A Periodic Inventory System of Intermittent Demand Items with Fixed Lifetimes

Abstract

Perishable items with a limited lifespan and intermittent/erratic consumption are found in a variety of industrial settings: dealing with such items is challenging for inventory managers. In this study, a periodic inventory control system is analysed, in which items are characterised by intermittent demand and known expiration dates. We propose a new inventory management method, considering both perishability and intermittency constraints. The new method is a modification of a method proposed in the literature, which uses a periodic order-up-to-level inventory policy and a compound Bernoulli demand. We derive the analytical expression of the fill rate and propose a computational procedure to calculate the optimal solution. A comparative numerical analysis is conducted to evaluate the performance of the proposed solution against the standard inventory control method, which does not take into account perishability. The proposed method leads to a bias that is only affected by demand size, in contrast to the standard method which is impacted by more severe biases driven by intermittence and periods before expiration.

Keywords: Intermittent demand, perishable items, forecasting, inventory control, periodic inventory system.

1. Introduction

Inventory systems for perishable goods have been the focus of much attention in the academic literature, particularly for their application in common sectors of goods (e.g., food and pharmaceutical products). The assumption that an item can be stored indefinitely in warehouses does not hold for perishable goods, and this complicates their inventory control. Perishability is a broad topic in the literature, as confirmed by the extensive reviews of the relevant research, such as those of Raafat (1991), Goyal and Giri (2001), and Bakker, Riezebos, and Teunter (2012). The taxonomy drivers in the latter two reviews establish that the first key element to investigate is the lifetime of the item, which may be fixed, distributed according to certain probability distributions, or characterised by a time-inventory dependent deterioration rate (White and Censlive, 2015; Kouki et al. 2014; Kouki et al. 2016a,b). It is worth to remark indeed that deterioration can occur in various ways, but it must be distinguished from obsolescence, which refers to the loss of value due to technological changes or the entry of new products into the market. However, obsolescence has attracted little attention because an obsolete good is simply not reordered (Goyal and Giri 2001). In our study we regard the lifetime of goods as deterministic (i.e., known *a priori*). The second key element is the demand, which can be either deterministic or stochastic. Our contribution deals with a stochastic demand.

When perishable goods also exhibit intermittent consumptions, their inventory control results in a further complication due to the ineffectiveness of inventory systems for non-intermittent demand generation processes. Nevertheless, intermittency is relevant in several industrial settings. Spare parts are typical items of intermittent consumption, but intermittency could be also the consequence of batching decisions in the supply chain. Hence, food and pharmaceutical multi-echelon supply chains are contexts in which perishable goods, in particular those with fixed lifetimes (decaying products are not considered here), may also exhibit intermittency. To the best of our knowledge, no inventory models for perishable goods with intermittent demand are provided in the literature. Addressing this research gap is the objective of our study.

In this study, the periodic inventory system proposed in Teunter, Syntetos, and Babai (2010) is adapted to perishable items with fixed lifetimes. This work represents an extension of Balugani et al. (2017). Our inventory control model is validated through a two-level full factorial design experiment around the most significant variables, whereas in Balugani et al. (2017) only a scenario analysis was presented. The experimental results are statistically analysed with a linear regression, proving that the variables do not impact its performance and suggesting that the underlying model is unbiased. In addition, differently from Balugani et al., we conduct in this paper a more realistic numerical investigation where the demand distribution parameters are forecasted (using an appropriate intermittent demand forecasting method) and this is integrated in the inventory model.

The paper is organised as follows. Section 2 contains an overview of the background of two research streams, i.e. stochastic demand of perishable goods with fixed lifetimes and intermittent demand. Section 3 details the forecasting and inventory control models and their assumptions; Section 4 outlines the experiment designed to validate the model and the obtained experimental results. Section 5 provides conclusions and the research agenda.

2. Research background

Recent contributions have referred to the stochastic demand of perishable goods with fixed lifetimes. Minner and Transchel (2010) dynamically determined replenishment quantities for perishable goods with fixed lifetimes that satisfy multiple service-level constraints during a specific period, and they extended their model to non-stationary demand. Xin, Pang, and Limeng (2014) addressed a joint pricing and inventory control problem for stochastic perishable inventory systems, in which both backlogging and lost-sales cases were studied. They provided an approach able to deal with both continuous and discrete demand distributions. Similarly, Duan, Cao, and Huo (2018) dealt with the dynamic pricing and production rate for stochastic and price-dependent demand of items with fixed-lifetime in a continuous-time environment. Pauls-Worm et al. (2014) addressed the production planning of perishable products with fixed lifetimes when the demand is non-stationary; they formulated an MILP model containing a service-level constraint. Pauls-Worm et al. (2016) proposed another MILP model for a fill-rate constraint. Muriana (2016) addressed the normally distributed demand of perishable items with fixed lifetimes to reach the optimum lot size. She evaluated the probability of a product remaining in stock beyond the end of its lifetime, and determined the best order size, the time at which the inventory level drops to zero, and the cycle time minimizing the expected total cost. Gutierrez-Alcoba et al. (2017) achieved the expected inventory level at different ages for the non-stationary stochastic demand of perishable items with fixed lifetimes. They also extended Silver's heuristic (Silver 1978) to deal with these conditions by means of analytical and simulation-based variants of the original heuristic. Janssen et al. (2018) adopted a periodic review system for the stochastic demand of items with fixed lifetimes, adding the closing days constraint as a typical feature of groceries. Kouki, Babai, and Minner (2016) showed the value of dual-sourcing in the context of perishable items with fixed lifetimes and a Poisson-distributed demand. They considered an age-based control with a base stock policy. Perishable items with stochastic demand and fixed lifetimes have also been studied by Kara and Dogan (2018), who proposed an aged-based replenishment policy solved by a reinforcement learning algorithm.

Intermittent demand can be characterised by two stochastic variables, the non-zero demand (i.e., demand size) and the time interval between two successive non-zero demands (i.e., the inter-demand interval). Croston's method (Croston 1972) is the seminal contribution to intermittent demand forecasting according to a normally distributed demand size and a Bernoulli probability of a demand occurrence. A simple exponential smoothing is applied to both variables when the demand occurs, and an estimator of the expected value of demand per period is then evaluated by the ratio of these estimators. Syntetos and Boylan (2005) proposed an approximately unbiased modification of Croston's method called the Syntetos-Boylan Approximation (SBA). Another modification of Croston's method was proposed by Levén and Segerstedt (2004). However, Boylan and Syntetos (2007) demonstrated that this leads to a more biased estimator than Croston's original method. Teunter and Sani (2009) compared several modifications of Croston's method, while Regattieri et al. (2005) compared other forecasting approaches for intermittent demand. Babai, Syntetos, and Teunter (2014) and Babai et al. (2018) addressed the intermittent demand forecasting issue for items with a risk of obsolescence. Machine learning techniques, in particular artificial neural networks, have also been used to forecast intermittent demand by exploiting their ability to deal with not linear processes without requiring any distributional assumptions (e.g., Gutierrez, Solis, and Mukhopadhyay 2008; Kourentzes 2013; Lolli et al. 2017). When the intermittent demand patterns also contain seasonal and trend components, Seasonal Auto Regressive Integrated Moving Average (SARIMA) modelling has shown promising results (e.g., Gamberini et al. 2010). Several researchers have recommended the use of the non-parametric bootstrapping approach to estimate the lead-time demand based on a large number of independent bootstrap replications from available data. A recent literature review on bootstrapping forecasting methods in the context of intermittent demand is presented in Hasni et al. (2018a). For a comparison between parametric and non-parametric approaches and a thorough investigation of bootstrapping, the reader can refer to Syntetos, Babai, and Gardner (2015), Sillanpää and Liesiö (2018) and Hasni et al. (2018b). Most of the above mentioned research has looked at the forecast accuracy of the forecasting methods and their inventory performance. A good discussion on the performance measures of intermittent demand forecasting methods is presented by Prestwich et al. (2014) and Petropoulos and Kourentzes (2015).

The literature provides a wide set of compound distributions for modelling the intermittent demand generation process and computing the parameters of the stock control policies. However, as emphasised by Babai, Ladhari, and Lajili (2015), as the data become more erratic the true demand size distribution may not comply with any standard theoretical distribution. Two demand generation processes are typically carried out. If time is treated as a discrete (integer) variable, demand can be assumed to be generated by a Bernoulli process, so that the inter-demand intervals are geometrically distributed. Otherwise, the Poisson demand generation is used, which leads to negative exponentially distributed intervals. When combining a Bernoulli or a Poisson demand arrival with a generic distribution of demand sizes, a compound distribution is obtained. These demand generation processes are generally conducted when modelling the re-order policies via statistical analysis. The statistical modelling of intermittent demand was conducted by Teunter, Syntetos, and Babai (2010) and Babai, Jemai, and Dallery (2011). An empirical goodness-of-fit investigation conducted by Syntetos, Babai and Altay (2012) showed the good fit of compound Poisson distributions to thousands of spare parts characterised with intermittent demand. Lengu, Syntetos, and Babai

(2014) combined issues of distributional assumptions for modelling purposes and item classification, while Syntetos and Boylan (2006) focused on the interaction between forecasting and stock control. They applied negative binomial distribution to model the demand, and completed a factorial experiment by simulating the behaviour of a periodic review system when combined with different forecasting methods (simple moving average, simple exponential smoothing, Croston's method, and SBA).

The proposed inventory model in this paper extends the Teunter, Syntetos, and Babai (2010)'s model by accounting for perishable items with fixed lifetimes. We also extend the work of Balugani et al. (2017) by conducting a numerical investigation of the inventory model where the demand distribution parameters are forecasted. Experimentally, an in-depth statistical validation is carried out to measure the method's performance.

3. Methodology

In this section, we first present the forecasting and inventory control assumptions considered, followed by the proposed inventory replenishment model.

3.1 Forecasting and inventory control assumptions

The standard compound Bernoulli intermittent demand model we consider in this study was proposed in Croston (1972). The demand d_i of a period i is defined as:

$$d_i = \begin{cases} \phi(d_i, 1) & with\,probability\;\; p \\ 0 & with\,probability\;\; 1-p \end{cases} \tag{1}$$

where p is the probability that a positive demand takes place in a period and $\phi(d_i, 1)$ is the probability that a positive demand with size d_i occurs during a single period. More generally, we denote by $\phi(x, y)$ the distribution function that a positive demand x occurs during y periods. The model is quite general and, as such, the demand size can assume any positive probability distribution, the most common ones being truncated Normal and Gamma.

Given the model expressed in Equation (1), the Teunter-Syntetos-Babai (2010) model, which is also considered in our work, estimates the demand using SBA (Syntetos and Boylan, 2005) as follows:

$$\hat{z}_i = \alpha \cdot z_{i-1} + (1-\alpha) \cdot \hat{z}_{i-1} \tag{2}$$

$$\widehat{in}_i = \alpha \cdot in_{i-1} + (1-\alpha) \cdot \widehat{in}_{i-1} \tag{3}$$

$$\hat{d}_i = \left(1 - \frac{\alpha}{2}\right) \cdot \frac{\hat{z}_i}{\widehat{in}_i} \tag{4}$$

where $\hat{z}_i$ is the estimated size of a positive demand, after the positive demand z_{i-1} in period $i-1$, and $\widehat{in}_i$ is the estimate inter-arrival between positive demands, calculated from the last inter-arrival in_{i-1} between positive demands. Both forecasts are used in Equation (4) to produce an estimate of the demand $\hat{d}_i$. Equations (2), (3) and (4) are updated only after a positive demand using the smoothing parameter $\alpha \in (0,1)$ defines how much new data has an effect on the previous estimates.

From Equation (3), an estimate of the probability p, denoted by $\hat{p}_i$, can also be obtained as follows:

$$\hat{p}_i = \frac{1}{\widehat{in}_i} \tag{5}$$

To compute an expected mean squared error $\widehat{MSE}(z_i)$ for the positive demand, an exponential smoother is applied to the squared error, as in Teunter, Syntetos, and Babai (2010):

$$\widehat{MSE}(z_i) = (1-\beta) \cdot \widehat{MSE}(z_{i-1}) + \beta \cdot (z_{i-1} - \hat{z}_{i-1})^2 \tag{6}$$

where the smoothing parameter $\beta \in (0,1)$ does not necessarily equal the smoothing parameter α.

As in Teunter, Syntetos, and Babai (2010), the inventory replenishment model considered in this study is the periodic order-up-to (T,S) policy where the order-up-to-level S is calculated to satisfy a target fill rate service level.

In the following subsection we propose a solution to calculate the order-up-to-level by taking into account the perishability constraint.

3.2 Development of the inventory replenishment model

To derive the fill rate expression when considering the perishability constraint, we first present the fill rate model that is considered. An order can be placed every t periods, collectively defining the constant review time, and requires a fixed lead time l to arrive, with $l \leq t$. At the beginning of a review time in this scenario, a stock $s \geq 0$ is available and an order o can be placed. The total amount $s + o$ is expected to cover the demand of t periods after the lead time, and a new order can in fact be placed only after t periods and requires l periods to arrive. The performance measure associated with this model is the fill rate over the specified t periods, defined as the probability of a positive demand taking place in one of those periods that is satisfied by $o + s$ and thus generates no stock-out.

Given a stock s and order quantity o, the fill-rate fr is:

$$fr = \frac{1}{t}\sum_{i=1}^{t}\sum_{k=0}^{i+l-1}\binom{i+l-1}{k}p^{k}(1-p)^{i+l-1-k}\cdot\Phi(o+s,k+1) \tag{7}$$

where $\Phi(x, y)$ is the cumulative distribution function that a positive demand x occurs during y periods while p is the probability a period yields a positive demand.

The replenishment model that takes into account the perishability constraint divides the stock s into two separate stocks:

- s_e the amount of goods that will expire at the end of one of the t periods after the lead time l.
- s_{ne} the amount of goods that will not expire in the given time frame.

These quantities are updated as in the Teunter, Syntetos, and Babai (2010)'s model at the beginning of each period, before the order o is placed. The expired stock is discarded and the expiring stock is moved from s_{ne} to s_e. The stock s_e is assumed to expire at the end of period t_e, calculated from the update period before the lead time, while the ordered quantity o is assumed not to expire in the time frame.

Given a hypothetical positive demand d_i at period i, two mutually exclusive cases can arise:

- The period i occurs before the expiration date.
- The period i occurs after the expiration date.

In the first case, the perishability has no effect, thus Equation (7) is used. In the second case s_e has expired and a different Equation is required. As in Section 2.1, given a positive demand d_i in period i after the lead time l, all the possible demands in the previous periods $\sum_{k=0}^{i+l-1} d_k$ must be considered. This leads to two scenarios:

- The demands before the expiration date partially or totally consumed the expiring stock, i.e., $\sum_{k=0}^{t_e+l-1} d_k \leq s_e$.
- The demands before the expiration date consumed more than the expiring stock, i.e., $s_e < \sum_{k=0}^{t_e+l-1} d_k \leq s_e + s_{ne} + o$.

The fill rate fr_{i1} of the first scenario is the probability that the demands before t_e are satisfied by s_e and the demands after t_e including d are satisfied by $s_{ne} + o$:

$$fr_{i1} = \sum_{k=0}^{t_e} \sum_{h=0}^{i+l-t_e-1} \binom{t_e}{k} g(k, t_e) \Phi(s_e, k) \cdot \binom{i+l-t_e-1}{h} g(h, i+l-t_e-1) \Phi(o + s_{ne}, h+1) \qquad (8)$$

where:

$$g(x, y) = p^x (1-p)^{y-x} \qquad (9)$$

is a notation shortcut for the probability that x periods over y present a positive demand, and:

$$\Phi(x, 0) = 1 \quad \forall x \geq 0 \qquad (10)$$

as in absence of positive demands no stock out can occur.

The fill rate fr_{i2} of the second scenario is the probability that the demands d_{be} before t_e are satisfied by $o + s$ and the demands after t_e including d_i are satisfied by the remaining stock $o + s - d_{be}$ with $d_{be} > s_e$:

$$fr_{i2} = \sum_{k=1}^{t_e} \sum_{h=0}^{i+l-t_e-1} \sum_{d_{be}=s_e+1}^{o+s} \binom{t_e}{k} g(k, t_e) \phi(d_{be}, k) \cdot \binom{i+l-t_e-1}{h} g(h, i+l-t_e-1) \Phi(o + s - d_{be}, h+1) \qquad (11)$$

These scenarios are mutually exclusive, thus the fill rate fr_i of period i is:

$$fr_i = fr_{i1} + fr_{i2} \tag{12}$$

The overhaul fill rate fr accounts for the individual fill rates of t periods after the lead time, as in Equation (7):

$$fr = \frac{1}{t}\sum_{i=1}^{t} fr_i \tag{13}$$

This methodology expands that defined at the beginning of this section, by considering a portion s_e of the stock as perishable. The calculations above refer to a single expiration date, but similar considerations can be applied to address multiple expiration dates in the frame of analysis.

From a computational perspective, the proposed methodology is more demanding than the original, and the calculation of fr_{i2} requires an analysis of $o + s_{ne}$ demands before t_e. This calculation is necessary as the last component of Equation fr_{i2}, defining the probability a demand after t_e does not produce a stock out, and requires the number of units $o + s - d_{be}$ left in stock.

3.3 Inventory replenishment model: computational solution

The model presented in Section 2.2 aims to define the order quantity o_{min} at the beginning of lead time l. o_{min} is the minimum order capable of achieving a target fill rate fr_{target} for t periods after the lead time l. In contrast, Equation (13) calculates the fill rate fr of t periods after the lead time l given a predefined order quantity o. Equation (13) is not easy to invert, thus no direct equation is available to solve the problem at hand. A common solution in the relevant literature involves a stepwise search:

Step 1. Start assuming an order quantity $o = 0$

Step 2. Calculate fr for the value of o under analysis.

Step 3. If $fr \geq fr_{target}$ then stop, $o_{min} = o$.

Step 4. If $fr < fr_{target}$ then increment o by one unit and go to Step 2.

This procedure is feasible if the computational cost for the fill rate calculation is limited. In our case such cost is significant and increases with o, thus the algorithm reactivity decreases as it goes on.

An alternative procedure, based on the secant method, is proposed to decrease the amount of calculations involved. The optimum is formally defined as:

$$o_{min} = inf\{o: fr(o, s_e, s_{ne}) \geq fr_{target}\} \tag{14}$$

where $fr(o, s_e, s_{ne})$ is the fill rate relative to the order quantity o and the stocks s_e and s_{ne}.

As for fixed stocks s_e and s_{ne} the fill rate can grow only if o increases, Equation (14) can be rewritten as:

$$fr(o_{min}, s_e, s_{ne}) = inf\{fr(o, s_e, s_{ne}): fr(o, s_e, s_{ne}) \geq fr_{target}\} \tag{15}$$

Two properties of the fill rate, as calculated in Equation (13), provide two extremes o_{sup} and o_{inf} to initialize the secant method. This initialization requires no initial calculation of Equation (13) itself:

- *Ceteris paribus*, a decrease in s_e reduces fr.
- *Ceteris paribus*, substituting part of s_e with stock not expiring in t increases fr.

From these properties, two quantities can be defined:

$$fr\left(o_{sup}, 0, s_{ne}\right) = inf\{fr(o, 0, s_{ne}): fr(o, 0, s_{ne}) \geq fr_{target}\} \tag{16}$$

$$fr\left(o_{inf}, 0, s\right) = inf\{fr(o, 0, s): fr(o, 0, s) \geq fr_{target}\} \tag{17}$$

with the property:

$$o_{inf} \leq o_{min} \leq o_{sup} \tag{18}$$

In Equation (16), starting from the optimum order quantity as defined in Equation (15), the elimination of s_e reduces fr. From this point, to achieve $fr(o, 0, s_{ne}) \geq fr_{target}$ fixed s_{ne}, the order quantity now defined as o_{sup} increases. A similar effect takes place in Equation (17) where the expiring stock is fully substituted by non-expiring stock. The substitution increases the fill rate, and for this new configuration the initial order quantity is no longer the minimum required to achieve $fr(o, 0, s_{ne}) \geq fr_{target}$. The order quantity now defined as o_{inf} decreases to reach the required minimum fill rate.

Equations (16) and (17) contain no expiring stock, and thus the computationally expensive calculations of Section 2.2 are not required. Equation (7) is iteratively applied to define both o_{sup} and o_{inf}.

To apply the bisection method, the fill rate expressed in Equation (13) is shifted by fr_{target}:

$$fr_{shifted} = fr - fr_{target} \tag{19}$$

The fill rate strictly increases, as a function of o, if s_e and s_{ne} are fixed. If Equation (19) has roots in the interval $[o_{inf}, o_{sup}]$ it has a single root, while if Equation (19) has no roots in the interval then $fr(o_{inf}, s_e, s_{ne}) > fr_{target}$. In this last scenario $o_{min} = o_{inf}$ and the algorithm terminates during the calculation of $fr(o_{inf}, s_e, s_{ne})$ in the first step, as described below.

Given o_{sup} and o_{inf} the calculation of their fill rate using Equation (13) is required at the beginning of the bisection algorithm. This defines the extreme values of $fr_{shifted}$ and makes possible the initial secant calculation:

$$fr_{sup} = fr(o_{sup}, s_e, s_{ne}) \tag{20}$$

$$fr_{inf} = fr(o_{inf}, s_e, s_{ne}) \tag{21}$$

During the generation of new fr_{sup} and fr_{inf}, and the respective o_{inf} and o_{sup}, the algorithm operates only on integer values of o. The new quantity o identified by the secant must be approximated by the nearest integer. If it falls over the current o_{sup} or under the current o_{inf}, the value is approximated by $\lfloor o \rfloor$ and $\lceil o \rceil$ respectively. The algorithm terminates when $fr(o, s_e, s_{ne}) = fr_{target}$, when a floor approximation reaches o_{min} or a ceil approximation reaches o_{sup}. In the last two scenarios the interval of analysis has unitary length and by construction $o_{inf} < o_{min}$, thus $o_{min} = o_{sup}$.

4. Experimental analysis

4.1 Probability distribution and estimations

To calculate Equation (13), both the distribution function $\phi(x, y)$ and the cumulative distribution function $\Phi(x, y)$ of a positive demand x during y periods must be known. The probability p is also required to be known, in which a positive demand occurs during a period. These three components of Equation (13) vary across time and must be indirectly forecast from the item time series. As suggested in Teunter, Syntetos, and Babai (2010), the positive demand distribution (both cumulative and not cumulative) is hard to determine over an arbitrary number of periods, unless the multiple periods distribution can be defined from the single period distribution. This experimental analysis assumes that the positive demand during a single period follows a negative binomial distribution. The sum of independent negative binomial distributions is a negative binomial distribution itself, with different parameters depending on the number of random variables added. In our case, the number of random variables is the number of periods. The use of a discrete random variable, instead of a continuous one as in Teunter, Syntetos, and Babai (2010), is coherent with Equation (11) where d_{be} moves through integer values.

To estimate the single period parameters of the negative binomial distribution, a time series analysis is required. The methodology used for this experimental analysis is the same applied in Teunter, Syntetos, and Babai (2010). The forecasting technique described in Section 2.1 is applied to define $\hat{p}$, $\hat{z_i}$ and $\widehat{MSE}(z_i)$ and the parameter α is optimized over the initial warmup periods. From $\hat{z_i}$ and $\widehat{MSE}(z_i)$ the negative binomial distribution parameters are derived using the method of moments.

4.2 Experiment settings

The experimental analysis consists of two experiments, carried out with different parameters, where the proposed methodology is tested on a generated series and compared against the case where the standard order-up-to-level (T,S) policy is applied without taking into account the perishability constraint. In these simulations both methodologies consume the stock following a FIFO policy, which is in line with a fixed number of periods before expiration.

Intermittent demands following Equation (1) are generated, and their positive demand size follows a negative binomial distribution, while the probability p that a positive demand occurs is time invariant. If the demand size distribution yields a null demand it is still considered a positive demand to avoid uncontrolled changes in p.

The fixed and variable parameters, varying in each simulation, for experiments 1 and 2 are summarized in Table 1. In both experiments the values for n_e are greater than those for t to avoid expirations before the end of a single cycle. All the possible combinations of the second set of parameters are tested 10 times to assess the method performance in different contexts.

	Parameter	Description	Value
Experiment 1 Fixed parameters	n_w	Number of warm-ups	100
	n_p	Number of simulated periods	1000
	t	Periods in a cycle	10
	l	Lead time	5
Experiment 1	p	Demand probability	0.1, 0.5

Variable parameters	$E(z)$	Positive demand expected value	10, 20
	$\frac{\sqrt{MSE(z)}}{E(z)}$	Relative positive demand mean squared error	0.5, 1.4
	n_e	Periods before expiration	10, 15
Experiment 2 Fixed parameters	n_w	Number of warm-ups	100
	n_p	Number of simulated periods	1000
	fr_{target}	Fill rate target	0.8
Experiment 2 Variable parameters	p	Demand probability	0.1, 0.5
	$E(z)$	Positive demand expected value	10, 20
	$\frac{\sqrt{MSE(z)}}{E(z)}$	Relative positive demand mean squared error	0.5, 1.4
	n_e	Periods before expiration	20, 25
	t	Periods in a cycle	10, 20
	l	Lead time	2, 5

Table 1. Fixed and variable parameters for experiments 1 and 2.

The parameters of experiment 1 are designed to cover extreme scenarios:

- Low intermittence ($p = 0.5$) vs. high intermittence ($p = 0.1$).
- Low demand ($E(z) = 10$) vs. high demand ($E(z) = 20$).
- Low lumpiness ($\frac{\sqrt{MSE(z)}}{E(z)} = 0.5$) vs. high lumpiness ($\frac{\sqrt{MSE(z)}}{E(z)} = 1.4$).
- Close expiration date ($n_e = 10$) vs. distant expiration date ($n_e = 15$).

The parameters of experiment 2 are designed to cover a wider range:

- Low intermittence ($p = 0.5$) vs. high intermittence ($p = 0.1$).
- Low demand ($E(z) = 10$) vs. high demand ($E(z) = 20$).

- Low lumpiness ($\frac{\sqrt{MSE(z)}}{E(z)} = 0.5$) vs. high lumpiness ($\frac{\sqrt{MSE(z)}}{E(z)} = 1.4$).
- Distant expiration date ($n_e = 20$) vs. very distant expiration date ($n_e = 25$).
- Short cycle ($t = 10$) vs. long cycle ($t = 20$).
- Short lead-time ($l = 2$) vs. long lead-time ($l = 5$).

The aim of these experiments was to identify a structure in the algorithms' behaviour in order to be able to define outperforming regions for the two methods.

The results are collected for each simulation period after the first lead time, when the first order has already arrived. Using this precaution, no initial level of backorders and stock is required to make the first measurements fair.

4.3 Performance metrics

Performance measurements are recorder after each simulation period. If the period presents a positive demand, then the total number of positive demand in the simulation is updated. Simultaneously, the performance record keeps track of the number of positive demand that have been satisfied from the stock, not adding to the backlog. The ratio between these two raw measurements is the fill-rate of the simulation fr_{sim}.

At the beginning of each cycle the optimal order quantity o_{min} is defined. In this context parameters s_e or s_{ne} cannot be changed and only positive values of o_{min} are produced. The fill rate is thus set to achieve $fr(o_{min}, s_e, s_{ne}) \geq fr_{obj}$. This goal-setting leads to difficulties when comparing fr_{sim} and fr_{obj} as, by construction, on average $fr_{sim} \geq fr_{obj}$ thus the difference $fr_{sim} - fr_{obj}$ is designed to be ≥ 0. To avoid this unfair comparison the values of $fr(o_{min}, s_e, s_{ne})$ are collected in each simulation as they are generated, and their average is compared with fr_{sim} instead of fr_{obj}:

$$\Delta fr = fr_{sim} - avg\big(fr(o_{min}, s_e, s_{ne})\big) \tag{22}$$

To measure the benefit of using the proposed method rather than the standard method that does not take into account perishability, we also calculate Δfr_{New}, which is expressed using Equation (23).

Note that by the standard method we mean the order-up-to-level (T,S) inventory policy without taking into account the perishability constraint as described in Teunter, Syntetos, and Babai (2010). Obviously the standard method is expected to be biased and to underachieve the target fill rate when the perishability is not taken into account. This is analysed in the following subsection.

$$\Delta fr_{New} = |\Delta fr_{pr}| - |\Delta fr_{st}| \tag{23}$$

where Δfr_{pr} is the Δfr of the proposed method and Δfr_{st} is the Δfr of the standard (T,S) method.

4.4 Results

Tables 2 and 3 summarize the average Δfr_{pr}, Δfr_{st} and Δfr_{New} results obtained for each combination of parameters. The values for $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ differ from those provided in Section 4.2 since the negative binomial the generated distribution, used for data generation, requires one of its parameters to be a natural number. Not any combination of $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ is allowed and their values end up changed when the parameter is rounded.

p	$E(z)$	$\frac{\sqrt{MSE(z)}}{E(z)}$	n_e	t	l	Δfr_{pr}	Δfr_{st}	Δfr_{New}
0.1	10.5	0.488	10	10	5	-0.024	-0.289	-0.236
0.1	10.5	0.488	15	10	5	-0.067	-0.153	-0.047
0.5	10.5	0.488	10	10	5	-0.023	-0.140	-0.110
0.5	10.5	0.488	15	10	5	-0.033	-0.064	-0.029
0.1	18.6	1.027	10	10	5	0.015	-0.242	-0.163
0.1	18.6	1.027	15	10	5	0.004	-0.088	-0.029
0.5	18.6	1.027	10	10	5	0.011	-0.130	-0.110
0.5	18.6	1.027	15	10	5	-0.013	-0.058	-0.028
0.1	20	0.5	10	10	5	-0.003	-0.238	-0.195
0.1	20	0.5	15	10	5	-0.020	-0.133	-0.086

0.5	20	0.5	10	10	5	-0.004	-0.135	-0.112
0.5	20	0.5	15	10	5	0.008	-0.023	-0.009
0.1	38.2	1.013	10	10	5	-0.003	-0.230	-0.179
0.1	38.2	1.013	15	10	5	0.012	-0.068	-0.047
0.5	38.2	1.013	10	10	5	-0.002	-0.177	-0.153
0.5	38.2	1.013	15	10	5	-0.008	-0.052	-0.032

Table 2. Average performance obtained for each combination of parameters in experiment 1.

p	$E(z)$	$\frac{\sqrt{MSE(z)}}{E(z)}$	n_e	t	l	Δfr_{pr}	Δfr_{st}	Δfr_{New}
0.1	10.5	0.488	20	10	2	-0.045	-0.158	-0.106
0.1	10.5	0.488	20	10	5	-0.019	-0.139	-0.095
0.1	10.5	0.488	20	20	2	-0.002	-0.196	-0.143
0.1	10.5	0.488	20	20	5	-0.009	-0.207	-0.153
0.1	10.5	0.488	25	10	2	0.016	-0.036	0.004
0.1	10.5	0.488	25	10	5	-0.011	-0.052	-0.03
0.1	10.5	0.488	25	20	2	-0.026	-0.168	-0.094
0.1	10.5	0.488	25	20	5	-0.045	-0.152	-0.089
0.1	18.6	1.027	20	10	2	0.033	-0.077	-0.024
0.1	18.6	1.027	20	10	5	0.018	-0.093	-0.058
0.1	18.6	1.027	20	20	2	-0.004	-0.23	-0.174
0.1	18.6	1.027	20	20	5	-0.088	-0.259	-0.13
0.1	18.6	1.027	25	10	2	0.048	0.006	-0.005
0.1	18.6	1.027	25	10	5	-0.001	-0.057	-0.019
0.1	18.6	1.027	25	20	2	0.006	-0.143	-0.116

0.1	18.6	1.027	25	20	5	0.035	-0.088	-0.054
0.1	20	0.5	20	10	2	-0.007	-0.135	-0.104
0.1	20	0.5	20	10	5	-0.028	-0.138	-0.097
0.1	20	0.5	20	20	2	0.01	-0.2	-0.149
0.1	20	0.5	20	20	5	-0.013	-0.209	-0.158
0.1	20	0.5	25	10	2	-0.003	-0.053	-0.004
0.1	20	0.5	25	10	5	-0.039	-0.091	-0.031
0.1	20	0.5	25	20	2	0	-0.143	-0.093
0.1	20	0.5	25	20	5	-0.001	-0.114	-0.036
0.1	38.2	1.013	20	10	2	0.011	-0.121	-0.083
0.1	38.2	1.013	20	10	5	0.067	-0.052	-0.031
0.1	38.2	1.013	20	20	2	0.011	-0.193	-0.145
0.1	38.2	1.013	20	20	5	0.049	-0.184	-0.131
0.1	38.2	1.013	25	10	2	0.047	0.002	0.001
0.1	38.2	1.013	25	10	5	0.03	-0.024	0.015
0.1	38.2	1.013	25	20	2	-0.015	-0.12	-0.09
0.1	38.2	1.013	25	20	5	-0.007	-0.15	-0.113
0.5	10.5	0.488	20	10	2	0.007	0.001	-0.001
0.5	10.5	0.488	20	10	5	-0.011	-0.031	-0.014
0.5	10.5	0.488	20	20	2	-0.021	-0.102	-0.065
0.5	10.5	0.488	20	20	5	-0.009	-0.079	-0.051
0.5	10.5	0.488	25	10	2	-0.019	-0.02	0
0.5	10.5	0.488	25	10	5	-0.02	-0.024	-0.004
0.5	10.5	0.488	25	20	2	0.008	-0.019	-0.017
0.5	10.5	0.488	25	20	5	-0.005	-0.039	-0.022

0.5	18.6	1.027	20	10	2	0.003	-0.032	-0.02
0.5	18.6	1.027	20	10	5	0.014	-0.018	-0.015
0.5	18.6	1.027	20	20	2	-0.015	-0.138	-0.106
0.5	18.6	1.027	20	20	5	0.003	-0.124	-0.101
0.5	18.6	1.027	25	10	2	0.023	0.009	0.004
0.5	18.6	1.027	25	10	5	-0.008	-0.025	0.001
0.5	18.6	1.027	25	20	2	-0.016	-0.087	-0.041
0.5	18.6	1.027	25	20	5	0.019	-0.053	-0.035
0.5	20	0.5	20	10	2	-0.01	-0.022	-0.009
0.5	20	0.5	20	10	5	0.007	-0.005	-0.006
0.5	20	0.5	20	20	2	0.005	-0.082	-0.068
0.5	20	0.5	20	20	5	0.004	-0.067	-0.052
0.5	20	0.5	25	10	2	-0.003	-0.005	-0.002
0.5	20	0.5	25	10	5	-0.004	-0.006	0.001
0.5	20	0.5	25	20	2	0.006	-0.025	-0.011
0.5	20	0.5	25	20	5	-0.001	-0.036	-0.018
0.5	38.2	1.013	20	10	2	0.006	-0.03	-0.019
0.5	38.2	1.013	20	10	5	0.001	-0.056	-0.034
0.5	38.2	1.013	20	20	2	-0.008	-0.126	-0.094
0.5	38.2	1.013	20	20	5	0.007	-0.13	-0.101
0.5	38.2	1.013	25	10	2	-0.005	-0.019	-0.007
0.5	38.2	1.013	25	10	5	0.002	-0.017	-0.004
0.5	38.2	1.013	25	20	2	0.018	-0.054	-0.026
0.5	38.2	1.013	25	20	5	-0.017	-0.093	-0.069

Table 3. Average performance obtained for each combination of parameters in experiment 2.

The standard (T,S) method is biased, leading to fill-rates that are always lower than the target ($\Delta fr_{st} < 0$) in experiment 1 (Table 2), and lower than the target 93.7% of the times in experiment 2 (Table 3). In the proposed method, this bias is corrected and, as a result, the Δfr_{pr} values are affected only by random error and thus characterized by varying signs. On average the new method produces a 9.78% fill-rate performance increase for perishable items in experiment 1 and a 5.53% increase in experiment 2.

A linear regression is fitted over the non-averaged simulation results using p, $E(z)$, $\frac{\sqrt{MSE(z)}}{E(z)}$ and n_e, in the first experiment, and p, $E(z)$, $\frac{\sqrt{MSE(z)}}{E(z)}$, n_e, t and l, in the second experiment, as features to separately predict Δfr and Δfr_{New}. The values of $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ used as features are obtained from the negative binomial distribution parameters used for data generation, they differ from those listed in Tables 2 and 3 as one of the distribution parameters must be an integer and is rounded when calculated from the original $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$. As a result, $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ are correlated ($r = 0.64$), while the others are not. The way this issue is handled is outline below. The linear regressions coefficients and their t-tests are summarized in Table 4.

		Coefficient	Squared error	T	p-value
Experiment 1 Δfr_{pr}	Intercept	-0.021	0.042	-0.490	0.625
	p	0.007	0.035	0.209	0.834
	$E(z)$	0.000	0.001	0.501	0.617
	$\frac{\sqrt{MSE(z)}}{E(z)}$	0.033	0.034	0.968	0.334
	n_e	-0.002	0.003	-0.744	0.458

Experiment 1 Δfr_{st}	Intercept	-0.520	0.044	-11.752	0.000
	p	0.207	0.037	5.653	0.000
	$E(z)$	0.000	0.001	0.506	0.614
	$\frac{\sqrt{MSE(z)}}{E(z)}$	0.020	0.036	0.552	0.582
	n_e	0.024	0.003	8.046	0.000
Experiment 2 Δfr_{pr}	Intercept	-0.016	0.025	-0.634	0.526
	p	-0.004	0.012	-0.365	0.715
	$E(z)$	0.000	0.000	1.582	0.114
	$\frac{\sqrt{MSE(z)}}{E(z)}$	0.021	0.012	1.733	0.084
	n_e	0.000	0.001	0.265	0.791
	t	-0.001	0.000	-1.423	0.155
	l	-0.001	0.002	-0.890	0.374
Experiment 2 Δfr_{st}	Intercept	-0.267	0.028	-9.492	0.000
	p	0.189	0.014	13.928	0.000
	$E(z)$	0.000	0.000	0.612	0.541
	$\frac{\sqrt{MSE(z)}}{E(z)}$	-0.007	0.013	-0.504	0.614
	n_e	0.011	0.001	10.003	0.000
	t	-0.008	0.001	-14.342	0.000
	l	-0.001	0.002	-0.553	0.580

Table 4. Linear regression coefficients for Δfr_{pr} and Δfr_{st} in experiment 1 and 2.

The results of experiment 1 indicate that the performance of the proposed method is not affected by the simulation parameters, while experiment 2 shows that $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ together influence Δfr_{pr}. The results of experiment 1 are highlighted by the t-tests in Table 4, which show no significance (a significant result would yield a p-value lower than 0.05). The correlation between $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ could hide a significant model behind non-significant t-tests. Consequently, to assess this scenario an F-test was performed which resulted in a non-significant p-value of 0.454. The results for experiment 2 are in line with findings from experiment 1. No t-test in experiment 2 reached a significant p-value, while the F-test in experiment 2 yielded a significant p-value of 0.005. Performing a PCA on standardized $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ and re-fitting the regression model revealed that those features in combination are the only ones that influence Δfr_{pr}. The findings of experiments 1 and 2 are not inconsistent. In fact the tests in experiment 2 leverage more simulations, resulting in a higher power and, as a result, the conclusions drawn from experiment 2 are more accurate.

Figure 1 highlights the coherence between experiments 1 and 2 and plots the confidence intervals (one std) for p, $E(z)$, $\frac{\sqrt{MSE(z)}}{E(z)}$ and n_e coefficients in experiment 1 (x) and experiment 2 (*). The coefficients for the most precise experiment are included in those for the least precise one.

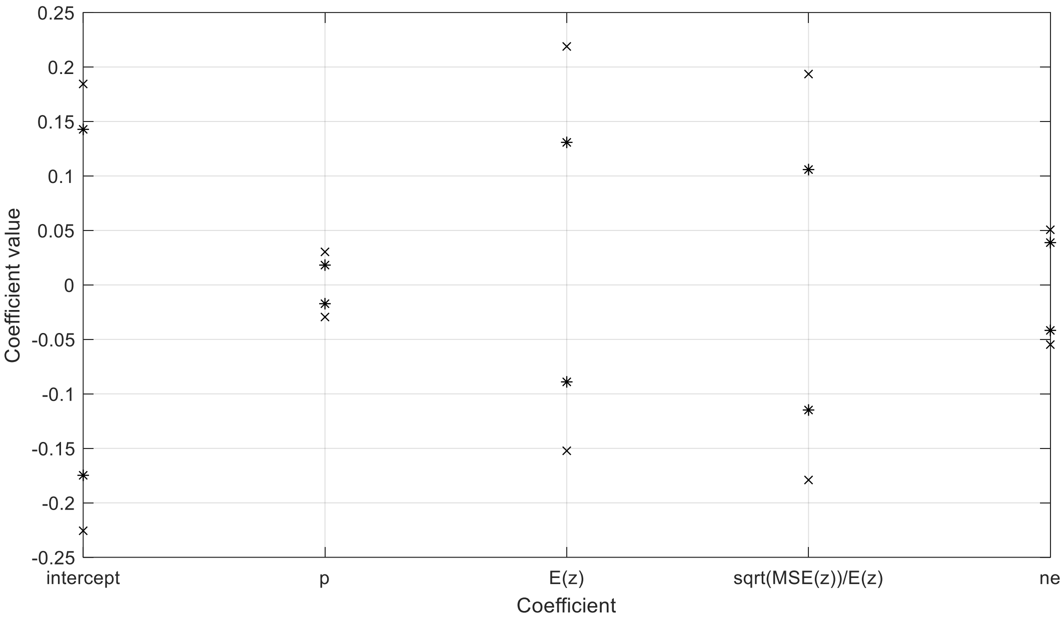

Figure 1. The confidence intervals of the coefficients in experiments 1 and 2.

According to experiment 1 (Table 4), the standard (T,S) is unaffected by demand size or variability as the p-values obtained are significantly higher than 0.05. The parameters affecting Δfr_{st}, in addition to the intercept, are intermittence and periods before expiration. Experiment 2 confirms these findings, moreover re-fitting both experiments regression models after a PCA on standardized $E(z)$ and $\frac{\sqrt{MSE(z)}}{E(z)}$ does not highlight any impact of such features over Δfr_{st}. The coefficients in experiments 1 and 2 are less coherent on their impact on Δfr_{st} than those for Δfr_{pr} depicted in Figure 1, suggesting more complex phenomena that cannot be defined with a simple linear model.

As a robustness test, the impact of the intermittent demand assumption on the proposed method is measured by re-running experiment 1 while using a SES and erroneously assuming $p = 1$. The results are found to be reliant on the intermittence assumption as the Δfr_{pr} obtained in this scenario is even higher than the Δfr_{st} achieved in experiment 1.

5. Conclusions and further research

Managing the inventories of perishable items is a key lever that enables inventory costs to be reduced by reducing waste, thus increasing the level of customer service. Managing the inventories of perishable items becomes a more challenging task when the demand for such items is intermittent. This study provides the first attempt in the literature to overcome this challenge. We have proposed a new methodology that modifies the standard order-up-to-level (T,S) policy for intermittent demand (Teunter, Syntetos, and Babai, 2010), which analytically derives the target fill rate for a compound binomial demand generation process in order to take into account the perishability constraint as well. We have also proposed an analytical expression of the fill rate under the new method, and due to the computational complexity to calculate the fill rate we have developed a procedure to obtain the optimal solution. We conducted a simulation experiment to analyse the performance of the standard and the proposed methods.

The results of this study show that when a proportion of the stock is affected by perishability, the proposed methodology leads to a considerable benefit by reducing the bias in the fill rate, unlike the standard method. The experiments reported in Section 3 demonstrate that the proposed methodology bias is only affected by demand size. On the other hand, intermittence, lumpiness, or number of periods before expiration do not impact its performance. The standard method is also proven to be unaffected by lumpiness, its effectiveness is only dictated by the number of periods before expiration and intermittence. From a computational standpoint the new methodology is significantly more expensive than the old one, as a combinatorial number of cases must be analysed, so practitioners are advised to apply the new methodology to scenarios characterized by high intermittence and low demand size. The use of simulation techniques to manage multiple expiration dates is also advised, to overcome the difficulty of determining analytical solutions in this case, since this can provide reliable results in exchange for a reasonable computational effort. Further research efforts are expected to gauge the effectiveness of simulation techniques and compare them with the available analytical solutions. Alongside this research avenue, efforts will be directed towards the characterization of different compound Bernoulli distributions, with the aim of encompassing both integer and continuous positive demand sizes (Syntetos, Babai and Altay, 2012; Syntetos, Lengu and Babai, 2013). Other distributions such as Compound Poisson distributions (Babai, Jemai and Dallery, 2011 ; Lengu, Syntetos and Babai, 2014) or Compound Erlang distributions (Saidan et al., 2013) have also been used to model intermittent demand and can be considered in future research. Once this characterization is achieved, comparisons between different distributions can take place and the effect of incorrectly selecting the demand size distribution can be quantified. The choice of an incorrect demand size distribution could seriously impact the performance of the inventory system. Another interesting avenue for further research would be to analyse the combined service and cost efficiency of the proposed methodology when compared to the standard one. Finally, it would be interesting to empirically show the benefits of the proposed model through an empirical investigation with real data, as it has been done in Teunter et al. (2010).

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