Renormalization-Group Constraints in Supersymmetric Theories

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We use the SU(3) ⊗ SU(2) ⊗ U(1) renormalization-group equations to constrain fermion masses and charged-scalar couplings in supersymmetric grand unified theories.

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Supersymmetric theories provide a promising framework for the solution of the fine-tuning and gauge-hierarchy problems.[1] They are the only known theories where elementary scalars are naturally light. The lightness of the Higgs boson can be understood if supersymmetry remains unbroken down to the weak scale M_W.

In spite of their enlarged symmetry, supersymmetric theories fail to provide any new information on the quark and lepton masses. The only model-independent predictions are those that follow from the infrared fixed points of the SU(3) ⊗ SU(2) ⊗ U(1) renormalization-group equations. In ordinary unified theories, this fixed-point structure implies that the masses and mixings of heavy quarks are independent of the details of the short-distance physics.[2,3] In this Letter we extend this analysis to supersymmetric grand unified theories. We find bounds on the spectrum of heavy fermions and restrictions on the couplings of the charged Higgs scalar.

Our fundamental hypothesis is that of a SU(3) ⊗ SU(2) ⊗ U(1) desert extending between the weak scale M_W and the unification scale M_X. We require all couplings to be small enough for perturbation theory to be valid, and we assume that supersymmetry is unbroken all the way down to M_W. These hypotheses are valid in most supersymmetric theories that address the gauge-hierarchy problem. This includes models where supersymmetry is broken in a hidden sector at an intermediate scale of about 10^{11} GeV. In these theories the effective scale of supersymmetry breaking in the visible sector is also M_W.

Supersymmetric theories contain two Higgs doublets, one giving mass to up-type quarks, and the other giving mass to their down-type partners. The Yukawa couplings are as follows:

$$\mathscr{L}_Y = \bar{u}\mathscr{U} Q\phi_u + \bar{d}\mathscr{D} Q\phi_d + \bar{e}\mathscr{E} L\phi_d, \tag{1}$$

where $\mathscr{U}$, $\mathscr{D}$, and $\mathscr{E}$ are the Yukawa matrices of the up-, down-, and electron-type fermions, Q and L are the quark and lepton isodoublets, and u, d, and e are the corresponding singlet fields.

Heavy-fermion masses are determined by the infrared-fixed-point structure of the SU(3) ⊗ SU(2) ⊗ U(1) renormalization-group equations. These equations do not receive contributions from soft supersymmetry-breaking terms. This follows from the fact that the mass splittings within supermultiplets are much smaller than the relevant desert momenta. The soft supersymmetry breakings, however, induce finite shifts in the fermion masses. These shifts are of magnitude $(\alpha/2\pi)M_W$, and we neglect them here.

In supersymmetric theories, the SU(3) ⊗ SU(2) ⊗ U(1) gauge couplings evolve as follows:[4]

$$dg_3/dt = (9-2N_F)g_3^3, \quad dg_2/dt = (5-2N_F)g_2^3,$$
$$dg_1/dt = -(\tfrac{3}{5}+2N_F)g_1^3, \tag{2}$$

where $t = -(1/16\pi^2)\ln(M/M_X)$, and N_F denotes the number of families. The requirement of perturbative unification restricts N_F to be less than or equal to 4. To one-loop order, the evolution of the Yukawa couplings are given by[5]

$$\mathscr{U}^{-1}\frac{d\mathscr{U}}{dt} = G_U - 3T_U - (3\mathscr{U}^\dagger\mathscr{U} + \mathscr{D}^\dagger\mathscr{D}),$$
$$\mathscr{D}^{-1}\frac{d\mathscr{D}}{dt} = G_D - 3T_D - T_E - (3\mathscr{D}^\dagger\mathscr{D} + \mathscr{U}^\dagger\mathscr{U}), \tag{3}$$
$$\mathscr{E}^{-1}\frac{d\mathscr{E}}{dt} = G_E - T_E - 3T_D - 3\mathscr{E}^\dagger\mathscr{E},$$

where

$$G_U = \tfrac{16}{3}g_3^2 + 3g_2^2 + \tfrac{13}{9}g_1^2,$$
$$G_D = \tfrac{16}{3}g_3^2 + 3g_2^2 + \tfrac{7}{9}g_1^2, \tag{4}$$
$$G_E = 3g_2^2 + 3g_1^2, \quad T_Y = \mathrm{Tr}\mathscr{Y}^\dagger\mathscr{Y},$$

with $\mathscr{Y}=\mathscr{U},\mathscr{D}$, or $\mathscr{E}$. Equation (3) has the following fixed points in the $t\to\infty$ limit[3]: (1) the quark fixed point, with

$$\mathscr{U}^\dagger\mathscr{U}=\mathscr{D}^\dagger\mathscr{D}=\frac{G_Q}{3N_F+4},\quad \mathscr{E}=0; \tag{5}$$

(2) the lepton fixed point, with

$$\mathscr{E}^\dagger\mathscr{E}=\frac{G_L}{N_F+3},\quad \mathscr{U}=\mathscr{D}=0. \tag{6}$$

Here G_Q denotes an appropriate average of G_U and G_D (with $g_1=0$), and G_L represents a similar average over G_E.

For physical gauge couplings, the quark fixed point determines the low-energy spectrum of quarks and leptons. Because of the fixed point, all quarks have the same Yukawa coupling as $t\to\infty$. All weak mixings and associated CP-nonconserving phases vanish as well. This implies that both isospin and family symmetry are restored in the infrared limit.

In previous work[2,3] it has been shown that infrared fixed points are not necessarily reached in realistic grand unified theories. This is because the physical range of t is rather short, $0\lesssim t\lesssim\frac{1}{5}$. In realistic theories, fixed points are approached only if the Yukawa couplings are sufficiently large. In what follows, we restrict our attention to fixed points that are reached in physical time.

We begin by considering the renormalization of the overall scale of heavy quarks, given by T_U and T_D. From the general equations (3) it is easy to show that T_U and T_D evolve as follows:

$$\begin{aligned}\frac{dT_U}{dt}&=2(G_U-3T_U)T_U-6\mathrm{Tr}(\mathscr{U}^\dagger\mathscr{U})^2\\&\quad-2\mathrm{Tr}(\mathscr{U}^\dagger\mathscr{U}\mathscr{D}^\dagger\mathscr{D}),\\ \frac{dT_D}{dt}&=2(G_D-3T_D-T_E)T_D-6\mathrm{Tr}(\mathscr{D}^\dagger\mathscr{D})^2\\&\quad-2\mathrm{Tr}(\mathscr{D}^\dagger\mathscr{D}\mathscr{U}^\dagger\mathscr{U}).\end{aligned} \tag{7}$$

These equations can be used to bound the scale of the heavy quarks,

$$\begin{aligned}dT_U/dt&\le 2(G_U-3T_U)T_U,\\ dT_D/dt&\le 2(G_D-3T_D)T_D.\end{aligned} \tag{8}$$

Equation (8) gives upper bounds on T_U and T_D at the weak scale M_W. Using the gauge couplings corresponding to $N_F=4$, we find[6]

$$T_U,T_D\lesssim 2.7. \tag{9}$$

A similar analysis in the lepton sector gives

$$T_E\lesssim 3.8. \tag{10}$$

The bounds for $N_F=3$ are even more stringent, so that the limits (9) and (10) are valid for any number of families.

To convert (9) into bounds on the quark masses, we introduce vacuum expectation values v_u and v_d for the scalar fields ϕ_u and ϕ_d. By using $\sum M_U^2=(v_u)^2T_U$ and $\sum M_D^2=(v_d)^2T_D$, we place limits on the quark mass spectrum:

$$\begin{aligned}&\sum M_U^2\lesssim (v_u/v)^2(290\ \mathrm{GeV})^2,\\&\sum M_D^2\lesssim (v_d/v)^2(290\ \mathrm{GeV})^2,\\&\sum M_Q^2\lesssim (290\ \mathrm{GeV})^2,\end{aligned} \tag{11}$$

where $v_u^2+v_d^2=v^2=(175\ \mathrm{GeV})^2$, and all masses are evaluated at the weak scale M_W. In Eq. (11), the sum over Q runs over both up- and down-type quarks.

Equation (9) can also be used to bound the ratio of the vacuum expectation values v_u/v_d. To see this, suppose that there is a fourth family, whose top- and bottom-type quarks have masses $m_{t'}$ and $m_{b'}$, respectively. The corresponding Yukawa couplings are given by $g_{t'}=m_{t'}/v_u$ and $g_{b'}=m_{b'}/v_d$. The fact that $g_{t'}$ and $g_{b'}$ satisfy (9) sets limits on v_u and v_d:

$$(v/v_u)^2\lesssim 160\quad(\text{or}\quad v_u\gtrsim 14\ \mathrm{GeV}); \tag{12a}$$

$$(v/v_d)^2\lesssim 160\quad(\text{or}\quad v_d\gtrsim 14\ \mathrm{GeV}). \tag{12b}$$

Here we have used the fact that $m_{t'}$ and $m_{b'}$ are greater than 23 GeV. [If there are only three families, the limit (12a) still holds.] The bounds on the vacuum expectation values are more stringent for heavier quarks. For quarks of mass $m_{b'}$ and $m_{t'}$, we find

$$\left(\frac{m_b'}{290\ \mathrm{GeV}}\right)^2\lesssim\left(\frac{v_d}{v_u}\right)^2\lesssim\left(\frac{290\ \mathrm{GeV}}{m_{t'}}\right)^2. \tag{13}$$

The ratio v_d/v_u governs the coupling of the charged Higgs boson to fermions. Therefore, our limits (12) and (13) bound the charged-Higgs couplings in supersymmetric theories. They suppress the one-loop charged-Higgs contributions to K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, and B^0-$\bar{B}^0$ mixing.

Finally, we examine the special case of a fourth family that is decoupled from its lighter counterparts. The evolution equations for $g_{t'}$ and $g_{b'}$ become

$$\begin{aligned}\frac{d}{dt}\ln g_{t'}&=G_U-3T_U-(3g_{t'}^2+g_{b'}^2),\\ \frac{d}{dt}\ln g_{b'}&=G_D-3T_D-T_E-(3g_{b'}^2+g_{t'}^2),\end{aligned} \tag{14}$$

where G_Y and T_Y are given in (4). In Fig. 1 we plot the evolution of $g_{t'}$ and $g_{b'}$. Figure 1(a) indicates that the fixed point is reached in physical time for a wide range of initial conditions. Figure 1(b) shows that the

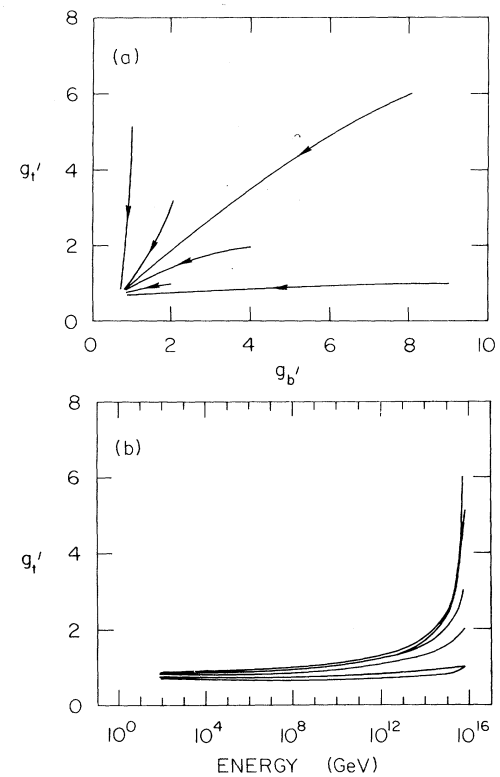

FIG. 1. (a) The evolution of $g_{t'}$ and $g_{b'}$ with energy for various initial conditions. We have neglected the contributions of the three light families to T_U and T_D. The arrows indicate the flow of increasing t (decreasing energy). (b) The evolution of $g_{t'}$ with energy for the same initial conditions.

fixed point is reached very quickly.

Equation (14) can be used to obtain tighter bounds for the masses of the fourth family. A bound on the value of $g_{t'}$ can be obtained by setting $T_U = g_{t'}^2$ and $g_{b'} = 0$ (and likewise for $g_{b'}$). This gives $g_{t'}, g_{b'} \lesssim 1.17$, which in turn implies

$$m_{t'} \lesssim (v_u/v) 205 \text{ GeV}, \quad m_{b'} \lesssim (v_d/v) 205 \text{ GeV}. \quad (15)$$

The bounds (15) imply that the lightest quark in the fourth family must have a mass of less than 150 GeV. This is in accord with the results of Tabata, Umemura, and Yamamoto.[7]

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