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A human-machine learning curve for stochastic assembly line balancing problems

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Abstract: The Assembly Line Balancing Problem (ALBP) represents one of the most explored research topics in manufacturing. However, only a few contributions have investigated the effect of the combined abilities of humans and machines in order to reach a balancing solution. It is well-recognized that human beings learn to perform assembly tasks over time, with the effect of reducing the time needed for unitary tasks. This implies a need to re-balance assembly lines periodically, in accordance with the increased level of human experience. However, given an assembly task that is partially performed by automatic equipment, it could be argued that some subtasks are not subject to learning effects. Breaking up assembly tasks into human and automatic subtasks represents the first step towards more sophisticated approaches for ALBP. In this paper, a learning curve is introduced that captures this disaggregation, which is then applied to a stochastic ALBP. Finally, a numerical example is proposed to show how this learning curve affects balancing solutions.

Keywords: Stochastic assembly line balancing problem; Learning curve; Human-machine interaction; Task time; Kottas-Lau heuristic; Rebalancing

1. INTRODUCTION

Human beings learn by doing. This empirical evidence was formalised early in manufacturing by the well-known power curve (Wright, 1936), where unitary task times decrease due to the increasing cumulative number of produced items. Learning curves have subsequently been used to represent the dynamics of different dependent variables (e.g. unitary costs, unitary task times, quality metrics) affected by experience, which in turn can be represented in terms of autonomous ('learning by doing') or induced learning sources (e.g. training hours, investments, equipment). Quality metrics (e.g. Lolli et al., 2016a; Ittner et al., 2001; Lolli et al., 2018), task times (e.g. Biskup, 1999; Bailey, 1989), and costs (Lolli et al., 2016b) are therefore the most common dependent variables used in learning curves in several operative fields.

Labour-intensive assembly lines are a typical example of a manufacturing environment in which the learning effects play a crucial role in assessing task time. The aim of balancing an assembly line is to allocate assembly tasks to workstations in order to optimize a given performance metric, while satisfying precedence constraints.

Most approaches refer to types I and II ALBPs. The type I ALBP (e.g. Li et al., 2017b; Gansterer and Hartl, 2017) aims to minimise the number of workstations in order to satisfy a given cycle time, whereas the type II ALBP (e.g. Tang et al., 2016; Li et al., 2017a) aims to minimise the cycle time with a given number of workstations (Baybars, 1986). Moreover, ALBPs have been classified into three families by Boysen et al. (2007): single-model ALPBs; mixed-model ALPBs, where one product is manufactured in multiple models on the same assembly line; and multi-model ALPBs, where multiple products are manufactured in batches. A further driver adopted for the taxonomy of ALBPs refers to the nature of task times, which may be either deterministic or stochastic. See Battaïa and Dolgui (2013) for a review of this topic. In this paper, a stochastic type I ALBP is considered.

Stochastic approaches for ALBPs fit labour intensive assembly lines well, where the task times are generally assumed to be normally distributed. However, the expected values of these task times decrease over time due to experience. That is to say, the higher the number of assembled items, the lower the task times. Variable task times due to experience lead to the optimal balancing solutions being modified over time. This is the focus of our proposal, which can be framed within the research stream of stochastic type I ALBP with learning effects.

The first contribution on learning effects into assembly lines was proposed by Cohen and Dar-El (1998), where a type I ALBP was solved analytically via makespan formulation

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with deterministic task times. Cohen et al. (2006) investigated the inverse II ALBP by adopting Wright's learning curve (1936) with homogenous learning slopes between workstations and deterministic task times. Toksari et al. (2008) dealt with a type I ALBP by adopting the position dependent learning curve proposed by Biskup (1999). They demonstrated that simple and U-type line balancing problems with homogenous learning effects are polynomially solvable. Toksari et al. (2010) dealt with type I ALBP also using a mixed nonlinear integer programming model. In this case, the Biskup's learning curve (1999) was coupled with a linear increase in the task time due to job deterioration. Hamta et al. (2013) proposed a meta-heuristic approach for a deterministic multi-objective ALBP with the Biskup's learning curve (1999) used to model the position-dependent task times. Task times were forced to vary between lower and upper bounds, in line with Hamta et al. (2011) who called them "flexible operation times".

To the best of our knowledge, only Lolli et al. (2017) have investigated a stochastic type I ALBP with learning effects, where the cost-based Kottas-Lau heuristic (1973) was coupled with the well-known Wright's curve (1936) with a plateau. Over time the best balancing solution is affected by learning, thus the assembly line needs to be rebalanced. In Lolli et al. (2017) the rebalancing problem was addressed solely as a consequence of the learning process involving each assembly workstation, whereas before it had been addressed in terms of changes in market conditions or product design (Gamberini et al., 2006; Gamberini et al., 2009)

In all the aforementioned contributions, learning occurs by operators, without considering that an assembly task may be furthermore broken up into human and machine subtasks. Humans are subject to learning, each with a specific learning rate, machines are not. The main idea is therefore to break up task times into subtask times, and include all such subtask times in a learning curve that represents the overall learning process. To the best of our knowledge, only Jaber and Glock (2013) focus on separating different types of subtasks in order to model the learning process more accurately. Jaber and Glock distinguished between cognitive and motor clusters of subtasks, and proposed a revised learning curve on the basis of those proposed by Wright (1936) and Dar-El and Rubinovitz (1991) where two learning rates are assigned to each cluster. However, the possibility that some subtasks, which are often automated, are not affected by learning, has not been taken into account, together with the fact that learning rates may vary between subtasks.

In this paper, the learning process was broken up into basic subtasks to propose a new learning curve, which was then applied to a stochastic type I ALBP. The Kottas-Lau heuristic (1973), which is a cost-oriented heuristic approach involving total labour cost and expected incompletion cost, was coupled with the aforementioned learning curve.

Section 2 introduces the novel learning curve; Section 3 details the Kottas-Lau heuristic (1973) with learning effects; Section 4 reports a numerical example; and Section 5 contains conclusions and the future research agenda.

2. LEARNING CURVE

Assembly workstations may contain equipment aimed at handling bulky and heavy parts, thus supporting the operator working at the station. Equipment can also be used for performing some assembly subtasks, either in hidden working time or not. We consider solely the subtask times that make up the overall task time, some due to operators and others to equipment. The assembly process is composed of a set of tasks to assign to workstations, and tasks can be broken up into subtasks $i = 1, \dots, I$ performed by operators, and thus affected by learning, and subtasks $j = 1, \dots, J$ performed by equipment.

The learning curve representing the behaviour of the task time with regard to the cumulative number of assembled units is as follows:

$$T_n = (1-r) \cdot \sum_{i=1}^{I} (T_i \cdot n^{-b_i}) + r \cdot \sum_{i=1}^{I} T_i + \sum_{j=1}^{J} T_j \quad (1),$$

where:

T_n is the task time after having assembled n items;
T_i is the time for performing subtask i for the first assembled item;
T_j is the time for performing subtask j not affected by learning;
b_i is the learning rate of subtask i, with $b_i > 0$;
r is the rate of the I subtask times not affected by learning; this varies between zero and one, is assumed to be dependent on the operator performing subtasks and thus is the same for all $i = 1, \dots, I$, and leads to the plateau of T_n equal to:

$$lim_{n \to \infty} T_n = r \cdot \sum_{i=1}^{I} T_i + \sum_{j=1}^{J} T_j \quad (2).$$

(1) may be rewritten as follows:

$$T_n = (1-r) \cdot T^* \cdot n^{-b^*} + r \cdot T^* + k \quad (3),$$

where $T^* = \sum_{i=1}^{I} T_i$, $b^*$is the learning rate that could be assigned to the task without breaking it up into subtasks, as in standard approaches applied to assembly tasks affected by learning, and $k = \sum_{j=1}^{J} T_j$.

From (1) and (3) and changing the logarithm bases from n to 10, it follows that:

$$b^* = \frac{-\ln \sum (\frac{T_i}{T^*} n^{-b_i})}{\ln(n)} \quad (4),$$

which is defined for $n > 1$.

Hence the learning rate b^* to assign to the task depends not only on the times T_i and learning rates b_i of the I subtasks, but crucially also on the cumulative number n of assembled items.

The learning curve is now rewritten from (3) as follows:

$$T_n = (1-r) \cdot T^* \cdot n^{\frac{\ln \sum (\frac{T_i}{T^*} n^{-b_i})}{\ln(n)}} + r \cdot T^* + k \quad (5).$$

It can be shown that $b^* \geq 0$. Let be $\frac{T_i}{T^*} = \alpha_i$ with $\sum_{i=1}^{I} \alpha_i = 1$, it follows that $\sum \frac{\alpha_i}{n^{b_i}} \leq 1$. Since $lim_{n \to +\infty} \frac{\alpha_i}{n^{b_i}} = 0$, then:

$$b^* = \frac{-\ln \sum (\frac{T_i}{T^*} n^{-b_i})}{\ln(n)} \geq 0 \quad \text{for all } n > 1.$$

Recall that (3) was inspired by Jaber and Glock (2013), whose learning curve is expressed by:

$$T_n = x \cdot T_1 \cdot n^{-b_c} + (1 - x) \cdot T_1 \cdot n^{-b_m} \quad (6),$$

where:
x is the portion of cognitive tasks, with $0 \leq x \leq 1$;
b_c is the learning rate of cognitive tasks;
b_m is the learning rate of motor tasks.

However, the learning curve expressed by (3) introduces a plateau by means of r, takes into account automated subtasks not affected by learning, and assigns a learning rate b_i to each subtask. Thus (3) may be seen as a generalisation of (6).

3. ASSEMBLY LINE BALANCING APPROACH

A cost-based stochastic type I ALBP is proposed by adopting task times subjected to learning in accordance with (3). The solving approach proposed by Lolli et al. (2017) is implemented.

The operative setting may be summarised as follows. The assembly line is paced, which means that operators have a fixed tack-time T to perform the assigned assembly tasks on a single product, whose execution must comply with precedence constraints. The uncompleted tasks are performed out of the line at a higher cost, along with all the tasks blocked due to the precedence constraints. It is assumed that task times are normal-distributed, with expected value and variance depending on the task. The expected value of the task times is given by (3) on the basis of the accumulated experience, while the relative standard deviation is fixed in order to avoid an unrealistic increase in the relative task variability as a result of the learning phenomenon. To the best of our knowledge, the effect of experience on the standard deviation has never been investigated, and thus could be part of the future research agenda. The Kottas-Lau heuristic (1973) represents the canonical cost-based solving approach for stochastic type I ALBP, with task times normal-distributed, but not varying with experience. Conversely, (3) provides task times affected by experience, thus leading to balancing solutions that vary with n. Given a workstation k and after n assembled items, the heuristic first calculates for each task s the probability that said task is not completed within T. Using this normalised variable:

$$z_{snk} = \frac{T - \sum_{l \in L} T_{lnk}}{\sqrt{\sum_{l \in L} \sigma_{lnk}^2}} \quad (7),$$

where L is the set of tasks already assigned to workstation k, each with expected time and standard deviation after n assembled items equal to T_{lnk} (3) and σ_{lnk} respectively, $F(z_{snk})$ is the probability of completing all the tasks, including s, within T. Given the cost C_s, arising due to precedence constraints if task s is not completed inline, and the cost L_s for the execution of task s, the heuristic allows tasks to be assigned to workstations in accordance with the precedence constraints by categorizing the tasks:

i. A task s is desirable if $L_s \geq (1 - F(z_{snk})) \cdot C_s$. In the case of more desirable tasks, the one with the highest C_s is assigned.
ii. A task s is safe if $F(z_{snk}) \geq 0.995$.
iii. A task s is critical if it is not desirable, and thus it should be assigned to an empty workstation. In the case of more critical tasks, the one with the highest number of subsequent tasks is assigned.

In reality, (7) depends on n through (3) leading to optimal balancing solutions that change while experience increases. The periodic re-balancing of the assembly line is therefore justified also due to learning effects. The solving approach is described in detail in Lolli et al. (2017).

4. NUMERICAL EXAMPLE

Note that, from (4), b^* also depends on n. This complicates the implementation of the aforementioned balancing approach. Before evaluating the effect of n on the balancing solutions, a possible approximation of b^* independent of n is tested, i.e. the mean $\mu(b_i)$ of b_i. It is then compared with the exact formula of b^* in terms of the mean absolute percentage error. The data used for this comparison are reported in Table 1, where five tasks (t1,…,t5), each composed of three subtasks (st1, st2, st3), show different sets of b_i.

Table 1. Approximation of b*

	b_i			T_i				
	st1	st2	st3	st1	st2	st3	T^*	$\mu(b_i)$
t1	0.140	0.200	0.175	25	10	13	48	0.172
t2	0.240	0.170	0.105	5	5	15	25	0.172
t3	0.030	0.105	0.100	22	8	2	32	0.078
t4	0.052	0.060	0.040	11	9	5	25	0.051
t5	0.115	0.200	0.100	10	10	10	30	0.138

Given these data, b^* may be calculated by applying (4) for different levels of accumulated experience, in particular for $n = \{2, 3, 4, 5, 10, 15, 20, 30, 50, 100, 150, 200\}$.

Table 2. Values of $b^*$by changing n

n	t1	t2	t3	t4	t5
2	0.162	0.144	0.053	0.052	0.138
3	0.162	0.143	0.052	0.052	0.137
4	0.161	0.143	0.052	0.052	0.137
5	0.161	0.142	0.052	0.052	0.137
10	0.161	0.142	0.052	0.052	0.136
15	0.161	0.141	0.051	0.052	0.136
20	0.161	0.141	0.051	0.052	0.136
30	0.161	0.140	0.051	0.052	0.135
50	0.161	0.139	0.051	0.052	0.135
100	0.161	0.139	0.050	0.052	0.134
150	0.160	0.138	0.050	0.052	0.134
200	0.160	0.138	0.050	0.052	0.133

The average over the five tasks of the mean absolute percentage error due to the use of $\mu(b_i)$ instead of b^* is 17%. This indicates a moderate error given by said approximation. Nevertheless, both $\mu(b_i)$ and b^* are used in the following for the implementation of the balancing approach in order to evaluate how they affect the optimal solutions.

The input data required for the implementation of the assembly line balancing approach are reported in Table 3. Eleven tasks (t1, t2,…, t11) have to be assigned to workstations, while complying with the precedence constraints ('Blocked tasks' column in Table 3). Each task has a cost C_s arising if it is not completed inline, while the inline hourly cost is 30 [€/h]. The tack-time T to perform the assigned assembly tasks is fixed at 10 minutes. Table 3 also reports the T^* and the variances σ^2 of the eleven tasks. Note that also σ changes with n, but $\sigma/T_n = \sigma/T^*$ is assumed to be fixed.

The eleven tasks are made up of subtasks, each with a specific b_i and an initial subtask time T_i. Table 4 reports the said values, along with r (i.e. the rate of the task times not affected by learning).

Table 3. Input data

Task	$T^*[min]$	$\sigma^2[min^2]$	C_s [€]	Blocked tasks
t1	6	1.2	31.5	t2 ,…, t11
t2	2	0.4	12.5	t6, t8, t10, t11
t3	4	1	9.5	t7, t9, t11
t4	9	5	14	t7, t9, t11
t5	2	0.4	8.5	t7, t9, t11
t6	2	0.4	10.5	t8, t10, t11
t7	3	0.6	7	t9, t11
t8	6	1.2	9	t10, t11
t9	5	1	5	t11
t10	5	1	5	t11
t11	3	1.8	2	-

Table 4. Input data

Task	Subtask	T_i	b_i	r
t1	st1	2.45	0.52	0.2
	st2	3.55	0.32	
t2	st1	1.5	0.2	0.52
	st2	0.5	0.4	
t3	st1	1.35	0.55	0.32
	st2	2.65	0.4	
t4	st1	5	0.25	6
	st2	4	0.2	
t5	st1	1.68	0.6	0.65
	st2	0.32	0.55	
t6	st1	1	0.7	0.15
	st2	1	0.3	
t7	st1	1.7	0.25	0.42
	st2	1.3	0.35	
t8	st1	1.9	0.18	0.56
	st2	4.1	0.4	
t9	st1	2.5	0.15	0.18
	st2	2.5	0.28	
t10	st1	1.2	0.4	0.6
	st2	3.8	0.6	
t11	st1	1.5	0.65	0.54
	st2	1.5	0.7	

The values of b^* (4), which depend on n, are used to calculate T_n by means of (3) and subsequently σ^2 for each task. To do this, seven values of n are chosen as a sample of different levels of the accumulated experience. In particular, $n = \{10, 20, 30, 50, 100, 150, 200\}$.

The calculation of these values for task t1 is reported in Table 5, along with the values of T_n^I and σ^{2I} achieved by using $\mu(b_i) = 0.42$ instead of the exact b^* formula.

Seven different balancing solutions are therefore achieved by implementing the balancing approach explained in Section 3. In reality, the case $n = 1$ represents the standard solution of the Kottas-Lau heuristic (1973) without any learning effect. Figure 1 details the seven different solutions along with those achieved by using $\mu(b_i)$ instead of the exact $\boldsymbol{b}^*$ formula.

Table 5. Task time and variance of t1 by changing $\boldsymbol{n}$

n	b^*	T_n	σ^2	T_n^I	σ^{2I}
10	0.391	3.151	0.331	3.025	0.305
20	0.388	2.702	0.243	2.564	0.219
30	0.386	2.491	0.207	2.350	0.184
50	0.384	2.268	0.172	2.128	0.151
100	0.381	2.029	0.137	1.894	0.120
150	0.380	1.916	0.122	1.785	0.106
200	0.379	1.846	0.114	1.719	0.098

Figure 1 highlights that the experience accumulated after two hundred assembled items changes the type I ALBP solution significantly. The number of workstations required changes from six to three, with a sharp reduction after just twenty assembled items. Such a result is case-sensitive, but nevertheless highlights the relevance of taking into account learning phenomena in assembly line balancing problems. In addition, the use of $\mu(b_i)$ instead of the exact formula leads to the same optimal numbers of workstations over the accumulated experience, but modifies the task assignment considerably.

5. CONCLUSIONS

Assembly task times are typically assumed to be normal-distributed in stochastic balancing problems. However, operators learn by doing, and consequently the unitary task times tend to decrease. Learning curves are therefore applied to represent this mechanism. Moreover, task times may be disaggregated into subtask times, some performed by human beings and therefore subject to learning, while others performed by automated equipment. In this paper, a learning curve based on said disaggregation was proposed, and then applied to a revised Kottas-Lau heuristic (1973) for stochastic balancing problems. The results indicate the strong effect of learning on the optimality of the balancing solutions.

The assumption that the relative standard deviation is fixed is strong, and therefore deserves to be addressed in future research. Laboratory settings aimed at investigating the effect of the accumulated experience on the variance could be useful to identify well-fitting learning curves. Moreover, the assembly line rebalancing process has not been thoroughly investigated. In fact, a trade-off between rebalancing and non-optimality costs arises, which indicates that the best review interval of the assembly line needs to be searched for. Finally, a possible extension of this proposal would be to apply our learning curve to the 'mirror' type II ALBP, which aims to minimise the cycle time for a given number of workstations.

n=1	Workstations					
	1	2	3	4	5	6
Task	1	4	2	8	7	9
	5		6		1	11
			3			

$\mu(b_i)$

n=10	Workstations				
	1	2	3	4	5
Task	1	3	10	4	7
	2	8			9
	6				11
	5				

n=20	Workstations			
	1	2	3	4
Task	1	8	4	9
	2	10	7	11
	6			
	3			
	5			

n=30	Workstations			
	1	2	3	4
Task	1	4	8	10
	2	7	9	11
	6			
	3			
	5			

n=50	Workstations			
	1	2	3	4
Task	1	4	8	10
	2	7	9	11
	6			
	3			
	5			

n=100	Workstations			
	1	2	3	4
Task	1	4	8	10
	2	7	9	11
	6			
	3			
	5			

n=150	Workstations			
	1	2	3	4
Task	1	4	8	9
	2	7	10	11
	6			
	3			
	5			

n=200	Workstations		
	1	2	3
Task	1	2	8
	4	6	10
	5	3	11
		7	
		9	

b^*

n=10	Workstations				
	1	2	3	4	5
Task	1	3	10	4	7
	2	8			9
	6				11
	5				

n=20	Workstations			
	1	2	3	4
Task	1	8	4	9
	2	5	7	11
	6	10		
	3			

n=30	Workstations			
	1	2	3	4
Task	1	4	8	10
	2	7	9	11
	6			
	3			
	5			

n=50	Workstations			
	1	2	3	4
Task	1	4	8	9
	2	7	10	11
	6			
	3			
	5			

n=100	Workstations			
	1	2	3	4
Task	1	4	8	9
	2	7	10	11
	6			
	3			
	5			

n=150	Workstations			
	1	2	3	4
Task	1	4	8	9
	2	7	10	11
	6			
	3			
	5			

n=200	Workstations		
	1	2	3
Task	1	4	8
	2	7	10
	6	9	11
	3		
	5		

Fig. 1. Optimal balancing solutions.

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