STOCHASTIC GRADIENT METHODS FOR ENERGY SAVING AND A CORRECT MANAGEMENT IN COMPLEX WATER SUPPLY SYSTEMS

***Jacopo Napolitano* [1*], *Alexei A. Gaivoronski* [2] *& Giovanni M. Sechi* [1]**

(1) DICAAR, Department of Civil and Environmental Engineering and Architecture, University of Cagliari (Italy)
(2) Department of Industrial and Technology Management, NTNU (Trondheim, Norway)

**email: jacopo.napolitano@unica.it*

KEY POINTS

- *Stochastic Gradient Methods*
- *Energy and water supply optimization*
- *Water Pumping Schedules*

1 INTRODUCTION

The management optimization of complex multi-source and multi-demand water resource systems under a high uncertainty level has been a subject of interest in the research literature (*Labadie*, 2004; *Cunha & Sousa*, 2010; *Yuan et al.*, 2016). In this context, energy saving in operation of water pumping plants and reduction of water deficit for users and activities are frequently conflicting issues. Dealing with these problems, the definition of optimal activation rules for emergency activation of pumping stations are a relevant topic recently treated in *Lerma et al.* (2015) and *Napolitano et al.* (2016).

In this study we want to define a trade-off between costs and risks considering the minimization of water shortage damages and the pumping operative costs, under different hydrological scenarios occurrences possibilities. Consequently, optimization results should provide the water system Authorities with a robust information about the optimal activation rules considering a large set of generated scenarios of hydrologic inputs to reservoirs. Using synthetic series it is possible to take into account the climate change impacts and balance the rules while also considering future behavior under the risk of the occurrence of shortages and the cost of early warning procedures to avoid water scarcity, mainly related to activation of emergency water transfers. Thereafter, this problem has been faced considering an efficient optimization tool based on the Stochastic Gradient method (SQG), see *Ermoliev & Wets* (1988) and *Gaivoronski* (2005). Testing the effectiveness of this proposal, an application of the modelling approach has been developed in a water shortage prone area in South-Sardinia (Italy).

2 OPTIMIZATION UNDER UNCERTAINTY AND STOCHASTIC GRADIENT METHODS SOLVING A WATER RESOURCE MANAGEMENT PROBLEM

Problems of water resource management are characterized by a high uncertainty level due to hydrologic variability and water demand behavior. To solve them efficaciously, we need special algorithms, designed for optimization under uncertainty: hence, the Stochastic Gradient method has been used here as an efficient tool in order to minimize uncertainty effects. This approach belongs to a class of methods specifically designed for problems with continuous distributions of random parameters and nonlinear optimization problems. These methods are suited for optimization and simulation models, where an analytical relationship between the objective function and parameters is difficult to obtain. The SQG could be described as the statistical estimate of the gradient of the objective function, providing an optimal direction for iterative updating of the current approximation to the solution of the optimization problem.

These methods solve the stochastic optimization problems of the following type:

$$\underset{x \in X}{Minimize}\, \mathrm{E}_{\omega} f_0(x,\omega) \tag{1}$$

where E_{ω} is an expected value considering $\omega \in \Re^{k}$ as a vector of random parameters and $x \in X \subseteq \Re^{n}$ is a vector of decision variables; where X represents the set of feasible solutions. The optimization algorithm starts from an initial point x^{0} and moves forward to the current approximation x^{s} of the optimal solution of the original problem (1) according to the following rule:

$$x^{s+1} = \pi_{X}(x^{s} - \rho_{s}\xi^{s}) \tag{2}$$

where ρ_{s} represents the size of the step in the direction opposite to the current estimate ξ^{s} of the stochastic gradient at the point x^{s}, while π_{X} is a projector operator.

Specifically, in this paper, using the SQG methods we want to define the optimal set of parameters q that describe the activation pumping rules while minimizing the average monthly costs, which are the sums of all costs supported in the water system management.

The network state v^{t} (water volumes in reservoirs) evolves in discrete time $t = 1,...,T$ (months). At each t, water demands d^{t} and inflows r^{t} arrives. Therefore, the pumping schedules are defined by pumping rules with parameters q and, at each t; the network flows x^{t} are obtained by minimization of costs (3):

$$C^{T}(q,v^{t},d^{t},r^{t}) = \underset{x \in X}{Minimize}\, C\left(x,q,v^{t},d^{t},r^{t}\right) \tag{3}$$

subject to constraints (4) (flow continuity, bounds, etc.).

$$\Phi\left(x,q,v^{t},d^{t},r^{t}\right) = 0 \tag{4}$$

The state v^{t+1} at the beginning of period $t+1$ is obtained from the state equation (5):

$$v^{t+1} = \Psi\left(x^{t},q,v^{t},d^{t},r^{t}\right) \tag{5}$$

where the functions $C(\cdot)$ $\Phi(\cdot)$ and $\Psi(\cdot)$ are linear with respect to (x,v).

The objective is to find the set of parameters $q = (q_{1},...,q_{n})$ that minimizes the average steady state costs, thus solving the optimization problem (3) with an infinite time horizon.

The solution approach resorts to a concurrent interrelation among simulation, optimization and evaluation steps. In the following, we give a description of the concurrent interaction among them.

The simulation process is referred to each single period t of the time horizon and is characterized by n-processes simultaneously. Each process has different sets of pumping activation threshold parameters q according to (6):

$$q^{t} + \delta e_{k} \tag{6}$$

where δ is a small positive value and e_{k} is a vector of zeros with value 1 in the *k*-th position. Here the objective function of costs referred to the single period t (3) is minimized in order to obtain an optimal configuration of the network water flows x^{t}.

The optimization process will be applied between two consecutive periods and the new parameter configuration will be evaluated according to equation (7):

$$q^{t+1} = \prod_{Q}\left(q^{t} - \rho_{t}\xi^{t}\right) \tag{7}$$

where ρ_{t} is the step size and $\prod_{Q}(\cdot)$ is the projection operator on feasible set $q \in Q$.

The *k*-th component of stochastic gradient $\xi^t = (\xi_1^t, \ldots, \xi_n^t)$ will be estimated as $\xi_k^t = \dfrac{C_k^t - C_0^t}{\delta}$.

Simultaneously, there is an estimation step based on a moving average.

$$C_0^{t+1} = (1-\alpha_t)\overline{C}_0^t + \alpha_t C_0^t \tag{8}$$

The estimated costs C_0^{t+1} are dependent on the average cost $\overline{C}_0^t$ referred to all periods, and the costs evaluated in the previous step C_0^t, the relationship between these terms is regulated by an averaging parameter α_t.

3 CASE STUDY: SOUTHERN SARDINIA WATER SUPPLY SYSTEM

In order to test its effectiveness, the SQG approach has been applied to a real case study of the water supply system of south-Sardinia (Italy), which is shown in Figure 1 and widely described in *Napolitano et al.* (2016).

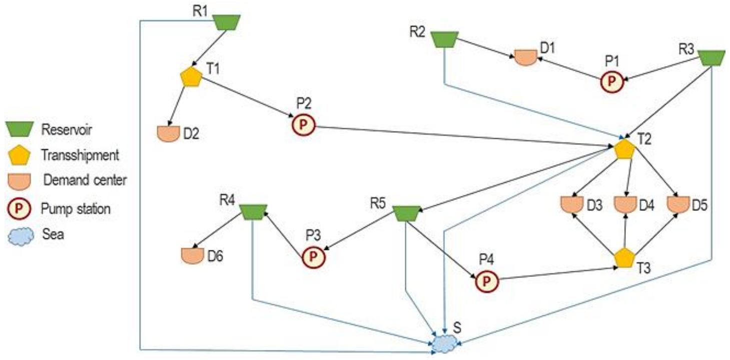

Figure 1. South-Sardinia multi-source water supply system

Synthetic hydrological inflows to reservoirs have been generated starting from this historical database (*RAS*, 2006) by a Monte Carlo procedure. Taking into account this synthetic database, the time horizon for optimization was equal to 6360 monthly periods for each reservoir.

3.1 Recursive simulation and optimization process

The SQG model was implemented in MATLAB, interfaced with CPLEX as optimization engine and with Excel being used for inputs and results representation. The set of optimized activation thresholds for pumping transfers are reported in Table 1 in terms of stored water volumes and volume fractions.

Activation Thresholds	q_1	q_2	q_3	q_4
Water Volumes [10^6 m^3]	0.271	72.480	2.775	21.316
Volume Fractions [-]	0.0201	0.1264	0.3061	0.0109

Table 1. Optimized activation thresholds values

These values have been used in order to evaluate a real economic response of the system: considering only the costs related to unplanned deficits (additional costs supported when demand requests can not be satisfied) and pumping operations.

Annual Average Costs	$C_{Deficit}$	C_{Pump}	C_{Tot}
[10^6 €/year]	0.277	2.578	2.855

Table 2. Economic post-processor

Almost the total costs amount (reported in Table 2) is due to energy contribution (pumping costs) while just a low contribution depends on the unplanned deficit occurrences, which means that almost the totality of system’s users have been satisfied during the referring time horizon. These results highlight a huge reduction in terms of costs supported managing this real water system respect the current configuration, saving more than 1 million of euros.

3.2 Sensitivity analysis

A sensitivity analysis has been performed in order to estimate the additional cost that can be expected if the management Authority modifies the proposed optimal activation rules reported in Table 1. Consequently, an additional simulation process has been implemented varying the volume fraction q_p between theoretical lower and upper bounds 0 and 1, for each pump $p \in P$. During this process, the volume fractions for all other pumps will be kept constant and equal to the optimized values.

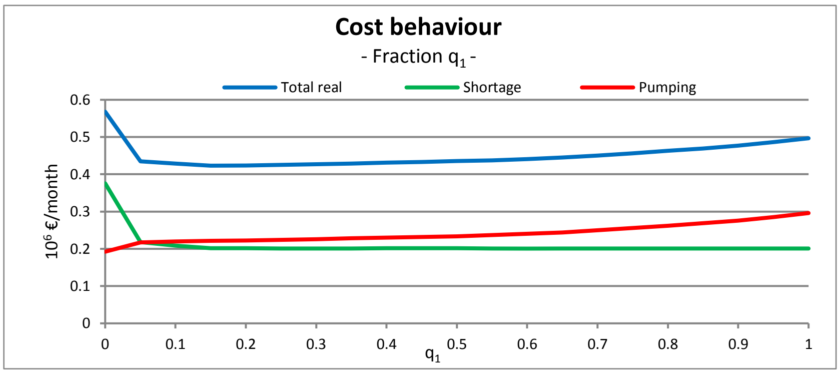

Figure 2. Real costs behaviour varying the pumping rules – Pump station P_1

The behaviour of costs is reported in Figure 2, where the blue line shows the dependence of the total costs on the changing parameter of pump activation rules. The total cost function has the general tendency to increase smoothly with increasing volume fraction values, compared to the optimal one. Instead, this increasing behaviour grow up much faster if we use a lower volume fraction compared to the optimal one. This increasing behaviour is important and highlights a criticality if the authority should decide to use a lower volume fraction values for the activation of pumps.

4 CONCLUSIONS

The SQG approach restricts the risks of deficit for the users and minimizes the costs of managing the system in the conditions of water shortage. The recursive simulation and optimisation process based on the SQG method confirms its potential when applied to the water system management. This methodology allows to consider substantially large and complex models and to provide optimal solutions under large uncertainty described by a significant number of synthetic scenarios.

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