Comment on "Do Earthquakes Exhibit Self-Organized Criticality?"

In a recent Letter, Yang *et al.* [1] study the interesting problem of the temporal structure of seismicity and its relation with self-organized criticality (SOC), finding that the reshuffling of earthquake magnitudes changes the shape of the earthquake recurrence time (or first-return-time) distribution when the low-magnitude bound, M_c, is raised. Subsequently, they conclude that it is not true that *an earthquake cannot "know" how large it will become.* First, we show that this implication is unjustified.

Yang *et al.* have in mind a fully uncorrelated temporal point process with independent magnitudes as a picture of SOC systems. It is obvious, by construction, that this model is invariant under random rearrangements of the data; as Yang *et al.* do not find this invariance in Southern California, they claim that "earthquakes do not happen with completely random magnitudes" and therefore they are not a SOC phenomenon. In fact, *the only conclusion that can be drawn from this is that the seismicity time series is not uncorrelated,* and there exists some dependence between magnitudes and recurrence times. [This conclusion can be obtained directly, from the fact that a scaling law exists for the recurrence-time distributions corresponding to different low-magnitude bounds, with a scaling function that is not a decreasing exponential (characteristic of a Poisson process, the only uncorrelated process that verifies a scaling law) [2,3].]

The existence of correlations means that, for a given event i, its magnitude M_i may depend on the magnitude of the immediate previous event, M_{i-1}, as well as on the backwards recurrence time, $T_i = t_i - t_{i-1}$, with t_i and t_{i-1} the time of occurrence of both events. This dependence can be extended as well to T_{i-1}, M_{i-2}, T_{i-2}, etc. But further, the recurrence time to the next event, T_{i+1}, may depend on the previous magnitudes M_j and recurrence times T_j, $j \le i$. The reshuffling of magnitudes performed in Ref. [1] breaks (if they exist) the possible correlations of M_i with the previous magnitudes and recurrence times, and the correlations of T_{i+1} with the previous magnitudes (but not with the previous recurrence times). Therefore, any of the influences $M_{i-1} \to M_i$, $T_i \to M_i$, or $M_i \to T_{i+1}$ may be responsible for the results of Yang *et al.*.

The most direct way to test the dependence of a given variable, in this case M_i, with another variable X, is to measure the probability density of X conditioned to different values of M_i, $P(X|M_i)$, and compare with the unconditioned probability density of X, $P(X)$. This is what Fig. 1 displays, using $X = T_i$ and $X = T_{i+1}$ [note that $P(T_{i+1}|M_i) \equiv P(T_i|M_{i-1})$], for Southern California [1], but restricted to periods of stationary seismicity (otherwise, strong aftershock sequences are more sensitive to catalog incompleteness). As $P(T_i|M_i)$ remains practically unchanged for different sets of values of M_i, temporal causality leads to the conclusion that M_i is independent of T_i.

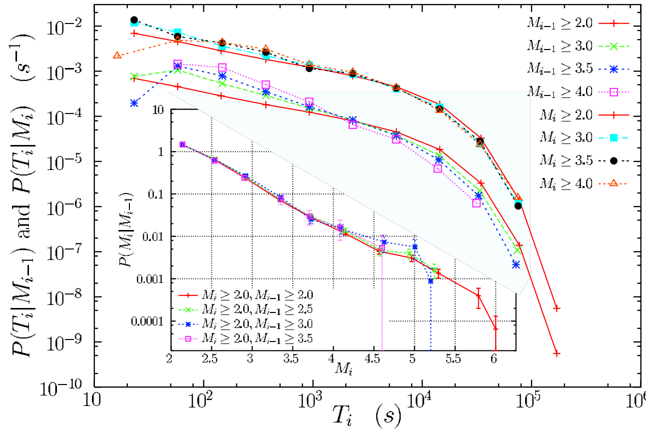

FIG. 1 (color online). (a) Probability densities $P(T_i|M_{i-1})$ (with $M_i \ge 2$) and $P(T_i|M_i)$ (with $M_{i-1} \ge 2$, shifted one decade upwards), compared to $P(T_i)$ (given by $M_i \ge 2$ and $M_{i-1} \ge 2$), for the period May 1994–July 1999. Inset: Probability densities $P(M_i|M_{i-1})$, with $M_i \ge 2$ and $T_i > 1800$ s, compared to $P(M_i)$ (given by $M_{i-1} \ge 2$) for several stationary periods.

In contrast, T_{i+1} clearly depends on M_i, as $P(T_{i+1}|M_i)$ changes for different sets of values of M_i. In other words, *the larger the magnitude* M_i*, the shorter the time to the next event* T_{i+1}*, but the value of this time has no influence on the magnitude of the event,* M_{i+1}. On the other hand, the inset of Fig. 1 shows that $P(M_i|M_{i-1})$ turns out to be not significantly different from $P(M_i)$, ensuring the independence of M_i and M_{i-1}, $\forall\ i$, if the T_i's are restricted to be larger than 30 min (shorter periods of time are not reliable, due to data incompleteness). So, *when an earthquake starts, its magnitude is undetermined* (from the information available at the catalogs).

A second point to clarify is the identification of SOC with the total absence of correlations. Indeed, the Bak-Tang-Wiesenfeld model displays an exponential distribution of recurrence times, but SOC is much more diverse than this model; other models have different recurrence-time distributions. Finally, the concept of SOC (as it happens with chaos) does not exclude the possibility of some degree of prediction, as some references in Yang *et al.* [1] show. So, nothing in Ref. [1] is against the SOC picture of earthquakes.

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[1] X. Yang, S. Du, and J. Ma, Phys. Rev. Lett. **92**, 228501 (2004).
[2] A. Corral, Phys. Rev. Lett. **92**, 108501 (2004).
[3] A. Corral, Phys. Rev. Lett. **95**, 028501 (2005).

