Cooling atomic motion with quantum interference

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We theoretically investigate the quantum dynamics of the center of mass of trapped atoms, whose internal degrees of freedom are driven in a Λ-shaped configuration with the lasers tuned at two-photon resonance. In the Lamb-Dicke regime, when the motional wave packet is well localized over the laser wavelength, transient coherent population trapping occurs, canceling transitions at the laser frequency. In this limit the motion can be efficiently cooled to the ground state of the trapping potential. We derive an equation for the center-of-mass motion by adiabatically eliminating the internal degrees of freedom. This treatment provides the theoretical background of the scheme presented in [G. Morigi *et al.*, Phys. Rev. Lett. **85**, 4458 (2000)] and implemented in [C.F. Roos *et al.*, Phys. Rev. Lett. **85**, 5547 (2000)]. We discuss the physical mechanisms determining the dynamics and identify parameters regimes, where cooling is efficient. We discuss implementations of the scheme to cases where the trapping potential is not harmonic.

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I. INTRODUCTION

The progress in laser cooling of atoms and ions has set the stage for coherent control of the dynamics of quantum-mechanical systems [1]. By means of laser cooling, states of the center-of-mass motion of trapped atoms with high purity have been prepared [1–4], allowing, for instance, for their coherent manipulation for quantum information processing [5]. Nevertheless, there is a continuous interest for new and efficient cooling methods, which solve experimental difficulties and increase the efficiency of the process. In this context, a laser-cooling scheme for trapped atoms has been recently proposed [6], which exploits the principles of coherent population trapping (CPT) [7] and allows to achieve almost unit probability of occupation of the trapping-potential ground state [6,8]. This method has been demonstrated to be an alternative to sideband [2] and Raman-sideband cooling [3,4], routinely used for the preparation of very pure states of the center-of-mass motion of trapped atoms and ions. Further applications of this cooling method (now labeled as ''EIT cooling'') have been discussed in several publications [9,10].

The focus of this work is to discuss theoretically the physical principles on which this method is based, and particularly the role of quantum coherence between atomic states on the mechanical effects of light on trapped atoms. Thus, in Sec. II we introduce the electronic level scheme composed of two stable or metastable states coupled by lasers to a common excited state, the Λ configuration, and discuss in general CPT when the transitions are driven by counterpropagating laser beams (the Doppler-sensitive case). Here, we observe that, in the presence of an external potential confining the center-of-mass motion, (transient) CPT is obtained when the lasers are set at two-photon resonance and the wave packet is well localized over the laser wavelength (the Lamb-Dicke regime). In Sec. III, starting from a general approach we develop the theoretical model, assuming that the atomic center of mass is confined by an external potential in the Lamb-Dicke regime: This allows one to adiabatically eliminate the internal degrees of freedom and derive an equation for the external degrees of freedom only [11]. We discuss this equation in detail when the potential is harmonic, and derive a set of rate equations for the occupation of the vibrational states. Thereby, we identify the parameter regime where cooling is effective. In some limits, these equations reduce to the ones used in Refs. [6,8,9]. Nevertheless, a result of this paper is the identification of the basic mechanism characterizing the dynamics, which allows us to determine parameter regimes where cooling can be efficient. We discuss the limit of the validity of the equations derived, give alternative interpretations of the dynamics, and consider possible extensions of the method to cases where the center of mass is confined by a potential that is not necessarily harmonic and whose functional form may depend on the electronic state.

We remark that the laser-cooling dynamics of trapped atoms, whose internal transitions are driven in a Λ configuration, have been investigated in several works, as for instance in Refs. [12–15]. These, however, focused on different cooling mechanisms. This work, together with Ref. [6], extends these previous analyses to other regimes, characterized by novel features of the center-of-mass dynamics, as we discuss below.

II. THE DARK RESONANCE AND THE MOTION

In this section, we first discuss the internal dynamics and steady state of an atom whose electronic bound states are driven by lasers in a resulting Λ configuration. We focus on the conditions for which CPT occurs. Then, we consider the center-of-mass degrees of freedom and discuss under which conditions the features characterizing the bare internal dynamics are preserved, when the motion is taken into account. The discussion in this section and throughout the paper is restricted to motion in one dimension, identified here with the $\hat{x}$ axis. This allows a simpler exposition without loss of generality.

A. The dark resonance

An exemplary atomic level configuration where the effects of quantum interference manifest is the Λ transition. It

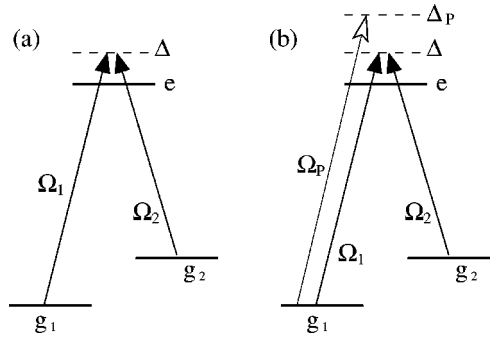

FIG. 1. (a) Level scheme: The solid arrows represent the lasers at Rabi frequencies Ω_1, Ω_2, that couple to the transitions $|g_1\rangle \rightarrow |e\rangle$, $|g_2\rangle \rightarrow |e\rangle$, respectively, and are detuned of Δ from atomic resonance. (b) Addition of a probe at Rabi frequency Ω_P and detuning Δ_P, coupling $|g_1\rangle \rightarrow |e\rangle$.

consists in two electronic transitions, formed by two stable or metastable states that we label $|g_1\rangle$, $|g_2\rangle$, which are coupled by lasers to the same excited state $|e\rangle$. For a closed transition, the atom stops to fluoresce when the states $|g_1\rangle$ and $|g_2\rangle$ are resonantly coupled (two-photon resonance), as shown in Fig. 1(a): The system evolves into the dark state, a stable atomic-states superposition that is decoupled from the excited state because of destructive interference between the excitation amplitudes. This phenomenon is called coherent population trapping [7], and the atoms are found in the coherence (dark state)

$$|\Psi_D\rangle = \frac{1}{\Omega}(\Omega_2|g_1\rangle - \Omega_1|g_2\rangle), \tag{1}$$

where $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$ and Ω_1 (Ω_2) is the Rabi frequency of the laser coupling to the transition $|g_1\rangle \rightarrow |e\rangle$ ($|g_2\rangle \rightarrow |e\rangle$). Here, without loss of generality, we have assumed Ω_1, Ω_2 to be real. The dark state is accessed by spontaneous emission, unless the system has been initially prepared in it. Thus, the density matrix $\rho_D = |\Psi_D\rangle\langle\Psi_D|$ is the steady-state solution of the master equation for the atomic density matrix ρ: $\partial\rho/\partial t = \mathcal{L}_0\rho$, where $\mathcal{L}_0$ is the Liouvillian defined as

$$\mathcal{L}_0\rho = \frac{1}{i\hbar}[H,\rho] + \mathcal{K}\rho. \tag{2}$$

Here, $H = H_0 + V_0$ is the Hamilton operator, and its terms have the form (in the rotating wave approximation and in the frame rotating at the laser frequencies)

$$H_0 = -\hbar\Delta(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|), \tag{3}$$

$$V_0 = \frac{\hbar}{2}(\Omega_1|e\rangle\langle g_1| + \Omega_2|e\rangle\langle g_2| + \text{H.c.}), \tag{4}$$

where $\Delta = \omega_1 - \omega_{L,1} = \omega_2 - \omega_{L,2}$ are the laser detunings, with the atomic resonance frequencies ω_j of the transition $|g_j\rangle \rightarrow |e\rangle$ and the frequencies of the corresponding driving laser $\omega_{L,j}$ ($j = 1,2$). The operator $\mathcal{K}$ is the Liouvillian describing spontaneous emission,

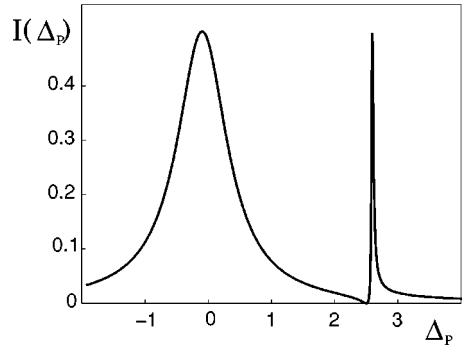

FIG. 2. Excitation spectrum $I(\Delta_P)$ in arbitrary units as a function of the probe detuning Δ_P in units of γ. Here, $\Omega = \gamma$, $\Delta = 2.5\gamma$, $\Omega_P = 0.05\gamma$.

$$\mathcal{K}\rho = -\frac{\gamma}{2}[|e\rangle\langle e|\rho + \rho|e\rangle\langle e|] + \sum_{j=1,2}\gamma_j|g_j\rangle\langle e|\rho|e\rangle\langle g_j|, \tag{5}$$

where γ_1, γ_2 are the rates of decay into $|g_1\rangle$, $|g_2\rangle$, respectively, and $\gamma_1 + \gamma_2 = \gamma$. It can be easily verified that the dark state is a dressed state of the system, i.e. an eigenstate of H. The other two dressed states read [16]

$$|\psi_+\rangle = \cos\theta|e\rangle + \sin\theta|\psi_C\rangle, \tag{6}$$

$$|\psi_-\rangle = \sin\theta|e\rangle - \cos\theta|\psi_C\rangle, \tag{7}$$

where

$$\tan\theta = \frac{\sqrt{\Delta^2 + \Omega^2} - \Delta}{\Omega}, \tag{8}$$

$$|\psi_C\rangle = \frac{1}{\Omega}(\Omega_1|g_1\rangle + \Omega_2|g_2\rangle), \tag{9}$$

and where we have introduced the state $|\psi_C\rangle$, orthogonal to $|e\rangle$ and $|\psi_D\rangle$. The states (6) and (7) are at eigenfrequencies $\delta\omega_\pm = (\Delta \mp \sqrt{\Delta^2 + \Omega^2})/2$, and since they possess a nonzero overlap with the excited state $|e\rangle$, they have a finite decay rate and are populated in the transient dynamics. We denote their linewidths with γ_+, γ_-. The steady state is accessed at the slowest rate of decay and, for later convenience, we introduce T_0, the time scale corresponding to the inverse of this rate.

The dressed-state picture is a useful tool for interpreting the atomic spectra in a pump-probe experiment, where, e.g., a weak probe at Rabi frequency Ω_P ($\Omega_P \ll \Omega_1, \Omega_2$) couples to the transition $|g_1\rangle \rightarrow |e\rangle$ as shown in Fig. 1(b), while its frequency is let sweep across the atomic resonance. Figure 2 shows the spectrum of excitation as a function of the detuning of the probe Δ_P, for a certain choice of the lasers parameters. Here, one can observe that the component of the spectrum at $\Delta_P = \Delta$ is zero, corresponding to the situation where the system is in the dark state $|\Psi_D\rangle$. Moreover, the spectrum exhibits two resonances centered at $\Delta_P = \delta\omega_\pm$, whose widths correspond approximately (when $|\Delta|, \Omega \gg \gamma$) to γ_+, γ_-, respectively, and can be identified with the dressed

states $|\psi_+\rangle$, $|\psi_-\rangle$ [17]. Note that these resonances have not a Lorentzian shape: The spectrum shares in fact many similarities with a Fano profile [17]. Typical excitation spectra, measured with a single ion in a trap, are reported in Refs. [18,19].

B. The motion

We consider now the center-of-mass motion in the presence of a conservative potential, of which for the moment, the form is not specified. Given the mass of the atom m, the momentum p, the position x, and the potential $U(x)$, the mechanical Hamiltonian is

$$H_{\rm mec}=\frac{p^2}{2m}+U(x). \tag{10}$$

We denote with $|\psi_\epsilon\rangle$ the eigenvectors of $H_{\rm mec}$ at the eigenvalues ϵ. The full dynamics are now described by the Master equation

$$\frac{\partial}{\partial t}\tilde{\rho}=\frac{1}{i\hbar}[\tilde{H},\tilde{\rho}]+\tilde{\mathcal{K}}\tilde{\rho}=\tilde{\mathcal{L}}\tilde{\rho}, \tag{11}$$

where $\tilde{\rho}$ is the density matrix for the internal and external degrees of freedom and

$$\tilde{H}=\tilde{H}_0+H_{\rm mec}+\tilde{V}. \tag{12}$$

Here, $\tilde{V}$ describes the coherent interaction of the atomic dipole with the lasers, and has the form

$$\tilde{V}=\frac{\hbar}{2}(\Omega_1\exp(ik_1x\cos\phi_1)|e\rangle\langle g_1| + \Omega_2\exp(ik_2x\cos\phi_2)|e\rangle\langle g_2|+\text{H.c.}), \tag{13}$$

where the lasers are traveling waves at wave vectors k_1 and k_2, propagating along the directions forming the angles ϕ_1, ϕ_2, respectively, with the $\hat{x}$ axis. In Eq. (13) the spatial dependence is explicitly included, which couples to the external degrees of freedom of the ion, while the Rabi frequencies Ω_1, Ω_2 are assumed to be constant over the spatial region where the ion is localized. The Liouvillian $\tilde{\mathcal{K}}$ describes the incoherent scattering processes, whereby a photon is spontaneously emitted under an angle ϕ with the axis of the motion. It has the form

$$\tilde{\mathcal{K}}\tilde{\rho}=-\frac{\gamma}{2}[|e\rangle\langle e|\tilde{\rho}+\tilde{\rho}|e\rangle\langle e|] + \sum_{j=1,2}\gamma_j\int_{-1}^{1}d\cos\phi\,\mathcal{N}(\cos\phi)|g_j\rangle \times\langle e|[\exp(ik_jx\cos\phi)\tilde{\rho}\exp(-ik_jx\cos\phi)]|e\rangle\langle g_j|, \tag{14}$$

where $\mathcal{N}(\cos\phi)$ is the probability distribution for the angles of photon emission with respect to the motional axis.

In this system, at a given instant of time perfect destructive interference between excitation amplitudes occurs for the state

$$|\tilde{\Psi}_{\rm D}\rangle=\frac{1}{\Omega}(\Omega_2|g_1,\Psi\rangle - \Omega_1\exp[i(k_1\cos\phi_1-k_2\cos\phi_2)x]|g_2,\Psi\rangle), \tag{15}$$

where $\exp[i(k_1\cos\phi_1-k_2\cos\phi_2)x]$ is the displacement operator, acting on the external degrees of freedom, and Ψ is a state of the center-of-mass motion. The state (15) is stable—and thus a dark state—if it is an eigenstate of $H_0+H_{\rm mec}$. This is always true when the lasers are copropagating and $k_1\cos\phi_1=k_2\cos\phi_2$ [or, for one-dimensional (1D) motion as in this case, when the direction of propagation of the lasers is orthogonal to the axis of the motion, $\cos\phi_1=\cos\phi_2=0$]: Then, the motional state factorizes out in Eq. (15). For $k_1\cos\phi_1\neq k_2\cos\phi_2$, on the contrary, one must consider the particular form of the confining potential. For instance, for free atoms $[U(x)=\text{const}]$ a perfect dark state exists for $k_1\cos\phi_1=-k_2\cos\phi_2=k$ and reads $|\tilde{\Psi}_{\rm D}\rangle=(\Omega_2|g_1,-\hbar k\rangle-\Omega_1|g_2,\hbar k\rangle)/\Omega$. This property has been used to prepare very cold atomic samples [20]. In the presence of a confining potential, on the other hand, there exists, in general, no state $|\tilde{\Psi}_{\rm D}\rangle$ that is perfectly dark. Approximate dark states have been discussed in Ref. [15] for a 1D flat bottom and for a 2D harmonic trap.

Nevertheless, transient CPT can be observed in trapping potentials and in Doppler-sensitive configurations when the atoms are in the Lamb-Dicke regime (LDR), i.e., when the size of their motional wave packet $\sqrt{\langle\Delta x^2\rangle}$ is much smaller than the wavelength of the light, $k_{1,2}\sqrt{\langle\Delta x^2\rangle}\ll 1$. In this limit, a hierarchy of processes in the excitation of the center-of-mass wave packet is established. At zero order in $\zeta=k_{1,2}\sqrt{\langle\Delta x^2\rangle}$, the effects due to the spatial gradient of the light-atom potential are neglected: the atoms behave as if they were pointlike, and the coherent transitions take place at the laser frequency (carrier). Then, after the transient time T_0 the atoms have accessed the internal dark state $|\Psi_{\rm D}\rangle$. At first order in ζ, effects due to the finite size of the motional wave packet become manifest, and transitions between different motional states (sidebands transitions) occur. On this longer time scale, which we denote with T_ζ, the atom is optically pumped out of the dark state into another state of the motion. In the Lamb-Dicke regime, the relation $T_\zeta\gg T_0$ allows for a coarse-grained description of the dynamics, where the internal state of the atom is assumed to be always the dark state $|\Psi_{\rm D}\rangle$.

These arguments suggest that for a trapped atom in the LDR, some of the properties of the excitation spectrum discussed for the Doppler-free case may also be applicable to the Doppler-sensitive one. Here, the carrier transition is predominant, whereas transitions which change the state of the motion (sidebands transitions) are of higher order in the Lamb-Dicke parameter, and can be interpreted as transitions due to a probe ($\Omega_{\rm P}$) set at the corresponding frequency in the bare atom, as illustrated for instance in Fig. 1(b). In the following section, we show that this interpretation is theoretically justified.

III. THEORY

Here, we derive the equations for the center-of-mass motion in the limit where the LDR applies and when the center-of-mass motion is confined by the same potential at all three electronic levels. The procedure consists in adiabatically eliminating the internal degrees of freedom from the dynamical equation at second order in the parameter ζ, and it corresponds to analyzing the coarse-grained evolution on the time interval Δt such that $T_\zeta \gg \Delta t \gg T_0$. The formalism we use has been first developed in Ref. [11] for a two-level transition driven by a running wave, and later applied to standing-wave drives and multilevel transitions in Ref. [21]. In the following, we outline the fundamental steps that are most general to all treatments, and refer the reader to Ref. [11,21] for details (we have used the same notation as in Ref. [21] when possible).

A. Lamb-Dicke limit

In the Lamb-Dicke limit $\zeta \ll 1$, the operators $\exp(ik_j x)$ appearing in Eqs. (13) (14) can be expanded in powers of ζ. At second order in this expansion, Eq. (11) can be rewritten as

$$\frac{\partial}{\partial t}\tilde{\rho} = [\tilde{\mathcal{L}}_0 + \tilde{\mathcal{L}}_1 + \tilde{\mathcal{L}}_2]\tilde{\rho}, \tag{16}$$

where the Liouvillians $\tilde{\mathcal{L}}_j$ describe processes at the jth order in the Lamb-Dicke parameter, and are defined as

$$\tilde{\mathcal{L}}_0\tilde{\rho} = \mathcal{L}_0\tilde{\rho} + \frac{1}{i\hbar}[H_{\rm mec}, \tilde{\rho}], \tag{17}$$

$$\tilde{\mathcal{L}}_1\tilde{\rho} = \frac{1}{i\hbar}[xV_1, \tilde{\rho}], \tag{18}$$

$$\tilde{\mathcal{L}}_2\tilde{\rho} = \frac{1}{i\hbar}[x^2V_2, \tilde{\rho}] + \tilde{\mathcal{K}}_2\tilde{\rho}. \tag{19}$$

Here, V_1, V_2 are the first- and second-order terms in the expansion of $\tilde{V}$ and read

$$V_1 = \frac{i\hbar}{2}\sum_{j=1,2} k_j \cos\phi_j \Omega_j (|e\rangle\langle g_j| - |g_j\rangle\langle e|), \tag{20}$$

$$V_2 = -\frac{\hbar}{4}\sum_{j=1,2} k_j^2 \cos^2\phi_j \Omega_j (|e\rangle\langle g_j| + |g_j\rangle\langle e|). \tag{21}$$

The Liouvillian $\tilde{\mathcal{K}}_2$ has the form

$$\tilde{\mathcal{K}}_2\tilde{\rho} = \alpha\sum_{j=1,2}\gamma_j k_j^2 |g_j\rangle\langle e|(2x\tilde{\rho}x - x^2\tilde{\rho} - \tilde{\rho}x^2)|e\rangle\langle g_j|, \tag{22}$$

where $\alpha = \int_{-1}^{1} d\cos\phi \mathcal{N}(\cos\phi)\cos^2\phi$.

At zero order in ζ, the internal and external degrees of freedom are decoupled: The state $\tilde{\rho}_{\rm St}$, solution of $\tilde{\mathcal{L}}_0\tilde{\rho} = 0$, is not uniquely defined, and has the form $\tilde{\rho}_{\rm St} = \rho_{\rm St} \otimes \mu(0)$, where $\rho_{\rm St} = \rho_{\rm D}$ is the internal steady state and $\mu(0) = {\rm Tr}_{\rm int}\{\mathcal{P}_0\tilde{\rho}(0)\}$ is the reduced density matrix, calculated from $\tilde{\rho}$ at $t=0$ by tracing over the internal degrees of freedom (${\rm Tr}_{\rm int}\{\}$) and applying the projector $\mathcal{P}_0$ acting over the external degrees of freedom. The latter is defined as $\mathcal{P}_0\tilde{\rho} = \Sigma_\epsilon \Sigma'_{\psi_\epsilon, \psi'_\epsilon} |\psi_\epsilon\rangle\langle\psi'_\epsilon|\langle\psi_\epsilon|\tilde{\rho}|\psi'_\epsilon\rangle$, where $|\psi_\epsilon\rangle$, $|\psi'_\epsilon\rangle$ are eigenstates of $H_{\rm mec}$ at ϵ. In general, at zero order, equation $\partial_t\tilde{\rho} = \tilde{\mathcal{L}}_0\tilde{\rho}$ admits an infinite number of stable solutions. They can be expanded in the basis of eigenvectors $\tilde{\rho}_{\epsilon,\epsilon'} = \rho_{\rm St} \otimes |\psi_\epsilon\rangle\langle\psi_{\epsilon'}|$ at the (imaginary) eigenvalues $\lambda_{\epsilon,\epsilon'} = -i(\epsilon - \epsilon')/\hbar$ of the Liouville operator $\tilde{\mathcal{L}}_0$, satisfying the secular equation $\tilde{\mathcal{L}}_0\tilde{\rho}_{\epsilon,\epsilon'} = \lambda_{\epsilon,\epsilon'}\tilde{\rho}_{\epsilon,\epsilon'}$ ($\tilde{\rho}_{\rm St}$ is eigenvector at $\lambda = 0$). The eigenspaces at the eigenvalues $\lambda_{\epsilon,\epsilon'}$ may be also infinitely degenerate, as it occurs, for instance, in the harmonic oscillator. For $\zeta \neq 0$, these subspaces are coupled by $\tilde{\mathcal{L}}_1$, $\tilde{\mathcal{L}}_2$. At second-order perturbation theory in ζ, for $\zeta_j\Omega_j \ll \min_{\epsilon,\epsilon'\neq\epsilon}(|\epsilon - \epsilon'|)$ (i.e., when the spectum of $\tilde{\mathcal{L}}_0$ is sufficiently spaced, to allow for nondegenerate perturbation theory), a closed equation for the dynamics in the subspace at $\lambda = 0$ can be derived. Denoting with $\tilde{\mathcal{P}}_0$ the projector onto this subspace, defined as $\tilde{\mathcal{P}}_0\tilde{\rho} = \rho_{\rm St} \otimes {\rm Tr}_{\rm int}\{\mathcal{P}_0\tilde{\rho}\}$, this equation has the form [11]

$$\frac{d}{dt}\tilde{\mathcal{P}}_0\tilde{\rho}(t) = \left[\tilde{\mathcal{P}}_0\tilde{\mathcal{L}}_2\tilde{\mathcal{P}}_0 + \int_0^\infty d\tau \tilde{\mathcal{P}}_0\tilde{\mathcal{L}}_1 e^{\tilde{\mathcal{L}}_0\tau}\tilde{\mathcal{L}}_1\tilde{\mathcal{P}}_0\right]\tilde{\rho}(t). \tag{23}$$

After substituting the explicit form of $\tilde{\mathcal{L}}_1, \tilde{\mathcal{L}}_2$ in the second term on the right-hand side of Eq. (23) and tracing over the internal degrees of freedom, we obtain

$$\frac{d}{dt}\mu = -\mathcal{P}_0\frac{1}{\hbar^2}\int_0^\infty d\tau ({\rm Tr}_{\rm int}\{V_1 \exp(\mathcal{L}_0\tau)V_1\rho_{\rm St}\}[\hat{x}, [\hat{x}(\tau), \mu]] + {\rm Tr}_{\rm int}\{V_1 e^{\mathcal{L}_0\tau}[V_1, \rho_{\rm St}]\}[\hat{x}, \mu\hat{x}(\tau)]). \tag{24}$$

Here, the matrix $\mu = {\rm Tr}_{\rm int}\{\mathcal{P}_0\tilde{\rho}\}$ is the reduced density matrix for the external degrees of freedom in the subspace at eigenvalue (at zero order) $\lambda = 0$. The operator $\hat{x}(\tau)$ is here defined as $\hat{x}(\tau) = \exp(-iH_{\rm mec}\tau/\hbar)\hat{x}\exp(iH_{\rm mec}\tau/\hbar)$.

It is remarkable that the term $\tilde{\mathcal{P}}_0\tilde{\mathcal{L}}_2\tilde{\mathcal{P}}_0 = 0$. This result is explained by looking at the form of Eq. (19). When tracing over the internal degrees of freedom, the first term in Eq. (19) gives rise to a contribution proportional to ${\rm Tr}_{\rm int}\{V_2\rho_{\rm St}\}$: This term usually gives rise to a shift to the eigenvalues $\lambda_{\epsilon,\epsilon'}$, it represents a renormalization of the harmonic-oscillator frequency due to the presence of the laser fields, and here it vanishes since there is no occupation of the excited state at steady state. The second term in Eq. (19) describes the diffusion arising from spontaneous emission into other mechanical states [21]. Again, since at steady state there is no excited-state occupation, it vanishes. Thus, the disappearance of $\tilde{\mathcal{P}}_0\tilde{\mathcal{L}}_2\tilde{\mathcal{P}}_0$ is due to quantum interference at zero order in the Lamb-Dicke expansion.

For a nondegenerate spectrum of eigenvalues ϵ, the reduced matrix μ is diagonal, and the equation for a matrix element has the form

$$\frac{d}{dt}\langle\psi_\epsilon|\mu|\psi_\epsilon\rangle=\sum_{\epsilon'} C_{\epsilon,\epsilon'}S(\omega_{\epsilon,\epsilon'}) \times[-\langle\psi_\epsilon|\mu|\psi_\epsilon\rangle+\langle\psi_{\epsilon'}|\mu|\psi_{\epsilon'}\rangle]+\text{H.c.}, \tag{25}$$

where the coefficient $S(\omega_{\epsilon,\epsilon'})$ is the value of the fluctuation spectrum of the operator V_1 at the frequency $\omega_{\epsilon,\epsilon'}=(\epsilon-\epsilon')/\hbar$, and reads

$$S(\omega_{\epsilon,\epsilon'})=\frac{1}{\hbar^2}\int_0^\infty d\tau\,\mathrm{Tr}_{\mathrm{int}}\{V_1\exp(\mathcal{L}_0\tau)V_1\rho_{\mathrm{St}}\}e^{i(\epsilon-\epsilon')\tau/\hbar}. \tag{26}$$

The coefficient $C_{\epsilon,\epsilon'}=|\langle\psi_\epsilon|x|\psi_{\epsilon'}\rangle|^2$ weights the coupling between the center-of-mass states $|\psi_\epsilon\rangle$ and $|\psi_{\epsilon'}\rangle$ due to the photon momentum at second order in the Lamb-Dicke expansion. The equations necessary for the derivation of the explicit form of Eq. (26) are reported in the Appendix. Equation (26) shows that the rate for the transition $|\psi_\epsilon\rangle\rightarrow|\psi_{\epsilon'}\rangle$ is given by the value of the excitation spectrum for a probe, whose interaction with the atomic transition is described by V_1 and which is detuned from the pump by $\omega_{\epsilon,\epsilon'}$ (sideband transition). Here, the form of the potential enters explicitly through the coefficients $C_{\epsilon,\epsilon'}$, and implicitly through the assumptions on the spectrum that have lead to Eq. (25).

B. Harmonic oscillator

We now let the potential be harmonic at frequency ν, $U(x)=\frac{1}{2}m\nu^2x^2$, and introduce the annihilation and creation operators a and $a^\dagger$ of a quantum of vibrational energy $\hbar\nu$, such that $x=x_0(a^\dagger+a)$, $p=ip_0(a^\dagger-a)$, with $x_0=\sqrt{\hbar/2m\nu}$ and $p_0=\sqrt{\hbar m\nu/2}$. The center-of-mass Hamiltonian reads

$$H_{\mathrm{mec}}=\hbar\nu(a^\dagger a+\tfrac{1}{2}). \tag{27}$$

Now, $|\psi_\epsilon\rangle=|n\rangle$ and $\epsilon=\hbar\nu(n+1/2)$, where $n=0,1,\ldots$ is the number of phonon excitations, and the mechanical energies are equidistantly spaced by $\hbar\nu$. The coefficients $C_{n,n'}=x_0^2[n\delta_{n',n-1}+(n+1)\delta_{n',n+1}]$, and thus at first order in the Lamb-Dicke expansion the relevant transitions between motional states are the blue sideband $|n\rangle\rightarrow|n+1\rangle$ at frequency $\omega_L-\nu$, and the red sideband $|n\rangle\rightarrow|n-1\rangle$ at frequency $\omega_L+\nu$. We define the Lamb-Dicke parameter $\eta_j=k_jx_0$, which fulfills the relation $\zeta_{1,2}=\eta_{1,2}\sqrt{2\langle n\rangle+1}$, with $\langle n\rangle$ being the average number of phonon excitations.

For the harmonic oscillator, the equations derived in the preceding section simplify notably: Equation (24) gets the form

$$\frac{d}{dt}\mu=x_0^2S(\nu)[-a^\dagger a\mu+a\mu a^\dagger] +x_0^2S(-\nu)[-aa^\dagger\mu+a^\dagger\mu a]+\text{H.c.}, \tag{28}$$

and for the probability $\mu_{n,n}=\langle n|\mu|n\rangle$ of the system to be in the number state $|n\rangle$, Eq. (28) turns to a rate equation, whose form is well known in laser cooling of single ions [23],

$$\frac{d}{dt}\mu_{n,n}=\eta^2[(n+1)(A_-\mu_{n+1,n+1}-A_+\mu_{n,n}) +n(A_-\mu_{n,n}-A_+\mu_{n-1,n-1})]. \tag{29}$$

In our case of a three-level atom, $\eta=\eta_1\cos\phi_1-\eta_2\cos\phi_2$, and

$$A_\pm=2\mathrm{Re}[S(\mp\nu)] =\frac{1}{4}\left(\frac{\Omega_1\Omega_2}{\Omega}\right)^2\frac{\gamma\nu^2}{[\Omega^2/4-\nu(\nu\pm\Delta)]^2+\gamma^2\nu^2/4}. \tag{30}$$

Equation (29) has the same structure as the rate equation derived for sideband cooling in a two-level system. Here, however, the rates $A_\pm$ describe the sideband excitation including the effect of quantum interference between the atomic transitions. Equation (29) allows for a steady state when $A_->A_+$, which is fulfilled when $\Delta<0$ (blue detuning) and $\Omega>2\nu$, or when $\Delta>0$ (red detuning) and $\Omega<2\nu$. The value of the trap frequency $\overline{\nu}=\Omega/2$ separates two regimes: for $\nu<\Omega/2$ it is the narrow resonance that determines relevantly the center-of-mass dynamics, whereas for $\nu>\Omega/2$ the sideband transitions are at the frequency range of the broad resonance [22]. We remark that $\eta=0$ for $k_1\cos\phi_1=k_2\cos\phi_2$, corresponding to the Doppler-free situation. Furthermore, the Lamb-Dicke parameter entering into the dynamics is the one determined by the laser wave vector. The Lamb-Dicke parameter connected to spontaneous emission events (i.e., recoils because of emission into other states of the motion) does not appear, since the diffusion term vanishes at second order in the Lamb-Dicke expansion.

In the following we assume $k_1\cos\phi_1\neq k_2\cos\phi_2$ and $\Delta<0$, $\Omega>2\nu$ $(A_->A_+)$. Some insight into the dynamics can be gained from the equation for the average number of phonon $\langle n(t)\rangle=\Sigma_{n=0}^\infty n\mu_{n,n}(t)$, which is derived from Eq. (29) and has the form [23]

$$\frac{d}{dt}\langle n\rangle=-W\langle n\rangle+\eta^2A_+, \tag{31}$$

where $W=\eta^2(A_--A_+)$ is the cooling rate. The steady state value $\langle n\rangle_\infty$ reads

$$\langle n\rangle_\infty=\frac{4[\Omega^2/4-\nu(\nu-\Delta)]^2+\gamma^2\nu^2}{4\nu|\Delta|(\Omega^2-4\nu^2)}, \tag{32}$$

and is minimum when $\Omega^2=4\nu(\nu-\Delta)$. This relation corresponds to setting the ac Stark shift $\delta\omega_+$ of the narrow resonance $|\Psi_+\rangle$ at the frequency of the first red sideband, $\delta\omega_+=\Delta-\nu$. For this value, $\langle n\rangle_\infty^{(\mathrm{min})}=(\gamma/4|\Delta|)^2$: Hence, low temperatures are achieved for lasers far detuned from atomic resonance. This corresponds to an enhanced asymmetry of the excitation spectrum, as that shown in Fig. 2, where the two resonances have very different widths.

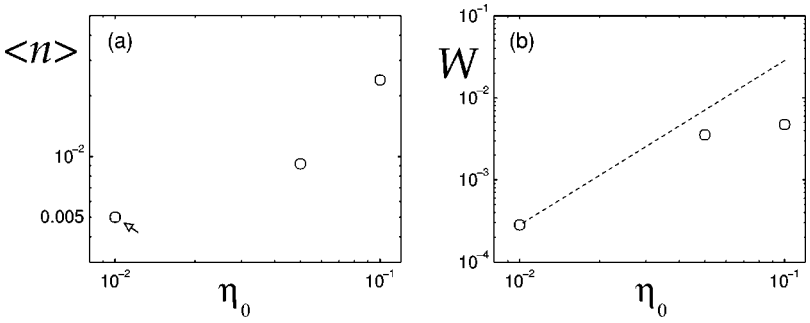

FIG. 3. Plot of (a) $\langle n\rangle_\infty$ and (b) W as a function of η_0. Dashed line, rate equation result; ○, numerical simulation, 500 trajectories with the quantum Monte Carlo method. Here, $\eta_1\cos\phi_1=\eta_2\cos\phi_2=\eta_0$ $(\eta=2\,\eta_0)$, $\nu=2$ MHz, $\gamma=20$ MHz, $\Omega_1=\Omega_2=17$ MHz, $\Delta=70$ MHz, $\gamma_1/\gamma_2=1$. In (a) the result of the simulation in agreement with the rate equation prediction ($\langle n\rangle=0.005$) is indicated by the arrow. In (b) the rate W is in units of $\gamma/2$.

For $\delta\omega_+=\Delta-\nu$ the cooling rate scales as

$$W^{\max}\sim\eta^2(\Omega_1\Omega_2/\Omega)^2/\gamma. \tag{33}$$

Thus, fast cooling is achieved for large Rabi frequencies and when $\Omega_1=\Omega_2$. The ultimate limit to W is set by the parameters that ensure the validity of the perturbative treatment applied here: This is valid for $\eta_j\Omega_j\cos\theta\ll\gamma_+$ $(j=1,2)$, with $\gamma_+\sim\gamma\cos^2\theta$ being the linewidth of the narrow resonance, corresponding in the bare atom to the situation where the probe (the sideband) does not saturate the transition to $|\Psi_+\rangle$. At $\delta\omega_+=\Delta-\nu$, one has $\gamma_+\sim\nu\gamma/4\sqrt{\Delta^2+\Omega^2}$, which sets the fastest rate at which efficient laser cooling can occur, $W^{\max}\sim\gamma_+/2$.

It is remarkable that these results do not depend on the branching ratio γ_1/γ_2. In fact, in this limit the branching ratio enters the problem only through T_0. Nevertheless, a too large branching ratio affects the time scale T_0 at which the transient steady state is reached.

In Fig. 3 we test the validity of the adiabatic elimination procedure for various values of the Lamb-Dicke parameter, by comparing the results predicted by Eq. (31) with a full numerical simulation. The parameters are reported in the caption. Full agreement between the two results is found for $\eta=0.02$ $(\eta_1\cos\phi_1=-\eta_2\cos\phi_2=0.01)$. It should be mentioned that in Ref. [6] full agreement has been found for η as large as 0.2. On the other hand, those results have been evaluated for the case $\Omega_1\ll\Omega_2$, and the small value of Ω_1 ensured the validity of the perturbative expansion.

IV. DISCUSSION

We have shown that, by properly choosing the lasers parameters, one can achieve almost unity ground-state occupation with this cooling method (EIT cooling). The state $|\psi_D\rangle|0\rangle$ is equivalent to the ground state in sideband cooling, since it is only off-resonantly (weakly) coupled to other states, and it satisfies the criteria of an approximate dark state as discussed in Ref. [15].

From Eq. (30) one recovers the rates of Eq. (4) in Ref. [6] in the limit $\Omega_1\ll\Omega_2$. We have shown that the same dynamics are encountered in more general situations, which do not impose a specific relation between the two Rabi frequencies. From the technical point of view, EIT cooling proves again to be more advantageous than Raman sideband cooling (see Refs. [6,9]). Such an advantage is mainly twofold. On one hand, in EIT cooling both lasers cool the atom, and a decay into one or the other channel does not affect the efficiency of the process, while in Raman sideband cooling a finite branching ratio gives rise to heating [24]. Another important feature of EIT cooling is the disappearance of the carrier absorption due to quantum interference. This effect implies the suppression of diffusive processes: Since in the coarse-grained evolution the excited state is effectively empty, processes, where the atom is scattered into other motional states by spontaneous emission, disappear at second order in the Lamb-Dicke expansion. This implies an improved efficiency with respect to Raman-sideband cooling, where instead such processes are present, as already discussed in Ref. [6].

It is instructive to compare the dynamics in EIT cooling with the dynamics of a trapped ion at the node of a standing wave, as studied, for example, in Ref. [21]. At the node of a standing wave the carrier absorption cancels, since here the value of the electric field is zero. Nevertheless, sideband absorption occurs because of the finite size of the motional wave packet. In the case of a Λ configuration driven by two traveling waves at two-photon resonance, the transient dark state (15) is a superposition of the states $|g_1\rangle$ and $|g_2\rangle$ whose relative phase is a function of the coordinate x, so that the finite size of the wave packet allows sideband absorption also in this case. Nevertheless, in the LDR the gradient of the phase over the wave packet is small, and the sideband transitions are excited on a longer time scale. This can be illustrated when writing the atom-laser interaction (13) at the first order in the Lamb-Dicke expansion and in the form

$$\tilde V\approx\hbar\frac{\Omega}{2}[|e\rangle\langle\Psi_C|+\mathrm{i}|e\rangle\langle\Psi_D|kx+\mathrm{H.c.}], \tag{34}$$

where we have made the simplifying assumptions $\Omega_1=\Omega_2=\Omega/\sqrt{2}$, $k_1\cos\phi_1=-k_2\cos\phi_2=k$. Here, we see that the dark state is coupled to the excited state at first order in the Lamb-Dicke expansion, for effects arising from the finite size of the motional wave packet.

The atom dynamics during the coarse-grained evolution can be interpreted in terms of field gradients over the size of the wave packet, which give rise to forces [25]. In this respect, one can say that this method uses the phase gradient of the dark state, due to the spatial gradient of the total field, for achieving cooling. In this context, we remark that the operator V_1 in Eq. (26) is the gradient of potential (13) at $x=0$, i.e., at the center of trap.

Finally, we apply the results obtained for the harmonic oscillator to the case of a generic potential $U(x)$. Several conclusions drawn in this section are applicable to the case

described in Eq. (25), when the mechanical Hamiltonian has a discrete spectrum, and the minimum distance between two neighboring energy levels is sufficiently large to allow for nondegenerate perturbation theory. Laser cooling is here achieved for the same parameters as for the harmonic oscillator. However, the narrow resonance enhances transitions in a finite range of frequencies ($<\overline{\nu}$), and $\delta\omega_+$ must be properly tuned, e.g., to the average value of the red sideband transitions frequencies. The process will thus be efficient under the condition that, for each motional state, there is a sufficient number of red sidebands inside this range, so that the rate of cooling for a given motional state is larger than the rate of heating.

An interesting question is how the dynamics are affected when the external potential depends on the electronic state, and thus when

$$U(x)=U_1(x)|g_1\rangle\langle g_1|+U_e(x)|e\rangle\langle e|+U_2(x)|g_2\rangle\langle g_2|.$$

We consider first the case $U_1(x)=U_2(x)$, while $U_e(x)$ is, say, constant, so that the center of mass of the excited atom is not spatially confined and the spectrum of $H_{\rm mec}$ at the state $|e\rangle$ is a continuum. Assuming that for U_1, U_2 the Lamb-Dicke regime holds, then at two-photon resonance and during the transient dynamics the atom is optically pumped into the (transient) dark state (1). However, during T_0 the center-of-mass wave packet changes, since each eigenstate of $U_1(x)$ $(U_2(x))$ may have a nonzero overlap with several eigenstates of $U_e(x)$. This effect constitutes a diffusion mechanism that lowers the cooling efficiency and, outside of some regimes, can make it even impossible. Formally, for $U_e\neq U_1,U_2$, the formalism applied in this section is not applicable, since one cannot separate the time scales characterizing the evolution of the internal and external degrees of freedom.

In the general case of three different confining potentials the presence of a dark state cannot be excluded: this however depends on the specific form of the functions $U_j(x)$.

V. CONCLUSIONS AND OUTLOOK

We have presented a systematic investigation of the center-of-mass dynamics of a trapped ion, the internal transitions of which are driven by lasers in a Λ-type configuration and set at two-photon resonance. Assuming that the center-of-mass wave packet is well localized over the laser wavelength (the Lamb-Dicke regime), we have adiabatically eliminated the internal degrees of freedom from the equation of the center-of-mass dynamics, and obtained a set of rate equations for the occupation of the motional states. We have identified the parameter regimes where efficient ground-state cooling can be achieved. The derivation here presented provides the theoretical background for the equations in Refs. [6,8,9] and extends the parameter regime to cases that have not been previously considered. As also discussed in Ref. [6], we have shown that diffusive processes, encountered in cooling with two-level atoms or with effective two-level systems (Raman sideband cooling), are suppressed because of quantum interference between the dipole transitions at zero order in the Lamb-Dicke expansion. Cooling takes place because of excitations due to the spatial gradient of the electric field over the width of the motional wave packet, which are due to the finite size of the wave packet itself and occur at first order in the Lamb-Dicke expansion. The motion can be said to be cooled by both lasers, while the branching ratio does not affect, in general, the efficiency of the process.

Finally, we have discussed the possibility to observe these dynamics for other types of potentials, which may depend on the electronic state.

This work opens interesting prospects in the manipulation of the quantum center-of-mass motion of atoms by using quantum interference in driven multilevel transitions, which is subject of on-going investigations.

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APPENDIX: CALCULATION OF $S(\nu)$

The term $S(\nu)$ in Eq. (26) is the Laplace transform at $i\nu$ of the correlation function $G(\tau)$, defined as $G(\tau)=\mathrm{Tr}_{\rm int}\{V_1(\tau)V_1(0)\rho_{\rm St}\}$, where $V_1(\tau)=V_1\exp(\mathcal{L}_0\tau)$. This is evaluated applying the quantum regression theorem [16,26]. In the following, we derive the equations that are essential for this calculation. For convenience, we introduce the vector-operator $\hat{\sigma}$ whose components are defined as $\hat{\sigma}_1=|g_1\rangle\langle g_1|$, $\hat{\sigma}_2=|g_2\rangle\langle g_2|$, $\hat{\sigma}_3=|g_1\rangle\langle e|$, $\hat{\sigma}_4=|e\rangle\langle g_1|$, $\hat{\sigma}_5=|g_2\rangle\langle e|$, $\hat{\sigma}_6=|e\rangle\langle g_2|$, $\hat{\sigma}_7=|g_2\rangle\langle g_1|$, $\hat{\sigma}_8=|g_1\rangle\langle g_2|$. The mean value $\langle\hat{\sigma}_j\rangle=\mathrm{Tr}\{\hat{\sigma}_j\rho\}$ obeys the equations $d\langle\hat{\sigma}_j\rangle/dt=M\langle\hat{\sigma}_j\rangle+B$, where M, B are a matrix and a column vector, respectively, and are defined through the equations

$$\sum_{j=1}^{8}M_{1,j}\langle\hat{\sigma}_j\rangle=-\gamma_1(\langle\hat{\sigma}_1\rangle+\langle\hat{\sigma}_2\rangle)-i\frac{\Omega_1}{2}(\langle\hat{\sigma}_3\rangle-\langle\hat{\sigma}_4\rangle),$$

$$\sum_{j=1}^{8}M_{2,j}\langle\hat{\sigma}_j\rangle=-\gamma_2(\langle\hat{\sigma}_1\rangle+\langle\hat{\sigma}_2\rangle)-i\frac{\Omega_2}{2}(\langle\hat{\sigma}_5\rangle-\langle\hat{\sigma}_6\rangle),$$

$$\sum_{j=1}^{8}M_{3,j}\langle\hat{\sigma}_j\rangle=-i\frac{\Omega_1}{2}(2\langle\hat{\sigma}_1\rangle+\langle\hat{\sigma}_2\rangle)-\left(\frac{\gamma}{2}+i\Delta\right)\langle\hat{\sigma}_3\rangle-i\frac{\Omega_2}{2}\langle\hat{\sigma}_8\rangle,$$

$$\sum_{j=1}^{8} M_{4,j}\langle\hat{\sigma}_j\rangle = \mathrm{i}\frac{\Omega_1}{2}(2\langle\hat{\sigma}_1\rangle+\langle\hat{\sigma}_2\rangle) - \left(\frac{\gamma}{2}-\mathrm{i}\Delta\right)\langle\hat{\sigma}_4\rangle + \mathrm{i}\frac{\Omega_2}{2}\langle\hat{\sigma}_7\rangle,$$

$$\sum_{j=1}^{8} M_{5,j}\langle\hat{\sigma}_j\rangle = -\mathrm{i}\frac{\Omega_2}{2}(\langle\hat{\sigma}_1\rangle+2\langle\hat{\sigma}_2\rangle) - \left(\frac{\gamma}{2}+\mathrm{i}\Delta\right)\langle\hat{\sigma}_5\rangle - \mathrm{i}\frac{\Omega_1}{2}\langle\hat{\sigma}_7\rangle,$$

$$\sum_{j=1}^{8} M_{6,j}\langle\hat{\sigma}_j\rangle = \mathrm{i}\frac{\Omega_2}{2}(\langle\hat{\sigma}_1\rangle+2\langle\hat{\sigma}_2\rangle) - \left(\frac{\gamma}{2}-\mathrm{i}\Delta\right)\langle\hat{\sigma}_6\rangle + \mathrm{i}\frac{\Omega_1}{2}\langle\hat{\sigma}_8\rangle,$$

$$\sum_{j=1}^{8} M_{7,j}\langle\hat{\sigma}_j\rangle = +\mathrm{i}\frac{\Omega_2}{2}\langle\hat{\sigma}_4\rangle - \mathrm{i}\frac{\Omega_1}{2}\langle\hat{\sigma}_5\rangle,$$

$$\sum_{j=1}^{8} M_{8,j}\langle\hat{\sigma}_j\rangle = -\mathrm{i}\frac{\Omega_2}{2}\langle\hat{\sigma}_3\rangle + \mathrm{i}\frac{\Omega_1}{2}\langle\hat{\sigma}_6\rangle,$$

and

$$B_j = \gamma_1\delta_{j,1} + \gamma_2\delta_{j,2} + \mathrm{i}\frac{\Omega_1}{2}(\delta_{j,3}-\delta_{j,4}) + \mathrm{i}\frac{\Omega_2}{2}(\delta_{j,5}-\delta_{j,6}),$$

with $j=1,\ldots,8$ and $\delta_{j,k}$ the Kronecker-delta. According to this definition, the steady-state vector is now $\sigma_{\mathrm{St}}=M^{-1}B$. Using this notation, we rewrite the operator V_1 in Eq. (20) as $V_1=\alpha_1(\hat{\sigma}_4-\hat{\sigma}_3)+\alpha_2(\hat{\sigma}_6-\hat{\sigma}_5)$, with $\alpha_j=\mathrm{i}\hbar k_j\cos\phi_j\Omega_j/2$, $j=1,2$. The Laplace transform $S(\nu)$ is then the sum of the Laplace transforms $s_j(\nu)$ of the individual terms $g_j(\tau)=\mathrm{Tr}\{\hat{\sigma}_j(\tau)V_1(0)\sigma_{\mathrm{St}}\}$, such that $S(\nu)=\alpha_1[s_4(\nu)-s_3(\nu)]+\alpha_2[s_6(\nu)-s_5(\nu)]$, where $s_j(\nu)$ are given by the equations

$$s_j(\nu)=\sum_k L_{jk}\left(\mathrm{Tr}\{\sigma_k V_1(0)\rho_{\mathrm{St}}\}+\frac{1}{\mathrm{i}\nu}B_k\mathrm{Tr}\{V_1(0)\rho_{\mathrm{St}}\}\right),$$

with L matrix, $L=[\mathrm{i}\nu-M]^{-1}$.

[1] H.J. Metcalf and P. van der Straten, *Laser Cooling and Trapping* (Springer-Verlag, New York, 1999).

[2] F. Diedrich, J.C. Bergquist, W.M. Itano, and D.J. Wineland, Phys. Rev. Lett. **62**, 403 (1989); Ch. Roos, Th. Zeiger, H. Rohde, H.C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, *ibid.* **83**, 4713 (1999).

[3] C. Monroe, D.M. Meekhof, B.E. King, S.R. Jefferts, W.M. Itano, D.J. Wineland, and P. Gould, Phys. Rev. Lett. **75**, 4011 (1995).

[4] M. Morinaga, I. Bouchoule, J.-C. Karam, and C. Salomon, Phys. Rev. Lett. **83**, 4037 (1999); A.J. Kerman, V. Vuletic, C. Chin, and S. Chu, *ibid.* **84**, 439 (2000); D.-J. Han, S. Wolf, S. Oliver, C. McCormick, M.T. DePue, and D.S. Weiss, *ibid.* **85**, 724 (2000).

[5] The proposals are several, and it goes beyond the scope of this paper to give a complete bibliography. Some, currently inspiring experiments with trapped ions and neutral atoms in optical lattices, are, respectively, J.I. Cirac and P. Zoller, Nature (London) **404**, 579 (2000); D. Jaksch, H.-J. Briegel, J.I. Cirac, C.W. Gardiner, and P. Zoller, Phys. Rev. Lett. **82**, 1975 (1999).

[6] G. Morigi, J. Eschner, and C.H. Keitel, Phys. Rev. Lett. **85**, 4458 (2000).

[7] E. Arimondo, *Progress in Optics XXXV,* edited by E. Wolf (North-Holland, Amsterdam, 1996), pp. 259–354; S.E. Harris, Phys. Today **50**(7), 36 (1997).

[8] C.F. Roos, D. Leibfried, A. Mundt, F. Schmidt-Kaler, J. Eschner, and R. Blatt, Phys. Rev. Lett. **85**, 5547 (2000).

[9] F. Schmidt-Kaler, J. Eschner, G. Morigi, C. Roos, D. Leibfried, A. Mundt, and R. Blatt, Appl. Phys. B: Lasers Opt. **73**, 807 (2001).

[10] F. Schmidt-Kaler, J. Eschner, R. Blatt, D. Leibfried, C. Roos, and G. Morigi, *Laser Cooling of Trapped Ions* in *Laser Physics at the Limits*, edited by H. Figger, D. Meschede, C. Zimmerman (Springer-Verlag, Berlin, 2001).

[11] J. Javanainen, M. Lindberg, and S. Stenholm, J. Opt. Soc. Am. B **1**, 111 (1984); M. Lindberg and S. Stenholm, J. Phys. B **17**, 3375 (1984).

[12] M. Lindberg and J. Javanainen, J. Opt. Soc. Am. B **3**, 1008 (1986).

[13] D.J. Wineland, J. Dalibard, and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **9**, 32 (1989).

[14] I. Marzoli, J.I. Cirac, R. Blatt, and P. Zoller, Phys. Rev. A **49**, 2771 (1994).

[15] R. Dum, P. Marte, T. Pellizzari, and P. Zoller, Phys. Rev. Lett. **73**, 2829 (1994).

[16] C. Cohen-Tannoudij, J. Dupont-Roc, G. Grynberg, *Atom-Photon Interactions* (Wiley, Toronto, 1992).

[17] B. Lounis and C. Cohen-Tannoudij, J. Phys. II **2**, 579 (1992).

[18] G. Janik, W. Nagourney, and H. Dehmelt, J. Opt. Soc. Am. B **2**, 1251 (1985).

[19] Y. Stalgies, I. Siemers, B. Appasamy, and P.E. Toschek, J. Opt. Soc. Am. B **15**, 2505 (1989).

[20] The first experiment where velocity selective coherent population trapping has been observed is reported in A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988); J. Opt. Soc. Am. B **6**, 2112 (1989); For a review, see C. Cohen-Tannoudji, Rev. Mod. Phys. **70**, 707 (1998) and [1].

[21] J.I. Cirac, R. Blatt, P. Zoller, and W.D. Phillips, Phys. Rev. A **46**, 2668 (1992).

[22] The frequency $\overline{\nu}=\Omega/2$ sets the upper bound to the modes that can be simultaneously cooled by means of this scheme, for example, when it is applied to cooling of all three motion axes of a trapped ion [8] or to the axial modes of a linear ion chain [9].

[23] S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).

[24] G. Morigi, H. Baldauf, W. Lange, and H. Walther, Opt. Commun. **187**, 171 (2001).

[25] G. Nienhuis, P. van der Straten, and S.-Q. Shang, Phys. Rev. A **44**, 462 (1991).

[26] L.M. Narducci, M.O. Scully, G.-L. Oppo, P. Ru, and J.R. Tredicce, Phys. Rev. A **42**, 1630 (1990); A.S. Manka, H.M. Doss, L.M. Narducci, P. Ru, and G.-L. Oppo, *ibid.* **43**, 3748 (1991).