Investigation of a CO_2 laser response to loss perturbation near period doubling

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We have observed, in experiments with a loss-driven CO_2 laser operating near the onset of a period-doubling bifurcation, strong amplification of a perturbation signal at half the driving frequency. However, deamplification has been observed above the bifurcation point. Using a two-level laser model we have obtained reasonably good agreement between numerical and experimental results.

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I. INTRODUCTION

Small signal amplification and deamplification phenomena near the onset of a period-doubling bifurcation (PDB) have attracted much attention recently because of their possible applications for signal amplification, in particular to increase sensitivity in modulation intracavity spectroscopy [1] and because of their relationship with the phenomenon of "squeezed light" in quantum optics [2]. The response of nonlinear dynamic systems to small perturbations near simple critical points has been studied theoretically and experimentally in diverse parametric systems [3–10] including lasers with modulated losses [9,10]. The authors of Ref. [9] observed that a CO_2 laser with modulated losses is able to increase or suppress noise perturbations near a PDB. They showed that the gain factor for the noise component in phase with the modulation could be larger than 100. Similar results were obtained in Ref. [10] where transient phenomena induced by short lived loss perturbations were investigated in a loss-modulated CO_2 laser.

In this paper we report the study of the effects of periodic loss perturbations at frequency f_p on the response of a CO_2 laser with losses modulated at a driving frequency $f_d \approx 2f_p$ that is close to a PDB, both from the experimental and theoretical points of view. Two features distinguish this work from previous ones in which the laser response to a periodic loss perturbation was studied. First, in this work we cannot consider the perturbation to be small. It is quite large in the sense that the laser response is not linear. Moreover, we show here under what conditions one can consider the perturbation to be small and how the perturbation amplitude affects the gain factor. The second feature of this work is the study of the laser response not only below but also above the PDB point. This allows us to observe perturbation "deamplification" experimentally, i.e., a very strong decrease in the laser intensity at the perturbation frequency, above the bifurcation point. This phenomenon is related to the effect of noise deamplification observed earlier by Bocko and Battiato in a phase-sensitive measurement in a nonlinear electrical circuit [11]. Wiesenfeld and McCarley [12] proposed an explanation of this effect based on a linear analysis. They found a fundamental connection between parametric amplification, deamplification, and the onset of bifurcations. Generally speaking, the behavior of the system being studied here is very similar to that of a parametric amplifier. As in Ref. [11], we use the term "deamplification" since we compare the output intensities of the laser at frequency f_p in the presence and in the absence of the main loss modulation at frequency f_d.

The paper is organized as follows. In Sec. II we report experimental results showing signal amplification below the PDB point, deamplification above it, and a strong dependence of the gain factor on the perturbation amplitude. To interpret these results theoretically, we use a simple two-level laser model (Sec. III) that exhibits a dynamic behavior in reasonably good agreement with the experiment. The model also predicts a strong dependence of the gain factor on the perturbation phase. The main conclusions of this work are summarized in Sec. IV.

II. EXPERIMENT

The experimental setup is schematically shown in Fig. 1. We used a cw, single mode, frequency stabilized, CO_2 laser. The cavity was formed by a totally reflecting, spherical, gold-coated, mirror with a radius of curvature of 1 m and a grating operating under the autocollimation

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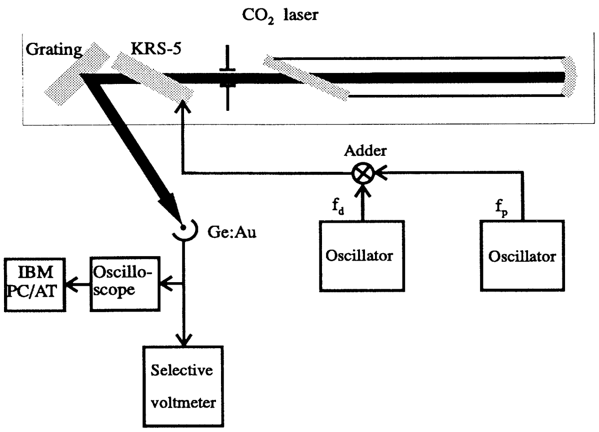

FIG. 1. Scheme of the experimental setup. The digital oscilloscope and the computer were used only for recording the results presented in Fig. 2. All the rest of the measurements were performed with the selective voltmeter.

condition. The coefficient of reflection of the grating in the first order was 0.92. The active medium consisted of a CO_2:N_2:He=1:1:5 gas mixture at a pressure of 27 Torr. The base line of the resonator was 95 cm, the active medium length was 35 cm, and the discharge tube diameter was 8 mm. The long-term power fluctuations were less than 2%. The laser operated on the 10P(22) line with a TEM_{00} mode.

The losses of this laser were modulated at frequency f_d by means of an acousto-optic modulator with a KRS-5 crystal (a cubic crystal of thallium bromoiodide) positioned inside the cavity at the Brewster angle. The modulator was able to produce a sinusoidal modulation of the laser intensity at frequencies up to 300 kHz. For the experiments reported here, we chose $f_d = 102.3$ kHz since it was one of the resonance frequencies of the crystal. By increasing the amplitude of the voltage supplied by the oscillator we found that the laser reached a PDB for amplitudes of the order of 7 V. The relaxation oscillation frequency f_R of the CO_2 laser was 180 kHz. Therefore the laser was operated in the regime $f_d < f_R$ in which it can follow the loss changes. A signal from a different oscillator at frequency $f_p = f_d/2 - \delta$ ($\delta \ll f_d$), very close to the frequency $f_p/2$ of the component of the laser intensity that appears at the PDB point, was fed to the same crystal. (In the following, for convenience, the signals at frequencies f_d and f_p will be called driving and perturbing signals, respectively.) The laser output was recorded by means of a Ge:Au photodetector with a time resolution smaller than 1 μs. The signal from the photodetector was fed to a selective voltmeter which was tuned to $f_d/2$ and whose bandwidth was of the order of 0.5 kHz.

Under the conditions mentioned above, our CO_2 laser can be considered a parametric amplifier with a central frequency equal to half the driving frequency. In general, if the perturbing signal has a detuning from $f_d/2$, an idler frequency appears in the system [4,6]. For illustration, Fig. 2(a) shows a typical time response of our loss-modulated CO_2 laser under the influence of a large perturbing signal with relatively large frequency detuning ($\delta \approx 2$ kHz). The corresponding Fourier spectrum, obtained with a digital oscilloscope, is shown in Fig. 2(b). One can clearly see the emergence not only of the idler frequency ($f_{\text{idler}} = f_p + 2\delta$) but also of a closely spaced set of other peaks in the spectrum of the laser intensity. In all the rest of the experiments we used very small detunings ($\delta \approx 10$ Hz). Since the bandwidth of our selective voltmeter was much larger than 2δ, the signal measured with the voltmeter was due to the laser response at both the perturbation and idler frequencies. Moreover, one can consider that the perturbing signal had a frequency $f_d/2$ (i.e., $\delta = 0$) but a random phase ($0 < \varphi < 2\pi$) since the phase of the perturbing oscillator changed on a time scale shorter than the time constant of the selective voltmeter that was of the order of a few seconds.

The temporal behavior of the laser response depends on the bifurcation parameter $\mu = V/V_{\text{bif}}$, where V is the driving voltage supplied by the oscillator at frequency f_d and V_{bif} is the driving voltage at which the PDB appears. To measure the influence of the perturbing signal on the response of the CO_2 laser near the PDB point we used the gain factor G defined as

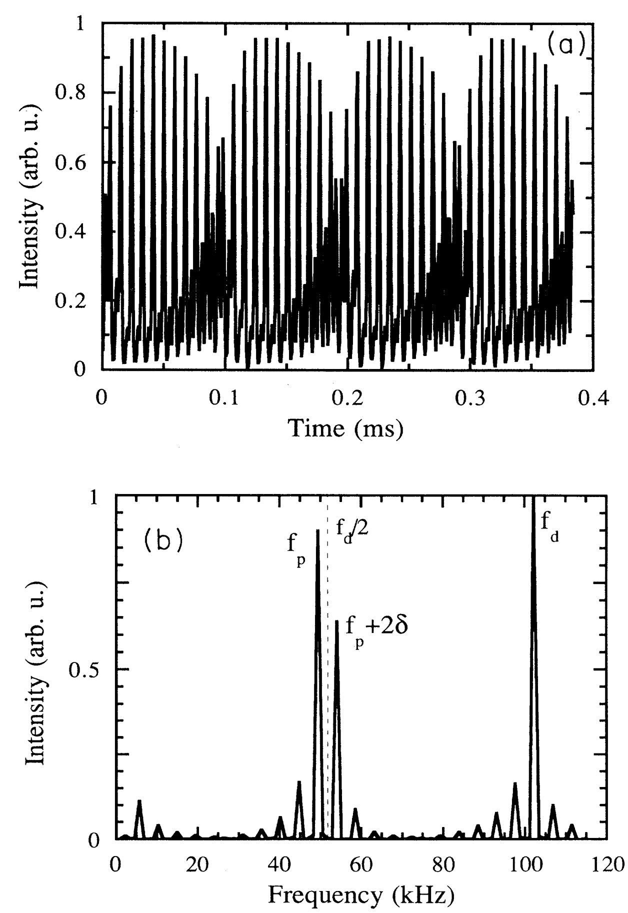

FIG. 2. Laser response to a large loss perturbation below the PDB point for $\delta \approx 2$ kHz: (a) time dependence and (b) frequency spectrum.

$$G=\frac{I_t(f_p)-I_d(f_p)}{I_p(f_p)} ,$$

where $I_p(f_p)$, $I_d(f_p)$, and $I_t(f_p)$ are, respectively, the laser intensities at frequency f_p due to only the loss perturbing signal, due to only the loss driving signal, and due to the total loss modulation, i.e., due to both the perturbing and driving signals acting simultaneously. Notice that $I_d(f_p)=0$ below the PDB point. $I_p(f_p)$, $I_d(f_p)$, and $I_t(f_p)$ were measured with the selective voltmeter. The experimental dependence of the gain factor on the perturbation amplitude [expressed in terms of $I_p(f_p)$ since it is proportional to the voltage, at frequency f_p, applied to the KRS-5 crystal] is shown in Fig. 3 in log-log scale for $\mu=1$ (PDB point). One sees there that the gain factor increases considerably with decreasing perturbation amplitude. We find that this dependence obeys the scaling law

$$G\propto[I_p(f_p)]^{-\alpha} , \tag{1}$$

where α, found from experimental data by a linear regression, is very close to $\frac{2}{3}$. The continuous line in Fig. 3 represents the fitting of the experimental data to Eq. (1) when $\alpha=0.675\,35$.

Figure 4 shows the gain factor obtained experimentally as a function of the bifurcation parameter μ for two different values of the perturbation amplitude. One can see that upon increasing μ from 0 to 1 the gain factor increases and then, above the PDB point ($\mu>1$), it decreases and takes negative values. This means that the addition of the perturbing signal reduces the component at frequency $f_d/2$ of the laser intensity obtained only with a driving modulation. (Further discussion about the negative-gain region is included in the next section.)

Similar behavior for signal amplification and deamplification has also been observed close to the $2T$-$4T$ bifurcation point T is defined as $T=1/f_d$, for a frequency of the perturbing signal approximately equal to $f_d/4$.

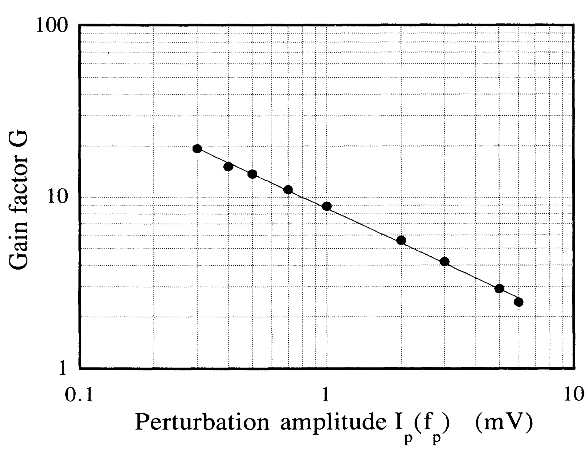

FIG. 3. Experimental gain factor G (for definition, see text) as a function of the perturbation amplitude, expressed in terms of $I_p(f_p)$, near the PDB point in log-log scale. $f_d=102.3$ kHz. Points represent experimental data. The solid line represents Eq. (1) for $\alpha=0.675\,35$.

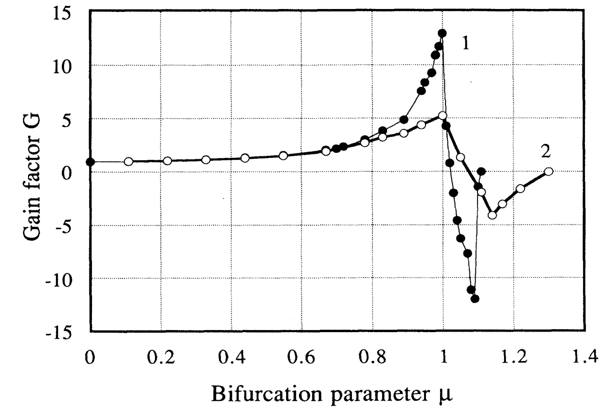

FIG. 4. Experimental gain factor G as a function of the bifurcation parameter μ for two different values of the perturbation amplitude measured in terms of $I_p(f_p)$. (1) $I_p(f_p)=0.2$ mV, and (2) $I_p(f_p)=2$ mV.

III. THEORY AND DISCUSSION

For many experimental situations, a CO_2 laser can be described using a simple two-level rate-equation model in the usual plane-wave approximation [13]. To simulate numerically the experiment described above we considered the following set of equations:

$$\frac{dI}{dt}=-2k(t)I(t)+2k_cPI(t)D(t) ,$$
$$\frac{dD}{dt}=-\gamma_{\parallel}[D(t)-1+I(t)D(t)] , \tag{2}$$
$$k(t)=k_c+k_d\cos(2\pi f_dt)+k_p\cos\left[2\pi\frac{f_d}{2}t+\varphi\right] .$$

In these equations P is the pump parameter and I and D are the laser intensity and the population inversion, respectively. $\gamma_{\parallel}$ is the population relaxation rate. $k(t)$ represents the time-dependent losses, k_c is the constant cavity damping rate mainly due to the out-coupling mirror. k_d and k_p are the amplitudes of the driving and perturbing signals, respectively. We consider the case where the perturbing signal has exactly half the frequency of the driving signal but a fixed phase difference φ.

The parameters used in the simulations are intended to describe as closely as possible the experimental situation described in the preceding section. Thus we took a cavity damping rate $k_c=7.5\times10^6$ s^{-1} and a population relaxation rate $\gamma_{\parallel}=10^4$ s^{-1}. The pump parameter P was fixed at 10. The frequency of the relaxation oscillations of a CO_2 laser with constant losses [$k_d=k_p=0$ in Eqs. (2)] can be calculated using the formula [13]

$$f_R=\frac{1}{2\pi}\sqrt{2\gamma_{\parallel}k_c(P-1)} .$$

For the parameters used in the calculations this gives the value 185 kHz, close to the relaxation oscillation frequency observed in the experiment. The modulation frequency f_d was fixed at 100 kHz. The three remaining parameters k_d, k_p, and φ were varied in the numerical simulations. These simulations were performed with a Runge-Kutta algorithm of seventh to eighth order. To calculate the gain factor G numerically, we had to obtain $I_p(f_p)$, $I_d(f_p)$, and $I_t(f_p)$, which are the components of the Fourier transform of $I(t)$ at frequency $f_d/2$ when $k_d=0$ and $k_p\neq 0$; $k_d\neq 0$ and $k_p=0$; $k_d\neq 0$ and $k_p\neq 0$, respectively. The Fourier transform was performed using a fast Fourier transform (FFT) algorithm with a time series of 65 536 points (128 periods of the driving modulation multiplied by 512 points per period).

With the parameters used, a PDB occurs for $k_d=k_d^{\text{bif}}=1.345\times 10^6\ \text{s}^{-1}$ and $k_p=0$. Figure 5 shows the calculated gain factor as a function of k_p at the bifurcation point, $\mu=k_d/k_d^{\text{bif}}=1$, for $\varphi=0$. One can distinguish three regions. In the first one (I) the gain factor is constant. This means that the perturbing signal is really small because the laser response is simply proportional to the amplitude of the perturbing signal. [One should notice, for this parameter range, that $I_p(f_p)$ is proportional to k_p. Moreover, since $I_d(f_p)=0$ below or at the bifurcation point, a constant gain factor implies that $I_t(f_p)$ is also proportional to k_p.] In particular, in our case, the perturbing signal is small if $k_p/k_d^{\text{bif}}<10^{-3}$ at the PDB point. In the second region (II) we have obtained the same scaling law, $\alpha\approx\frac{2}{3}$, as in the experiment (compare with Fig. 3) although here φ equals zero while the experimental gain was phase averaged. In the third region (III), the perturbing signal is of the same order of magnitude as the driving signal and there is no significant amplification or deamplification.

Since in all the numerical simulations the frequency f_p of the perturbing signal was considered equal to half the frequency f_d of the driving signal, the relative phase between the two signals plays an important role. It is not difficult to show that in general there exist two solutions of Eqs. (2) with a phase difference of π. In other words, if the gain factor $G(\varphi)$, as a function of φ, is a solution of the equations for a certain set of parameters, then, for the same set of parameters, $G(\varphi+\pi)$ is also a solution. We have found numerically that these two solutions are equal for $\mu<1$ (i.e., below the PDB point) and, thus, there is a single solution with period π. However, these two solutions are different for $\mu>1$ (above the PDB point) and both have a period 2π. In addition to these two π-dephased solutions, we have found another solution, which corresponds to a laser output with a very small component at frequency $f_p=f_d/2$ [i.e., $I_t(f_p)\approx 0$], that gives for $\mu>1$ a very large deamplification $G<0$. Moreover, we have found that for this solution $I_t(f_p)<I_p(f_p)$. This means that even though there is a period-doubling bifurcation, and thus the half driving frequency $f_d/2$ appears naturally, the laser output has a component at this frequency smaller than the one obtained with only a perturbing signal, without having a PDB. However, we have to emphasize that this third solution is qualitatively different from the previous two because it is associated with a period-4 attractor whereas the other two correspond to period-2 attractors of similar shape. The coexistence of different attractors is a phenomenon known as generalized multistability and was described for the first time in a CO_2 laser system by Solari *et al.* [14].

Figures 6(a)–6(c) show the phase dependence of the gain factor near the PDB for three values of the amplitude of the driving signal. Figure 6(a) corresponds to $\mu=1$. The gain factor is always positive in this case, leading to a large value of the phase-averaged gain, just as in the experiments (see Fig. 4) and, as explained above, the two π-dephased solutions have merged into a single one with a period of π. In Fig. 6(b) the gain factor versus φ is represented for $\mu=1.04$. The two π-dephased solutions and the third one with $I_t(f_p)\approx 0$ are shown. One can observe a discontinuity in the solutions 1 and 2 within the regions defined by the two dashed rectangles. This discontinuity is explained with the help of Fig. 7. In this figure we have represented $I_d(f_p)$ and $I_t(f_p)$, for three different values of φ, as a function of μ near the PDB point. One sees that for $\mu<1.02$ there is only one solution of $I_t(f_p)$ for the three values of φ represented in the figure and that when $\mu>1.046$ there is a second solution $I_t(f_p)$ for each value of φ. These second solutions appear in saddle-node bifurcations of period-2 orbits when $\mu=1.024$, 1.04, and 1.046 for $\varphi=0°$, 32°, and 57°, respectively. One can understand now the origin of the discontinuity of $G(\varphi)$. For example, looking at Fig. 7 one sees that the saddle-node bifurcation of the second solution for $\varphi=57°$ occurs at $\mu=1.046$, which is larger than $\mu=1.04$. Thus, when $\mu=1.04$, $G(\varphi)$ takes only one value for $\varphi=57°$ [cf. Fig. 6(b)]. The gain factor dependence on φ when $\mu=1.06$ is shown in Fig. 6(c). Again, the two π-dephased solutions are clearly seen and now, for each value of φ, there is one solution with amplification and another one with deamplification without any discontinuities. This is due to the fact that the saddle-node bifurcations at which the second solutions originate occur for $\mu<1.06$ for all values of φ. In Fig. 6(c) the third solution is not presented because it was seldom obtained. It seems that this third solution, the one with $I_t(f_p)\approx 0$, is less likely to appear as we increase

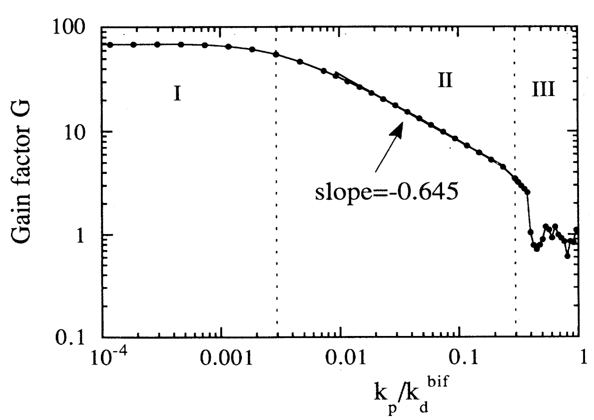

FIG. 5. Numerically calculated gain factor G as a function of the perturbation amplitude k_p at the bifurcation point ($\mu=1$) for $\varphi=0$. Three different regions (I, II, and III) can be distinguished as explained in the text.

the bifurcation parameter μ.

As explained in Sec. II, one can consider that the gain factor obtained experimentally is averaged over the phase φ. According to the difficulties encountered in obtaining numerically the lower-branch solution [i.e., that with $I_t(f_p)\approx 0$], for $\mu > 1.06$ one can make the assumption that only the two upper branches are significant. Furthermore, assuming that these two branches are equally probable (for each value of φ), one obtains an average gain that is negative, the modulus of which increases as μ decreases, approaching $\mu = 1$, just as in the experiments. For $\mu < 1.06$ the situation is different: the lower branch is now significant and the two upper branches are discontinuous. This implies that now one cannot make assumptions, as was done for $\mu > 1.06$, about the probability with which the real laser system, with its inherent experimental noise, will visit each solution. Consequently, for $\mu < 1.06$ we do not calculate the average gain. The general features observed in Figs. 6(a)–6(c) when the bifurcation parameter μ is varied are qualitatively similar to those found by Bryant and Wiesenfeld [4] in the model they considered. In Fig. 8 we show the numerically calculated gain factor as a function of the bifurcation parameter μ, for $\varphi = 0$. For $\mu > 1$ there are two branches, shown by open and closed circles, which correspond to the two π-dephased solutions. To make a comparison with the experiment we have argued that a phase-averaged gain factor would be more appropriate. For $\mu < 1$ this phase-averaged gain almost coincides with the $\varphi = 0$ gain plotted in Fig. 8. In the boxed region of this figure, however, the phase-averaged gain cannot be calculated, as discussed in the preceding paragraph. The squares in Fig. 8 denote the phase-averaged gain for large values of μ. When μ decreases these squares go closer to the lower than to the upper $\varphi = 0$ gain branch, thus behaving as in the experiment. In the experiment, strong fluctuations were found just above the bifurcation point (negative slope region in Fig. 4 where the points shown correspond to very long time averages, longer than 3 s), which might be related with the complexity of the three solutions given by the theoretical model in that region [see Fig. 6(b)]. For $\mu > 1.15$ we have drawn error bars

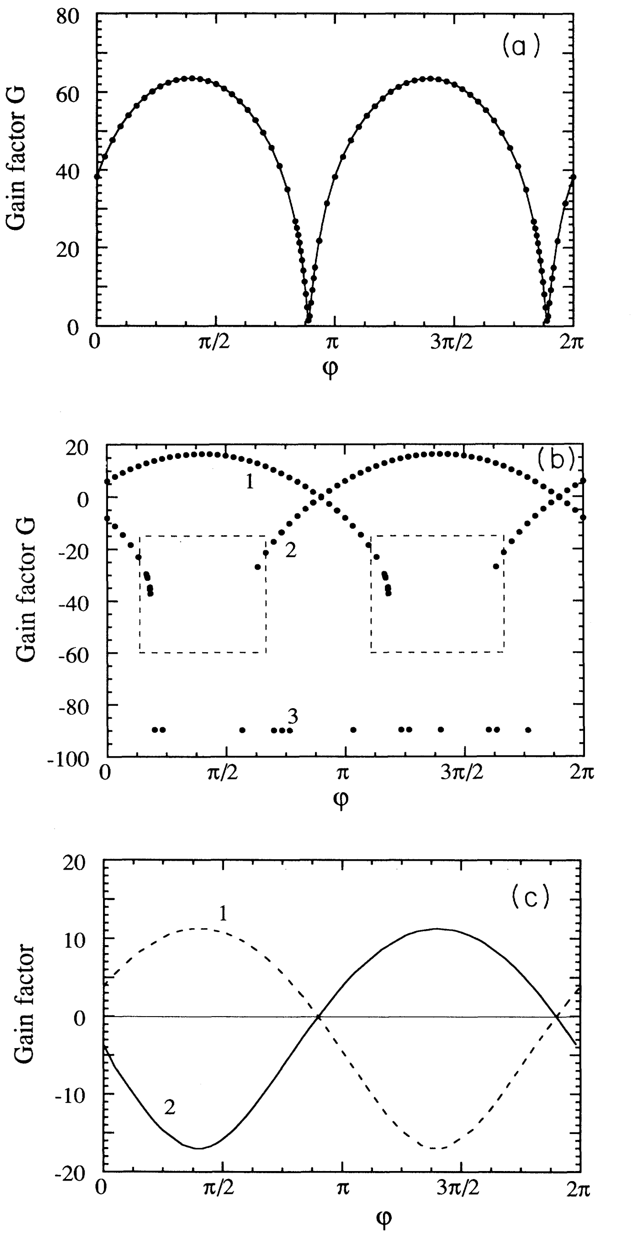

FIG. 6. Phase dependence of the numerical gain factor G near the PDB point and $k_p = 0.0074 k_d^{\text{bif}}$ for three different values of the bifurcation parameter: (a) $\mu = 1$, (b) $\mu = 1.04$, and (c) $\mu = 1.06$. In (a) only one solution is found and it has a period π. In (b) two π-dephased solutions associated with period-2 attractors of similar shape, together with a solution associated with a period-4 attractor, are found. In (c) only the two π-dephased solutions associated with period-2 attractors are found. Solutions 1 and 2 are discontinuous within the dashed rectangles of Fig. 6(b).

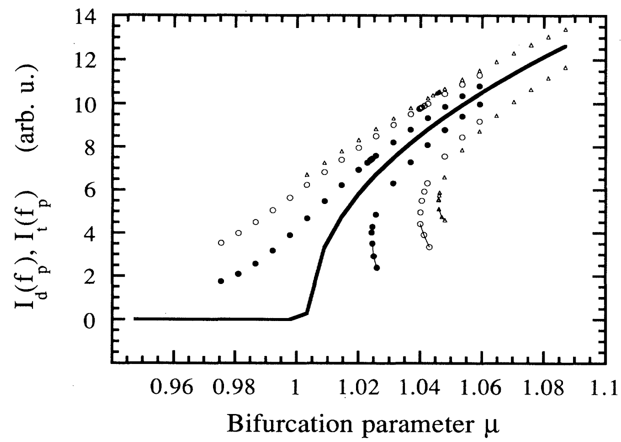

FIG. 7. Numerically calculated laser intensities $I_d(f_p)$ (solid line) and $I_t(f_p)$ (open and closed circles and triangles) as a function of the bifurcation parameter μ for different values of φ. ●, $\varphi = 0°$; ○, $\varphi = 32°$; △, $\varphi = 57°$. ●——●, ○——○, △——△ are unstable branches.

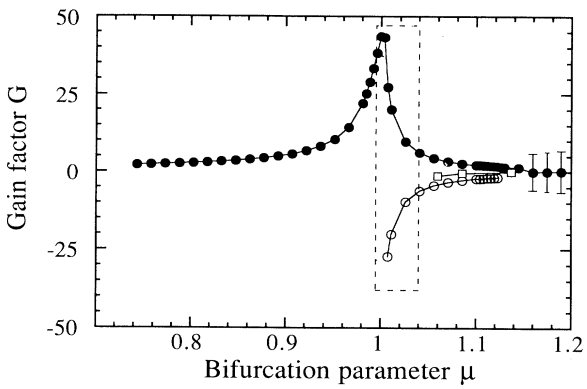

FIG. 8. Numerical gain factor as a function of the bifurcation parameter μ for $k_p=0.0074k_d^{\text{bif}}$. Dots and circles: calculated with $\varphi=0$. Squares: averaged gain factor calculated as explained in the text. Within the dashed rectangle drawn in the figure, we did not define the averaged gain factor (see text).

around the mean values obtained with a long time series of data, to indicate that it is difficult to evaluate in this case the gain factor because new solutions, different from all the solutions discussed previously, appear. This is the phenomenon of generalized multistability mentioned above.

IV. CONCLUSIONS

In this paper, the amplification and deamplification of a loss perturbation in a cw CO_2 laser with modulated losses have been investigated below and above a period-doubling bifurcation, both from the experimental and theoretical points of view.

The frequency of the perturbing signal used in the experiments was very close to half the driving modulation frequency. Strong amplification of this perturbing signal below the bifurcation point and deamplification above it have been observed. We have found that the dependence of the gain factor on the perturbation amplitude obeys a $-\frac{2}{3}$ power law at the bifurcation point.

A simple two-level laser model has been used for computer simulations. The values of the parameters chosen for the simulations were very close to the experimental ones. This model has shown the strong influence of the relative phase between the driving and perturbing signals on the gain factor. Although in the model the phase was assumed constant in time while in the experiment it decreased steadily, good agreement between numerical and experimental results has been obtained when the numerical results were averaged over the phase. An interesting feature of our numerical results is the existence of more than one solution for the gain factor above the bifurcation point. This is the phenomenon of generalized multistability described previously in Ref. [14] for a parameter range different from the one explored in this work.

We believe that the results presented here can be useful in some applications. In particular, signal amplification near the bifurcation point is very promising for increasing the sensitivity of intracavity measurements in modulation spectroscopy with narrow band cw lasers. Near the onset of a PDB, a parametrically modulated laser may be used as a detector of weak input signals at half the driving frequency [8]. The strong dependence of the gain factor on the phase of the perturbing signal gives the possibility of controlling the amplification or deamplification of the laser intensity at half the driving frequency by changing the phase. Moreover, the results of this paper might be of interest in connection with parametric amplification, when the fundamental amplifying mechanism is based on the PDB [5].

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