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The properties of a skewness index and its relation with volatility and returns

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Abstract: The objectives of this study are threefold. First, we investigate the properties of a skewness index in order to determine whether it captures fear (fear of losing money), or greed in the market (fear of losing opportunities). Second, we uncover the contemporaneous linear relationship among skewness, volatility and returns. Third, we provide evidence on the information content of skewness on future returns, which is highly debated in the literature. Fourth, we investigate the Italian market, where a skewness index is not traded yet. The methodology we propose for the construction of the Italian skewness index is applicable also to other European and non-European countries characterized by a limited number of option traded. Several results are obtained. First, in the Italian market the skewness index acts as measures of market greed, as opposed to market fear, namely that it measures more investors' excitement than investors' fear. Second, for almost 70% of the daily observations, the implied volatility and the skewness index move together but in opposite directions. Increases (decreases) in volatility and decreases (increases) in the skewness index are associated with negative (positive) returns. Last, we find strong evidence that positive returns are reflected both in a decrease in the implied volatility index and in an increase in the skewness index the following day. Implications for investors and policy makers are drawn.

*Keywords***:** *skewness index, risk-neutral moments, implied volatility, returns, Italian market.*

JEL classification: G12, G13, G17

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1. Introduction

The Volatility Index of the Chicago Board Options Exchange (CBOE), *VIX,* has been called the "investor fear gauge" (Whaley (2000)) because it measures the investors' consensus view about expected future stock market volatility (market sentiment). The higher the value of the *VIX* index, the greater the investors' fear is considered to be about losing money. The *VIX* index spikes during periods of market turmoil (bad news) because when expected volatility of returns increases, investors demand higher rates of return, resulting in lower stock prices (leverage effect[1]). However, the correlation between the *VIX* index and market returns is neither exact nor always negative as in some cases the *VIX* index spikes and stock prices also rise, manifesting a positive relationship. In these cases, it can be said that the *VIX* index captures investors' "excitement" or "greed"[2], reflecting the fear of losing profitable opportunities.

The *CBOE SKEW* index has been listed on the CBOE since February 2011 to measure the tail risk not fully captured by the *VIX* index. While *VIX* measures the overall risk in the 30-day S&P500 log-returns without disentangling the probabilities attached to positive and negative returns; the skewness index *(CBOE SKEW)* is intended to measure the perceived tail risk, i.e. the probability that investors attach to extreme negative returns. The *CBOE SKEW* index is intended to vary around a value of 100. Values above the threshold level 100 point to a negative risk-neutral skewness and a distribution skewed to the left. These values indicate that negative returns are more often expected than positive returns. The opposite is true for values below 100. These values suggest that positive returns are more often expected than negative ones. Moreover, a high value of the *CBOE SKEW* index indicates that buying protection against downturn (put options) is more expensive. From January 1990, the *CBOE SKEW* index has varied between 101.23 and 154.34, indicating that the S&P500 implied distribution has been always left-skewed during this period. The role of the *CBOE SKEW* index as an indicator of "market fear" has been questioned since it moves frequently in the same direction as returns (Liu and Faff (2017)). Moreover, the power of a skewness index[3] in predicting future realized returns is still debated in the literature, and

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[1] The leverage effect refers to the observed tendency that rising asset prices are accompanied by declining volatility, and vice versa, pointing to a negative relation. The economic rationale is that, when asset prices decline, companies become more leveraged since the relative value of their debt increases with respect to that of equity. Therefore, the stock is perceived as riskier and as a result it is expected to be more volatile.

[2] Whaley (2000) defines greed as the investors' excitement in a market rally.

[3] These authors investigate risk-neutral skewness, which is embedded in the skewness index with a negative sign. Therefore if risk-neutral skewness has a positive relation with returns, the skewness index has a negative one. We recall that moments of an asset return distribution can be obtained by means of two different approaches: first, they can be estimated through the historical series of the underlying asset (physical or realized moments); second, they can be obtained from the prices of options listed on the underlying asset (implied or risk-neutral moments).

its combined effect with volatility has still to be clarified. Bali and Murray (2013) and Conrad et al. (2013) find a negative relation between risk-neutral skewness and future stock returns. On the other hand, several other studies find a positive relation between the same variables, suggesting that informed investors trade first in options and only subsequently the information is embedded in asset prices (Xing et al. (2010), Yan (2011), Cremers and Weinbaum (2010), Rehman and Vilkov (2012), Stilger et al. (2017)).

Our contributions include the following. First, along the guidelines used to construct the *CBOE SKEW* index, we delineate a skewness index for an important European market: the Italian stock market, that we call *ITSKEW*. While the implied volatility of the aggregate Italian market is currently measured by the implied volatility index (the *IVI* index),[4] a measure of the asymmetry in the return distribution and tail risk has yet to be adopted for this market. We fill this void. The lower liquidity of the Italian option market, compared to the US market, is mitigated using an interpolation-extrapolation procedure among implied volatility of call and put prices as proposed in Muzzioli (2010). This procedure can be used also for other European and non-European countries characterized by a limited number of option traded. This is very important since it enlarges the number of countries for which a skewness index can be computed.

Second, we uncover the role of a skewness index as an indicator of fear (fear of losing money) or greed (fear of losing opportunities) in the market. The literature has deeply addressed the role of the volatility index, but is silent on the role of a skewness index. Third, we provide further evidence and possible explanations for the relationship between volatility, skewness and future returns, which is highly debated in the literature. This issue is of interest for investors who can combine the information of both skewness and volatility indices in order to form expectations about future rises and falls in the FTSE MIB index.

We obtain several findings. First, the *ITSKEW* index attains an average value of 103.78, pointing to a left-skewed risk-neutral distribution in the FTSE MIB[5] index returns. Second, the market index FTSE MIB is asymmetrically associated to the *ITSKEW* index. Specifically, if the *ITSKEW* index falls by 100 basis points, the FTSE MIB index is expected to decrease by 0.35% (35bp), while if it increases by 100 basis points, the FTSE MIB index is expected to increase by 0.14% (14bp), manifesting an asymmetric behavior. Investors could exploit this result in order to form expectations about FTSE MIB index fluctuations.

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[4] The *IVI* index is computed by FTSE Russell, by using the *CBOE VIX* index formula adapted to the Italian market.

[5] Financial Times Stock Exchange Milano Indice di Borsa is a capital-weighted index composed of 40 major stocks listed on the Italian market.

Unlike the inverse relationship between volatility and FTSE MIB returns (see e.g. Muzzioli (2013b)), a positive relationship prevails between *ITSKEW* and FTSE MIB returns, qualifying the Italian skewness index more as a measure of market greed (fear of losing opportunities), than market fear (fear of losing money). In other words, high returns are in general associated with low levels of the volatility index but with high levels of the skewness index. In terms of magnitude, the higher the volatility index, the greater the fear and the higher the skewness index, the greater the greed (fear of losing opportunities. This is important for investors who could exploit the properties of the *ITSKEW* index as an indicator of market greed to implement trading strategies and for regulators to monitor the prevailing sentiment in the market and intervene with the necessary policy measures.

By investigating the combined effect of volatility and skewness, we highlight that for almost 70% of the daily observations, the implied volatility (*IV*) and the skewness index (*ITSKEW*) move together but in opposite directions. In particular, when volatility increases and the *ITSKE*W index decreases, the contemporaneous market return is negative (-1.24%) and significant at the 1% level. The same relation is detected when the volatility decreases and the *ITSKEW* index increases: the daily FTSE MIB return is on average positive and significant at the 1% level. This suggests that the combined effect of a negative change in the volatility index and a positive change in the skewness index, represents a buy opportunity with an average daily return of 1.18%.

We provide evidence that a positive return is reflected both in a decrease in the implied volatility index and in an increase in the *ITSKEW* index the following day. A possible explanation for the positive relationship is that in a framework characterized by high returns and low market volatility, investors are willing to pay in order to hedge their gains, and they become more concerned about future market returns.

The plan of the paper is as follows: in Section 2, we describe the data set and the methodology adopted in order to obtain the initial input of option data. In Section 3, we provide a detailed description of the construction of the Italian skewness index. In Section 4, we analyze the properties of the *ITSKEW* index, such as its relation with volatility and market returns and its role as an indicator of greed or fear in the market. In Section 5 we analyze the dynamic interactions between changes in the *ITSKEW* index, changes in volatility, and returns. Finally, in Section 6, we conduct a robustness test, investigating the *ITSKEW* index behavior in both volatile and calm market period. The last section concludes.

2. The Italian data

The data set consists of closing prices on FTSE MIB index options (MIBO), recorded from 3 January 2011 to 28 November 2014. MIBO are European options on the FTSE MIB index. As for the underlying asset, closing prices of the FTSE MIB-index recorded in the same time-period are used. The FTSE MIB adjusted for dividends ($\hat{S}_t$) is computed as follows:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \qquad (1)$$

where S_t is the FTSE MIB index value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option. As a proxy for the risk-free rate, Euribor rates with maturities of one week, one month, two months, and three months are used: the appropriate yield to maturity is computed by linear interpolation. The data set for the MIBO is kindly provided by Borsa Italiana S.p.A; the time series of the FTSE MIB index, the dividend yield and the Euribor rates are obtained from Datastream.

Our dataset is a suitable framework for investigating the behavior of tail risk measures such as the *SKEW* index because while the first part of the sample (2011-2012) is characterized by an extreme level of volatility due to the European debt crisis, in which Italy has played a key role, the second part (2013-2014) is characterized by a tranquil market and a bullish trend in the stock prices. This allows us to investigate the proprieties of the skewness index under different market conditions.

Several filters are applied to the option data set in order to eliminate arbitrage opportunities and other irregularities in the prices. First, consistently with the computational methodology of other indices (such as the *CBOE SKEW*), we eliminate options near to expiry (options with time to maturity of less than eight days). Second, following Ait-Sahalia and Lo (1998) only at-the-money and out-of-the-money options are retained. These include put options with moneyness (*X/S*, where *X* is the strike price and *S* the index value) lower than 1.03 and call options with moneyness higher than 0.97. In order to have a one-to-one mapping between strikes and implied volatilities, we compute the average of implied volatilities that correspond to the same strike price. Finally, option prices violating the standard no-arbitrage constraints are eliminated.

3. The construction of the Italian skewness index

In order to compute the Italian skewness index, in line with the CBOE procedure (CBOE (2010)), we use the Bakshi et al. (2003) model-free skewness formula:

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\left\{\left(R(\mathrm{t},\ \tau) - E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^3\right\}}{\left\{\left(E_t^q(R(\mathrm{t},\ \tau) - E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^2\right\}^{3/2}} . \quad (2)$$

In this specification, $R(\mathrm{t},\ \tau)$, $\left[R(\mathrm{t},\ \tau)\right]^2$ and $\left[R(\mathrm{t},\ \tau)\right]^3$ are the payoffs of the contracts at time t with maturity τ, based on first, second and third moment of the distribution, respectively. The prices of these contracts are obtained under the risk-neutral expectation (E_t^q) (for a more detailed discussion of the contracts see the Appendix).

Formula (2) relies on the assumption of the existence of a continuum of strike prices ranging from zero to infinity, assumption which is not fulfilled in the reality of the options market. If for the US market this assumption can be mitigated by the high number of option prices traded (usually around 120 per day), truncation and discretization errors can be expected to be very high in the Italian market, which is characterized by an average number of 15 strike prices traded per day. Therefore, in order to adapt the formula to the Italian market, we adopt the following steps. First, after having applied the filters described in Section 2, we create a table of around 15 entries of strike prices and implied volatilities, which is our initial input (see Table 1). Second, in order to generate a sufficient number of strike prices, we follow an interpolation-extrapolation methodology. We interpolate between two adjacent knots by means of cubic splines that keep the function smooth in the knots. We extrapolate outside the traded domain of strike prices by supposing constant volatility. In order to ensure continuity, the constant volatility used in the left (right) part of the extended smile is equal to the volatility of the lowest (highest) strike price traded. Last, from the interpolated-extrapolated smile, we compute a matrix of strike prices and implied volatilities of almost 6500 rows per day, by using a space interval of 10 basis points between strikes ($\Delta K = 10$) in the interval $S/(1+u) \leq K \leq S(1+u)$, where S is the underlying asset value and u is equal to 3. The parameters u and ΔK have been chosen in order to ensure insignificant truncation and discretization errors (in line with Muzzioli, (2013a)). The obtained implied volatilities are then converted into option prices and used in equation (2) (See the Appendix for further details on the Bakshi et al. (2003) formula). The interpolation-extrapolation methodology greatly improves the precision of the skewness estimate. We can see in Figure 1 in the top panel the initial input of strike prices and implied volatilities and in the bottom panel the interpolated-extrapolated smile.

In line with the CBOE procedure (CBOE (2010)), we use two option series, a first option series with a maturity of less than 30 days and a second option series with time to maturity greater than 30 days in order to obtain a constant 30-day measure for implied skewness:

$$SK = wSK_{near} + (1-w)SK_{next} \tag{3}$$

with $w = (T_{next} - 30)/(T_{next} - T_{near})$, and T_{near} (T_{next}) are the time to expiration of the near and next term options, and SK_{near} and SK_{next} are the skewness measures, which refer to the near and next term options, respectively.

Moreover, in line with the CBOE procedure (CBOE, 2010), we compute the Italian skewness index as:

$$ITSKEW = 100 - 10 \times SK \tag{4}$$

where *SK* is obtained in equation (3). Given that the risk-neutral skewness attains typically negative values for equity indices, formula (4) enhances the interpretation of the *ITSKEW* index. For a symmetric distribution, risk-neutral skewness is equal to zero and the *ITSKEW* index will be equal to 100. This value is a threshold level for the skewness index, since values greater (lower) than 100 mean that the risk-neutral distribution is asymmetric to the left (right).

Therefore, our procedure in order to compute the Italian skewness index follows as much as it can the CBOE methodology, departing from it in the interpolation-extrapolation step fundamental to cope with the paucity of strike prices traded in the Italian market.

4. Properties of the Italian skewness index

In this section we discuss the properties of the proposed skewness index for the Italian market. First, we investigate the descriptive statistics of *ITSKEW* in order to have a comparison with the existing measures of skewness (*CBOE SKEW*) and we propose a preliminary analysis, based on correlation, on the relation between the *ITSKEW* index on one side and market returns and volatility on the other. Second, in order to understand if the skewness index can be considered as a measure of fear (fear of losing money) or greed (fear of losing opportunities) in the market, we assess the relation between changes in the *ITSKEW* index on one side, and FTSE MIB returns on the other. Third, we uncover the relation between changes in skewness and volatility. Last, we investigate in encompassing regressions the explanatory power of volatility and skewness on returns.

4.1 Descriptive analysis

Table 2 provides the summary statistics for the FTSE MIB index returns, the implied volatility index[6] (*IV*), the risk-neutral (*ITSKEW*) skewness index, daily changes in model-free implied volatility (*IV*), and daily changes in the *ITSKEW* index ($\Delta ITSKEW$). $\Delta ITSKEW^{+}$ and $\Delta ITSKEW^{-}$ are the positive and negative changes in the *ITSKEW* index, respectively, which will be discussed later. For each variable, the last rows provide the Jarque-Bera test for normality and the p-value of the test. A high Jarque-Bera statistic value indicates that the null hypothesis of a normal distribution for the variable is rejected. A few observations are in order in this connection.

First, the physical returns (*R*) display slightly negative skewness and pronounced excess kurtosis (the Jarque-Bera test rejects the normality hypothesis). Also for model-free implied volatility (*IV*), the hypothesis of a normal distribution is strongly rejected, indicating the presence of extreme movements in volatility in the form of fat tails (column 2). Second, the skewness index for the Italian stock market is on average higher than the threshold level of 100 (103.78 for *ITSKEW*), suggesting that the risk-neutral skewness is in general negative during the sample period. Similar results were obtained for the S&P500 index option market by, e.g., Neumann and Skiadopoulos (2013) and Kozhan et al. (2013). These values indicate that extreme price decreases are more likely to occur, than extreme price rises. Third, the risk-neutral skewness index (*ITSKEW*) displays positive skewness and excess kurtosis and the hypothesis of a normal distribution is strongly rejected by the Jarque-Bera statistic (column 1). This indicates the presence of extreme movements also in the skewness index.

The correlation coefficients between the *ITSKEW* index and the other moments of the return distribution are shown in Table 3, both in terms of levels and daily changes. The *ITSKEW* index also presents a positive and significant correlation (0.208) with daily returns and has a negative and significant correlation with model-free implied volatility (*IV*). Therefore, according to the *ITSKEW* index, the risk-neutral distribution of the FTSE MIB index returns is less negatively skewed when model-free implied volatility is high.

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[6] The implied volatility index for the Italian market is computed by exploiting the model-free methodology as in Muzzioli (2013b), with an extrapolation outside the existing domain of strike prices with a constant volatility function. The obtained model-free volatility measure is multiplied by 100 (CBOE (2009) methodology) in order to compute the Italian volatility index (*IV*).

The correlation between the daily changes of the *ITSKEW* index and the daily changes in model-free implied volatility is negative, suggesting that a positive change in model-free implied volatility is associated with a negative change in the skewness index (i.e. less negative asymmetry in the distribution). The correlation between the daily changes of the *ITSKEW* index and the returns is positive, suggesting that a positive return is associated with a positive change in the *ITSKEW* index (more negative asymmetry in the distribution).

4.2 The *ITSKEW* index as a measure of fear or greed in the market

The introduction of a skewness index in the US market (*CBOE SKEW*) is meant to complement the information provided by the volatility index (*VIX*) about the probability that investors attach to extreme negative returns. Therefore we expect the skewness index to be a measure of fear in the market. Moreover, given the key role attained by the skewness in assessing the riskiness of the return distribution, we expect innovations in the *ITSKEW* index to be strongly related with innovations in implied volatility and with market returns. Theoretical literature provides a little help in understanding the expected relationship between changes in the *ITSKEW* index and market returns. The traditional CAPM model assumes normality and does not take into account the investors' preferences for higher order moments of return distribution. Extension of the intertemporal capital asset pricing model (ICAPM, Merton (1973)) proposed in the literature (see e.g. Chabi-Yo (2012)) account for uncertainty in higher order moments. The price of market skewness depends on the fourth derivative of the utility function, that is hard to sign. Therefore, as stated in Chang et al. (2013), the relation between returns and changes in skewness, remains largely an empirical question.

Many papers find that positive returns are associated with declines in volatility, while negative returns are associated with increases in volatility (e.g., Simon (2003) and Giot (2005)). In particular, peaks in the *VIX* index, that occur during market downturns, can be considered as indicators of investors' fear (market stress), suggesting that the *VIX* is indeed a barometer of the investors' fear. On the other hand, as far as we know, there are no studies investigating the role of a skewness index as an indicator of current fear or greed in the market. Therefore, in order to investigate whether the *ITSKEW* index can be considered as an indicator of market fear or market greed, i.e. whether it measures more investors' excitement than investors' fear, we estimate the following regression:

$$R_t = \alpha + \beta \Delta ITSKEW_t + \varepsilon_t \qquad (5)$$

where R_t is the daily FTSE MIB log-return and $\Delta ITSKEW$ is the daily change in the *ITSKEW* index defined in logarithmic terms as follows: $\Delta x_{t+1} = \ln\left(x_{t+1} / x_t\right)$, where *x* is the series under investigation.

Results are presented in Table 4, Panel A. The slope coefficient of changes in the *ITSKEW* is positive and significant, indicating that an increase in the *ITSKEW* index (the risk-neutral distribution becoming more negatively skewed), is associated with positive returns. This suggests that the *ITSKEW* index acts more as a measure of market greed (fear of losing opportunities) than as a measure of market fear (fear of losing money). This unexpected result, should lead to a deep consideration on the role of a skewness index both in absolute terms an in relation with the volatility index. If the skewness index were designed to measure tail risk (the one not fully captured by the volatility index, that, by construction, reacts symmetrically both to positive and negative returns) then there is something wrong either in the construction of the skewness index, or in the matching between the asymmetry concept and the measurement of tail risk. We believe in this last explanation. In fact, the one to one mapping between asymmetry and tail risk is far from being understood. We can think about situations where the negative asymmetry is pronounced, but the tail risk is little and situations in which the distribution is symmetric, but the tail risk is high. These situations are depicted in Figure 2. Therefore we claim that what is measured by the skewness index (asymmetry of the distribution) is not tail risk and that in order to measure tail risk, new measures have to be produced in the literature.

The latter point becomes more clear if we split the changes in the *ITSKEW* index into positive and negative ones as follows:

$$\Delta ITSKEW_t^+ = \Delta ITSKEW_t \text{ if } \Delta ITSKEW_t > 0 \text{, otherwise } \Delta ITSKEW_t^+ = 0 \tag{6}$$

$$\Delta ITSKEW_t^- = \Delta ITSKEW_t \text{ if } \Delta ITSKEW_t < 0 \text{, otherwise } \Delta ITSKEW_t^- = 0 \tag{7}$$

We then estimate the following model:

$$R_t = \alpha + \beta_1 \Delta ITSKEW_t^+ + \beta_2 \Delta ITSKEW_t^- + \varepsilon_t \tag{8}$$

The regression results are reported in Table 4, Panel B. Both positive and negative changes in the *ITSKEW* index are highly significant as both slope coefficients are positive. In terms of magnitude, the coefficient of negative changes in the *ITSKEW* index is more than twice as large as that of the positive changes, indicating an asymmetric effect. Moreover, the Wald test statistics strongly rejects the hypothesis *H0:* $\beta_1 = \beta_2$ (at the 1% level). In terms of magnitude, a 100 basis points decrease in the *ITSKEW* index (risk-neutral skewness increases) is associated with a strong decrease in returns (0.349%, 34bp), while a 100 basis points increase in the *ITSKEW* index (risk-neutral skewness become more negative) is associated with a less pronounced increase in returns (0.140%, 14bp). Therefore, the market reacts more negatively to decreases in the *ITSKEW* index than it reacts positively to increases in the *ITSKEW* index. Interestingly, the positive sign of both coefficients still suggest that in this setting the *ITSKEW* index acts more as a measure of market greed (fear of losing opportunities) than as a measure of market fear (fear of losing money). Positive peaks in *ITSKEW* can be considered as indicators of high investor greed, negative peaks in *ITSKEW* can be considered as indicators of high investor fear.

This effect can be compared to the finding in Whaley (2000) for the volatility index VIX. Whaley (2000) finds an asymmetric relation between positive and negative changes in the VIX index and the S&P100 index returns: the US stock market reacts more negatively to an increase in the volatility index than it reacts positively to a decrease of the same magnitude in the same index.

Putting together the reaction of returns to both changes in the volatility index and changes in the skewness index, we can draw the picture summarized in Table 5. We can see that for almost 70% of the daily observations, the implied volatility (*IV*) and the skewness index (*ITSKEW*) move together but in opposite directions. In particular, when volatility increases and the *ITSKE*W index decreases, the contemporaneous market return is negative (-1.24%) and significant at the 1% level. When the volatility decreases and the *ITSKEW* index increases the daily FTSE MIB return is on average positive (1.18%) and significant at the 1% level.

On the other hand, when the volatility index and the skewness index move in the same direction, the relation is less clear. More specifically, when both the volatility and the skewness index decrease, the effect associated to the lower volatility (lower uncertainty in the market) seems to be prevalent, since returns are positive (0.30%) and significant at the 1% level. Finally, when both the skewness and the volatility indices increase (the least recurring cases), market returns are not statistically different from zero, suggesting that the effect of the innovations in volatility and skewness are offset. The relation between the volatility index and the skewness index is further investigated in the next subsection.

4.3 Relation between changes in the *ITSKEW* and changes in volatility

Given the well-known role of implied volatility as a measure of market fear (Whaley (2000)), we extend the analysis to examine the relation between changes in the *ITSKEW* index and changes in the Italian implied volatility index (*IV*) in order to better investigate the role and the properties of the *ITSKEW* index. In particular, if the *ITSKEW* index acts as a measure of market greed, we expect changes in the *ITSKEW* index to be negatively related to those of the volatility index. In theory, since implied volatility appears in the denominator of the *CBOE SKEW* formula, an increase in implied volatility, may result in a decrease in the skewness index. Given this result and the well-documented negative relation between changes in volatility and market return, the *ITSKEW* index and market returns are expected to move in the same direction. To investigate this issue, we estimate the relationship between changes in the *ITSKEW* index and changes in the Italian volatility index (*IV*) as described by equation (9), where ΔIV shows the daily changes in the Italian volatility index (in logarithmic terms):

$$\Delta ITSKEW_t = \alpha + \beta \Delta IV_t + \varepsilon_t \tag{9}$$

The estimation results for this model, presented in Table 4, Panel C, show a negative and significant relation between volatility changes and changes in the *ITSKEW* index. This indicates that an increase in model-free implied volatility is associated with a decrease in the *ITSKEW* index, or a less negative risk-neutral distribution (the distribution becomes less skewed towards left).

The results for the *ITSKEW* index are consistent with the findings of Chang et al. (2013) for the S&P500 index options market. Neuberger (2012) also finds a positive correlation coefficient between model-free variance and skewness (0.297 in the period 1997-2009) for the S&P500 index, implying that the higher the variance, the less left-skewed the distribution is. Recall that the correlation between volatility and returns is negative (leverage effect). A possible explanation is that when volatility is high, returns are low (for example, in a stressed market or after a market crash) and also the greed (fear of losing opportunities) is low. On the other hand, when volatility is low, returns are high (complacent market) and the greed (fear of losing opportunities) is high.

4.4 Encompassing regressions

As a final step, in order to assess if the relation between market returns and changes in the *ITSKEW* index remains positive and significant after accounting for changes in implied volatility as an explanatory variable, we estimate the following encompassing regression:

$$R_t = \alpha + \beta_1 \Delta ITSKEW_t + \beta_2 \Delta IV_t + \varepsilon_t \tag{10}$$

Estimation results for the model are shown in Table 4, Panel D. We can see that the coefficient of changes in model-free implied volatility (ΔIV) is negative and highly significant, as established in the literature (see e.g. Muzzioli (2013b)). Moreover, the coefficient of the changes in the *ITSKEW* index remains positive and statistically significant, as in the univariate model (equation (5)). This positive relation has been also confirmed in the US market (e.g., Neuberger (2012), Chang et al. (2013), and Liu and Faff (2017)). In terms of magnitude, if the *ITSKEW* index (which accounts for market greed), increases by

100 basis points, the FTSE MIB index goes up by 0.126% or 12.6 basis points. On the other hand, if the fear in the market (measured by the *IV* index) increases by 100 basis points, the FTSE MIB index decreases by 0.103% or 10.3 basis points. This result could be of interest for investors who can combine the information of both skewness and volatility indices in order to form expectations about FTSE MIB index fluctuations.

5. Dynamic interactions between changes in the ITSKEW index, changes in volatility and returns

In order to investigate the existence of a lagged relation between changes in the *ITSKEW* index and market returns and to account for the interaction among returns, volatility and skewness changes, we estimate a vector autoregression (VAR) model described as follows:

$$R_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t \quad (11)$$

$$\Delta ITSKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t \quad (12)$$

$$\Delta IV_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t \quad (13)$$

The lag length *K* is chosen equal to *1* according to both the Schwarz and Hannan-Quinn information criteria. We include changes in the volatility index in the VAR model due to its significant relation with both market returns and changes in the *ITSKEW* index. This specification is supported by Muzzioli (2013b) who found that changes in implied volatility (as measured by both Black-Scholes (1973) implied volatility and model-free implied volatility) can serve as an early-warning indicator of market stress, and that the returns have explanatory power in forecasting future implied volatility.

The estimation output of the VAR model is shown in Table 6, Panel A. The VAR coefficient estimates show that market returns can be explained only by past changes in the implied volatility index. Changes in the *ITSKEW* index are found to be explained by past return and past changes in the *ITSKEW* index with insignificant contribution coming from changes in volatility. Changes in the implied volatility index are influenced by a more comprehensive set of variables: past market returns, past changes in the volatility index and, marginally, by lagged values of changes in the *ITSKEW* index. In order to determine whether one variable in the model is useful in forecasting the other variables, we perform a Granger causality test for the null hypothesis of zero effect on each variable from the other two variables. The results of the Granger causality tests are reported in Table 6, Panel B.

The Granger causality highlights three main results. First, a positive return is reflected both in a decrease in the implied volatility index and in an increase in the *ITSKEW* index the following day (column 2, rows 3,5). A possible explanation for the positive relationship is that in a framework characterized by high returns and low market volatility, investors are willing to pay in order to hedge their gains, and they become more concerned about future market returns. This phenomenon is called the "bubble theory". This theory is investigated in Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative. Therefore, we may suppose that increases in the *ITSKEW* index are a consequence of high past returns (high significance in the Granger causality test), while the opposite is not necessarily true (significant only at the 10% level in the Granger causality test).

Second, we find evidence that an increase in the volatility index is associated with a positive market return the following day. This evidence is consistent with the prediction of the capital asset pricing model: investors perceive an increase in market volatility as a negative shock for the investment opportunity set and, as a consequence, they require higher returns on such assets in order to be compensated for their higher exposure to volatility risk. Third, we find weak evidence that positive changes in the *ITSKEW* index are reflected in an increase in the Italian volatility index the following day. This result is important for volatility traders, who can improve their strategies by taking into account the information provided by the skewness index.

Finally, in Table 6, Panel C, we report the results of the variance decomposition for the VAR model under investigation. Looking at the first row of Panel C, we can see that shocks in market returns (R) are entirely explained by fluctuations in returns (own shock). Similarly, innovations in the *ITSKEW* index are mainly explained by fluctuation in the *ITSKEW* index itself (81.61%), while innovations in returns cause the 18.39% of fluctuation in the *ITSKEW* index (row 2). On the other hand, the results for the variance decomposition of ΔIV (reported in the third row of Panel C) suggest that shocks in implied volatility are able to explain only the 64.21% of the total variation in volatility. Specifically, a significant part of the variation in volatility is explained both by market returns (32.02%) and by innovations in the *ITSKEW* index (3.77%), suggesting that both skewness and market returns should be taken into account to enhance the forecasting of volatility fluctuations.

In order to further investigate the direction and the persistence of the interaction between the variables, we compute the impulse-response function for the vector auto-regression (VAR) model under investigation and present the results in Figure 3. Several observations are noteworthy. First, for each of the variables, the response function attains values close to zero for the days after the first, suggesting that the effects are in general exhausted in one day. Second, by looking at the panels on the top of the figure, we can see that returns are affected (positively) only by past return (own shock). Third, the response of *ΔITSKEW* to both shocks in returns and innovations in the skewness index is positive (panels in the middle). Finally, we can see that both shocks in market returns and in skewness index affect the implied volatility negatively (panels on the bottom). These results support previous evidence about the interaction between returns, innovations in skewness and innovations in volatility. Therefore, we can conclude that the skewness index can only marginally be used in order to forecast returns, but returns can be used in order to forecast the skewness index.

6. Sample Disaggregation: Analysis of skewness index during calm and volatile periods

In Figure 4 we compare the behavioral patterns of the FTSE MIB index, implied volatility and the *ITSKEW* index. From the graph we can observe two different medium-term trends: a negative trend (bearish market) characterized by higher volatility between February 2011 and the end of July 2012, and a positive trend (bullish market) in the second part of the sample period between the end of July 2012 and November 2014, which is characterized by lower volatility. We may attribute the reversal in trend to the positive effect of the "whatever it takes" London Speech of the ECB President Mario Draghi (26 July 2012) that put an end to the acute phase of the European sovereign debt crisis. Proclaiming that the ECB would do "whatever it takes" to save the Euro was the incipit to the Outright Monetary Transactions[7] policy, putting an end to speculation on government bonds of the peripheral countries. The speech was followed by an immediate rise in the market indices of the European stock markets. In order to contrast the behavior of the skewness indices in high and low volatility periods, we split the data set according to these volatility periods and report the descriptive statistics for the *ITSKEW* index and the implied volatility index in the two sub-periods in Table 7.

The results confirm that implied volatility is on average higher in the first part of the sample (the average volatility = 40.32) relative to the second (average implied volatility = 29.42). Risk-neutral

[7] Outright Monetary Transactions, announced on 2 August, 2012, is a programme which allows the ECB to purchase, under certain conditions, sovereign bonds issued by Eurozone member-states. http://www.ft.com/cms/s/0/448a6f28-f822-11e1-828f-00144feabdc0.html

skewness index attains a value higher than 100 in both sub-periods, pointing to an overall negative skewness. The *ITSKEW* index is high in the bullish period and low in the bearish period, pointing to a more negatively skewed distribution in the period characterized by a stable market and low volatility (the second period), characterized by a higher greed (fear of losing opportunity) in the market.

In Table 7, Panel C, we report the results of the test for $H_0: \beta_1 = \beta_2$ in order to assess whether the mean values for both implied volatility and *ITSKEW* index attained in the first period (reported in Panel A, first row) are statistically different from the ones obtained in the latter (reported in Panel B, first row). The results of the test suggest that the mean values of the *ITSKEW* index in the two sub-periods are statistically different (at the 5% level). These findings are consistent with those of Han (2008) and Liu and Faff (2017) based on the S&P500 options market. Neuberger (2012) also finds that in the S&P500 options market during the 2003-2007 period, when index volatility was low, negative skewness was relatively high, whereas during the volatility spike of 2008, skewness was rather low. Risk-neutral skewness tends to be more negative during bullish market periods, when market returns are positive, and more positive during bearish market periods, since investors are expecting an inversion of the tendency. In fact, when the market is bearish, investors may purchase out-of-the-money calls instead of buying the underlying asset, shifting the risk-neutral distribution to the right (Simon (2003)).

In order to assess whether the properties of the *ITSKEW* index as a measure of fear or greed for the Italian market are affected by high or low volatility levels, we replicate the analysis based on equations (5)-(10) in each of the two sub-periods. The results, not reported here to save space, are very similar to the ones obtained using the whole sample, both regarding the relation between the *ITSKEW* index and market returns, and the relation between the *ITSKEW* index and implied volatility. This analysis further supports our conclusion about the *ITSKEW* index serving as an indicator of market greed (fear of losing opportunities). When volatility is low, market returns are high (positive) and the high value of the *ITSKEW* index suggests that investors are greedy (or complacent). On the other hand, when volatility is high, returns are low and the investors are fearful.

7. Conclusions

In this paper we pursue three main objectives. First, we delineate, for the first time, a skewness index for the Italian stock market. Second, we investigate the properties of the *ITSKEW* index in order to assess whether it captures fear or greed in the Italian stock market. Third, we shed light on the debated relation between skewness, volatility and future market returns.

In order to compute a skewness index for the Italian market (we call it *ITSKEW*), we exploit the CBOE methodology, which is based on the Bakshi et al. (2003) skewness formula. The lower liquidity of the Italian option market compared to the US one is mitigated using an interpolation-extrapolation procedure among implied volatility of call and put prices as proposed in Muzzioli (2010). This procedure can be used also for other European and non-European countries characterized by a limited number of option prices traded. This is very important since it enlarges the number of countries for which a skewness index can be computed.

In terms of properties, the *ITSKEW* is found to be higher than the threshold level of 100 (103.78) during the 2011-2014 period, pointing to a left-skewed risk neutral distribution for the Italian index market. By investigating the relation between the skewness index and market returns, we find that an increase in the *ITSKEW* index (i.e. the risk-neutral distribution becoming more negatively skewed), is associated with an increase in returns indicating that the market index and the skewness index move in the same direction. Interestingly, this suggest that in this setting the *ITSKEW* index acts more as a measure of market greed (fear of losing opportunities) than as a measure of market fear (fear of losing money). Positive peaks in *ITSKEW* can be considered as indicators of investor greed, negative peaks in *ITSKEW* can be considered as indicators of investor fear. We also find that the effects of positive and negative changes in *ITSKEW* are asymmetric in nature. In other words, a decrease in the *ITSKEW* index, indicating that the distribution becomes more skewed to the right, is associated with a strong decrease in the returns, while an increase in the *ITSKEW* index is associated with a less pronounced increase in returns. The market reacts more negatively, in terms of magnitude, to decreases in the *ITSKEW* index than it reacts positively to increases in the *ITSKEW* index.

This unexpected result, should lead to a deep consideration on the role of a skewness index both in absolute terms an in relation with the volatility index. If the skewness index was designed to measure additional tail risk (the one not captured by the volatility index, which by construction, reacts symmetrically both to positive and negative returns) then there is something wrong either in its construction, or in the matching between the asymmetry concept and the measurement of tail risk. We claim that what is measured by the skewness index (asymmetry of the distribution) is not tail risk and that in order to measure tail risk, new measures have to be produced in the literature. This highlights the importance of investigating other tail risk measures that may better complement the implied volatility information in explaining market returns.

By uncovering the combined effect of volatility and skewness on returns, we find that for almost 70% of the daily observations, the implied volatility (*IV*) and the skewness index (*ITSKEW*) move together but in opposite directions. In particular, when volatility increases and the *ITSKE*W index decreases, the contemporaneous market return is negative (-1.24%) and significant at the 1% level. The same relation is detected when the volatility decreases and the *ITSKEW* index increases: the daily FTSE MIB return is on average positive (1.18%) and significant at the 1% level.

When we look to the dynamic interactions between changes in the *ITSKEW* index, changes in volatility and returns, we find weak evidence of a negative relation between changes in the *ITSKEW* index and future market returns (i.e. a positive relation between skewness and returns), consistent with the theory that informed investors trade in options, rather than in the underlying asset. On the other hand, we find strong evidence that positive returns are reflected both in a decrease in the implied volatility index and in an increase in the *ITSKEW* index the following day. This is in line with Harvey and Siddique (2000), who find that when past returns have been high, the investors' forecast of skewness becomes more negative, in line with the so-called "bubble theory". The theory states that if past returns have been high, a bubble has been inflating and, therefore, a large drop can be expected when the bubble bursts. In particular, the combination of a high skewness index and a low implied volatility may indicate an overly complacent market, and signal the creation of speculative bubbles. On the other hand, market declines may also give rise to an investors' preference for the limited downside risk associated with buying out-of-the-money calls instead of buying the underlying asset (Simon (2003)), shifting the risk-neutral distribution to the right.

As investors are averse to volatility, the *VIX* index has been called the investor fear gauge, since it has been found to spike mainly during high levels of market turmoil. From the findings of the current paper, the *ITSKEW* index should be considered an investor greed gauge, given its positive relation with market returns. The higher the volatility, the greater the fear; the higher the *ITSKEW* measure, the greater the greed.

Given the possibility of using the Italian *ITSKEW* index for settling portfolio strategies and the properties of the *ITSKEW* index as an indicator of market greed, we believe that the results of the paper can be of importance to both investors and regulators, who could monitor the information embedded both in volatility and skewness indices. In particular, a large negative change in skewness indices, combined with an increasing implied volatility, may be regarded as an early warning of a strong fall in the stock market.

This analysis may be extended in many directions. Further research is needed in order to assess the relationship among implied moments and the study of other asymmetry measures which are able to capture changes in the implied distribution coming from the different tails. Moreover, as the skewness coefficient is a normalized measure which is divided by variance, the study of non-normalized measures which react only to asymmetry, and not to both asymmetry and variance, will be useful to better understand the properties of the skewness indices.

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Appendix A. The Bakshi et al. (2003) model-free skewness formula

We provide in this section further details about the model-free formula proposed in Bakshi et al. (2003) in order to compute higher moments of the option implied return distribution. According to Bakshi et al. (2003) model-free skewness is obtained from the following equation as:

$$SK(\mathrm{t},\tau) \equiv \frac{E_t^q\left\{\left(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^3\right\}}{\left\{\left(E_t^q(R(\mathrm{t},\ \tau)-E_t^q\left[R(\mathrm{t},\ \tau)\right]\right)^2\right\}^{3/2}} = \frac{e^{r\tau}W(\mathrm{t},\tau)-3e^{r\tau}\mu(\mathrm{t},\tau)V(\mathrm{t},\tau)+2\mu(\mathrm{t},\tau)^3}{\left[e^{r\tau}V(\mathrm{t},\tau)-\mu(\mathrm{t},\tau)^2\right]^{3/2}} \quad \text{(A1)}$$

where $\mu(\mathrm{t},\tau)$, $V(\mathrm{t},\tau)$, $W(\mathrm{t},\tau)$ and $X(t,\tau)$ are the prices of the contracts, at time t with maturity τ, based on first, second, third and fourth moment of the distribution, respectively; their value are computed as:

$$\mu(t,\tau) \equiv E^q \ln\left[S(t+\tau)/S(t)\right] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau) \quad \text{(A2)}$$

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2\left(1-\ln\left[K/S(t)\right]\right)}{K^2}C(t,\tau;K)\,dK + \int_0^{S(t)} \frac{2(1+\ln\left[S(t)/K\right]}{K^2}P(t,\tau;K)\,dK \quad \text{(A3)}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\ln\left[K/S(t)\right]-3\ln\left[K/S(t)\right]^2}{K^2}C(t,\tau;K)\,dK - \int_0^{S(t)} \frac{6\ln[S(t)/K]+3\ln[S(t)/K]^2}{K^2}P(t,\tau;K)\,dK \quad \text{(A4)}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\ln\left[K/S(t)\right]-3\ln\left[K/S(t)\right]^2}{K^2}C(t,\tau;K)\,dK - \int_0^{S(t)} \frac{6\ln[S(t)/K]+3\ln[S(t)/K]^2}{K^2}P(t,\tau;K)\,dK \quad \text{(A5)}$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\ln\left[K/S(t)\right]^2-4\ln\left[K/S(t)\right]^3}{K^2}C(t,\tau;K)\,dK + \int_0^{S(t)} \frac{12\ln[S(t)/K]^2+4\ln[S(t)/K]^3}{K^2}P(t,\tau;K)\,dK \quad \text{(A6)}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of a call and a put option at time t with maturity τ and strike K, respectively, $S(t)$ is the underlying asset price at time t.

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Table 1 - Strike prices and implied volatilities used as initial input in the *ITSKEW* index calculation

Strike price, K	Moneyness (K/S)	Implied Volatility (σ)
15500	0.77	64.09%
16000	0.80	57.02%
16250	0.81	53.54%
16500	0.82	50.10%
16750	0.84	46.68%
17000	0.85	44.11%
17250	0.86	41.40%
17500	0.87	39.15%
17750	0.89	36.63%
18000	0.90	34.66%
18250	0.91	32.83%
18500	0.92	31.35%
18750	0.94	29.44%
19000	0.95	28.44%
19250	0.96	27.42%
19500	0.97	27.31%
19750	0.99	26.31%
20000	1.00	25.48%
20250	1.01	24.90%
20500	1.02	24.38%
20750	1.04	24.94%
21000	1.05	24.93%
21250	1.06	24.92%
21500	1.07	25.03%
21750	1.09	25.27%
22000	1.10	25.59%
22250	1.11	25.69%
22500	1.12	26.74%
22750	1.14	26.91%

Note: The table provides an example on a single date, January 10, 2011, of the initial grid of input for the interpolation-extrapolation method, after the application of the filters described in Section 2. The underlying asset is worth 20035.16, the maturity of the options is 11 days, the dividend yield and the risk-free rate are equal to 3.81% and 0.59%, respectively.

Table 2 – Descriptive statistics for the Italian market.

	ITSKEW	*IV*	*R*	*ΔIV*	*ΔITSKEW*	$\Delta ITSKEW^{+}$	$\Delta ITSKEW^{-}$
Mean	103.78	33.83	0.00	0.00	0.00	0.01	-0.01
Median	103.84	31.20	0.00	0.00	0.00	0.00	-0.00
Maximum	126.36	75.43	0.06	0.30	0.15	0.15	0.00
Minimum	89.11	14.95	-0.07	-0.52	-0.20	0.00	-0.20
Std. Dev.	4.56	9.73	0.02	0.08	0.03	0.02	0.02
Skewness	0.39	1.28	-0.24	-0.81	0.31	3.13	-3.10
Kurtosis	4.92	4.51	4.44	6.82	7.70	15.39	20.13
Jarque-Bera	174.53	360.88	92.98	700.75	913.73	7833.07	13493.71
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note: The table reports the descriptive statistics of the risk-neutral skewness index, the model-free implied volatility, FTSE MIB returns and daily changes in volatility and skewness measures. *ITSKEW* is the index we compute using the CBOE method, *IV* is the model-free implied volatility multiplied by 100 (VIX methodology), *R* is the FTSE MIB daily return (continuously compounded); $\Delta ITSKEW^{+}$ and $\Delta ITSKEW^{-}$ are the positive and negative changes in the *ITSKEW* index, respectively. The p-value refers to the Jarque-Bera test for normality.

Table 3 – Correlation table for the Italian market.

	ITSKEW	*IV*	*R*	*ΔIV*	*ΔITSKEW*	$\Delta ITSKEW^{+}$	$\Delta ITSKEW^{-}$
ITSKEW	1.000						
IV	-0.284^{***}	1.000					
R	0.208^{***}	-0.117^{***}	1.000				
ΔIV	-0.145^{***}	0.134^{***}	-0.573^{***}	1.000			
ΔITSKEW	0.354^{***}	-0.059^{*}	0.439^{***}	-0.435^{***}	1.000		
$\Delta ITSKEW^{+}$	0.352^{***}	-0.060^{*}	0.286^{***}	-0.432^{***}	0.830^{***}	1.000	
$\Delta ITSKEW^{-}$	0.214^{***}	-0.034	0.431^{***}	-0.264^{***}	0.788^{***}	0.310^{***}	1.000

Note: The table reports the correlation coefficients between the measures used in the study of the Italian market. For the definition of the measures see Table 1. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 4 - Regression output for linear regression models in equations (5-10)

	α	β_1	β_2	*Adj. R^2*
Panel A	0.000 (0.108)	0.237*** (9.051)		0.191
Panel B	-0.002*** (3.407)	0.140*** (4.561)	0.349*** (6.770)	0.210
Panel C	-0.000 (-0.431)	-0.177*** (-6.645)		0.188
Panel D	-0.000 (-0.077)	0.126*** (4.525)	-0.104*** (-8.851)	0.371

Note: The table presents the estimated output of the following regressions:

Panel A Model: $R_t = \alpha + \beta \Delta ITSKEW_t + \varepsilon_t$

Panel B Model: $R_t = \alpha + \beta_1 \Delta ITSKEW_t^+ + \beta_2 \Delta ITSKEW_t^- + \varepsilon_t$

Panel C Model: $\Delta ITSKEW_t = \alpha + \beta \Delta IV_t + \varepsilon_t$

Panel D Model: $R_t = \alpha + \beta_1 \Delta ITSKEW_t + \beta_2 \Delta IV_t + \varepsilon_t$

where $\Delta ITSKEW_t$ is the daily change in *ITSKEW* index; $\Delta ITSKEW_t^+$ and $\Delta ITSKEW_t^-$ are the positive and negative daily changes in *ITSKEW* index, respectively. R_t is the daily FTSE MIB log-return, ΔIV_t are the daily changes in model-free implied volatility; t-stats in parentheses. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 5 – Combined effect of innovations in volatility and skewness on contemporaneous market return

Number of occurrences	Innovation in volatility	Innovation in skewness	Market reaction	Average return (t-stat)
343 (35.11%)	*IV* ↑	*ITSKEW* ↓	FTSE MIB ↓ (Return -)	-1.24% *** (-11.85)
315 (32.24%)	*IV* ↓	*ITSKEW* ↑	FTSE MIB ↑ (Return +)	1.18% *** (13.71)
155 (15.86%)	*IV* ↑	*ITSKEW* ↑		0.03% (0.32)
164 (16.79%)	*IV* ↓	*ITSKEW* ↓	FTSE MIB ↑ (Return +)	0.30% *** (2.76)

The table summarizes the combined effect of innovations in the volatility index and in the skewness index on contemporaneous market returns. During the sample period January 3, 2011- November 28, 2014, we report in the first column the number of occurrences of the states described in columns 2 and 3, in column 4 the market reaction and in the last column the average return of the FTSE MIB index. We test whether the average return is statistically different from zero (t-stats in parentheses). Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

Table 6 – Analysis of a lagged relationship between changes in the *ITSKEW* index, changes in the implied volatility index (*IV*), and market returns.

Panel A: VAR Estimation output	R_t	$\Delta ITSKEW_t$	ΔIV_t
R_{t-1}	0.055	0.227^{***}	-0.991^{***}
	(1.377)	(3.131)	(-5.513)
$\Delta ITSKEW_{t-1}$	-0.031	-0.272^{***}	0.172^{*}
	(-1.545)	(-7.642)	(1.951)
ΔIV_{t-1}	0.021^{**}	0.024	-0.213^{***}
	(2.378)	(1.506)	(-5.388)
C	0.000	-0.000	-0.001
	(0.019)	(-0.199)	(-0.272)
Panel B: Granger causality test		X^2	*p-value*
Null Hypotheses:			
$\Delta ITSKEW$ does not Granger cause R		3.425	0.065
ΔIV does not Granger cause R		6.307	0.012
R does not Granger cause $\Delta ITSKEW$		7.392	0.007
ΔIV does not Granger cause $\Delta ITSKEW$		1.749	0.186
R does not Granger cause ΔIV		22.992	0.000
$\Delta ITSKEW$ does not Granger cause ΔIV		3.884	0.049
Panel C: Variance decomposition of	R	$\Delta ITSKEW$	ΔIV
R	100.000	0.000	0.000
$\Delta ITSKEW$	18.387	81.613	0.000
ΔIV	32.016	3.768	64.21

Note: The table reports in Panel A the estimation output (t-stat in parentheses) of the VAR model:

$$R_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t$$

$$\Delta ITSKEW_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t$$

$$\Delta IV_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta ITSKEW_{t-l} + \sum_{l=1}^{K} d_l \Delta IV_{t-l} + u_t$$

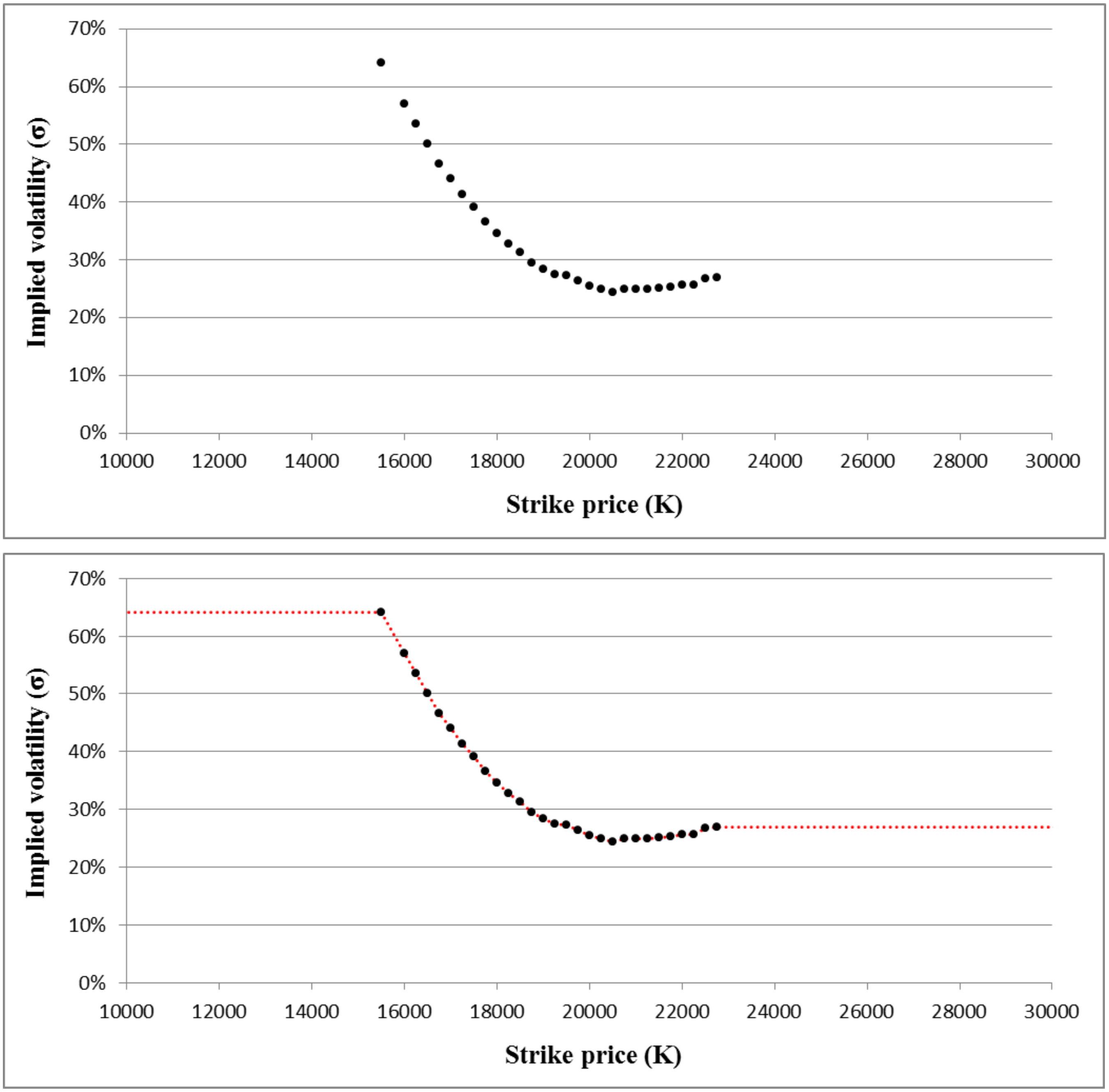
In Panel B we report the results for the Granger causality test between daily returns on the FTSE MIB and daily changes in *ITSKEW* index and in the Italian implied volatility index. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *. Finally, Panel C reports the VAR model Variance Decomposition for one period.

Table 7 – Descriptive statistics of skewness index and implied volatility in the bearish and bullish sub-periods.

	Panel A: *Bearish market* 03/01/2011-25/07/2012		Panel B: *Bullish market* 26/07/2012 – 28/11/2014		Panel C: Mean difference	
	ITSKEW	*IV*	*ITSKEW*	*IV*	*ITSKEW*	*IV*
Mean	103.35	40.32	104.16	29.42	-0.77	10.91
Maximum	114.77	75.43	126.36	46.43	(-2.34)	(18.24)
Minimum	93.68	14.95	89.11	18.97		
Median	103.65	37.54	104.10	28.69		
Std. Dev.	3.70	11.09	5.03	5.23		
Skewness	-0.09	0.58	0.41	0.60		
Kurtosis	0.00	2.67	1.77	2.89		

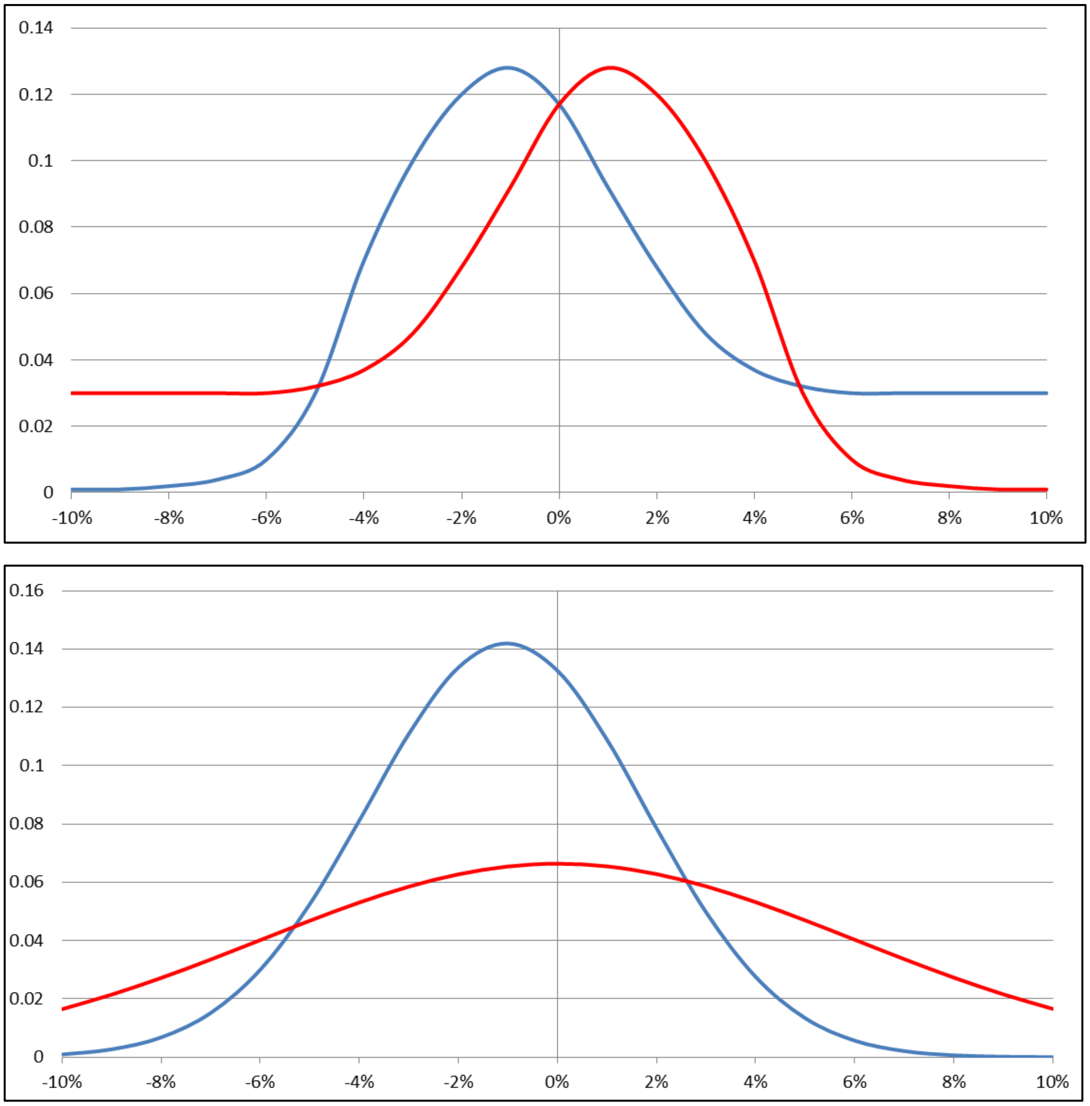
Note: for the definition of the measures see Table 1. In Panel C we report the results of the test for H_0: $\beta_1 = \beta_2$, where β_1 and β_2 are the mean values of the measures reported in Panel A and Panel B, respectively; t-stats are reported in parenthesis.

Figure 1 – The interpolation-extrapolation method.



We report in the top panel of the figure the initial input of strike prices and implied volatilities reported in Table 1. In the bottom panel the dashed line represents the implied volatility obtained through the interpolation-extrapolation method described in Section 3.

Figure 2 – Stylized examples of the return distribution



We report in the top panel of the figure an example of two distribution of returns with equal skewness, but different probability assigned to left tail events. In the bottom panel we depict a distribution characterized by negative skewness and low probability assigned to left tail events (blue line) and a symmetrical distribution that assigns higher probability to left tail events (red line).

Figure 3 – Response to Cholesky One S.D. Innovations

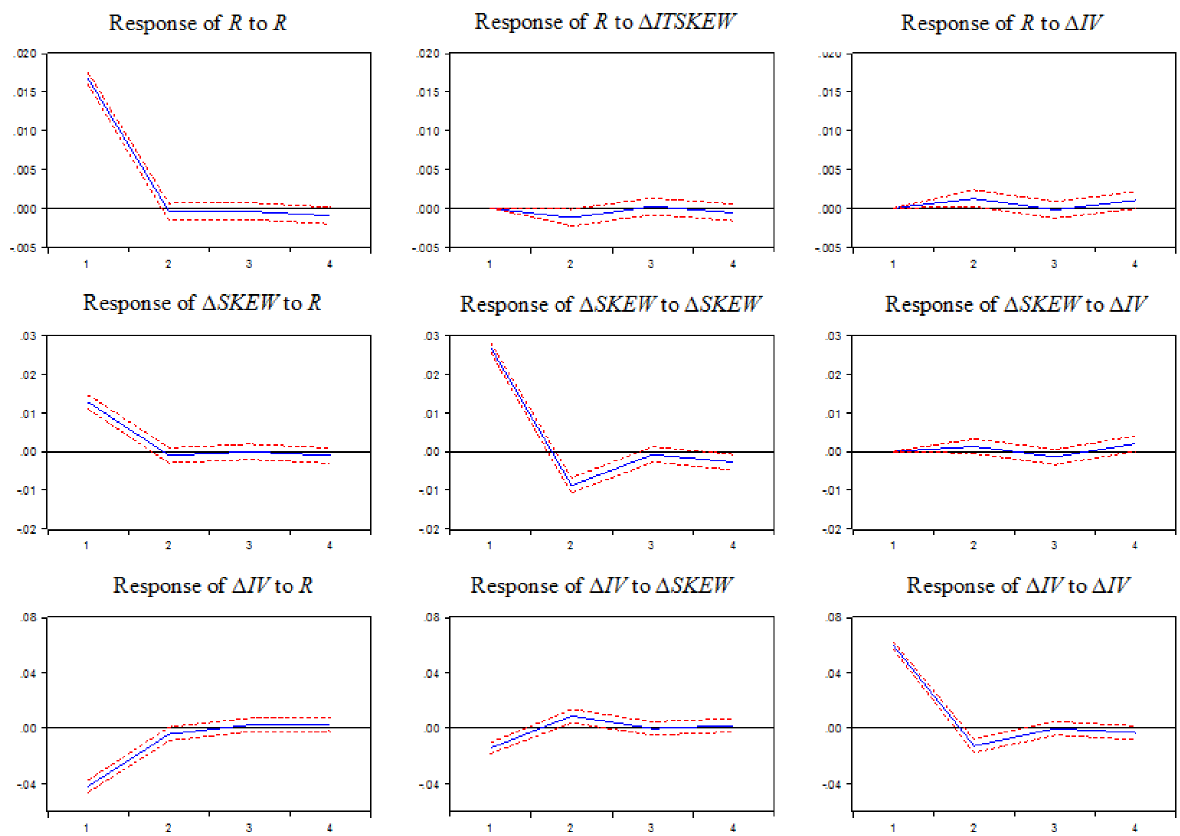
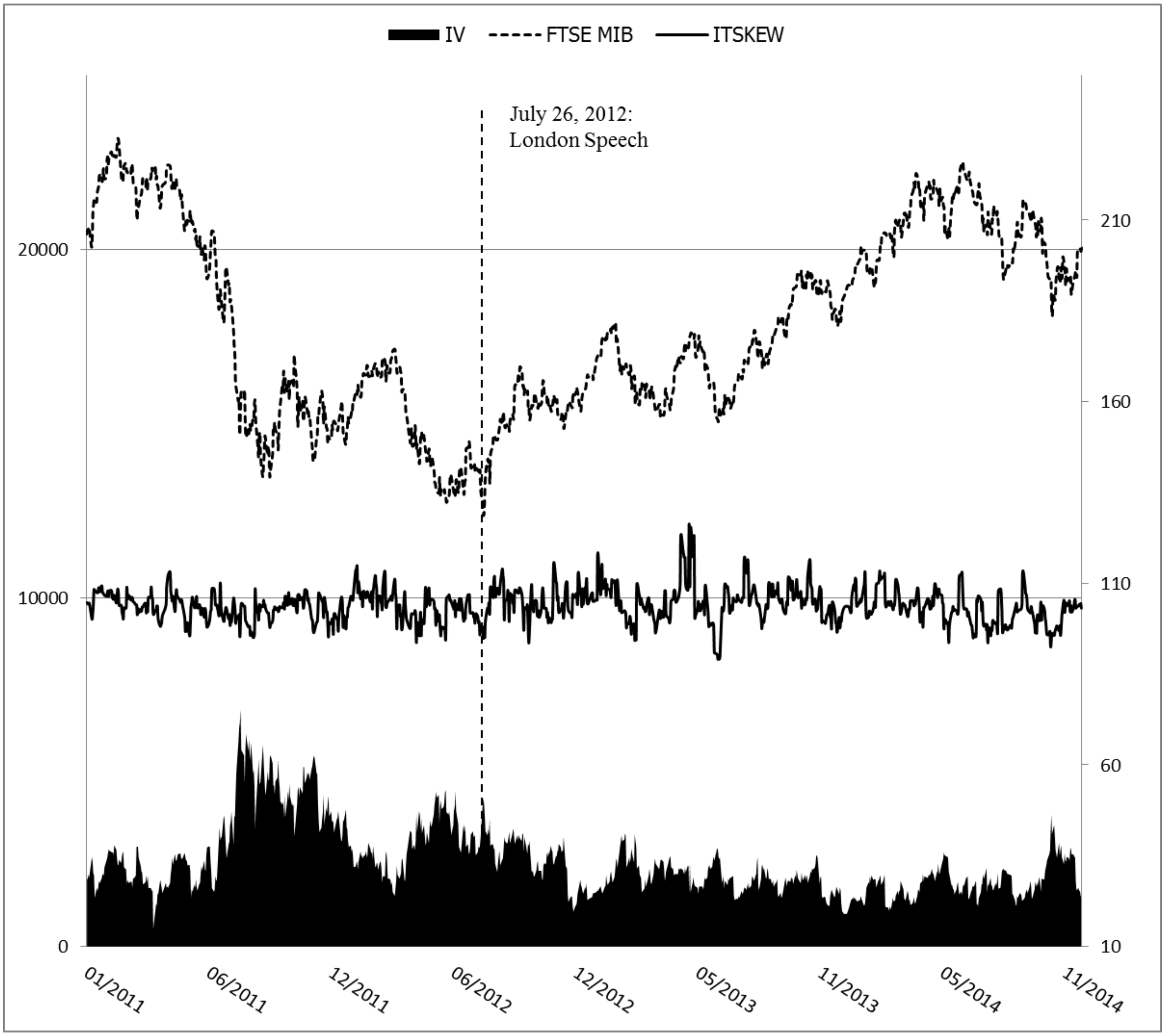


Figure 4 – Comparison between FTSE MIB index, implied volatility index and *ITSKEW* index



Note: FTSE MIB index refers to the values on the left, while implied volatility and *ITSKEW* index refer to the values on the right. Implied volatility values are obtained as the model-free implied volatility multiplied by 100 (VIX methodology).