Probing the Desert with Fermion Masses

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We use the $SU(3) \otimes SU(2) \otimes U(1)$ renormalization-group equations to place *upper* limits on the scale where new physics beyond the standard model must emerge. Our bounds rely solely on the structure of the renormalization-group equations and on the magnitudes of heavy-quark masses. For example, if $\sum m_q^2 \geq (450 \text{ GeV})^2$, new physics must be found below 10^5 GeV.

The standard $SU(3) \otimes SU(2) \otimes U(1)$ model provides a very successful description of the strong, weak, and electromagnetic interactions. It is, however, in no sense a fundamental theory. The $SU(3) \otimes SU(2) \otimes U(1)$ model contains at least eighteen parameters, each of which must be adjusted by hand to account for low-energy phenomenology.

What lies beyond the standard model? A variety of extensions have been discussed in recent years. Most of these proposals predict new physics above a characteristic scale M_N. This threshold ranges from a few hundred gigaelectronvolts, in hypercolor scenarios, to about 10^{10} GeV, in axion models, and all the way up to 10^{15} GeV, in typical grand unified theories.

It is very important to constrain the scale M_N. Experiments place lower limits on M_N that are typically of the order of a few hundred gigaelectronvolts. The precise bounds depend on the model under consideration and on the values of various free parameters. In this Letter we describe a new *upper* bound of the scale M_N. We shall show that heavy quarks provide a *model-independent* probe of the $SU(3) \otimes SU(2) \otimes U(1)$ desert.

Our idea is based on the following two observations:

(1) Heavy fermions have large Yukawa couplings at the weak scale M_W. The structure of the $SU(3) \otimes SU(2) \otimes U(1)$ renormalization-group equations implies that such couplings blow up at a relatively low energy M_B.

(2) At the scale M_B, where the Yukawa couplings diverge, new physics should be found. For example, large Yukawa couplings induce strong couplings in the Higgs sector. Such a strongly coupled Higgs sector should give rise to a new spectrum of bound states and to new effective interactions. Of course, it is always possible that the gauge group might change at an ener-

gy $M_N < M_B$. In this case, new gauge bosons of mass M_N should then be found. No matter what happens, new physics must emerge before the Yukawa couplings diverge.

The idea that diverging couplings imply new physics is not new. In quantum electrodynamics, it is associated with the presence of "Landau ghosts." In QED, the electromagnetic coupling blows up at an astronomically high energy, about $M_B \gtrsim e^{137}$ GeV. If QED were a fundamental theory, it would—at the very least—become strongly coupled at the scale M_B. New physics would arise, and quantum electrodynamics as we know it would break down. Of course, we can never see this new physics because QED is only an effective theory. At the scale $M_N \simeq 80$ GeV, QED loses its separate identity and becomes absorbed into the standard electroweak theory.[1]

In what follows we apply these ideas to the standard $SU(3) \otimes SU(2) \otimes U(1)$ model. For ease of presentation, we first examine a three-family model with a heavy top quark (and a single Higgs doublet). We denote the top Yukawa coupling by g_t. Its evolution with energy is given by[2]

$$dg_t/d\tau = \tfrac{1}{2} g_t (9 g_t^2 - 2G), \qquad (1)$$

where $\tau = \ln(E/M_W)/16\pi^2$, $M_W = 80$ GeV, and $G = 8g_3^2 + 2.25g_2^2$. Here g_3 and g_2 are the strong and weak gauge couplings, and we neglect small hypercharge effects. As can be seen in Eq. (1), the evolution of g_t has two types of contributions. The first comes from Higgs boson exchanges and tends to increase the Yukawa coupling at high energies. The second comes from the gauge bosons; it tends to decrease the Yukawa coupling. For large values of g_t, the first term dominates, and the Yukawa coupling grows exponentially with energy.
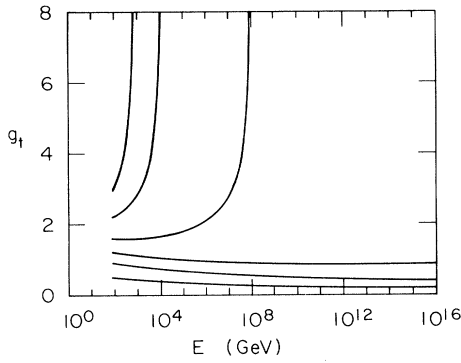
FIG. 1. The evolution of the top Yukawa coupling g_t as a function of the energy E. We have evolved the color gauge coupling using $\Lambda_{\overline{MS}} = 250$ MeV.

In Fig. 1 we show the evolution of g_t with energy for a variety of initial conditions g_t^0 (at the weak scale M_W). For $g_t^0 > 2G/9$, the coupling g_t increases with energy and diverges at an energy M_B.[3] Because of Eq. (1), it diverges faster and faster for larger and larger initial conditions.

In Fig. 2 we plot the scale M_B as a function of the top mass m_t (evaluated at the weak scale M_W). With our conventions, $m_t = g_t^0 \langle \phi \rangle$, where $\langle \phi \rangle$ is normalized to 175 GeV. For large initial conditions, the scale M_B is quite small. A top mass of 250 GeV implies new physics below $M_B = 10^{11}$ GeV. For $m_t = 300$ GeV, this tightens to just 10^6 GeV. If m_t is as large as 400 GeV, new physics should appear below 10 TeV! The top mass provides a very sensitive probe of the SU(3) $\otimes$ SU(2) $\otimes$ U(1) desert.

The value of M_B depends sensitively on the value of g_3 at the weak scale M_W. The larger the value of g_3, the larger the scale M_B. Since we are bounding M_N from above, we wish to choose the maximum value of g_3 consistent with experiment. Therefore, we take g_3

to be given by $\Lambda_{\overline{MS}} = 250$ MeV, in the four-flavor regime ($\overline{MS}$ denotes the modified minimal-subtraction scheme).

More general bounds on M_N can be found by examination of the renormalization of an arbitrary number of heavy families. The appropriate generalization of the top Yukawa coupling is the so-called quark "radius" $T_Q^2 = \Sigma g_q^2$. Here the g_q denote the (real) eigenvalues of the quark Yukawa matrices, and the sum runs over all up- and down-type quarks. The renormalization of T_Q has been investigated elsewhere.[4] For large Yukawa couplings, the quark radius renormalizes even more quickly than the individual couplings themselves.

As with the top Yukawa coupling, the evolution of T_Q can be used to bound the scale of new physics. Instead of m_t, we now plot $(\Sigma m_q^2)^{1/2}$, where the index q runs over all up- and down-type quarks. The results are shown in Fig. 3. When $\Sigma m_q^2 > (355 \text{ GeV})^2$, $M_N < 10^{15}$ GeV. The bound tightens very quickly. For $\Sigma m_q^2 = (450 \text{ GeV})^2$, the scale M_N lowers to just 10^5 GeV. Because of the renormalization group, the heavy-quark spectrum provides a model-independent probe of the SU(3) $\otimes$ SU(2) $\otimes$ U(1) desert. If the sum Σm_q^2 is large, the desert must bloom.

In Fig. 3, we have evolved g_2 and g_3 with the beta functions appropriate for $N_F = 8$, 9, 10, and 11 families. The limits for $N_F < 8$ are not shown because they are even tighter. At any given energy, the bound on M_N is given by the curve with the greatest value of N_F. In Fig. 3, the graphs for $N_F > 8$ are terminated when the color coupling blows up.

Certain extensions of the standard model have two Higgs doublets at low energies. These models usually have one Higgs doublet coupled to up-type quarks, and the other coupled to their down-type partners. This
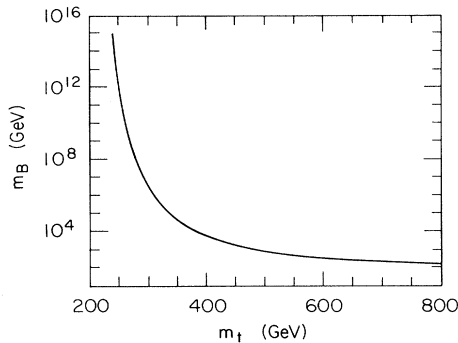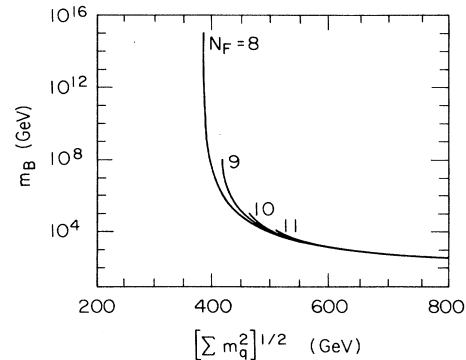

FIG. 2. The upper limit on the scale of new physics, as a function of the top mass m_t in a three-family model. As in Fig. 1, we take $\Lambda_{\overline{MS}} = 250$ MeV. Smaller values of $\Lambda_{\overline{MS}}$ lead to tighter bounds.

FIG. 3. The upper limit on the scale of new physics, as a function of the quark radius $(\Sigma m_q^2)^{1/2}$. The bound is given by the graph with the greatest number of families in a given energy region. We have included results for $N_F = 8$, 9, 10, and 11. The graphs terminate when the color coupling diverges. As before, we take $\Lambda_{\overline{MS}} = 250$ MeV.
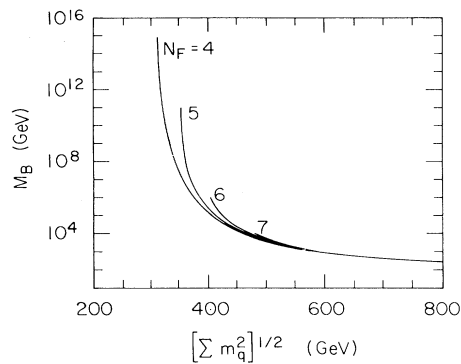
FIG. 4. The same as in Fig. 3, for the supersymmetric version of the standard model. We include the limits for $N_F = 4, 5, 6$, and 7.

type of coupling naturally suppresses flavor-changing neutral currents. Our analysis is easily extended to include these two-Higgs-doublet models. The evolution of T_Q differs from the one-Higgs-doublet case, but it is not hard to see that the bounds on M_N remain the same.[5,6] Therefore, the limits in Fig. 3 are also valid in two-Higgs-doublet extensions of the standard $SU(3) \otimes SU(2) \otimes U(1)$ model.

Finally, we consider the supersymmetric extension of the standard model. We shall see that if supersymmetry is found below the weak scale M_W, the bounds on M_N can be tightened still further. This follows from the fact that the superpartners of the quarks, leptons, and gauge bosons all contribute to the $SU(3) \otimes SU(2) \otimes U(1)$ beta functions.[7] The final results are shown in Fig. 4, with the assumption that the superpartners are all lighter than 80 GeV. As before, we take $\Lambda_{\overline{MS}} = 250$ MeV. Because of the effects of the superpartners, the color beta function blows up very quickly, and so Fig. 4 includes only the four cases $N_F = 4, 5, 6$, and 7.

In this Letter we have demonstrated that heavy quarks provide a model-independent probe of the desert. They extend the range of teraelectronvolt accelerators to well beyond the center-of-mass energies. The discovery of new heavy quarks would indicate that new physics waits to be found in the $SU(3) \otimes SU(2) \otimes U(1)$ desert.

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[1]The hypercharge factor in the $SU(3) \otimes SU(2) \otimes U(1)$ model also suffers from Landau ghosts. In grand unified theories, this is cured by embedding $SU(3) \otimes SU(2) \otimes U(1)$ in an asymptotically free semisimple gauge group G.

[2]T. Cheng, E. Eichten, and L. Li, Phys. Rev. D 9, 2255 (1974).

[3]We run the gauge couplings with their two-loop renormalization-group equations. M_B is defined to be the scale where $\alpha_t = g_t^2/4\pi = 10$.

[4]B. Pendleton and G. Ross, Phys. Lett. 98B, 291 (1981); C. Hill, Phys. Rev. D 24, 691 (1981); E. Paschos, Z. Phys. C 26, 235 (1984); J. Bagger, S. Dimopoulos, and E. Massó, Nucl. Phys. B253, 397 (1985).

[5]J. Bagger, S. Dimopoulos, and E. Massó, Phys. Lett. 156B, 357 (1985).

[6]The second doublet gives a small contribution to the renormalization of g_2. We neglect this effect; it leaves our bounds essentially unchanged.

[7]The evolution of T_Q in the supersymmetric case is discussed in J. Bagger, S. Dimopoulos, and E. Massó, Phys. Rev. Lett. 55, 920 (1985).