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Citation: *Applied Physics Letters* **98**, 253504 (2011); doi: 10.1063/1.3602318

View online: <http://dx.doi.org/10.1063/1.3602318>

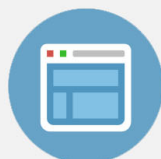
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Progressive breakdown dynamics and entropy production in ultrathin SiO₂ gate oxides

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(Received 2 May 2011; accepted 2 June 2011; published online 24 June 2011)

The progressive breakdown of ultrathin (≈ 2 nm) SiO₂ gate oxides subjected to constant electrical stress is investigated using a simple equivalent circuit model. It is shown how the interplay among series, parallel, and filamentary conductances that represent the breakdown path and its surroundings leads under certain hypothesis to a sigmoidal current-time characteristic compatible with the experimental observations. The dynamical properties of the breakdown trajectories are analyzed in terms of the logistic potential function, the Lyapunov exponent, and the system's attractor. It is also shown that the current evolution is compatible with Prigogine's minimum entropy production principle. © 2011 American Institute of Physics. [doi:10.1063/1.3602318]

Ultrathin (oxide thickness < 2 nm) SiO₂ dielectric films exhibit a gradual change in their insulating properties when subjected to constant electrical stress.¹ This is called progressive breakdown (BD) and is the result of the aggregation of defects and the subsequent formation of a localized leakage path across the oxide layer. Several letters have revealed that after the BD onset, the current flowing through the structure increases, and reaches a saturation level determined by the final resistance of the BD path circuit.² The current increment is related to the enlargement of the BD spot size and therefore to an increase in its conductance. However, the way the current increases is still controversial, yet a power-² and an exponential-law³ seem to have the widest consensus. If a self-limiting mechanism for the current is invoked, the logistic curve can be fitted to the experimental current-time (I - t) characteristic.⁴ This is the solution of the Verhulst equation, which is the standard model for population growth with limited resources. In a subsequent work, the sigmoidal behavior was linked to the physics of mesoscopic conductors and to the power dissipated inside the BD path,⁵ but a derivation of the logistic model was still missing. Even though the model proposed here relies on some of these earlier ideas, the resulting closed-form expressions provide a more clear picture of what's going on inside the system under study. The fundamental difference with previous approaches is the starting hypothesis which involves a self-limited current increase driven by the "free-motion" conductance of the BD spot. As it will be demonstrated in this letter, this basic premise leads to a sigmoidal I - t characteristic. Moreover, thanks to the oversimplified treatment, classical mathematical tools mainly developed to characterize the trajectory of mechanical systems, such as the potential function, the Lyapunov exponent (LE), and the attractor space can be brought into play. Since we are dealing with the formation of an out-of-equilibrium dissipative structure, a thermodynamic interpretation for the system's evolution based on Prigogine's minimum entropy production principle is also provided.

Figure 1 shows the equivalent electrical circuit considered for the broken down gate oxide. G_S is the series con-

ductance (external or internal), G_P is the tunneling conductance (in an extended model it comprises information about the oxide thickness and device area), G_{BD} is the BD path conductance, and V the applied bias. For the sake of simplicity, it is assumed that all circuit elements obey Ohm's law and any possible classical residual⁶ or contact resistances associated with quantum mismatches at the two ends of the constriction⁵ is disregarded. While G_P and G_S provide the physically necessary lower and upper bounds for the BD trajectory, $G_{BD}(t)$ represents the evolution of the spot's transmission properties without constraints. In particular, for the exponential dynamics, G_{BD} reads

$$G_{BD}(t) = G_{BD0}e^{\alpha t} = e^{\alpha(t-\tau)}, \quad (1)$$

where $\alpha > 0$ is a constant and G_{BD0} the conductance at $t=0$. Notice that G_{BD0} is equivalent to a time shift $\tau = -\ln(G_{BD0})/\alpha$. For practical purposes, τ will also include the time-to-BD. Eq. (1) expresses that the BD constriction evolves toward a ballistic filamentary path. Then the current at any time is given by the relationship

$$I(t) = \frac{I_\infty}{1 + G_S[e^{\alpha(t-\tau)} + G_P]^{-1}}, \quad (2)$$

where $I_\infty = G_S V$ is the current for $t \rightarrow \infty$. Notice that Eq. (2) yields a sigmoidal I - t characteristic as proposed in Refs. 4 and 5. Figure 2(b) shows experimental and simulation results obtained from Eq. (2). Detailed information about the devices used in this study and stress conditions can be found in

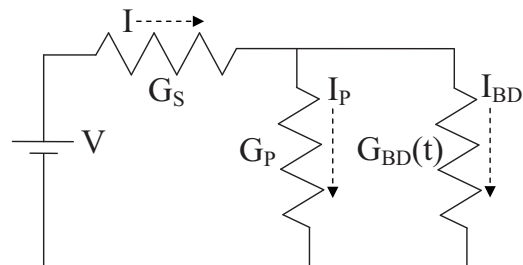


FIG. 1. Equivalent circuit model for the broken down gate oxide. G_S is the series conductance, G_P the tunneling conductance, G_{BD} is the BD path conductance, and V the applied bias. I , I_P , and I_{BD} are the currents flowing through the system. t is the time.

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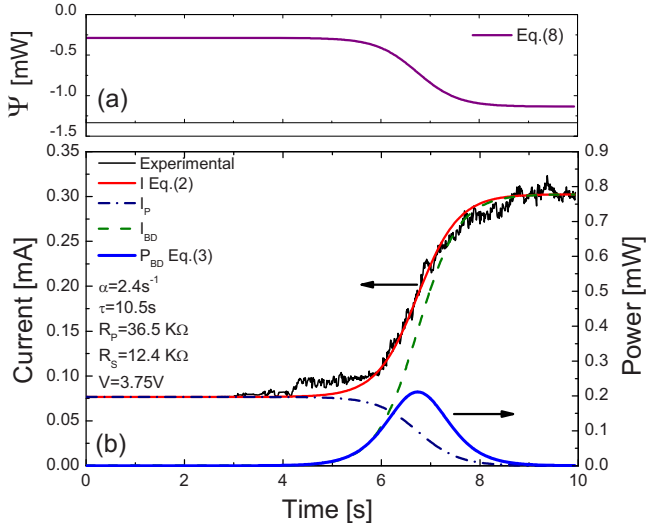


FIG. 2. (Color online) (a) Dissipation function Ψ as function of time [see Eq. (8)]. (b) I-t characteristics during a constant voltage stress ($V=3.75$ V) for a 2-nm-thick oxide layer in a metal-oxide-semiconductor structure. The noisy solid line is the experimental data, the thin solid line the simulated current using Eq. (2), the dashed lines are the currents flowing through the branches of the circuit illustrated in Fig. 1. The heavy solid line is the power dissipated in the BD spot Eq. (3).

Ref. 5. The same figure shows the evolution of the power dissipated in the BD spot

$$P_{BD}(t) = (V - G_S^{-1}I)I_{BD} = e^{\alpha(t-\tau)}(V - G_S^{-1}I)^2. \quad (3)$$

Notice that, as pointed out in Ref. 5, as the degradation proceeds, the power dissipated in the BD spot goes through a maximum and finally decreases to zero, indicating that, at this point, the power is entirely dissipated in G_S . When this occurs, the degradation stops and the system enters into an stationary phase compatible with the external constraint (applied voltage). This is consistent with the mesoscopic viewpoint for a ballistic conductor in which power dissipation takes place at the electron reservoirs.⁷

The logistic differential equation associated with Eq. (2) can be expressed as

$$F(I) = \frac{dI}{dt} = \alpha I_\infty \frac{G_P}{G_S} \left(\frac{I}{I_\infty} - 1 \right) \left(1 - \frac{I}{I_\infty} \right), \quad (4)$$

where $I_\infty = (G_S G_P / G_S + G_P)V$ is the current in the limit $t \rightarrow -\infty$. Equation (4) has a positive equilibrium at I_∞ that is stable, which, as shown below, is the attractor for all current initial conditions. On the contrary, I_∞ is an unstable equilibrium. This can be clearly seen from the potential equation $F(I) = -d\Phi(I)/dI$, where

$$\Phi(I) = \alpha \frac{G_P}{G_S} I_\infty \left[I - \left(\frac{1}{I_\infty} + \frac{1}{I_\infty} \right) \frac{I^2}{2} + \left(\frac{1}{I_\infty I_\infty} \right) \frac{I^3}{3} \right] \quad (5)$$

and which is plotted in Fig. 3. The minimum in this plot corresponds to the positive equilibrium situation and $F(I)$ can be regarded as the field force in which the system evolves. The region $I > I_\infty$ is only accessible through system's fluctuations (not considered here). Within this dynamical context, a measure of the divergence or convergence of nearby trajectories is given by the LE, a key tool of chaotic dynamics.⁸ LE is an average over the deterministic attractor and can be calculated from the Jacobian of the logistic model

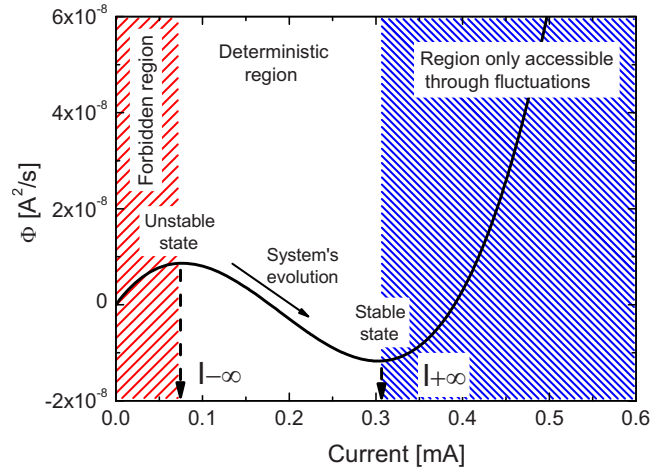


FIG. 3. (Color online) Logistic potential function vs the current flowing through the circuit. Three regions are identified: a forbidden region for currents lower than $I(-\infty)$ (the current cannot be lower than the tunneling current I_P), the deterministic region in which the system evolves, and a region above $I(+\infty)$ only accessible through current fluctuations (not included in the reported treatment). The stable state is the minimum of the function.

$$LE = \left. \frac{dF}{dI} \right|_{I=I_\infty} = -\alpha < 0. \quad (6)$$

The negative sign of LE confirms the absence of deterministic chaos, as it is expected for a one-dimensional autonomous differential equation.⁹ However, it is worth mentioning that the eigenvalue of the derivative $dF(I^*)/dI=0$, at which maximum dissipation in the BD spot occurs, is the average current $I^* = (I_\infty + I_{-\infty})/2$. I^* separates the state space of currents I-t into two regions as illustrated in Fig. 4. This means that two initial conditions satisfying $I(t=0) < I^*$ separated by a small distance will have trajectories that diverge from each other for a short time period. For larger injection times both currents will reach the region $I > I^*$ and their distance will eventually decrease as the trajectories approach I_∞ . In other words, the system is insensitive to the initial conditions in the long term run, where G_S plays a dominant role. This behavior is illustrated in Fig. 4 for two experimental curves with different time-to-BD. A common practice in connection

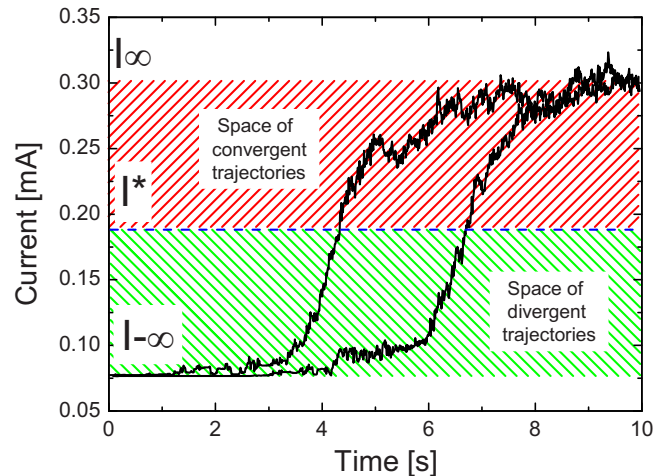


FIG. 4. (Color online) I-t characteristics measured in two different devices. Two regions are considered: below I^* , the region of divergent trajectories, above I^* , the region of convergent trajectories. I^* separates the space into these two regions.

with progressive BD is to calculate the degradation rate DR from the slope of the current in the rapidly growing phase.³ From Eq. (4), DR can be calculated as

$$DR = F(I^*) = \frac{\alpha G_S^2 V}{4(G_S + G_P)}, \quad (7)$$

which occurs at a time $t^* = \alpha^{-1} \ln(G_S + G_P) + \tau$. Equation (7) shows that DR is not an inherent property of the BD spot (like α) but that it is affected by the series and parallel resistances in the post-BD circuit. For the data shown in Fig. 2(b), $DR = 1.35 \times 10^{-4}$ A/s while $dI_{BD}/dt = 1.81 \times 10^{-4}$ A/s.

The above dynamical analysis would be incomplete if the irreversibility of the oxide degradation process is not taken into consideration. Let us now calculate the entropy production associated with the conversion of electrical energy into heat for the circuit illustrated in Fig. 1. To this end, we invoke Prigogine's or Onsager-Prigogine's (OP) minimum entropy production principle, which establishes that "in the linear regime, the total entropy production rate dS/dt in a system subject to flow of energy and matter reaches a minimum value at the stationary state."¹⁰ This principle is also valid for mesoscopic systems.¹¹ Notice that since we are exclusively dealing with a resistor network and the circuit is assumed isothermal, OP's principle is equivalent to Maxwell's minimum dissipation theorem,¹² from which Kirchhoff's laws for circuits can be derived. As claimed several times, caution should be exercised with the application of these "general" principles to more complex circuits since some restrictions apply.¹³ For a resistor network with a generator V (see Fig. 1), the dissipation function Ψ ^{14,15} reads

$$\Psi = TdS/dt = G_S^{-1}(I_P + I_{BD})^2 + G_P^{-1}I_P^2 + G_{BD}^{-1}I_{BD}^2 - 2V(I_P + I_{BD}) = -VI, \quad (8)$$

where T is the absolute temperature and I the current given by Eq. (2). Clearly, the global minimum of Eq. (8) is reached for $t \rightarrow \infty$ as shown in Fig. 2(a), in total consistency with OP principle. Notice also that $\partial\Psi/\partial I_P = \partial\Psi/\partial I_{BD} = 0$ yields the mesh equations for the circuit in Fig. 1. It is worth pointing out that a resistive network with constant sources is always

in the steady state, since there are no dynamic elements associated with energy storage. In our case, the way the system approaches the attractor is ultimately ruled by $G_{BD}(t)$.

In summary, a simple analytic model for the progressive BD dynamics was presented. The ideas exposed here can be easily extended to more complex dynamics including nonlinear circuit elements (i.e., for the tunneling conductance) or using other dynamical models for the filament conductance (such as a power-law). Moreover, an stochastic model can also be generated including a noise term in α . We have shown that an alternative description based on well-known dynamical concepts can open a new road to the interpretation of oxide BD phenomenology.

The authors acknowledge the support of the Spanish Ministry of Science and Technology under Contract No. TEC2009-09350 (partially funded by the European Union FEDER program), and the DURSI of the Generalitat de Catalunya under Contract No. 2009SGR783.

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