversion toward ambiguity.

So far, we have discussed how ambiguity attitude evolves when the decision maker becomes better off in terms of *utility*. Alternatively, one may want to predict changes in ambiguity attitude

[1] Consider a maxmin expected utility maximizer, who has a set of priors in mind and evaluates a bet by the lowest expected utility he may get. If he thinks there may be between 3 and 7 yellow balls, then his initial winning probability is between 0.15 and 0.35. He will be indifferent between the bet on yellow and a bet on $p = 0.15$ (the worst case). When he can also win with red, his winning probability now belongs to $[0.65, 0.85]$ and he is now indifferent between the new bet and winning with probability $p + \frac{1}{2} = 0.65$.

[2] The maxmin expected utility maximizer of footnote 1 now has in mind a probability between 0.30 and 0.70, and will be indifferent between betting on yellow and $p \times 2 = 0.30$.

when the decision maker becomes better off in terms of *wealth*. This approach requires to account for risk attitudes, as shown by Cerreia-Vioglio et al. (2017). If utility is linear, then our results remain: a CAAA decision maker will display the same preferences irrespective of whether he faces a change in wealth or in utility. If the decision maker is risk averse and satisfies expected utility, then his preferences will remain unchanged at higher wealth levels if he is CARA and CRAA (Cerreia-Vioglio et al., 2017, see 1.3 for further details). Combining our results about risk and ambiguity, we demonstrate that an increase of wealth can have mixed effects on ambiguity aversion.

The following section formally introduces our tests of CAAA and CRAA. Section 2 describes the experiment, and the results are reported in section 3. Section 4 concludes.

1. Conceptual background

1.1. Absolute and relative risk aversion

We briefly recall the definitions of constant, decreasing, and increasing absolute risk aversion (referred to as CARA, DARA, and IARA) and their relative counterparts (CRRA, DRRA, and IRRA). Risk attitude can be characterized by comparing how much an agent values a lottery with the expected value of the lottery. Let $M = [0, \overline{m}]$, an interval of the reals, represent all possible *outcomes*. We denote by $\mathcal{L}$ the set of all finite *lotteries* ℓ over M. The *binary lottery* $x_p y$ yields x with probability p and y otherwise. The outcome z such that $z \sim \ell$ is called the *certainty equivalent (CE)* of ℓ and is denoted $ce(\ell)$. *Risk aversion* holds if $ce(\ell) \leq E(\ell)$ with $E(\ell)$ being the *expected value* of ℓ. *CARA [DARA, IARA]* is characterized by

$$ce(\ell + W) = [\geq, \leq]\ ce(\ell) + W \tag{1.1}$$

where $\ell + W$ is obtained by adding $W > 0$ to all outcomes of ℓ (assuming $\ell + W \in \mathcal{L}$). *CRRA [DRRA, IRRA]* is defined by

$$ce(\alpha\ell) = [\leq, \geq]\ \alpha ce(\ell) \tag{1.2}$$

where $\alpha\ell$ is obtained by multiplying all outcomes of ℓ by $\alpha \in (0, 1)$.

1.2. Absolute and relative ambiguity aversion

Just as CEs are useful to characterize risk attitude, probability equivalents (PEs) are key to study ambiguity attitude (Dimmock et al., 2016).[3] In the following, we show how to characterize constant, decreasing, and increasing absolute ambiguity aversion (CAAA, DAAA, and IAAA) and their relative counterparts (CRAA, DRAA, and IRAA).

Uncertainty is introduced through a *state space* S, which is a finite set of *states of nature* s. As usual in the Anscombe and Aumann (1963) framework, an *act* f maps S to the set of lotteries $\mathcal{L}$. An act yielding the same lottery for all $s \in S$ is referred to as the lottery itself. $\mathcal{F}$ is the set of all acts. The decision maker has preferences $\succsim$ over $\mathcal{F}$. The *mixture* $\alpha f + (1 - \alpha) g$ is the act that assigns the lottery $\alpha f(s) + (1 - \alpha) g(s)$ to $s \in S$. Let $\overline{\ell} = \overline{m}_1 0$ and $\underline{\ell} = \overline{m}_0 0$ be the best and the worst lotteries. We say that preferences satisfy *monotonicity* if $f(s)$ being first-order stochastically dominated by lottery ℓ for all s implies $f \precsim \ell$.

[3] PEs are also often called matching probabilities.

Consider the lottery $\overline{m}_p 0$ such that $f \sim \overline{m}_p 0$. If we scale the utility over M between 0 and 1, virtually all ambiguity models interpret p as the utility of act f. We call such p the *probability equivalent* of f and denote it $pe(f)$. If f is such that $f(s) = \overline{m}_{p_s} 0$, we define the *complementary act* f^c of f by $f^c(s) = \overline{m}_{1-p_s} 0$.[4]

Schmeidler's (1989) defined *ambiguity aversion* as follows: for all $f, g \in \mathcal{F}$ and $\alpha \in (0,1)$, $f \sim g$ implies $\alpha f + (1-\alpha) g \succsim f$. This definition implies Siniscalchi's (2009, axiom 10) *complementary ambiguity aversion*, which states that, in our notation, $f \sim \overline{m}_{pe(f)} 0$ and $f^c \sim \overline{m}_{pe(f^c)} 0$ imply $\frac{1}{2} f + \frac{1}{2} f^c \succsim \overline{m}_{\frac{1}{2} pe(f) + \frac{1}{2} pe(f^c)} 0$. Using $\frac{1}{2} f + \frac{1}{2} f^c = \overline{m}_{\frac{1}{2}} 0$, and assuming that preferences over lotteries satisfy first-order stochastic dominance, complementary ambiguity aversion implies $pe(f) + pe(f^c) \leq 1$. Hence, comparing the sum of the probability equivalents of two complementary acts with 1 is a test of complementary ambiguity aversion and of stronger ambiguity-aversion conditions.

We use the definition of CAAA proposed by Grant and Polak (2013):

Definition 1 *(CAAA).* For all act f in $\mathcal{F}$, lotteries ℓ_1, ℓ_2, and ℓ_3, and $\alpha \in (0,1)$, $\alpha f + (1-\alpha)\ell_1 \succsim \alpha \ell_2 + (1-\alpha)\ell_1 \Rightarrow \alpha f + (1-\alpha)\ell_3 \succsim \alpha \ell_2 + (1-\alpha)\ell_3$.

Grant and Polak (2013) showed that this condition is a weakening of Schmeidler's (1989) comonotonic independence, Gilboa and Schmeidler's (1989) certainty independence, and Maccheroni et al.'s (2006) weak certainty independence.[5] All these axioms require invariance to translations of utility profiles. Hence, all ambiguity models relying on one of these axioms and listed in the introduction predict constant absolute aversion toward ambiguity. In terms of PEs and using the best and the worst lotteries, CAAA can be tested with the condition:

$$pe(\alpha f + (1-\alpha)\overline{\ell}) = pe(\alpha f + (1-\alpha)\underline{\ell}) + (1-\alpha) \tag{1.3}$$

with $\alpha \in (0,1)$. In words, increasing the probability of obtaining the best outcome $\overline{m}$ by $(1-\alpha)$ for all states of nature increases the PE by $(1-\alpha)$.

Observation 1. *Assume that preferences satisfy weak ordering and monotonicity. Then CAAA implies Eq. (1.3).*

Proof. Assume $pe(\alpha f + (1-\alpha)\underline{\ell}) = p$, that is $\alpha f + (1-\alpha)\underline{\ell} \sim \overline{m}_p 0$. Any f must yield lotteries that are (weakly) dominated by getting the best outcome for sure and therefore, act $\alpha f + (1-\alpha)\underline{\ell}$ yields lotteries that are (weakly) first-order stochastically dominated by $\overline{m}_\alpha 0$. Hence, $p \leq \alpha$ (by monotonicity) and we can define $\ell_2 = \overline{m}_{\frac{p}{\alpha}} 0$. We thus have $\alpha f + (1-\alpha)\underline{\ell} \sim \alpha \ell_2 + (1-\alpha)\underline{\ell}$. Then CAAA implies $\alpha f + (1-\alpha)\overline{\ell} \sim \alpha \ell_2 + (1-\alpha)\overline{\ell} = \overline{m}_{p+1-\alpha} 0$. □

Observation 2. *Gilboa and Schmeidler's (1989) maxmin expected utility, Schmeidler's (1989) Choquet expected utility, Ghirardato et al.'s (2004) invariant biseparable preferences and alpha-maxmin expected utility, Maccheroni et al.'s (2006) variational preferences, Siniscalchi's (2009) vector expected utility and Grant and Polak's (2013) mean dispersion preferences imply Eq. (1.3).*

[4] The pair (f, f^c) is complementary according to definition 3 of Siniscalchi (2009).

[5] Trautmann and Wakker (2018) showed that these axioms were violated because ambiguity attitude changes between gains ans losses.

Proof. Grant and Polak (2013) showed that all these models imply CAAA. Furthermore, they all assume weak ordering and monotonicity. Hence, by Observation 1, these models imply Eq. (1.3). □

We will further classify decision makers as *DAAA [IAAA]* using:

$$pe(\alpha f + (1-\alpha)\,\overline{\ell}) \geq [\leq]\, pe(\alpha f + (1-\alpha)\,\underline{\ell}) + (1-\alpha) \tag{1.4}$$

with $\alpha \in (0, 1)$. Under the same assumptions as before, the conditions for DAAA and IAAA are implications of recent definitions proposed by Chambers et al. (2014, Definition 9). The DAAA condition is also implied by an axiom used by Ghirardato and Siniscalchi (2015) and Xue (2018, Axiom A.2.1).

Maxmin expected utility (Gilboa and Schmeidler, 1989) is invariant to shifts of utility profiles but also to multiplication or rescaling. Its core axiom, certainty independence, implies CAAA and Chateauneuf and Faro's (2009) *worst independence* axiom, a form of homotheticity or invariance to mixture with the worst lottery. We propose to use this latter axiom to define CRAA because it is similar to CRRA (the only type of homothetic preferences under expected utility).

Definition 2 *(CRAA).* For all acts f, g in $\mathcal{F}$, and $\alpha \in (0, 1)$, $f \sim g \Rightarrow \alpha f + (1-\alpha)\underline{\ell} \sim \alpha g + (1-\alpha)\underline{\ell}$.

In terms of PEs, we can observe CRAA by

$$pe(\alpha f + (1-\alpha)\,\underline{\ell}) = \alpha pe(f) \tag{1.5}$$

with $\alpha \in (0, 1)$. In words, multiplying the probability of obtaining the high outcome by α for all states of nature multiplies the PE by α.

Observation 3. *Gilboa and Schmeidler's (1989) maxmin expected utility and Chateauneuf and Faro's (2009) confidence preferences imply CRAA, which implies Eq. (1.5).*

Proof. The first implication comes from Chateauneuf and Faro (2009). Furthermore, take $g = \overline{m}_p 0$ where p is $pe(f)$. We have $f \sim g$. CRAA implies $\alpha f + (1-\alpha)\,\underline{\ell} \sim \alpha g + (1-\alpha)\,\underline{\ell} = \overline{m}_{\alpha p} 0$. □

Decision makers deviating from CRAA can be classified as *DRAA [IRAA]* by

$$pe(\alpha f + (1-\alpha)\,\underline{\ell}) \leq [\geq]\, \alpha pe(f) \tag{1.6}$$

with $\alpha \in (0, 1)$.

In the introduction, we listed the ambiguity models satisfying CAAA and CRAA, including special cases of Klibanoff et al.'s (2005) smooth ambiguity model. Let $\Delta(S)$ be the set of probability measures on S. According to the smooth ambiguity model, an act f is evaluated by $\int_{\Delta(S)} \mu(Q)\varphi\left(\sum_{s\in S} Q(s)Eu\,(f\,(s))\right) dQ$, where μ is a second-order belief measure over the possible probability distributions on S. The smooth ambiguity model also satisfies CAAA and CRAA if the smooth ambiguity function φ is exponential or power, respectively (see Appendix B for details).

In this section, we rely on the Anscombe-Aumann setting, where acts assign lotteries to events, and where risk independence is assumed (Cerreia-Vioglio et al., 2011), ensuring expected utility

Table 1
Equivalence between the definitions in terms of utility and in terms of wealth (for CARA decision makers).

	Risk seeking	Risk neutral	Risk averse
W-DAAA	DRAA	DAAA	IRAA
W-CAAA	CRAA	CAAA	CRAA
W-IAAA	IRAA	IAAA	DRAA

under risk. However, if expected utility under risk does not hold, adding the same likelihood of winning to all states of nature might not have the same impact, depending on the initial lottery assigned to the state. After introducing the experimental design, we will assess the robustness of the implementation of the CAAA and CRAA tests to deviations from expected utility.

1.3. Impact of wealth

The definitions of CAAA and DAAA given above are common in the literature (Grant and Polak, 2013; Ghirardato and Siniscalchi, 2015; Xue, 2018) and in line with Klibanoff et al.'s (2005) smooth ambiguity model and its applications (Berger, 2014; Cherbonnier and Gollier, 2015; Berger, 2016). However, one may prefer to study the impact of changes of wealth instead of changes of utility, as recently done by Cerreia-Vioglio et al. (2017).

Let f_W be the act assigning lottery $f(s) + W$ to state s for $W > 0$. Further define $\succsim_W$ by $f \succsim_W g$ whenever $f_W \succsim g_W$. The relation $\succsim_W$ represents the preferences at a higher wealth level. *Constant absolute ambiguity aversion in terms of wealth* (*W-CAAA*) holds if $\succsim$ and $\succsim_W$ fully agree. To define changes in ambiguity attitudes, Cerreia-Vioglio et al. (2017) used Ghirardato and Marinacci's (2002) comparative ambiguity aversion: $\succsim_1$ is *more ambiguity averse than* $\succsim_2$ if, for all $f \in \mathcal{F}$ and $\ell \in \mathcal{L}$, $f \succsim_1 \ell \Rightarrow f \succsim_2 \ell$. In words, if the more ambiguity averse decision maker 1 prefers an act to a lottery, then the less ambiguity averse agent 2 should also prefer the act to the lottery. *Decreasing absolute ambiguity aversion in terms of wealth* (*W-DAAA*) is defined as $\succsim$ being more ambiguity averse than $\succsim_W$ for all $W > 0$. Symmetrically, *increasing absolute ambiguity aversion in terms of wealth* (*W-IAAA*) is defined as $\succsim_W$ being more ambiguity averse than $\succsim$.

The first observation of Cerreia-Vioglio et al. (2017) is that preferences satisfy one of the three conditions (W-DAAA, W-CAAA, or W-IAAA) only if they are also CARA. CARA guarantees that the decision maker's risk attitude remains constant when wealth increases, and therefore, that changes in ambiguity attitudes cannot be confounded with changes in risk attitudes. Furthermore, it is obvious that W-CAAA and CAAA agree (as do W-DAAA and W-IAAA with DAAA and IAAA) for risk neutral decision makers because their utility is linear in money.

Cerreia-Vioglio et al. (2017) established the following results for CARA utility ($u(x) = -e^{-\rho x}$). A wealth increase of $+W$ multiplies the utility of each state of nature by $e^{-\rho W}$. Hence, exponential utility transformed additive shifts into multiplicative shifts. Ambiguity aversion will therefore remain constant when wealth increases if it is multiplication invariant when utility increases, that is, if CRAA holds. Aversion will decrease if DRAA holds and the multiplication factor $e^{-\rho W}$ is more than 1 ($\rho < 0$, risk seeking) or if IRAA holds and $e^{-\rho W}$ is less than 1 ($\rho > 0$, risk averse). The results of Cerreia-Vioglio et al. (2017) are summarized in Table 1.

2. Experimental design

The experiment consisted of two types of tasks: CE tasks under risk and PE tasks under ambiguity.

2.1. *CE tasks*

Subjects were asked to make a series of decisions between a lottery and sure amounts (see Fig. 2.1). We define the CE as the midpoint between the lowest amount preferred to the lottery and the highest amount for which the lottery was preferred.

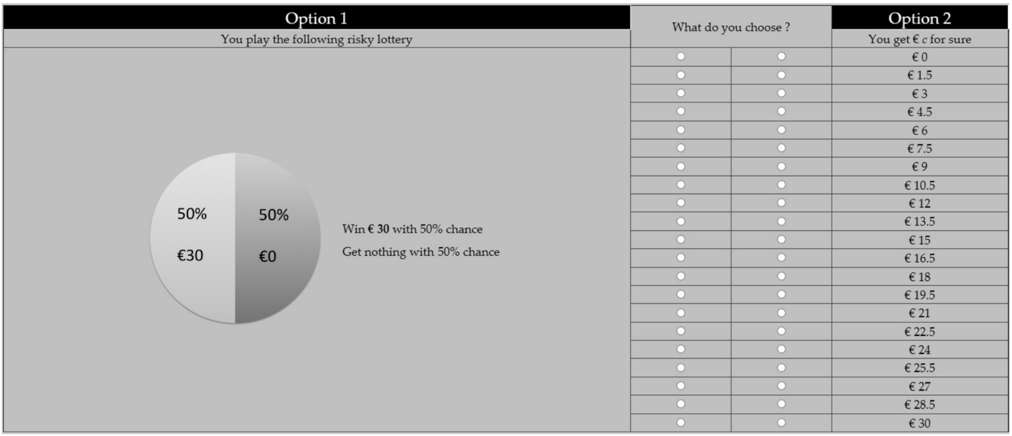


Fig. 2.1. CE-task.

Table 2 presents the list of lotteries that the subjects were asked to evaluate. Risk attitude can be obtained by comparing the CEs with the expected values (reported in the third column). The tests for CARA and CRRA are reported in the fourth and fifth columns, respectively. For instance, lottery ℓ_2 is obtained by adding 10 euros to both outcomes of lottery ℓ_1, which allows us to test CARA.[6] The outcomes of lottery ℓ_1 are one-third of those of ℓ_4, which allows us to test CRRA.

2.2. *PE tasks*

The second type of tasks our subjects were asked to complete were PE tasks. The best and worst outcomes were 30 and 0 euros. We measured PEs for bets on the color of a ball drawn from an Ellsberg urn. The urn contained balls of various colors (red, black, green, yellow, and blue), but the proportions of yellow and green balls were unknown. Subjects were asked to make a series of decisions between a given act and lotteries yielding 30 euros with probability p (see

[6] In Table 2, we can see that CARA predicts $ce(\ell_2) = ce(\ell_1) + 10$ and $ce(\ell_3) = ce(\ell_1) + 20$. Obviously, it also predicts $ce(\ell_3) = ce(\ell_2) + 10$, but we do not mention this test in the Table because it would be redundant. Throughout the paper, redundant tests are omitted.

Table 2
Lotteries and tests.

Lottery		Risk neutral $[ce(\ell_i)=]$ averse $[\leq]$ seeking $[\geq]$	CARA $[ce(\ell_i)=]$ DARA $[\geq]$ IARA $[\leq]$	$[ce(\ell_i)=]$ DRRA $[\leq]$ IRRA $[\geq]$
ℓ_1	$10_{1/2}0$	5		$\frac{1}{3}ce(\ell_4)$
ℓ_2	$20_{1/2}10$	15	$ce(\ell_1)+10$	
ℓ_3	$30_{1/2}20$	25	$ce(\ell_1)+20$	
ℓ_4	$30_{1/2}0$	15		
ℓ_5	$15_{1/2}10$	12.5		$\frac{1}{2}ce(\ell_3)$
ℓ_6	$10_{1/4}0$	2.5		$\frac{1}{3}ce(\ell_8)$
ℓ_7	$20_{1/4}10$	12.5	$ce(\ell_6)+10$	
ℓ_8	$30_{1/4}0$	7.5		
ℓ_9	$10_{3/4}0$	7.5		
ℓ_{10}	$30_{3/4}20$	27.5	$ce(\ell_9)+20$	
ℓ_{11}	$15_{3/4}10$	13.75		$\frac{1}{2}ce(\ell_{10})$

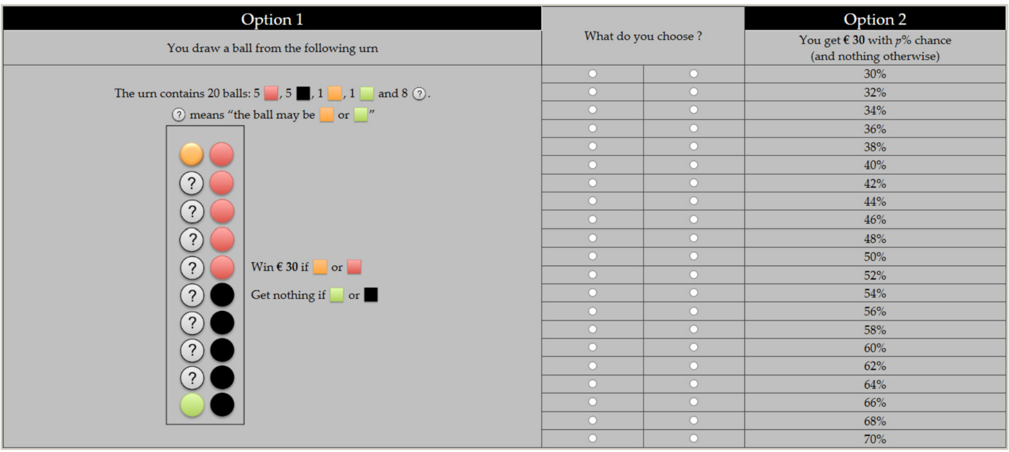


Fig. 2.2. PE-task.

Fig. 2.2 for a screenshot). We define the PE as the midpoint between the lowest p preferred to the act and the highest p to which the act was preferred.

Table 3 describes the twelve acts and urns that we used to conduct the different tests for ambiguity neutrality, CAAA and CRAA. For instance, act f_1 wins 30 euros if a yellow ball is drawn from an urn containing 20 balls, 5 being red, 5 being black, and the other 10 being yellow or green (with at least one of each color).[7] Ambiguity arose from the unknown proportions of yellow and green balls in the urns. We specified that the proportions of yellow and green balls

[7] The presence of at least one green and one yellow ball in the urn ensured that acts yielding $\overline{\ell} = 30_1 0$ and $\underline{\ell} = 30_0 0$ never occurred. This prevented certainty and impossibility effects from distorting our CAAA and CRAA tests. See subsection 2.3.

Table 3
Acts and tests for ambiguity.

Act	Winning color	*Known*				Unknown	Tests		
	win 30 euros, 0 otherwise	*# of balls*	*R*	*B*	*L*	Y (≥ 1) or G (≥ 1)	Ambiguity neutral [$pe(f_i)$=] averse [≤] seeking [≥]	CAAA [$pe(f_i)=$] DAAA [≥] IAAA[≤]	CRAA [$pe(f_i)=$] DRAA [≤] IRAA[≥]
f_1	*Y*	*20*	*5*	*5*	–	10			$\frac{1}{2}pe(f_6)$
f_2	*Y&R*	*20*	*5*	*5*	–	10		$pe(f_1)+\frac{1}{4}$	
f_3	*Y&R&B*	*20*	*5*	*5*	–	10		$pe(f_1)+\frac{1}{2}$	
f_4	*Y&R&B*	*60*	*5*	*5*	*40*	10			$\frac{1}{3}pe(f_3);\frac{1}{2}pe(f_5)$
f_5	*Y&R&B*	*30*	*5*	*5*	*10*	10			
f_6	*Y*	*10*	–	–	–	10			
f_7	*G*	*20*	*5*	*5*	–	10	$1-pe(f_3)$		$\frac{1}{2}pe(f_{12})$
f_8	*G&R*	*20*	*5*	*5*	–	10	$1-pe(f_2)$	$pe(f_7)+\frac{1}{4}$	
f_9	*G&R&B*	*20*	*5*	*5*	–	10	$1-pe(f_1)$	$pe(f_7)+\frac{1}{2}$	
f_{10}	*G&R&B*	*60*	*5*	*5*	*40*	10			$\frac{1}{3}pe(f_9);\frac{1}{2}pe(f_{11})$
f_{11}	*G&R&B*	*30*	*5*	*5*	*10*	10			
f_{12}	*G*	*10*	–	–	–	10	$1-pe(f_6)$		

Y (R, B, L and G) indicates that the color of the ball is “yellow”, “red”, “black, “blue” or “green,” respectively.

were the same for all acts.[8] It allows us to model the ambiguous (part of the) urn by the state space $S=\{1,...,9\}$ representing the number of yellow balls in the urn.[9]

Act f_6 offers $\frac{s}{10}$ chances of obtaining 30 euros for a given state s, and act f_{12} offers $\frac{10-s}{10}$ chances of obtaining 30 euros. Hence, the two acts are complementary, which enables us to test ambiguity aversion (see column 8 in Table 3). Comparing f_1 and f_2, observe that f_2 adds red to yellow as a winning color and therefore increases the winning probability for all s by $\frac{1}{4}$. Under the CAAA assumption, the PE should also increase by $\frac{1}{4}$ (see column 9 in Table 3 for the other CAAA tests). Comparing f_1 and f_6, observe that the winning color remains the same but that the urn for act f_1 contains 10 more balls than the urn for act f_6. The probability of winning has been halved in f_1 with respect to f_6 and thus should be the PE under the CRAA hypothesis (see the last column of Table 3 for the other CRAA tests).

[8] Ellsberg urns create ambiguity because subjects do not know the composition of the urn. We also implemented a variation of these urns by relating the urn composition to naturally occurring events, namely, whether the Dutch stock index (AEX) during the experiment would increase or decrease. For these acts, subjects were ambiguity neutral (at the aggregate level). It could either be that they were not averse towards this particular source of uncertainty (Baillon and Bleichrodt, 2015 and Baillon et al., 2018 found little to no ambiguity aversion for resembling sources of uncertainty and similar subjects) or that they did not perceive that this source generated ambiguity. As a consequence, our tests could not be applied. For the sake of completeness, details on this part of the experiment are reported in supplementary material.

[9] Alternatively, the state space could describe the color of the ball drawn from the urn. Yet, a state space describing the number of yellow balls, as used here, is simpler to present, remains the same for all acts, and models a uniform source of ambiguity. We describe the alternative state space(s) in Appendix A and show that our tests remain valid for a general class of uncertainty-averse preferences if the subjects incorporates the objective information in their perception of uncertainty.

2.3. Robustness to non-expected utility under risk

The theoretical section of this paper is based on the Anscombe-Aumann framework, assuming expected utility under risk, but this assumption is usually violated in empirical studies. For instance, Allais (1953) famously showed that people tend to be *too* attracted by certainty (or impossibility). In the experiment, we avoided certainty and impossibility effects by excluding degenerate lotteries from the acts. This does not solve everything though and we need to assess the robustness of our experiment to non-expected utility. Many non-expected utility models exist but we will focus here on the most used one, Quiggin's (1981) rank dependent utility (equivalently, Tversky and Kahneman's (1992) prospect theory for gains). In this model, probabilities are weighted, which can bias tests that rely on PEs and shifts of probabilities. We will consider several forms of probability weighting and study the biases they imply.

First, we can identify forms of weighting to which the tests are robust. Denote $f_i(s) = \overline{m}_{p_{i,s}}0$ the lottery assigned by act f_i to state s. With at least one green ball and one yellow ball in the urn, we ensured $p_{i,s} \in (0, 1)$ for all i and s. If subjects have a *neo-additive* weighting function, defined as a function that is linear on $(0, 1)$, then the CAAA tests are still valid, as shown in Appendix C - Observation 4. The appendix also shows that the CRAA tests are robust to power weighting functions.

Second, we can estimate the impact of other form of probability weighting. We focus on the popular weighting function proposed by Prelec (1998), $w(p) = exp(-(-ln(p))^{\rho})$, with ρ a curvature parameter. Expected utility corresponds to $\rho = 1$. Eliciting Prelec's weighting function in many different countries, l'Haridon and Vieider (2019) found values of ρ ranging from 0.5 to 1 (with the exception of Nigerian students, for whom the value was 0.27). We assumed the smooth ambiguity model of Klibanoff et al. (2005) and studied the impact of ρ for values between 0.4 and 1.2. For each test described in Table 3, we computed how much the obtained probability equivalent differed from the predicted one (assuming CAAA or CRAA). For instance, if $pe(f_2)$ was 1.05 times $pe(f_1) + \frac{1}{4}$, we said that it had a 5% bias. All computational details and assumptions are reported in Appendix C.

We assessed two cases: (i) if we had run the experiment with no restrictions on the number of green and yellow balls; (ii) with our restriction that there was at least one ball of each color ($Y \geq 1$ & $G \geq 1$). We found that the restrictions on Y and G more than halved the biases generated by probability weighting. For our CAAA tests, even extreme probability weighting ($\rho = 0.4$), rarely observed, would not create a bias of more than 5%. For CRAA, one type of tests is sensitive (comparing f_6 and f_{12} to f_1 and f_7), the second type is fair, with biases of less than 5%, and the last one is very robust to Prelec's probability weighting. This most robust type of tests compares an act with chance of winning between 11/30 and 19/30 to an act with chance of winning between 11/60 and 19/60. For such values, probability weighting seems to be negligible.

To account for the risk of probability weighting to affect our results, we will report results in a conservative manner, requiring a deviation of more than 5% of the PEs to classify subjects as non-CAAA or non-CRAA. To classify subjects as CRAA / IRAA / DRAA, we will focus on the robust tests, excluding the comparisons of f_6 and f_{12} to f_1 and f_7.

2.4. Participants and organization

To conduct the experiment, 78 participants were recruited at Erasmus University Rotterdam (mean age is 21.5; 60% are male). The ordering of the parts (risk and ambiguity) was counterbalanced between participants, and choice tasks were randomized within each part. We ran 8 sessions on the same day, with 8 to 12 subjects each. A session began with general instructions, which were read to all subjects who then entered their cubicles. The CE and PE tasks lasted approximately 30 minutes. Afterward, subjects were paid as described in the next section.

2.5. Incentives

We used the random incentive system with the slight modification that the choice that would be played out was determined before the experiment began. For each session and before the beginning of the experiment, a subject was asked to draw two envelopes in front of the other subjects and to sign them. The first envelope was drawn from a pile of envelopes containing all lotteries and acts of the experiment (as described in Tables 2, 3, and 7). The second envelope was drawn from a pile of 21 envelopes, each containing a different number from 1 to 21, corresponding to a row in the choice lists depicted in Fig. 2.1 or 2.2. At the end of the experiment, the signed envelopes were opened and the corresponding choice was played out for real money. The subjects received a show-up fee of 5 euros and an additional amount of up to 30 euros depending on their choices. On average, the subjects earned 21.50 euros for approximately one hour of participation. Lotteries and acts were implemented with physical devices (a pair of 10-sided dice for the lotteries and an urn for the acts). Subjects were informed that the proportions of yellow and green balls were the same for all acts. In practice, an urn with only yellow and green balls was prepared before the experiment, and depending on the act that was supposed to be played for real, the corresponding number of red, black, and blue balls was added.

Random incentives provided subjects with a mixture over acts and therefore, provided them with a way to hedge against ambiguity. Overall, in our experiment, being paid for a choice involving Y as winning color was as likely as being paid for a choice involving G as winning color. Ambiguity averse subjects who would perceive the whole experiment as one choice may then behave as if they were ambiguity neutral (Oechssler and Roomets, 2014; Bade, 2015). Hence, random incentives may lead to underestimate the prevalence of ambiguity aversion. Baillon et al. (2014) argued that performing the randomization before the resolution of the uncertainty (and even before choices are made) can mitigate this problem. We followed this procedure, even though there is no guarantee that it eliminates hedging concerns.

3. Results

From our initial sample of 78 subjects, eight who violated dominance in the choice lists (choosing dominated lotteries or acts) at least three times were removed. In the aggregate analysis, we report the results of two-tailed t-tests. Wilcoxon tests produced similar results.

3.1. Risk

In a first step, we report the aggregate results of our tests of risk neutrality and constant absolute and relative risk aversion. We measured risk attitude by the difference between the expected value of the lottery $E(\ell_i)$ and the average certainty equivalent $ce(\ell_i)$. Table 4 shows that, at the aggregate level, the subjects were risk seeking for lotteries with winning probability $\frac{1}{4}$ and averse for lotteries with winning probability $\frac{3}{4}$. For probability $\frac{1}{2}$, they were risk averse or neutral. This pattern suggests that our subjects would be better represented by rank-dependent utility than by expected utility. Appendix D reports the results of maximum likelihood estimation of expected

Table 4
Tests of risk neutrality, CARA and CRRA.

Test for Risk neutrality [=0]	p	Result	Conclusion
$E(\ell_1) - ce(\ell_1)$		0.46^{***} (0.14)	aversion
$E(\ell_2) - ce(\ell_2)$		0.21 (0.16)	neutral
$E(\ell_3) - ce(\ell_3)$	$\frac{1}{2}$	−0.1 (0.15)	neutral
$E(\ell_4) - ce(\ell_4)$		2.71^{***} (0.52)	aversion
$E(\ell_5) - ce(\ell_5)$		−0.02 (0.08)	neutral
$E(\ell_6) - ce(\ell_6)$		-0.49^{***} (0.15)	seeking
$E(\ell_7) - ce(\ell_7)$	$\frac{1}{4}$	-0.74^{***} (0.17)	seeking
$E(\ell_8) - ce(\ell_8)$		-0.90^{**} (0.35)	seeking
$E(\ell_9) - ce(\ell_9)$		1.11^{***} (0.21)	aversion
$E(\ell_{10}) - ce(\ell_{10})$	$\frac{3}{4}$	1.44^{***} (0.21)	aversion
$E(\ell_{11}) - ce(\ell_{11})$		0.93^{***} (0.10)	aversion
Test for CARA [=0]			
$ce(\ell_2) - [ce(\ell_1) + 10]$		0.23 (0.16)	CARA
$ce(\ell_3) - [ce(\ell_1) + 20]$		0.55^{***} (0.17)	DARA
$ce(\ell_7) - [ce(\ell_6) + 10]$		0.29 (0.19)	CARA
$ce(\ell_{10}) - [ce(\ell_9) + 20]$		−0.31 (0.21)	CARA
Test for CRRA [=0]			
$ce(\ell_1) - \frac{1}{3}ce(\ell_4)$		1.33^{***} (0.42)	IRRA
$ce(\ell_5) - \frac{1}{2}ce(\ell_3)$		−0.05 (0.17)	CRRA
$ce(\ell_6) - \frac{1}{3}ce(\ell_8)$		0.56 (0.38)	CRRA
$ce(\ell_{11}) - \frac{1}{2}ce(\ell_{10})$		−0.42 (0.29)	CRRA

***, **, and * indicate that the test is significant at 1, 5, and 10%, respectively. Not rejecting the null hypothesis is interpreted as risk neutrality, CARA and CRRA. Standard errors are in parentheses.

utility and of rank-dependent utility for neoadditive and Prelec probability weighting. Introducing probability weighting substantially increases the fit of the model and neo-additive weighting fits the data slightly better than the Prelec weighting function. This result should be interpreted with caution because the experiment was not designed to compare weighting functions but it is reassuring for the CAAA tests because they are not affected by neo-additive weighting.

Neither CARA nor CRRA was rejected in three out of four tests. Only one of the CARA tests is rejected in favor of DARA, and one of the CRRA tests is rejected in favor of IRRA. For comparison, previous empirical results from the literature are mixed. Levy (1994) reported experimental evidence in favor of DARA but not IRRA, but Eisenhauer (1997) found evidence for IARA in an empirical study on life insurance. More comparable to our work, Holt and Laury (2002) found that their experimental data conformed well to IRRA together with DARA (expo-power utility function).

In a second step, we classified our subjects according to their risk behavior. For all classifications, we used a 5% error margin to account for the (im)precision of the choice lists. A subject was classified as CARA (CRRA) if the conditions described in Table 2 were satisfied within a 5% error margin on average. As seen in the aggregate results, the subjects were risk seeking for small probabilities and risk averse for large probabilities. To classify subjects as, overall,

Table 5
Classification of subjects depending on their risk attitude.

	IARA	CARA	DARA	Total
Risk seeking	2	6	2	10
Risk neutral	5	22	1	28
Risk averse	7	22	3	32
Total	14	50	6	70

(a) Risk attitude and absolute risk aversion

	IRRA	CRRA	DRRA	Total
Risk seeking	6	2	2	10
Risk neutral	5	17	6	28
Risk averse	14	11	7	32
Total	25	30	15	70

(b) Risk attitude and relative risk aversion

	IRRA	CRRA	DRRA	Total
IARA	2	6	6	14
CARA	18	23	9	50
DARA	5	1	0	6
Total	25	30	15	70

(c) Absolute and relative risk aversion

	IRRA	CRRA	DRRA	Total
IARA	2	1	4	7
CARA	9	10	3	22
DARA	3	0	0	3
Total	14	11	7	32

(d) Absolute and relative risk aversion (risk averse subjects only)

more risk averse or seeking, we only considered lotteries ℓ_1 to ℓ_5, which involved a $\frac{1}{2}$ chance of winning. A subject was considered risk neutral if his CE was, on average, within 5% of the expected value of the lotteries. Subjects whose CEs were lower (higher) than the expected values by more/than 5% on average were classified as risk averse (seeking).

The results of the classification are reported in Table 5. A large majority of subjects (71%) displayed CARA (panel (a)). CARA was satisfied by a majority of risk averse subjects (panel (d)). In terms of relative risk attitude, CRRA was the most common pattern (43%), followed by IRRA (36%).

3.2. Ambiguity

Table 6 reports the results of the tests for ambiguity neutrality and for constant absolute and relative ambiguity aversion. At the aggregate level, all tests yielded results in favor of ambiguity aversion. CAAA and CRAA were systematically rejected in favor of DAAA and DRAA but the effect sizes are relatively small and still within the range of possible biases due to non-expected utility under risk such as Prelec-style probability weighting. It is therefore crucial to explore individual-level data to identify whether the rejection of CAAA and CRAA arises from a small bias, possibly due to probability weighting and that all subjects exhibit, or from clear and strong deviations of CAAA and CRAA for part of the sample.

Fig. 3.1 displays the PEs of complementary acts, whose sum should be 1 under ambiguity neutrality. Many subjects are close to ambiguity neutrality but we see much more and much stronger deviations in the direction of ambiguity aversion than in the direction of ambiguity seeking. The sum of PEs of f_6 and f_{12} is further away from 1 for many subjects than the sum of other PEs for other complementary acts because f_6 and f_{12} concerned the fully ambiguous urns.

Figs. 3.2 and 3.3 depict the PEs of all subjects. They illustrate the magnitude of the violations of the CAAA and CRAA conditions at the individual level. In Fig. 3.2.a, the (green) circles represent $pe(f_2)$ as a function of $pe(f_1)$, with the size of the circle representing the number of subjects with this combination. The dashed line $pe(f_1) + \frac{1}{4}$ represents the CAAA hypothesis, and a circle above (below) this line indicates DAAA (IAAA). The surrounding dark gray area represents a ±5% error margin (which could be due to probability weighting, as illustrated in Fig. C.1), and the light gray area represents a ±10% error margin. Similarly, the (red) squares

Table 6
Tests of ambiguity neutrality, CAAA and CRAA.

Test for ambiguity neutrality [=1]	Result	Conclusion
$pe(f_6) + pe(f_{12})$	0.96^{*} (0.023)	aversion
$pe(f_1) + pe(f_9)$	0.95^{***} (0.013)	aversion
$pe(f_2) + pe(f_8)$	0.97^{***} (0.012)	aversion
$pe(f_3) + pe(f_7)$	0.94^{***} (0.013)	aversion
Test for CAAA [=0]		
$pe(f_2) - \left[pe(f_1) + \frac{1}{4}\right]$	0.03^{***} (0.009)	DAAA
$pe(f_3) - \left[pe(f_1) + \frac{1}{2}\right]$	0.02^{*} (0.012)	DAAA
$pe(f_8) - \left[pe(f_7) + \frac{1}{4}\right]$	0.03^{***} (0.009)	DAAA
$pe(f_9) - \left[pe(f_7) + \frac{1}{2}\right]$	0.04^{***} (0.012)	DAAA
Test for CRAA [=0]		
$pe(f_1) - \frac{1}{2} \times pe(f_6)$	-0.03^{***} (0.009)	DRAA
$pe(f_4) - \frac{1}{3} \times pe(f_3)$	-0.01^{**} (0.004)	DRAA
$pe(f_4) - \frac{1}{2} \times pe(f_5)$	-0.01^{***} (0.003)	DRAA
$pe(f_7) - \frac{1}{2} \times pe(f_{12})$	-0.03^{***} (0.009)	DRAA
$pe(f_{10}) - \frac{1}{3} \times pe(f_9)$	-0.02^{***} (0.004)	DRAA
$pe(f_{10}) - \frac{1}{2} \times pe(f_{11})$	-0.01^{***} (0.004)	DRAA

***, **, and * indicate that the test is significant at 1, 5, and 10%, respectively. Not rejecting the null hypothesis is interpreted as ambiguity neutrality, CAAA and CRAA. Standard errors are in parentheses.

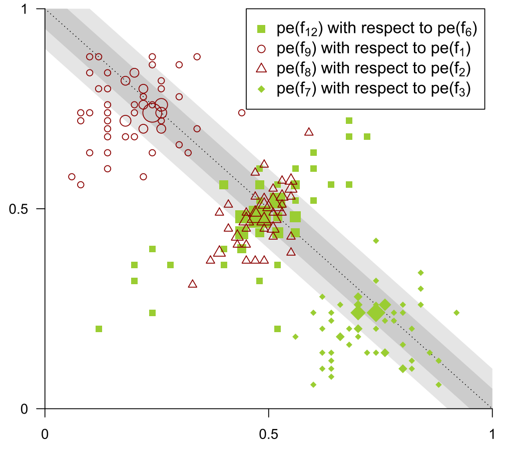
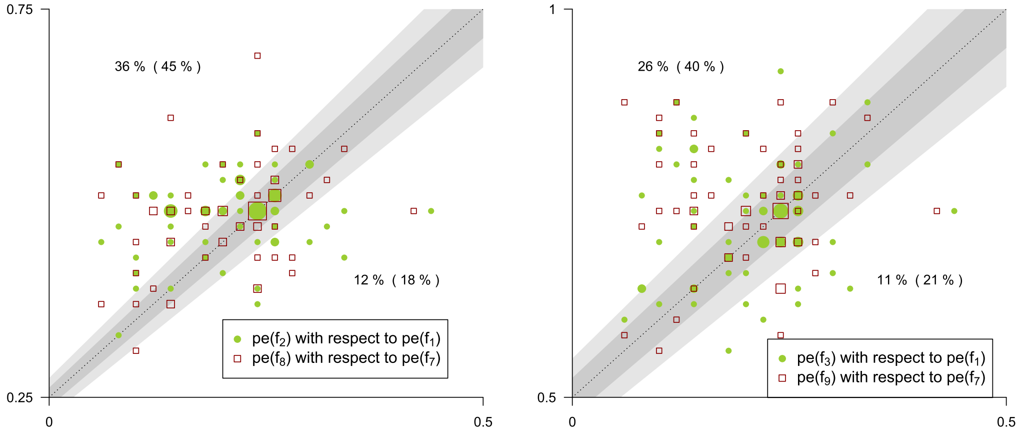


Fig. 3.1. Magnitude of ambiguity aversion. *Notes:* Both axes describe PEs. The line represents ambiguity neutrality and the light (dark) gray areas a 10% (5%) error margin.

in panel (a) represent $pe(f_8)$ as a function of $pe(f_7)$. Panel (a) shows that CAAA is a good approximation of the behavior of many subjects but that a substantial mass of subjects lays above the gray area. Assuming CAAA for those subjects would imply an error of more than 10% when predicting their behavior. The circles and squares in panel (b) represent the two CAAA



(a) PEs when chances are increased by 1/4 (b) PEs when chances are increased by 1/2

Fig. 3.2. Magnitude of violations of the CAAA conditions. *Notes:* Both axes describe PEs. The line represents CAAA and the light (dark) gray areas a 10% (5%) error margin (in terms of ordinates). The percentages indicate the proportion of subject who deviate from CAAA by more than 10% (more than 5% between brackets).

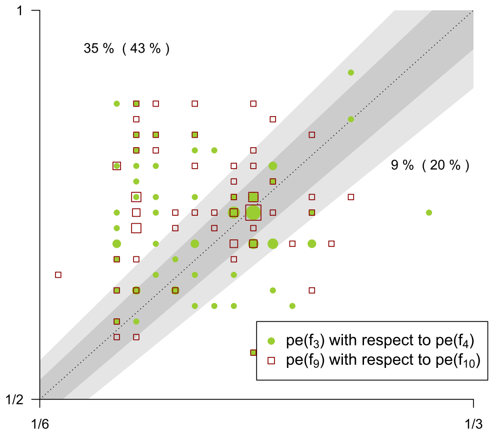
conditions when chances are increased by 1/2. For these PEs, CAAA does not seem to be a bad approximation for a vast majority of subjects if we are willing to accept errors of up to 10%.

In Fig. 3.3, the dashed lines represent the CRAA conditions. The PEs should be multiplied, which is represented by a line crossing the origin. Subjects above the CRAA line are DRAA, and those below are IRAA. The dark (light) gray areas again represent a 5% (10%) error margin. In all panels, a number of subjects approximately satisfy CRAA but a substantial mass of subjects is located above the CRAA line, with a deviation of more than 10%. The results in panel (c) should be taken with caution, because they correspond to the tests that were the least robust to probability weighting according to our robustness analysis.

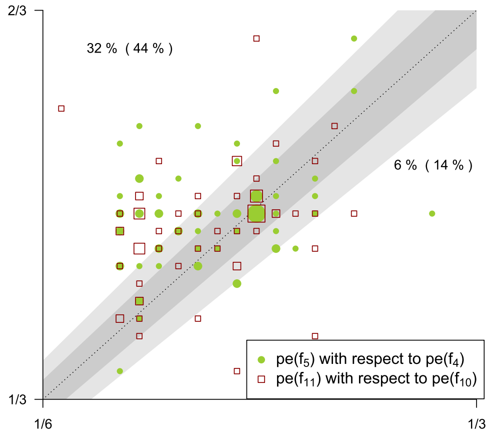
Table 7 reports the classification of subjects according to their ambiguity behaviors. We used classification rules similar to those used for risk attitude and compatible with our robustness analysis, with an error margin of 5% to reflect the possible impact of probability weighting. For each subject, we computed the average deviation from the conditions given in Table 3 but excluded the two CRAA tests that were especially sensitive to probability weighting. Subjects were almost equally distributed between CAAA and DAAA (panel (a)). DRAA was found for the majority of ambiguity-averse subjects, and most DAAA subjects were also DRAA. Some subjects could be classified as ambiguity neutral (if they were slightly ambiguity averse for some acts and slightly ambiguity seeking for others) and still classified as DAAA if the switch from averse to seeking was consistent across tests. The DAAA-DRAA patterns was also confirmed for ambiguity averse subjects (panel (d)).

3.3. *Impact of wealth on ambiguity attitudes*

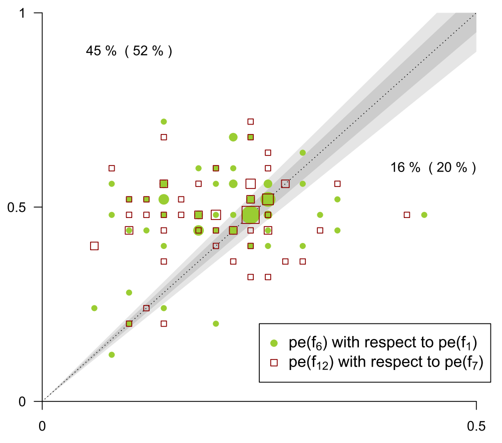
Combining our results about risk and ambiguity, we could classify CARA subjects with the definitions of Cerreia-Vioglio et al. (2017). A total of 50 subjects were identified as CARA. As



(a) PEs when chances are divided by 3



(b) PEs when chances are divided by 2



(c) PEs when chances are divided by 2

Fig. 3.3. Magnitude of violations of the CRAA conditions. *Note:* Both axes describe PEs. The line represents CRAA and the light (dark) gray areas a 10% (5%) error margin (in terms of ordinates). The percentages indicate the proportion of subject who deviate from CAAA by more than 10% (more than 5% between brackets).

explained in Section 1.3, for risk neutral subjects, W-CAAA [W-DAAA,W-IAAA] agrees with CAAA [DAAA,CAAA]. Indeed, for subjects whose utility is linear, if the degree of ambiguity aversion remains constant when utility increases (CAAA) then it also remains constant when wealth increases (W-CAAA). A majority of risk neutral subjects were classified as W-CAAA, followed by W-DAAA (Table 8, panel (a)).

For risk averse subjects, the equivalence between CAAA and W-CAAA does not hold anymore. For such subjects, W-CAAA is equivalent with CRAA. We therefore classified them using their relative ambiguity aversion, as described by Table 1. Risk averse CARA subjects exhibited mostly W-IAAA and W-CAAA in equal share. Table 8 reports the complete results.

Table 7
Classification of subjects depending on their ambiguity attitude.

	IAAA	CAAA	DAAA	Total
Ambiguity seeking	0	1	5	6
Ambiguity neutral	6	18	11	35
Ambiguity averse	4	10	15	29
Total	10	29	31	70

(a) Ambiguity attitude and absolute ambiguity aversion

	IRAA	CRAA	DRAA	Total
Ambiguity seeking	0	0	6	6
Ambiguity neutral	5	17	13	35
Ambiguity averse	5	9	15	29
Total	10	26	34	70

(b) Ambiguity attitude and relative ambiguity aversion

	IRAA	CRAA	DRAA	Total
IAAA	5	2	3	10
CAAA	5	18	6	29
DAAA	0	6	25	31
Total	10	26	34	70

(c) Absolute and relative ambiguity aversion

	IRAA	CRAA	DRAA	Total
IAAA	2	1	1	4
CAAA	3	4	3	10
DAAA	0	4	11	15
Total	5	9	15	29

(d) Absolute and relative ambiguity aversion (ambiguity-averse subjects only)

Table 8
Classification of CARA subjects in terms of W-CAAA, W-DAAA, and W-IAAA.

	W-IAAA	W-CAAA	W-DAAA	Total
Risk seeking	1	1	4	6
Risk neutral	3	8	11	22
Risk averse	10	9	3	22
Total	14	18	18	50

(a) Risk attitude and impact of wealth

	W-IAAA	W-CAAA	W-DAAA	Total
Ambiguity seeking	2	0	2	4
Ambiguity neutral	4	14	9	27
Ambiguity averse	8	4	7	19
Total	14	18	18	50

(b) Ambiguity attitude and impact of wealth

Overall, the impact of wealth on ambiguity generates a rich variety of behavior. Sadly, the restriction to CARA decreases the sample size by almost a third and many CARA subjects were also ambiguity neutral. For non-CARA subjects, we only know that the way their ambiguity attitudes depend on wealth is irregular, and therefore, that their behavior is non-classifiable in terms of W-CAAA, W-DAAA or W-IAAA. By contrast, studying the impact of changes of utility, as in the previous subsection, has the advantage of identifying regularities that are useful in applications about saving and prevention for instance.

4. Conclusion

We designed simple tests of CAAA and CRAA based on variations of the Ellsberg examples. At the aggregate level, we found evidence for DAAA and DRAA. The magnitude of the deviations from CAAA suggests that relying on the common CAAA assumption to predict behavior at higher utility levels would lead to errors of more than 10% for a substantial proportion of subjects. CAAA and DAAA coexisted in almost equal shares in our sample of subjects. Our findings seem to encourage the use of ambiguity models that are flexible enough to accommodate changes in ambiguity attitudes at increased utility levels, such as the smooth ambiguity model (but excluding exponential and power smooth-ambiguity functions) and the new models of Ghirardato and Siniscalchi (2015) and Xue (2018). Finally, combining our results about ambiguity with those obtained for risk showed that an increase of wealth can have mixed effects on ambiguity aversion.

Appendix A. Alternative specification of the state space

Let $S_1 = \{Y, G\}$ be the state space for acts 6 and 12 and $S_2 = \{Y, G, R, B\}$ be the state space for acts 1-3 and 7-9. $\mathcal{F}_1$ and $\mathcal{F}_2$ are the sets of acts, and Δ_1 and Δ_2 the set of all measures over S_1 and S_2, respectively. We assume that agents have *uncertainty averse preferences (UAP)* as defined by Cerreia-Vioglio et al. (2011). Such preferences encompass many ambiguity models in the literature satisfying ambiguity aversion (but that need not satisfy CAAA or CRAA). According to UAP, $v_1(f_1) = min_{P\in\Delta_1} G_1\left(\int Eu(f_1)\, dP, P\right)$ for $f_1 \in \mathcal{F}_1$ and $v_2(f_2) = min_{\delta\circ P\in\Delta_2^o} G_2\left(\int Eu(f_2)\, d\delta\circ P, \delta\circ P\right)$ for $f_2 \in \mathcal{F}_2$, where G_1 and G_2 are quasiconvex (reflecting ambiguity aversion) and increasing in their first variable (reflecting monotonicity). Subjects were informed that, for acts 1-3 and 7-9, 5 red balls and 5 black balls would be added to the urn. For consistency, we make the following assumptions:

- Subjects took the objective information into account; therefore, if $P(R) \neq \frac{1}{4}$ or $P(B) \neq \frac{1}{4}$, then $G_2(t, P) = +\infty$.
- Subjects understood that adding red and black balls to the urn did not change the ambiguity about the number of yellow (green) balls in the urn; therefore, for P satisfying $P(R) = P(B) = \frac{1}{4}$, $G_2(t, P) = G_1(t, \delta\circ P)$ where $\delta\circ P$ is uniquely defined by $\delta\circ P(Y) = 2\times P(Y)$. Note that we also assume here that G_1 and $G_2$are scaled in the same way.

This consistency assumption implies that $v_2(f_2) = min_{P\in\Delta_1} G_1\left(\int Eu(f_2)\, d\delta\circ P, P\right)$. We fix $u(30) = 1$ and $u(0) = 0$.

- If $f_1 \in \mathcal{F}_1$, $f_2 \in \mathcal{F}_2$, $f_2(Y) = f_1(Y)$, $f_2(G) = f_1(G)$, and $f_2(R) = f_2(B) = 30_0 0$, then $v_2(f_2) = min_{P\in\Delta_1} G_1\left(\frac{P(Y)}{2} Eu(f_1(Y)) + \left(\frac{1}{2} - \frac{P(Y)}{2}\right) Eu(f_1(Y)), P\right) = v_1\left(\frac{1}{2} f_1 + \frac{1}{2} 30_0 0\right)$.
- If $g_1 \in \mathcal{F}_1$, $g_2 \in \mathcal{F}_2$, $g_2(Y) = g_1(Y)$, $g_2(G) = g_1(G)$, and $g_2(R) = g_2(B) = 30_1 0$, then $v_2(g_2) = min_{P\in\Delta_1} G_1\left(\frac{P(Y)}{2} Eu(g_1(Y)) + \left(\frac{1}{2} - \frac{P(Y)}{2}\right) Eu(g_1(Y)) + \frac{1}{2}, P\right) = v_1\left(\frac{1}{2} g_1 + \frac{1}{2} 30_1 0\right)$.

As a consequence, under the assumption of subjects' understanding and incorporating the objective information into their decisions, the valuations (and, therefore, the probability equivalents) obtained for f_2 and g_2 are the same as those that would have been obtained for $\frac{1}{2} f_1 + \frac{1}{2} 30_0 0$ and $\frac{1}{2} g_1 + \frac{1}{2} 30_1 0$. Hence, we can still use them to test constant relative and absolute ambiguity aversion. The same exercise could be performed for the state spaces of acts 4, 5, 10, and 11.

Appendix B. Application to the smooth ambiguity model

Under risk, CARA and CRRA correspond to exponential and power utility, respectively. We show here that the definitions (and tests) that we use in this paper for ambiguity allow us to characterize the curvature of the smooth ambiguity function of Klibanoff et al. (2005). Recall that $f(s) = \overline{m}_{p_s} 0$ for all s. We set $u(\overline{m}) = 1$ and $u(0) = 0$, which implies that $Eu(f(s)) = p_s$. Under the smooth ambiguity model, the PEs satisfy the condition $\varphi(pe(f)) = \int_{\Delta(S)} \mu(Q)\varphi\left(\sum_{s\in S} Q(s) p_s\right) dQ$, with φ the smooth ambiguity function and μ second order beliefs over $\Delta(S)$. This implies

$$\varphi\left(pe\left(\alpha f+(1-\alpha)\,\overline{\ell}\right)\right)=\int\limits_{\Delta(S)}\mu(Q)\varphi\left((1-\alpha)+\alpha\sum_{s\in S}Q(s)p_s\right)dQ$$

and

$$\varphi\left(pe(\alpha f+(1-\alpha)\,\underline{\ell})\right)=\int\limits_{\Delta(S)}\mu(Q)\varphi\left(\alpha\sum_{s\in S}Q(s)p_s\right)dQ.$$

It follows that we can apply the usual results for CEs under expected utility to our PEs under the smooth model. CAAA is thus equivalent to φ being an exponential function and CRAA to φ being a power function. This also shows that, unlike ambiguity models assuming CAAA, the smooth model can accommodate a broader range of ambiguity attitudes if φ is not exponential.

Klibanoff et al. (2005, definition 6) defined CAAA as invariance of preferences to increases in utility. Implementing a direct test of their definition would require observing utility first. Our test does not rely on such additional measurements but still enables us to study the implication of their definition (φ being exponential).

Appendix C. Deviations from expected utility under risk

Mixtures of acts and lotteries such as $\alpha f+(1-\alpha)\,\overline{\ell}$ gives corresponding mixtures of expected utility values. In our design, we only use acts such that $f(s)=\overline{m}_{p_s}0$. Hence, with u normalized such that $u(\overline{m})=1$ and $u(0)=0$, we obtain $Eu\left(f(s)\right)=p_s$, $Eu\left(\alpha f(s)+(1-\alpha)\,\overline{\ell}\right)=\alpha p_s+(1-\alpha)$, and $Eu\left(\alpha f(s)+(1-\alpha)\,\underline{\ell}\right)=\alpha p_s$. It is therefore crucial that Eu is linear in probabilities to obtain the properties about probability equivalents introduced in the previous subsections.

Now assume that expected utility under risk is replaced by rank-dependent utility (Quiggin, 1981). According to that model, with the same normalization of u, a lottery $f(s)=\overline{m}_{p_s}0$ is evaluated by $w(p_s)$. The function w is the probability weighting function and is increasing with $w(0)=0$ and $w(1)=1$. If w is nonlinear, then $w\left(\alpha p_s+(1-\alpha)\right)=\alpha w(p_s)+(1-\alpha)$ may not hold.

However, if w is linear on one interval of the probability domain, then the CAAA test based on acts yielding probabilities within that interval are still valid. As noted by Cohen (1992) and Webb and Zank (2011) (see also Chateauneuf et al., 2007), certainty effects can be accounted for by rank-dependent utility models with w being *neo-additive*, i.e., $w(p)=\frac{a-b}{2}+(1-a)*p$ for all $p\in(0,1)$. The neo-additive weighting functions generate jumps at 0 and 1, thus modeling impossibility and certainty effects. However, it is linear on $(0,1)$ and we can make use of it to test invariance to utility shifts.

The following observation will apply to models that combine rank-dependent utility for lotteries with an ambiguity model. It can for instance be applied to a sort of "maxmin rank-dependent utility", that could be written as

$$\min_{Q\in C}\left(\sum_{s\in S}Q(s)w\left(p_s\right)\right)\tag{C.1}$$

when f is of the form $f(s) = \overline{m}_{p_s}0$ and with $C \subset \Delta(S)$, the set of priors. Another example would be Klibanoff et al.'s (2005) smooth ambiguity model, with non-expected utility for lotteries where f is valued:

$$\int_{\Delta(S)} \mu(Q)\varphi\left(\sum_{s\in S} Q(s)w(p_s)\right) dQ. \tag{C.2}$$

Observation 4. *Consider an ambiguity model that values risky lotteries by rank-dependent utility with a neo-additive weighting function and that is invariant to utility shifts. Then Eq. (1.3) still holds for f of the form $f(s) = \overline{m}_{p_s}0$ with $0 < p_s < (1-\alpha)$ (i.e. neither $\alpha f + (1-\alpha)\underline{\ell}$ nor $\alpha f + (1-\alpha)\overline{\ell}$ assigns a sure outcome to any state.)*

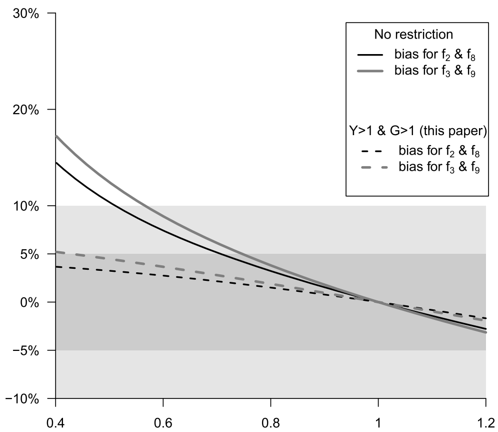
Proof. Under neo-additive rank-dependent utility for risk, an act f assigning a lottery $m_{p_s}0$ yields utility $w(p_s) = \frac{a-b}{2} + (1-a)p_s$ on state s. Act $\alpha f + (1-\alpha)\overline{\ell}$ yields utility $\frac{a-b}{2} + (1-a)(\alpha p_s + (1-\alpha))$ on state s whereas act $\alpha f + (1-\alpha)\underline{\ell}$ only yields utility $\frac{a-b}{2} + (1-a)(\alpha p_s)$ on state s. Hence, the utility on each state is higher by $(1-a)(1-\alpha)$ for the former act than for the latter. We obtain a constant increase of utility across the state space. Now consider an ambiguity model assigning $pe(f)$ to f. The value $w(pe(f)) = \frac{a-b}{2} + (1-a) \times pe(f)$ can be interpreted as the subjective value of the act, expressed in the unit of the risk model (rank-dependent utility). Invariance to utility shifts means that adding $(1-a)(1-\alpha)$ to each state increases the value of the act by exactly $(1-a)(1-\alpha)$. It must therefore imply $w(pe(\alpha f + (1-\alpha)\overline{\ell})) = w(pe(\alpha f + (1-\alpha)\underline{\ell})) + (1-a)(1-\alpha)$. Solving $\frac{a-b}{2} + (1-a) \times pe(\alpha f + (1-\alpha)\overline{\ell}) = \frac{a-b}{2} + (1-a) \times pe(\alpha f + (1-\alpha)\underline{\ell}) + (1-a)(1-\alpha)$ gives $pe(\alpha f + (1-\alpha)\overline{\ell}) = pe(\alpha f + (1-\alpha)\underline{\ell}) + (1-\alpha)$. □

Our CRAA tests are also robust to some weighting functions. The next observation establishes it.

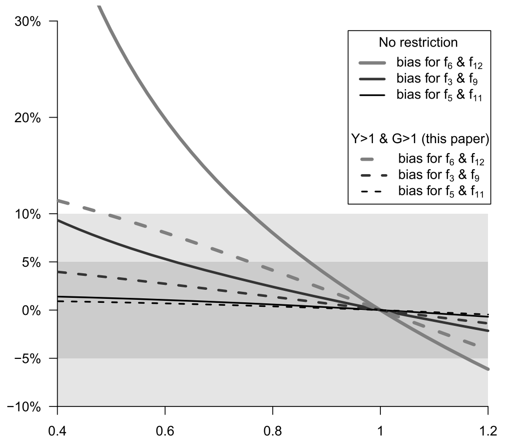
Observation 5. *Consider an ambiguity model that values risky lotteries by rank-dependent utility with $w(p) = bp^c$ defined on $[0, 1)$ and that is invariant to utility multiplication. Then Eq. (1.5) still holds for f of the form $f(s) = \overline{m}_{p_s}0$ with $p_s < 1$.*

Proof. Act f yields utility bp_s^c on state s whereas act $\alpha f + (1-\alpha)\underline{\ell}$ yields utility $b\alpha^c p_s^c$ on state s. Hence, the utility on each state is multiplied by α^c for the latter act with respect to the former. Consider a model that is invariant to utility multiplication such as (C.1), i.e. multiplying the utility by α^c on each state multiplies the value of the act by the same factor. It must therefore imply $\alpha^c \times w(pe(f))^c = w(pe(\alpha f + (1-\alpha))\underline{\ell}))$. Solving $\alpha^c \times b(pe(f))^c = b(pe(\alpha f + (1-\alpha)\underline{\ell}))^c$ gives $\alpha \times pe(f) = pe(\alpha f + (1-\alpha)\underline{\ell}))$. Note that this reasoning holds as long as all probabilities are strictly less than 1, that is, certainty is never reached on any state of the world. □

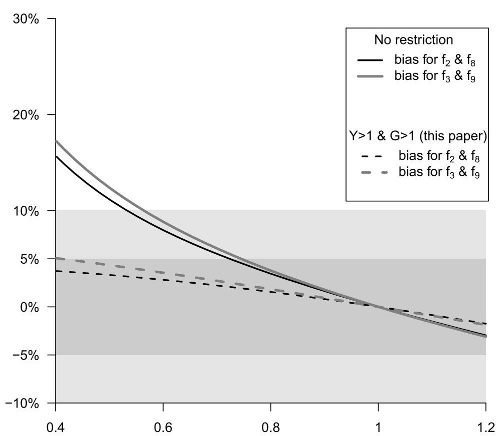
We do not have formal results for the Prelec weighting function but can compute how much it biases the tests for given parameter values. Assume that the subjects' behavior can be represented by the smooth ambiguity model as in Eq. (C.2). For further tractability, we assume that $\mu(Q) =$



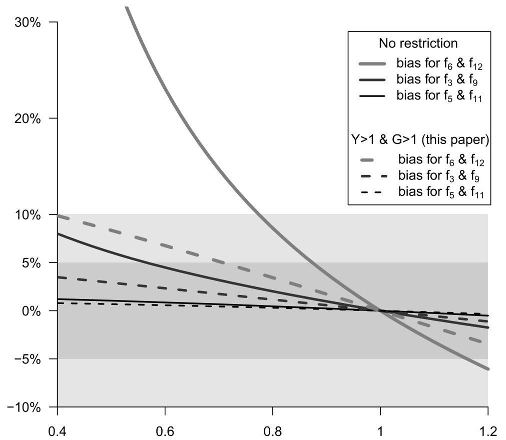
(a) CAAA tests and $\varphi(x) = x$



(b) CRAA tests and $\varphi(x) = x$



(c) CAAA tests and $\varphi(x) = -exp(-0.5x)$



(d) CRAA tests and $\varphi(x) = x^{0.5}$

Fig. C.1. Bias as a function of the probability weighting parameter.

$\frac{1}{|S|}$ if there is s such that $Q = 1_s$ and $\mu(Q) = 0$ otherwise. We obtain $\sum_{s \in S} \frac{1}{|S|} \varphi(w(p_s))$. With this formula, we can compute the probability equivalent of each act. We did so assuming φ linear (such that it should satisfy both CAAA and CRAA), exponential (such that it should satisfy CAAA), and power (such that it should satisfy CRAA). For the exponential and power functions, we chose an arbitrary parameter (0.5) to illustrate the effect of the curvature of φ. Finally, for each test described in Table 3, we computed how much the obtained probability equivalent differed from the predicted one (assuming CAAA or CRAA). For instance, if $pe(f_2)$ was 1.05 times $pe(f_1) + \frac{1}{4}$, we said that it had a 5% bias.

Fig. C.1 displays the biases for all probability equivalents we could predict as a function of ρ, the weighting function parameter. Continuous lines represent biases if we had run the experiment with no restrictions on the number of green and yellow balls; dashed lines represent biases with our restriction that there was at least one ball of each color ($Y \geq 1$ & $G \geq 1$).

Table 9
Estimates of rank-dependent utility under risk.

	Model 1	Model 2	Model 3	Model 4	Model 5
utility curvature γ	0.10**	0.16***	0.10***	0.01	0.11***
	(0.04)	(0.03)	(0.04)	(0.04)	(0.03)
Neo: insensitivity a		0.44***	0.42***		
		(0.03)	(0.03)		
Neo: pessimism b			0.04		
			(0.02)		
Prelec: insensitivity $1-\rho$				0.41***	0.46***
				(0.03)	(0.03)
Prelec: pessimism $1-\theta$					0.10***
					(0.03)
σ	0.17***	0.15***	0.14***	0.15***	0.15***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
n	770	770	770	770	770
Pseudo log-likelihood	−1539.18	−1437.50	−1435.28	−1443.48	−1436.78
AIC	3082.36	2881.01	2878.55	2892.96	2881.55

Risk neutrality is equivalent to 0 for all parameters.
Standard errors in parentheses.
* $p < 0.10$.
** $p < 0.05$.
*** $p < 0.01$.

Appendix D. Parametric fitting of weighting functions under risk

We used the CEs obtained under risk to estimate several specifications of expected utility and rank-dependent utility, using maximum likelihood and clustering standard errors at the subject level. We assumed power utility $u(x) = x^{(1-\gamma)}$ to follow the literature (e.g., Bruhin et al., 2010) even though one of our tests of CRRA rejected it (another test also rejected CARA). The neo-additive model was expressed as $w(p) = \frac{a-b}{2} + (1-a) * p$ such that a and b match the insensitivity and pessimism indices defined by Abdellaoui et al. (2011). The Prelec function, we used $w(p) = exp(-\theta(-ln(p))^{\rho})$ with ρ the insensitivity parameter and θ capturing pessimism. We followed Bruhin et al. (2010) and assumed that the error term (the difference between the observed CE and the predicted CE) followed a normal distribution with a standard deviation equal to $\sigma * |y - x|$ for lottery $x_p y$. We estimated expected utility (Model 1) and the two rank-dependent utility models with and without pessimism. Model 2 is the neo-additive model with insensitivity only, Model 3 the full neo-additive model, Model 4 the Prelec model with insensitivity only, and Model 5 the full Prelec model.

Table 9 reports the estimates expressed such that risk neutrality is equivalent to 0 for all parameters (so we report $1-\rho$ and $1-\theta$ for the Prelec function). First, note that the estimates of ρ are between 0.54 and 0.59 (Models 4 and 5), falling within the range such that the bias of the CAAA and CRAA tests does not exceed 5%. Second, expected utility (Model 1) is clearly rejected in favor of rank-dependent utility as can be seen by the significant weighting function parameters

(Models 2 to 5).[10] Third, the highest pseudo log-likelihood and the lowest AIC were obtained for the full neo-additive model (Model 3). If anything, this analysis supports the neo-additive model.

There are a few caveats to this conclusion though. We only had three probability levels and none of them was very low or very high. Many probabilities, especially extreme ones, would be necessary to properly compare the two weighting functions. Moreover, the difference in terms of pseudo-likelihood between Models 2 to 5 remains mild, compared to the difference with expected utility. We can only conclude that the weighting function between 0.25 and 0.75 was close to linear but this is already reassuring for our main results about CAAA.

Appendix E. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2019.02.006.

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[10] We also ran likelihood ratio tests for nested models on the estimates obtained without clustering standard errors and rejected expected utility in favor of the all other models.

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