Nambu-Poisson Gauge Theory

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Abstract

We generalize noncommutative gauge theory using Nambu-Poisson structures to obtain a new type of gauge theory with higher brackets and gauge fields. The approach is based on covariant coordinates and higher versions of the Seiberg-Witten map. We construct a covariant Nambu-Poisson gauge theory action, give its first order expansion in the Nambu-Poisson tensor and relate it to a Nambu-Poisson matrix model.

Keywords: Nambu-Poisson structures, noncommutative gauge theory, matrix models, M-Theory

1. Introduction

In this letter, we introduce a higher analogue of noncommutative (abelian) pure gauge theory. What we consider here is a deformation, in the presence of a background $(p+1)$-rank Nambu-Poisson tensor, of an abelian gauge theory with a p-form gauge potential, i.e., a $(p-1)$-gerbe connection. Our approach, for $p > 1$, is similar to that of [1] which deals with the more familiar case of $p = 1$. A Nambu-Poisson gauge theory was pioneered by P.-M. Ho et al. in [2] as the effective theory of M5-brane for a large longitudinal C-field background in M-theory. Related work can be found in their papers [3–5].

We formulate the theory independently of string/M-theory. Nevertheless, the motivation comes from M-theory branes; more explicitly from an effective DBI-type theory proposed for the description of multiple M2-branes ending on a M5-brane, where the Nambu-Poisson 3-tensor enters as a pseudoinverse of the 3-form field C [6, 7]. We develop the theory at a semiclassical level, briefly commenting on the issue of quantization at the end.

The paper is organized as follows: After discussing conventions in Sec. 2, we introduce in Sec. 3 covariant coordinates, which transform nontrivially with respect to gauge transformations parametrized by a $(p-1)$-form, the gauge transformation being described in terms of a $(p+1)$-bracket arising from a background Nambu-Poisson $(p+1)$-tensor. Based on these covariant coordinates, we introduce Nambu-Poisson gauge fields in Sec. 4. In Sec. 5, we construct Nambu-Poisson gauge fields explicitly, using a suitable generalization [6–8] of the Seiberg-Witten map [9], starting form an ordinary $(p-1)$-form gauge potential. We give explicit expressions for all components of the Nambu-Poisson field strength. In Sec. 6, we give the corresponding (semiclassically) "noncommutative" action and its first order expansion in the Nambu-Poisson tensor. Up to this order the the result is unambiguous, because quantum corrections from any type of quantization of the Nambu-Poisson structure will only affect higher orders. We conclude the letter by relating the action to (the semiclassical version of) a Nambu-Poisson matrix model.

We only briefly comment on deformation quantization of Nambu-Poisson structures in this letter. A satisfactory description of Nambu-Poisson noncommutative gauge theory beyond the semiclassical level will require a suitable analogue of Kontsevich's formality, solving in particular the deformation quantization problem for an arbitrary Nambu-Poisson structure.

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2. Conventions

We assume that n-dimensional space-time M is equipped with a rank $p+1$ Nambu-Poisson structure Π, with $1 < p < n$.[1] The corresponding Nambu-Poisson bracket is denoted by $\{\cdot, \ldots, \cdot\}$. In order to keep notation close to the familiar $p=1$ case, we write $\{f, \lambda\} := \Pi(df, d\lambda) = \frac{1}{p!}\Pi^{ij_1\cdots j_p}\partial_i f (d\lambda)_{j_1\cdots j_p}$ for a $(p-1)$-form λ and a function f. In the special case, where $d\lambda$ factorizes as a product $d\lambda = d\lambda_1 \wedge \cdots \wedge d\lambda_p$, we have $\{f, \lambda\} \equiv \{f, \lambda_1, \ldots, \lambda_p\}$. We consider a set of local coordinates $(x^1, \ldots, x^n)$ on M and denote the corresponding indices by lower case Latin characters i, j, k, etc.. Upper case Latin characters I, J, K, etc. denote strictly ordered p-tuples of indices, i.e. $J = (j_1, \ldots, j_p)$ with $1 \leq j_1 < \cdots < j_p \leq n$. With this notation, $\Pi(df, d\lambda) = \Pi^{iJ}\partial_i f (d\lambda)_J$. Often, we will omit indices altogether, implicitly implying matrix multiplication of the underlying rectangular matrices as in $(\Pi F^T)^i_j = \Pi^{iK} F_{Kj}$. We use Roman characters a, B, etc. for indices and multi-indices taking values only in the "noncommutative" directions $1, ..., p+1$.

3. Covariant coordinates

Before we introduce in the next section the Nambu-Poisson gauge potential[2] $\widehat{A}$ and field strength $\widehat{F}$, let us define "covariant coordinates"[3] as functions $\widehat{x}^i(x)$, $i = 1, \ldots, n$ of the space-time coordinates $\{x^i\}_{i=1}^n$, which transform under gauge transformations parametrized by a $(p-1)$-form Λ as

$$\delta_\Lambda \widehat{x}^i = \{\widehat{x}^i, \Lambda\} \, , \tag{1}$$

where the bracket is a $p+1$ Nambu-Poisson bracket (cf. Sec. 2 for notation). We assume our fixed (but arbitrarily chosen) coordinates x^i to be invariant under gauge transformations. We also assume that they can be expanded around any point $x \in M$, at least in the sense of formal power series, as $\widehat{x}^i = x^i + \ldots$. Hence, at least formally, we can always solve for x^i as functions of covariant coordinates $\widehat{x}^i$, i.e. $x^i = \widehat{x}^i + \ldots$. We denote by ρ the (formal) diffeomorphism on M corresponding to this change of local variables on M and write $\widehat{x}^i = \rho^*(x^i)$ for the respective local coordinate functions. The change of coordinates defined by ρ^* is also called "covariantizing map". The diffeomorphism ρ can be used to define a new Nambu-Poisson structure Π' with bracket $\{\cdot, \ldots, \cdot\}'$:

$$\rho^*(\{x^{i_1}, \ldots, x^{i_{p+1}}\}') := \{\rho^* x^{i_1}, \ldots, \rho^* x^{i_{p+1}}\} \equiv \{\widehat{x}^{i_1}, \ldots, \widehat{x}^{i_{p+1}}\}. \tag{2}$$

4. Nambu-Poisson gauge fields

Here and in the subsequent sections, we follow closely the semiclassical parts of [10, 11], where the $p=1$ case is described. Using covariant coordinates $\widehat{x}^i$, we define the Nambu-Poisson ("noncommutative") gauge potential with components labeled by upper indices $i = 1, \ldots, n$ as[4]

$$\widehat{A}^i = \widehat{x}^i - x^i = \rho^*(x^i) - x^i. \tag{3}$$

Its gauge transformation follows from (1)

$$\delta_\Lambda \widehat{A}^i = \{\widehat{A}^i, \Lambda\} + \{x^i, \Lambda\}. \tag{4}$$

Next, we introduce the contravariant tensor F' with components $F'^{i_1\cdots i_{p+1}}$ as the difference of the Nambu-Poisson structures Π', see equation (2), and Π:

$$F'^{i_1\cdots i_{p+1}} = \Pi'^{i_1\cdots i_{p+1}} - \Pi^{i_1\cdots i_{p+1}}. \tag{5}$$

[1] The discussion could be extended to include also the well known case $p=1$, but for clarity and brevity we concentrate here on $p>1$ and refer to [7] for $p=1$.

[2] This is the higher analog of the $p=1$ noncommutative gauge potential.

[3] Covariant with respect to the gauge transformation (4). For $p=1$ they correspond to background independent operators of [9]; they are actually dynamical fields.

[4] See [12–14] for an alternative approach related to area-preserving diffeomorphisms.

Covariantizing the individual components of this tensor using the diffeomorphism ρ, we obtain the Nambu-Poisson ("noncommutative") field strength $\widehat{F}'$ with components

$$\widehat{F}'^{i_1 \cdots i_{p+1}} := \rho^*(F'^{i_1 \cdots i_{p+1}}). \tag{6}$$

Using (5) and a hat to denote the application of ρ^*,

$$\widehat{F}'^{i_1 \cdots i_{p+1}} = \widehat{\Pi}'^{i_1 \cdots i_{p+1}} - \widehat{\Pi}^{i_1 \cdots i_{p+1}} = \rho^*(\Pi'^{i_1 \cdots i_{p+1}}) - \rho^*(\Pi^{i_1 \cdots i_{p+1}}). \tag{7}$$

Rewriting this with the help of (2) as

$$\widehat{F}'^{i_1 \cdots i_{p+1}} = \{\widehat{x}^{i_1}, \ldots, \widehat{x}^{i_{p+1}}\} - \{x^{i_1}, \ldots, x^{i_{p+1}}\}(\widehat{x}), \tag{8}$$

the gauge transformation of $\widehat{F}'$ can be easily determined:

$$\delta_\Lambda \widehat{F}'^{i_1 \cdots i_{p+1}} = \{\widehat{F}'^i, \Lambda\}. \tag{9}$$

From now on we will assume without loss of generality that the local coordinates x^i are adapted to the Nambu-Poisson structure Π, i.e., $\{x^i\}$ are local coordinates around some point M, where Π is non-zero, such that[5]

$$\Pi = \partial_1 \wedge \cdots \wedge \partial_{p+1}. \tag{10}$$

With this choice of coordinates, we find

$$\widehat{F}'^{i_1 \cdots i_{p+1}} = \{\widehat{x}^{i_1}, \ldots, \widehat{x}^{i_{p+1}}\} - \{x^{i_1}, \ldots, x^{i_{p+1}}\}, \tag{11}$$

where the second bracket is in fact either zero or equal to the $p+1$ epsilon symbol in the noncommutative directions $1, \ldots, p+1$. Roman indices $a_1, \ldots, a_{p+1}$ shall henceforth denote these directions. Furthermore, we will focus on the case where for the covariantizing map ρ^* acts trivially (i.e. $\widehat{x}^i = x^i$) on coordinates x^i with indices in the commutative directions $p+2, \ldots, n$. It follows from (1) that only the covariant coordinates in the noncommutative directions transform non-trivially under gauge transformations and that the gauge fields $\widehat{A}^i$ are trivial for $i = p+2, \ldots, n$. Also, all the field strengths, except those indexed solely by noncommutative indices $i = 1, \ldots, p+1$, will automatically be zero. With these conventions, we can use the $p+1$ epsilon tensor to lower the index on $\widehat{A}^a$ and introduce another kind of gauge potential uniquely determined by complete antisymmetrization of its non-zero components $\widehat{A}_B$ labeled by strictly ordered p-tuples of indices, with individual indices taking values in the labels of the noncommutative directions

$$\widehat{A}_B := \epsilon_{aB} \widehat{A}^a. \tag{12}$$

The components $\widehat{A}_B$ transform in a more familiar looking manner (but recall that we are still dealing with a $p+1$ Nambu-Poisson bracket and a $(p-1)$-form gauge parameter Λ):

$$\delta_\Lambda \widehat{A}_B = (d\Lambda)_B + \{\widehat{A}_B, \Lambda\}. \tag{13}$$

Similarly, we define the corresponding field strength with components $\widehat{F}'_{aB}$ by

$$\widehat{F}'_{aB} = \epsilon_{aC}(\widehat{\Pi}'^{bC} - \Pi^{bC})\epsilon_{bB}. \tag{14}$$

The components $\widehat{F}'_{aB}$ transform as expected

$$\delta_\Lambda \widehat{F}'_{aB} = \{\widehat{F}'_{aB}, \Lambda\}. \tag{15}$$

[5]Here we ignore, for simplicity, points where Π could possibly be zero and focus on globally non-degenerate Nambu-Poisson structures.

A straightforward check reveals that $\widehat{F}'_{aB}$ can be consistently extended to be antisymmetric in all of its indices. Finally, $\widehat{F}'_{aB}$ can be expressed in terms of the gauge potential $\widehat{A}_B$. For this, we need to a $(p+1-q)$-ary Nambu bracket defined as[6]

$$\{\cdot,\ldots,\cdot\}^{i_1\ldots i_q} := \{x^{i_1},\ldots,x^{i_q},\cdot,\ldots,\cdot\}.$$

Now, using (3), (11), (12) and (14) we obtain

$$\widehat{F}'_{1\ldots p+1} = (d\widehat{A})_{1\ldots p+1} + \sum_{r=0}^{p-1}\sum_{\sigma\in S(r,n-r)}(-1)^{\sum_{k=r+1}^{p+1}(\sigma(k)-1)}\mathrm{sgn}(\sigma)\{\widehat{A}_{[\sigma(r+1)]},\ldots,\widehat{A}_{[\sigma(p+1)]}\}^{\sigma(1)\ldots\sigma(r)}, \qquad (16)$$

where $\sigma\in S(r,n-r)$ is an $(r,n-r)$ shuffle, and $[a]$ is the multi-index $1\cdots(a-1)(a+1)\cdots(p+1)$. This formula is a generalization to $p>1$ of the well-known $p=1$ formula for the (noncommutative) field strength that involves the 2-bracket ("commutator") of gauge fields.

In the next section we will use a higher analog of the Seiberg-Witten map in order to construct explicit expressions for the covariant coordinates and noncommutative gauge fields. This will allow us to also supplement the remaining components of the Nambu-Poisson gauge field strength (14), i.e., the ones with at least one index in a commutative direction.

5. Nambu-Poisson gauge fields via Seiberg-Witten map

We start with a brief summary of the relevant facts concerning the Seiberg-Witten map as it applies in the present context. We refer the reader to a detailed exposition in [7]. All order solution to the Seiberg-Witten map related to Nambu-Poisson M5-brane theory can be found in [8].

Let us consider a p-form gauge potential a on M with corresponding field strength $F=da$. Infinitesimally, under a gauge transformation given by a $(p-1)$-form λ,

$$\delta_\lambda a = d\lambda, \qquad \delta_\lambda F = 0. \qquad (17)$$

Using the $(p+1)$-form F we construct from a given Nambu-Poisson tensor Π the F-gauged tensor which we denote for now by Π_F,[7]

$$\Pi_F := (1-\Pi F^T)^{-1}\Pi = \Pi(1-F^T\Pi)^{-1}. \qquad (18)$$

These expressions are to be interpreted as matrix equations for the corresponding maps sending p-forms to 1-forms, cf. Sec. 2. The superscript T stands for the transposed map. For $p>1$, the $(p+1)$-tensor Π_F is always a Nambu-Poisson one,[8] furthermore, we also have due to factorizability of Π,

$$\Pi_F = \left(1-\frac{1}{p+1}\langle\Pi,F\rangle\right)^{-1}\Pi, \qquad (19)$$

where $\langle\Pi,F\rangle = \Pi^{iJ}F_{iJ} \equiv \mathrm{Tr}(\Pi F^T)$.

Now we define a 1-parametric family of Nambu-Poisson tensors $\Pi_t := (1-t\Pi F^T)^{-1}\Pi$, cf. Footnote 7, interpolating between Π and Π_F. Differentiation of Π_t with respect to t gives:

$$\partial_t\Pi_t = \Pi_t F^T\Pi_t. \qquad (20)$$

This equation can be rewritten as

$$\partial_t\Pi_t = -\mathcal{L}_{A_t^\sharp}\Pi_t, \qquad (21)$$

[6] With some abuse of notation we allow also for the case $p=q$, i.e., the "1-ary" bracket, which will become useful later.
[7] We assume that $1-\Pi F^T$ is invertible. In a more formal approach we also could treat Π_F as a formal power series in Π..
[8] Even for a non-closed F.

where the time-dependent vector field $A_t^\sharp$ is defined as $A_t^\sharp = \Pi_t^\sharp(a) = \Pi_t^{iJ} a_J \partial_i$ and $\mathcal{L}_{A_t^\sharp}$ is the corresponding Lie derivative. Equation (21) implies that the flow ϕ_t corresponding to $A_t^\sharp$, together with the initial condition $\Pi_0 = \Pi$, maps Π_t to Π, that is,

$$\phi_t^*(\Pi_t) = \Pi. \tag{22}$$

We have thus found the map $\rho_a := \phi_1$, such that $\rho_a^*(\Pi') = \Pi$. This is the higher form gauge field ($p > 1$) analogue of the well known semiclassical Seiberg-Witten map. We emphasize the dependence of this map on the p-form a by an explicit addition of the subscript a. The following observation is important: The Nambu-Poisson tensor Π_t is gauge invariant (because it depends on the p-potential a only via the gauge invariant $p+1$ form field strength $f = da$), but the Nambu-Poisson map ρ_a is not: The infinitesimal gauge transformation $\delta_\lambda a = d\lambda$, with a $(p-1)$-form gauge transformation parameter λ, induces a change in the flow, which is generated by the vector field $X_{[\lambda,a]} = \Pi^{iJ} d\Lambda_J \partial_i$, where the $(p-1)$-form Λ, explicitly given by

$$\Lambda = \sum_{k=0}^{\infty} \frac{(\mathcal{L}_{A_t^\sharp} + \partial_t)^k(\lambda)}{(k+1)!}\Big|_{t=0}, \tag{23}$$

is the semiclassically noncommutative $(p-1)$-form gauge parameter. This leads to the following rule for the gauge transformation of coordinates $\widehat{x}_a^i := \rho_a^*(x^i)$, cf. (1):

$$\delta_\lambda \widehat{x}_a^i = \{\widehat{x}_a^i, \Lambda\}. \tag{24}$$

Hence, the generalized Seiberg-Witten map provides us with an explicit construction, based on ordinary higher gauge fields, of the covariant coordinates $\widehat{x}^i$ that we introduced in Sec. 3. As a consequence, we can identify $\widehat{x}^i \equiv \widehat{x}_a^i$ and $\Pi' \equiv \Pi_F$. Moreover, $\widehat{x}^i = \widehat{x}_a^i = x^i$, for the "commutative" directions $i = p+2, \dots n$. All discussion of the previous sections 3 and 4 applies directly.

Having the ordinary p-form gauge field a at our disposal we can now define the full Nambu-Poisson field strength $\widehat{F}'$ with all components (in noncommutative as well as in commutative directions), such that that its components in the noncommutative directions $x^1, \dots, x^{p+1}$ coincide with those of $\widehat{F}'_{aB}$ (14).

For this let

$$F' := F(1 - \Pi^T F)^{-1} = (1 - F\Pi^T)^{-1} F \tag{25}$$

and define

$$\widehat{F}'_{iJ} := \rho_A^* F'_{iJ}, \tag{26}$$

i.e., the components of F' evaluated in the covariant coordinates. It is a rather straightforward check to see that for all indices $i_1, \dots, i_{p+1}$ taking values only in the set $\{1, \dots, p+1\}$ we get exactly the $\widehat{F}'_{aB}$ of (14).

Now we turn our attention to the remaining components of $\widehat{F}'$ (including commutative directions). Starting from (25) and (26), we can with the help of (7) and the explicit expression for Π in coordinates (10) use a construction very similar to the one leading to (16). We find that the resulting expressions involve a covariant scalar function that depends on $\widehat{A}$ (and hence via the generalized Seiberg-Witten map also on the ordinary p-form gauge potential a):

$$f[\widehat{A}] := 1 + \sum_{r=0}^{p} \sum_{\sigma \in S(r,n-r)} (-1)^{\sum_{k=r+1}^{p+1}(\sigma(k)-1)} \mathrm{sgn}(\sigma) \{\widehat{A}_{[\sigma(r+1)]}, \dots, \widehat{A}_{[\sigma(p+1)]}\}^{\sigma(1)\dots\sigma(r)}.$$

Firstly, let us consider $\widehat{F}'_{aK}$ with the index a taking on values in $\{1, \dots, p+1\}$, and K containing at least one index in one of the commutative directions $p+2, \dots, n$. We find

$$\widehat{F}'_{aK} = f[\widehat{A}] \widehat{F}_{aK}, \tag{27}$$

where $\widehat{F}_{aK} = \rho^* F_{aK}$ is the component F_{aK} of the ordinary (commutative) field strength evaluated at the covariant coordinates $\widehat{x}^i$. Secondly, for the components of $\widehat{F}'$ with index k taking value in $\{p+2, \dots, n\}$, and A containing only the indices lying in the set $\{1, \dots, p+1\}$,

$$\widehat{F}'_{kA} = f[\widehat{A}] \widehat{F}_{kA}, \tag{28}$$

Finally, for the components $\widehat{F}'_{kL}$, where k takes value in the set $\{p+2, \ldots, n\}$ and L contains at least one index of the same set, we have

$$\widehat{F}'_{kL} = \widehat{F}_{kL} + f[\widehat{A}] \sum_{a=1}^{p+1} (-1)^{a+1} \widehat{F}_{k[a]} \widehat{F}_{aL}. \tag{29}$$

Under (ordinary) infinitesimal gauge transformations δ_λ, all components of $\widehat{F}'$ transform as

$$\delta_\lambda \widehat{F}' = \{\widehat{F}', \Lambda\}, \tag{30}$$

justifying calling it "Nambu-Poisson" or "(semiclassically) noncommutative" field strength.

Note that unlike for the noncommutative components, the full tensor $\widehat{F}'$ cannot be extended to be a totally antisymmetric one.

6. Action

For simplicity, we assume Euclidean space-time signature.[9] The action

$$\frac{1}{g} \int_M d^n x \widehat{F}'_{iJ} \widehat{F}'^{iJ} \tag{31}$$

is by construction invariant under ordinary commutative as well as under Nambu-Poisson (semiclassically noncommutative) gauge transformations. This can easily be verified using partial integration. The coupling constant g is dimensionless in $n = 2(p+1)$ spacetime dimensions, i.e. for example for $p = 1$, $n = 4$ (NC Maxwell) and for $p = 2$, $n = 6$ (M2-M5 system). In the following we will set $g = 1$.

We expand $\widehat{F}'$ in a power series in Π

$$\widehat{F}'_{iJ} = F_{iJ} + A_L \Pi^{kL} F_{iJ,k} + F_{iL} \Pi^{kL} F_{kJ} + o(\Pi^2). \tag{32}$$

The corresponding expansion of the action (31) is

$$\int_M d^n x \widehat{F}'_{iJ} \widehat{F}'^{iJ} = \int_M d^n x \left\{ F_{iJ} F^{iJ} - \frac{1}{p+1} F_{iJ} F^{iJ} F_{kL} \Pi^{kL} + 2 F^{iJ} F_{iL} \Pi^{kL} F_{kJ} \right\} + o(\Pi^2). \tag{33}$$

A quantization of the underlying Nambu-Poisson structure will not add quantum corrections to the action at the given order of expansionr.

Shifting the components $\widehat{F}'_{1\ldots p+1}$ of the Nambu-Poisson field strength by the constants $\epsilon_{1\ldots p+1}$, will not affect the gauge invariance of the the action (31). Using (11) and (14) we see that the action (31) with shifted $\widehat{F}'$ takes the form of a semiclassical version of a Nambu-Poisson matrix model:

$$S_M = \int d^n x \{\widehat{x}^a, \widehat{x}^A\}\{\widehat{x}_a, \widehat{x}_A\} = \int d^n x \frac{1}{p!} \{\widehat{x}^{a_1}, \ldots, \widehat{x}^{a_{p+1}}\}\{\widehat{x}_{a_1}, \ldots, \widehat{x}_{a_{p+1}}\}, \tag{34}$$

where the summation in the second expression runs over all (not strictly ordered) $(p{+}1)$-indices $(a_1, \ldots, a_{p+1})$ and $(b_1, \ldots, b_{p+1})$, with all of them in the noncommutative direction. We could actually drop the a priori restriction of the summation to noncommutative directions, since the Nambu-Poisson bracket automatically takes care of this. For a more detailed discussion of the (semiclassical) matrix model we refer to [7].

Given an appropriate quantization $[\cdot, \ldots, \cdot]$ of the Nambu-Poisson bracket and trace of the quantized Nambu-Poisson structure, the Nambu-Poisson matrix model takes the form

$$\widetilde{S}_M = \frac{1}{p!} \operatorname{Tr}[\widehat{x}^{a_1}, \ldots, \widehat{x}^{a_{p+1}}][\widehat{x}_{a_1}, \ldots, \widehat{x}_{a_{p+1}}]. \tag{35}$$

[9] Another simple possibility would be consider the Minkowskian space-time, with Π extending in the spatial directions only. In case of a general metric g we would have to use the inverse metric matrix elements evaluated in the covariant coordinates to rise the indices of $\hat{F}'$ and the density defined by the metric also evaluates in the covariant coordinates.

There have been several attempts to find a consistent quantization of Nambu-Poisson structures. One of these [15] is in fact suitable for our purposes (at least in the case $p = 2$): It is an approach based on nonassociative star product algebras on phase space, whose Jacobiator defines a quantized Nambu-Poisson bracket on configuration space. Let us mention without going into details that this approach can be adapted to provide a consistent quantization of the Nambu-Poisson gauge theory described in this letter, including a quantization of the generalized Seiberg-Witten maps. Details of this construction are beyond the scope of the present letter and will be reported elsewhere.

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