Economic order quantity and storage assignment policies

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Abstract: The basic Harris's lot size model dates back to 1913 (Harris, 1913), hence one century from its publication has been recently celebrated. Starting from the seminal work of Harris, a wide plethora of contributors has faced with the lot-sizing problem for fitting the basic model of the economic order quantity to several environments. In fact, the three key parameters constituting the basic model, i.e. the demand rate, the ordering costs, and the inventory holding costs, have been widely explored in order to relax the assumptions of the original model. However, to the best of the authors' knowledge, the liaison between holding costs and warehouse management has not been completely addressed. The holding costs have been early considered for simplicity as primarily given by the cost of capital, and thus dependent solely on the average inventory on stock. Conversely, by including a more detailed supply chain costs contribution, the economic order quantity calculus appears depending on a recursive calculus process and on the storage assignment policy. In fact, different approaches of warehouse management, e.g. shared and dedicated storage, lead to highly variable distances to be covered for performing the missions. This leads to a total cost function, and consequently to optimum lot sizes, that are affected by the warehouse management. In this paper, this relationship has been made explicit in order to evaluate an optimal order quantity taking into account storage assignment policies.

Keywords: **Inventory Management; Economic Order Quantity; Storage Assignment Policy.**

1. Introduction

Ford Whitman Harris founded the inventory management in 1913, when he first calculated the economic order quantity (*EOQ*). In the original form the *EOQ* minimizes the overall inventory management cost (*CT*), usually assumed over an annual time horizon, taking into account the purchase/production cost (*CAA*), the cost of placing orders (*CAEO*) and the inventory holding cost (*CAMS*). The Harris model introduced the well-known management style defined *to pull*, which requires a continuous control of the item availability; it is dedicated to logistic scenarios characterized by steady state demand, with oscillations that are negligible if compared to the average value, and of high value like what characterizes mass distribution products.

The Harris model also observes the following hypotheses: the purchase/production cost is constant as regard to the order quantity as well as the unit cost of placing orders; item [illegible] safety stocks do not belong to the problem.

Without considering *CAA*, the general equation of *CT* enables finding the economic order quantity through the balance between *CAEO* and *CAMS*.

Starting from the seminal work of Harris, a wide plethora of contributors has faced with the lot sizing problem for fitting the basic model of the *EOQ* to real environments. For a review, readers can refer to Andriolo et al. (2014), who adopted an original classification framework for reviewing 219 papers on the *EOQ* concepts. Given the relevance of the inventory management, it is not surprising that the lot-sizing problem has received a great attention from researchers and practitioners. In fact, the reviews on this research topic, as well as the review of the reviews (i.e. tertiary study) of Glock et al. (2014), showed the outstanding amount of contributions on the lot-sizing problem. Despite the early introduction of the *EOQ* model, Andriolo et al. (2014) underlined that the most part of the 219 reviewed papers has been published in the last years, reinforcing by this way the belief that this topic is relevant in current enterprises.

Without loss of generality, all the *EOQ* models proposed in literature may be analysed along three constituting key parameters: i) the demand rate; ii) the unitary ordering cost; iii) and the unitary holding cost. It is remarkable that the unitary purchasing/production cost is not taken into account in the original *EOQ* model because it is considered constant over time and thus it does not affect the optimal solution.

With regard to the demand rate, the basic model assumes a [illegible] rate, hence attaining a static solution. These two assumptions have been relaxed over time by several authors through different perspectives. The first deterministic lot-sizing approach allowing time-varying demand has been proposed by Wagner and Whitin (1958), who introduced an optimizing algorithm for establishing the best lot sizes over a predefined planning horizon. They revised the basic deterministic *EOQ* model by means of a dynamic programming approach, with time-varying and deterministic demand and costs. With the same hypothesis, Silver and Meal (1973) introduced the first well-known heuristic on the basis of the average cost for solving the dynamic lot-sizing problem, with the advantage of reaching a near-optimal solution with a lower time consumption. Other models dealing with deterministic demand have been proposed up to the last years for gaining a well-fitting representation of real features both of purchasing and of production processes. Quantity discounts (e.g. Taleizadeh and Pentico, 2013), goods perishability (e.g. Önal et al., 2015), imperfect quality (e.g. Khan et al., 2014), and finite production rate (e.g. Grubbström, 2014) represent some of the most stressed topics for alternative formulations of *EOQ* models for deterministic demand.

Also stochastic demand has been tackled by a large amount of authors. The canonical stochastic single-period lot-sizing model is the newsvendor problem, whose name derives from the decision of how many newspapers the vendor should buy for the incoming day, when shortage and overage costs are known. The fractile solution of the newsvendor problem has been explicitly provided by Whitin (1953), but stochastic models are out of scope.

The second constituting key parameter, i.e. the unitary ordering cost leading to *CAEO*, can be exactly determined; this depends on the number of orders which, in turns, depends on the expected demand over the time horizon, on the quantity to be ordered, and on the unit cost of placing orders.

Conversely, the third constituting key parameter (i.e. the inventory holding cost, *CAMS*) requires a more detailed formulation as being the focus of our proposal; in the standard formulation, it considers the annual physical and figurative depreciation of inventories by means of the annual stock-keeping rate (i_m), which multiplies the value of the average stock (namely the average on hand quantity); that is to say, i_m can be defined as the portion of the value of inventory that is lost, on average and along the established time horizon, due to all the costs that are consequent to the decision to make inventories. Harris (1913) considers inventory holding costs deriving from capital interest and physical storage costs, and assigns a constant value to the annual stock-keeping rate $i_m = 10\%$ *€/€y*. From Harris (1913) to Andriolo et al. (2014), passing through Azzi et al. (2014), the inventory holding costs are evaluated as a percentage of the cost of the item, supposing that a large proportion is represented by the cost of capital. The assumption that the holding cost is a linear function of the length of time over which the item is stored has been used by the most of the authors in order to simplify the total cost modelling. Total cost formulations addressing holding costs non-linear with respect to time, as well as to the quantity ordered, appeared in recent contributions (e.g. Alfares and Ghaithan 2016 and San-José et al., 2016).

Nevertheless, the breakdown and the increasingly accounting of logistic costs thanks to the growing contribution of information and traceability systems allows a more accurate assessment of annual stock-keeping rate. In fact, as several authors have stated, some cost items to be included into inventory costs are related to the value of inventory, others to physical properties, such as handling, controlling, warehousing, and so on, often named "out-of-pocket holding costs" (Azzi et al., 2014). The breakdown of holding costs has been already performed by other authors (e.g. Torkul et al., 2016), under the hypotheses that warehousing costs are independent from the stock level. This is the assumption that our work intends to relax.

The calculation of the economic order quantity is therefore recursive because the average stock is itself a function of the quantity to be ordered, and can be better analysed considering how the costs of the supply chain are formed, and in particular those of handling and storage. The latter depend significantly on the storage assignment strategies. Furthermore, it is not possible to define them exhaustively without analysing the entire inventory holding dynamics simultaneously. Two storage assignment strategies criteria are usually applied. On the one hand, it is usual to make warehouse space dedicated to each item (dedicated storage); the assigning rules allocate items to warehouse locations, far from the point from which the pick-up mission originates, on the base of a decreasing and expected probability with which the items will be requested. On the other hand, it is possible to share the space, allocating items where it is allowed (shared storage). In this case the probability of visiting each warehouse location is constant and, consequently, the application of any material handling strategies is useless.

The above mentioned storage assignment strategies have opposite features and performances which depend on boundary conditions, most of all on item mass flows. Hence, a general rule cannot be defined. While the dedicated storage assignment enables implementing optimal material handling strategies, minimizing the single/dual command time cycle, the same assignment strategy requires a greater warehouse space mitigating considerably the former advantages deriving from the allocation based on the probability of picking.

This paper therefore attempts to improve the calculation of the economic order quantity taking into account the contribution of storage assignment strategies on the stock-keeping rate and how this modifies the quantities to be ordered.

2. **Methodology**

Let be:

Q: order quantity [u] or [u/order];

D: annual demand [u/y];

K: cost of placing one order [€] or [€/order];

c: unit purchase/production cost per item [€/u];

h: unit stock holding cost per item per year, including interest and depreciation in stock [€/uy]. Harris (Harris,

1913) defined the unit charge for interest and depreciation on stock [\$/u] by means of the following equation:

$$I = \frac{i \cdot c \cdot A}{D} + \frac{i \cdot S}{2D} = \frac{i \cdot c \cdot Q}{2D} + \frac{i \cdot S}{2D} = \frac{h \cdot Q}{2D} + \frac{i \cdot S}{2D} \quad (1)$$

where:

i was defined as an interest rate which takes into account the annual capital interest and stock depreciation;

A is the average stock;

S is the setup cost;

$h=i \cdot c$ (not defined in the original notation of Harris' work), can be defined as the component of the annual charge for interest and depreciation on stock;

$I \cdot D$, which depends on the inventory holding; hereinafter it will be named unit inventory holding cost [€/(uy)];

EOQ: optimal order quantity; [u] or [u/order]

CQ: annual inventory management cost; [€/y]

The well-known Harris' square root equation is based on the assumptions that the demand rate is known and constant, backorders are not allowed, and replenishments are instantaneous.

Under these assumptions, the annual inventory management cost can be defined as follows:

$$CT = \left(\frac{hQ}{2}\right) + \frac{KD}{Q} + cD \text{ [€/y]} \quad (2)$$

The annual inventory management cost is a continuous convex function of the order quantity; it can be differentiated to minimize the total cost. This operation leads to the well-known square root formula:

$$EOQ = \sqrt{\frac{2K}{h}} \text{ [u] or [u/order]} \quad (3)$$

Equation 3 is constituted by three key parameters: the unit inventory holding cost *h*, the order placement cost *K* and the demand rate *D*. Inventory holding costs are usually defined as the cost of holding inventory for one year.

Obviously, Harris took for granted that a good approximation for the aggregate costs should be an annual interest percentage charged on the value of the average physical level. Despite the vast amount of literature on lot sizing developed during the last 100 years, the major part of contributions has been concerned with a total cost function definition from an economic point of view, following Harris' basic approach which makes use of a direct costing method and fixes the annual stock keeping rate.

Here we aim to relax this latter assumption assessing how the economic order quantity depends on the assignment policies. The economic order quantity, in the original Harris' formulation, minimizes the total and annual cost of ownership (*CT*), according to the following equation:

$$CT(Q) = CAA(Q) + CAEO(Q) + CAMS(Q) \quad (4)$$

The inventory holding cost can be, in turn, modelled by means of the following product:

$$CAMS(Q) = i_m \, c \, G_m \quad (5)$$

where:

c is the item purchase cost, which does not change with the ordered quantity *Q* (i.e. discount not allowed), G_m is the average on hand quantity, and i_m assumes the general meaning of stock-keeping rate.

The unit purchase cost *c*, the average on hand quantity, G_m, and the inventory dynamics from which it derives, do not depend on the assumed storage assignment policy. On the contrary, the stock-keeping rate i_m can be calculated by means of the following general equation:

$$i_m = i_p + \sum_i C_i / R_m \quad (6)$$

where:

i_p is the capital interest which captures the figurative nature of inventory cost in term of working capital;

R_m is the economic value of the average on hand quantity, usually evaluated by the product $R_m = c\, G_m$;

$\sum_i C_i$ counts the relevant annual supply chain costs among which it is possible to cite the following: the annual cost of in-transit inventories (*CAST*) the annual cost of material handling (*CAHM*), the annual cost of items storage (*CAS),* the annual cost of material depreciation (*CAD)*, the annual cost of item obsolescence (*CAO)*, the annual cost of the process Quality assurance (*CAQ)*, and the annual cost of the logistic distribution system (*CSD).*

Hereinafter we consider not negligible the annual cost of material handling and the annual cost of items storage. The latter is evaluated as follows:

$$CAS = c_{stock} O_m \quad (7)$$

where:

c_{stock} is the annual storage cost per location, [€/slot·y];

O_m is the average number of warehouse locations involved in the item storage dynamics and depends on the applied assignment policy; it corresponds with the *EOQ* when the *dedicated storage* policy is applied while the same number O_m is equal to *EOQ/2* in case of *shared storage* policy. The annual cost of material handling can be modelled by means of the following notation:

$$CAHM = c_{mov} PA + c_s N_p \quad (8)$$

where:

c_{mov} is the specific handling cost, [€/m];

PA is the expected mileage which depends on the assumed storage assignment policy. This variable cannot be evaluated without considering the whole warehouse dynamics:

$$PA = N_p d(\cdot) \quad (9)$$

where:

d() is the distance between the origin of material handling cycles (i/o; figures 1-2) and the mass centre of the storage locations (CoM, figures 1-2) which are assigned to each item.

The distance *d()* depends also on the storage assignment policy; c_s is the setup cost of the average material-handling mission; N_p is the expected number of retrieval material handing missions. Hereinafter we assume the annual demand *D* as the sum of customer orders with discrete unit of load ordered quantity and the material handling is managed according to *order picking* rules (a retrieval mission is dedicated to each customer order).

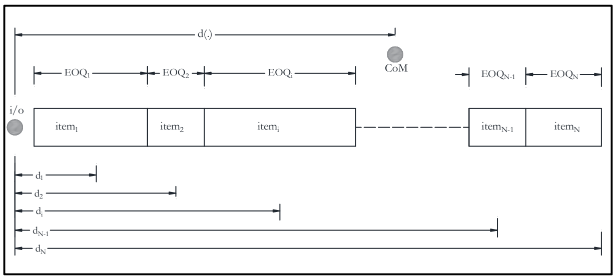

Fig.1 Warehouse model of the *dedicated storage* assignment policy.

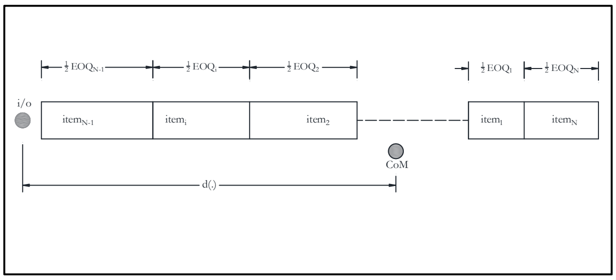

Fig.2 Warehouse model of the *shared storage* assignment policy (unsteady state item location).

According to the previous assumptions (Equations 6-9), the annual stock keeping rate can be written according to the following model:

$$i_m = i_p + \frac{[c_{stock} O_m + c_{mov} N_p d(\cdot) + c_s N_p]}{\left(\frac{c_a EOQ}{2}\right)} \quad (10)$$

$$EOQ = \sqrt{\left(\frac{2DK}{c_a i_m}\right)} \quad (11)$$

The system of equations (10) and (11) shows the implicit link between stock-keeping rate and economic order quantity. Furthermore, the simulation of the material handling dynamics of all the items in stock is required in order to assess material handling and storage costs for each storage assignment scenario, and thus to evaluate the optimal quantities to be ordered.

3. Experimental analysis

The system of equations (10) and (11) was applied as regard to a case simple study involving the inventory management of N=10 items.

Table 1 shows the structure of the MS Excel® spreadsheet model, which was *ad hoc* codified. The model enable the simulation of the following variables:

1. annual mass flows per item (D_i); D_i can be simulated according to a specific Pareto curve, fitting a particular logistic scenario; the Pareto index shows which share of annual mass demand, Do, is generated by a certain share of items.
   For example, let $Do = \sum_1^N Di$ be the annual demand, an 80-20 Pareto index implies that a little share of items (20%) causes a great share of annual mass flow (80%); on the other hand, a 20-20 Pareto index implies that all items are requested with the same probability.
2. inventory holding costs which consider the capital interest, the storage and material handling costs, depending on the application of the above mentioned assignment policies (*shared storage or dedicated storage*). As regard to the dedicated storage assignment policy, item allocation is performed assigning a location to a material as far from the origin point of material handling cycles (i/o) as the probability to enter the same location decreases. Item allocation is performed according to the IA_i access index, which is defined as the probability to enter each dedicated storage location:

$$IA_i = NP_i / O_i \quad (12)$$

where NP_i is the number of material handling cycles per item, and O_i is the average number of locations which the applied assignment policy requests.

The following further assumptions are defined: material handling involves only single command cycles dedicated to each customer order (*order picking* condition); a unit time differs each warehouse location from the next one; item mass flows are constant and the contribution of safety stocks, as regard to the inventory management, is negligible.

The assignment policy based on materials *duration of stay* is not simulated because it is optimal only when the system is perfectly balanced (a perfectly balanced system is characterized by the equality of the incoming and outgoing flows, for each item and each class of expected duration of stay); it is too far from real field conditions which are always affected by demand and supplying uncertainties.

Tab.1 Main results of the simulation process (only item 1 and 10 behaviours are reported; uol: warehouse unit of load).

		Item			
Variable	**Symbol**	**1**	**..**	**10**	**um**
Annual demand	D_i	4.000,0	..	7,8	uol/y
Cost of placing order	K	100	..	100	€/or
Item cost	c_a	1000	..	1000	€/uol
Capital interest	i_p	10,0%	..	10,0%	€/€y
Annual stock keeping rate	i_m	50,5%	..	35,1%	€/€y

Economic order quantity	*EOQ*	40	..	2	uol/or
On hand quantity	G_m	20	..	1	uol
Annual storage cost per location	c_{stock}	100	..	100	€/uol y
Specific material handling cost	C_{mov}	0,05	..	0,05	€/m
Annual storage cost (dedicated storage policy)	CAS_{DED}	4000	..	200	€/y
Stock keeping rate (dedicated storage policy)	$i_{cas\text{-}DED}$	20,0%	..	20,0%	€/€y
Annual storage cost (shared storage policy)	CAS_{SHA}	2000,0	..	100,0	€/y
Stock keeping rate (shared storage policy)	$i_{cas\text{-}SHA}$	11,4%	..	6,7%	€/€y
Access index	*IA*	100,0	..	3,9	mov/uol
Simulated material handling distance	$d(\cdot)$	20,5	..	129,5	m
Annual material handling cost (dedicated storage policy)	$CAHM_{DED}$	4100	..	50,59	€/y
Annual material handling rate (dedicated storage policy)	i_{movDED}	20,5%	..	5,1%	€/€y
Annual material handling cost (shared storage policy)	$CAHM_{SHA}$	7800,00	..	15,23	€/y
Annual material handling rate (shared storage policy)	i_{movSHA}	44,6%	..	1,0%	€/€y

3.1 Findings

Figure 3 shows the annual inventory management cost which is realized after applying the *dedicated* (*DED*) or *shared* (*SHA*) storage assignment policy. The case study, under the latter assignment policies (*DED-H*, *SHA-H*), is also evaluated taking into account a fixed stock keeping rate i_m=0,10 €/€y - as suggested by Harris' in his original work - and without performing the circular computing process above proposed which enables to consider the material handling and the storage costs contribution. The *shared* storage assignment policy (*SHA*) minimizes operations costs if compared with the dedicated storage one (*DED*). Table 2 shows the comparison between inventory management costs resulting after the application of the above-mentioned strategies to each demand scenario.

The *DED* assignment policy appears optimal only when the demand mass flow is highly focused (Pareto index equal to 80-20). This is due to the greater warehouse space that this policy requires: the trade-off between inventory holding and material handling costs is optimal only and if the reduction of the material handling costs overtakes the increasing of inventory holding costs.

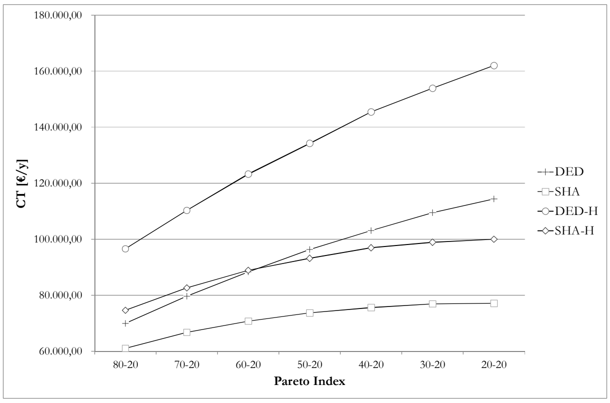

Fig.3 Annual and overall inventory management cost *CT* (*DED*: dedicated storage assignment policy; *SHA*: shared assignment policy and random item allocation; *DED-H*: dedicated storage assignment policy; item allocation by means of access index *AI*; Harris' economic order quantity; *SHA-H*: shared storage assignment policy; renadom items allocation; Harris' economic order quantity)

Tab.2 Inventory management cost comparison.

$\Delta CT=(CT_{DED} - CT_{SHA})/CT_{DED}$					
	Item				
Pareto index	**1**	**2**	**3**	**4**	**5**
80-20	19%	-24%	-29%	-32%	-34%
70-20	25%	-20%	-27%	-31%	-33%
60-20	28%	-17%	-25%	-31%	-34%
50-20	29%	-13%	-24%	-30%	-36%
40-20	25%	-10%	-22%	-29%	-34%
30-20	20%	-6%	-19%	-28%	-32%
20-20	13%	-4%	-15%	-24%	-32%
	Item				
Pareto index	**6**	**7**	**8**	**9**	**10**
80-20	-36%	-35%	-37%	-37%	-37%
70-20	-36%	-39%	-41%	-40%	-41%
60-20	-37%	-39%	-42%	-42%	-45%
50-20	-38%	-40%	-42%	-46%	-47%
40-20	-38%	-42%	-43%	-47%	-47%
30-20	-37%	-41%	-45%	-47%	-50%
20-20	-37%	-40%	-45%	-49%	-51%

Figure 4 highlights significantly how much the economic order quantity depends on the logistic boundary conditions as the distribution of the mass flows among the ten inventories and on the way in which the management costs are accounted.

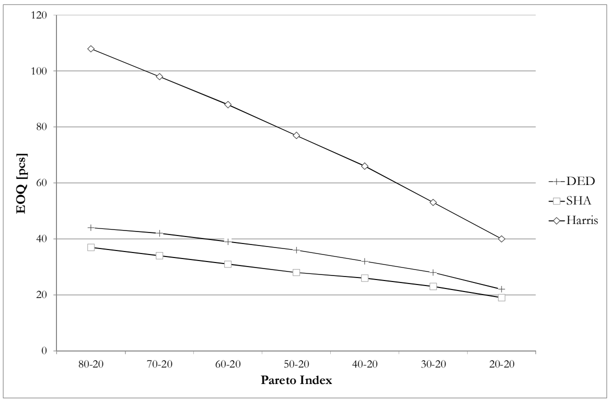

Fig.4 Item 1 economic order quantity per assignment policy.

4. Conclusions

The paper attempts to deepen the method of calculating the Harris' economic order quantity, taking into account a more detailed assessment of the annual stock keeping rate which depends on physical, figurative and most of all management styles as well as the storage assignment policies; the topic is of particular interest because it aims to take into account all costs associated with supply chain management that are generated once the decision of holding inventories is made.

The recursive procedure proposed enables a better definition of the optimal order quantity to be purchased and sheds new light on storage assignment policies and relevant performances.

Despite the limited nature of the case study considered, the application appears to be suitable for further and easy extensions to the management of a wider range of items and to model demand and supplying uncertainties.

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