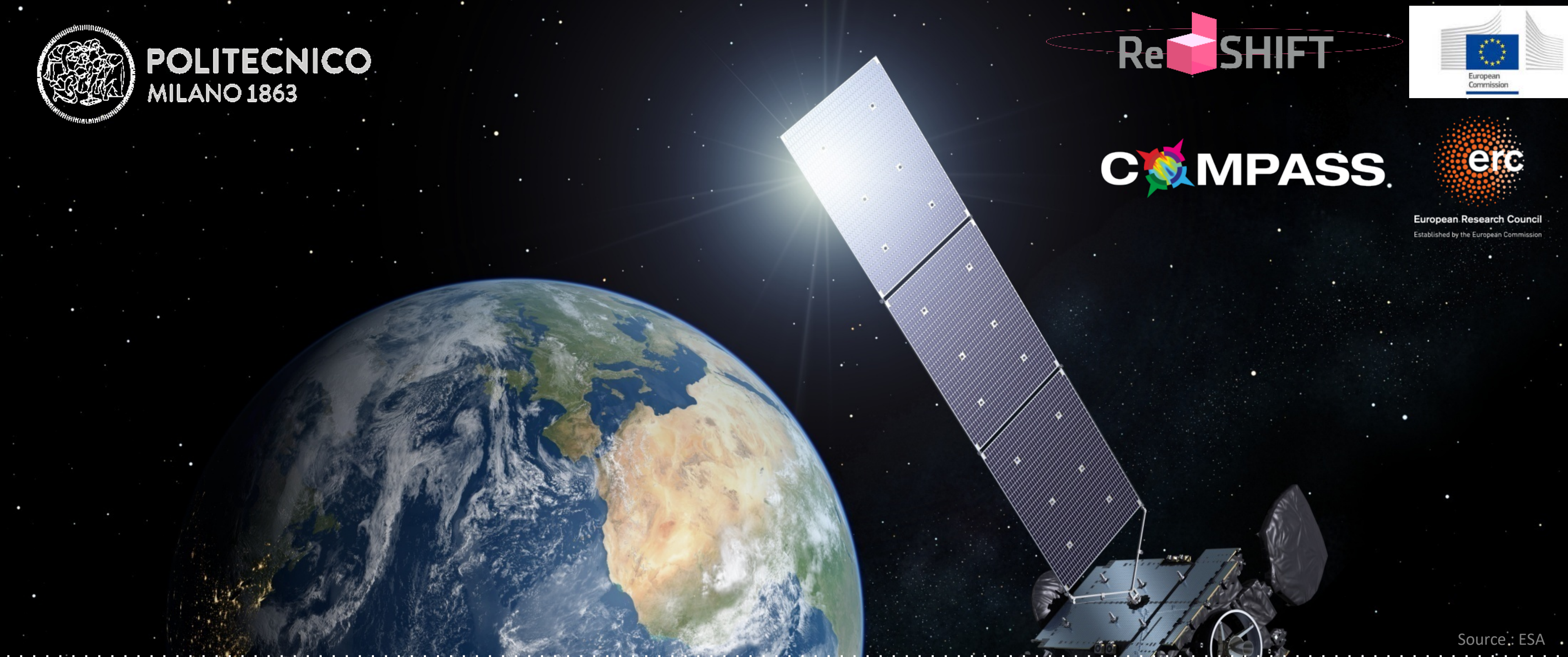

Towards a sustainable exploitation of the geosynchronous orbital region

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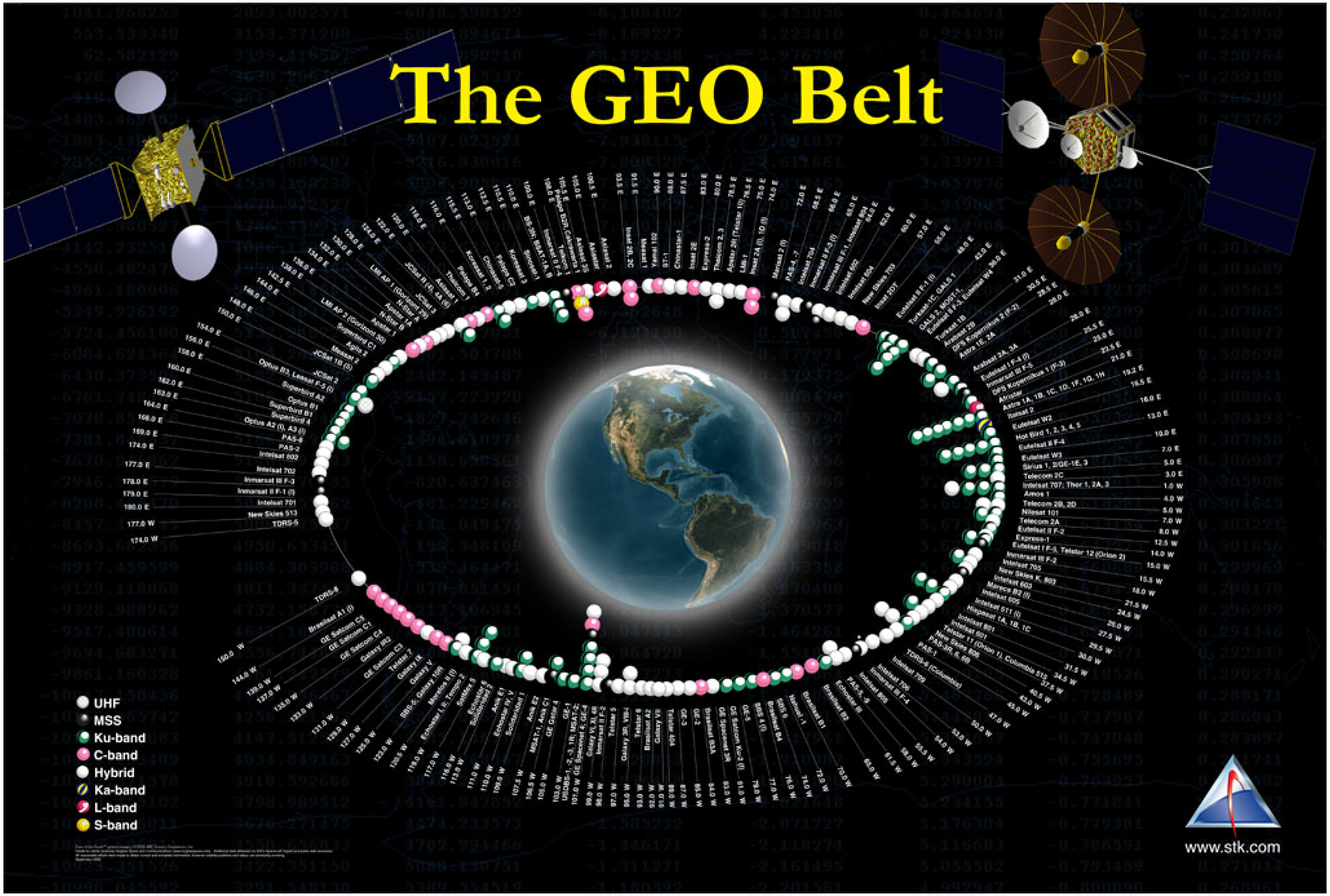
University of Arizona, August 2018

Introduction

Geostationary belt

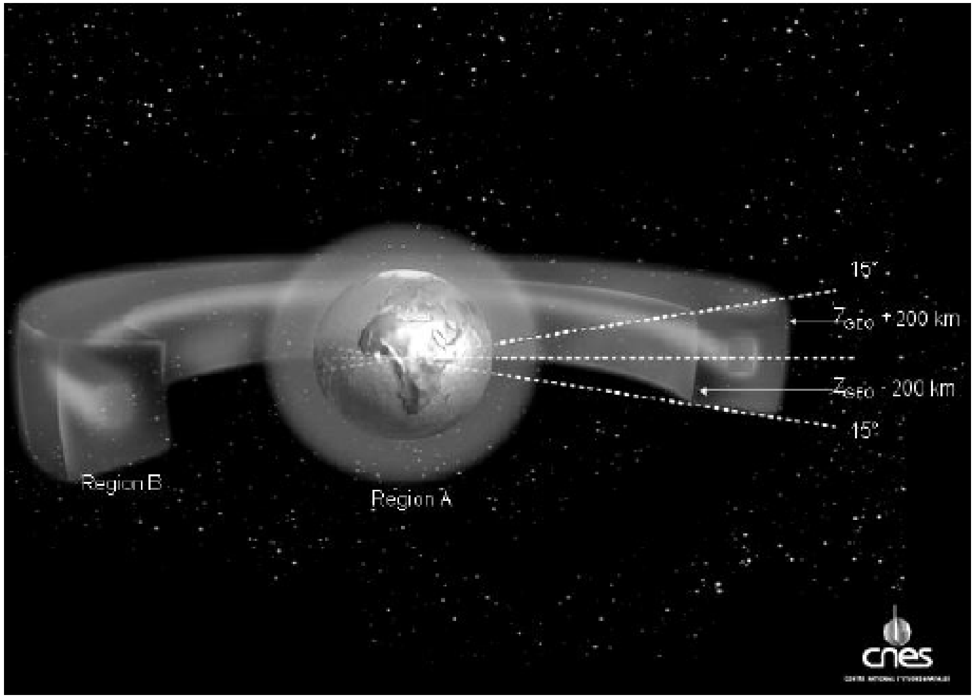


Introduction

Current guidelines

Conformance with the GEO disposal requirement can be ensured by using a disposal orbit with the following characteristics:

- Eccentricity ≤ 0.005,
- Min perigee altitude above the GEO altitude $\Delta h_p \geq 235+1000\ c_R\ A/m$



GEO protected region (GEO region): segment of spherical shell

- lower altitude boundary = geostationary altitude minus 200 km,
- upper altitude boundary = geostationary altitude plus 200 km,
- latitude sector: 15 degrees South ≤ latitude ≤ 15 degrees North

Introduction

Why revisit GEO disposal?

- Many people believe that the debris situation in GEO is shorted out, but is it really and in which timescale?
- Population models predict on average 1 GEO collision in the next 100 years.
- Satellites in graveyard orbits act as debris sources, even without collisions (e.g. HARM GEO population).
- From planetary defence point of view, if we keep the same rate of populating GEO, we will detectable by an equivalently advanced civilization by the year 2200.

Questions:

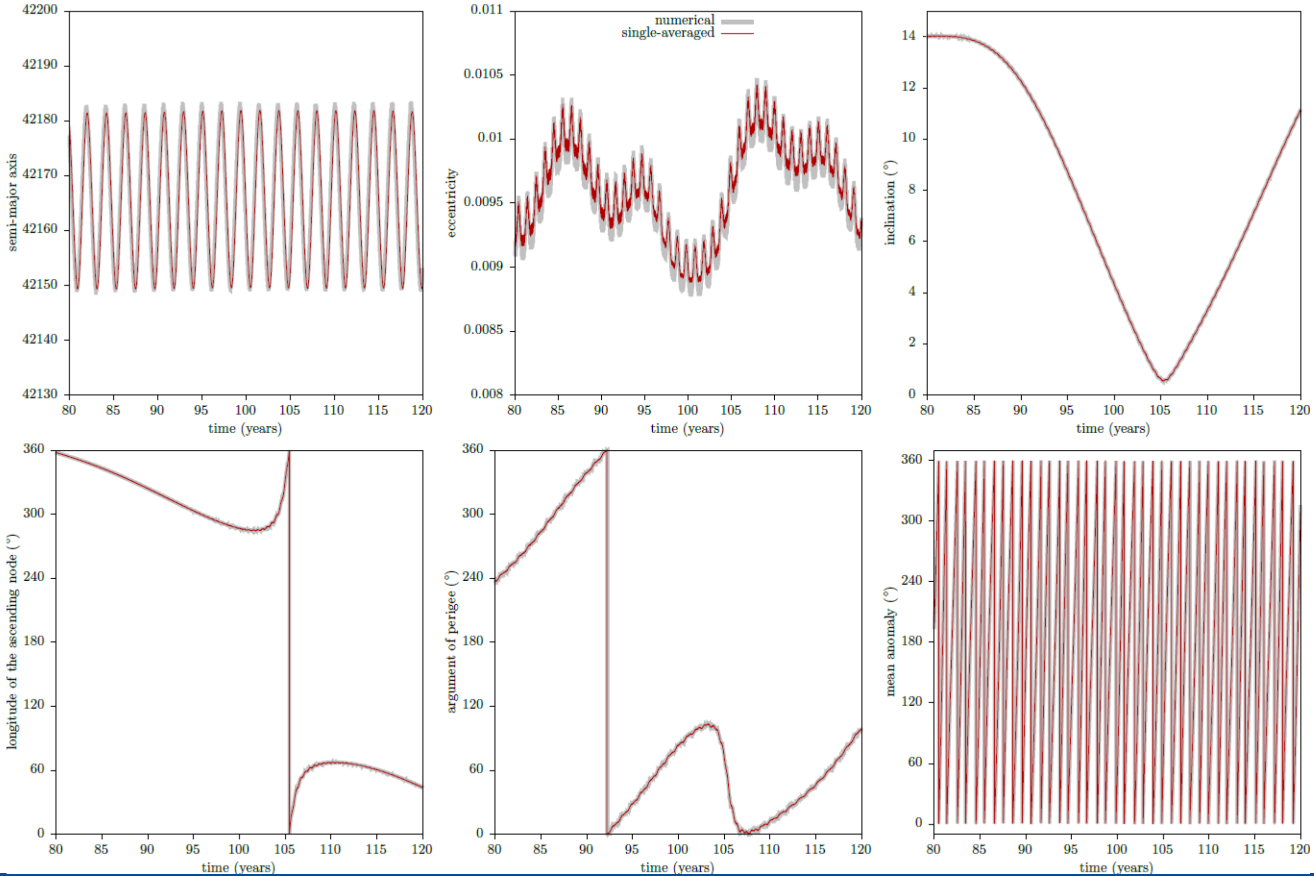
- Are current guidelines enough to ensure long-term GEO sustainability?
- Are there alternative ways to exploit the geosynchronous orbital region?

GEO DYNAMICAL MAPPING

Semi-analytical modelling

PlanODyn (semi-analytical orbit propagation)

Force model: 4x4 geopotential, 3rd body perturbations (up to 5th order in the parallax factor), solar-radiation pressure, Earth's precession



Grid definitions

Orbit propagation for 120 years

- Tesseral Maps

  Main grid: a - λ (201x201) > 4 Million

  Parameters: e, i, A/m (5x11x2)
- Disposal Maps

  Main grid: ω - Ω (201x201) > 36 Million

  Parameters: e , i, A/m (5x91x2)
- Action Maps

  Main grid: e - i (201x201) > 12 Million

  Parameters : a, (Ω, ω), A/m (3x50x2)

Orbits propagated > 50 Million

Dynamical indicators: $Diam(e) = |e_{max} - e_{min}|$

$$\Delta e = \frac{|e_{max} - e_0|}{|e_{re-entry} - e_0|}$$

$\Delta e \rightarrow 0$ Bounded

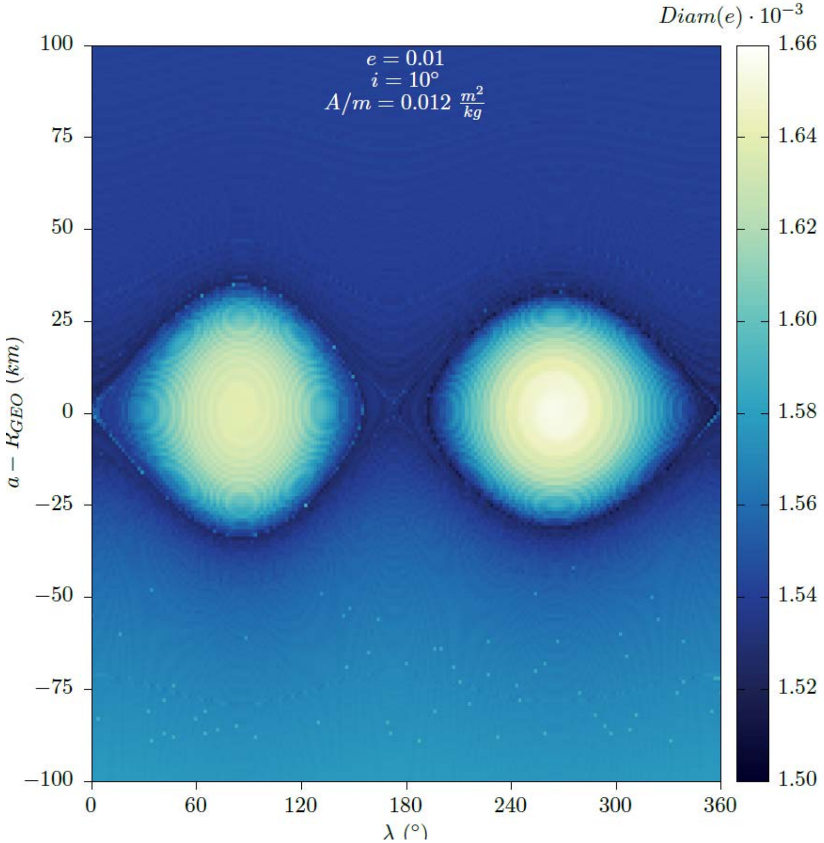
$\Delta e \rightarrow 1$ Re-entry

Tesseral maps

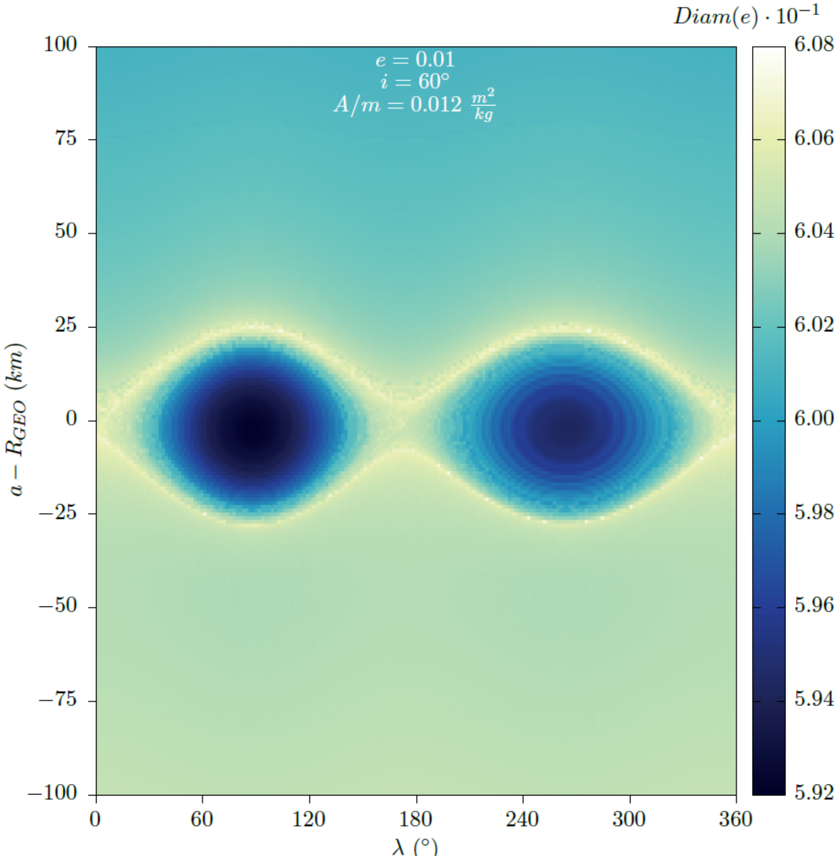
Standard s/c, initial circular orbit

$$A/m = 0.012\ m^2/kg$$
$$e_0 = 0.01$$

low inclination $i_0 = 10^\circ$



high inclination $i_0 = 60^\circ$

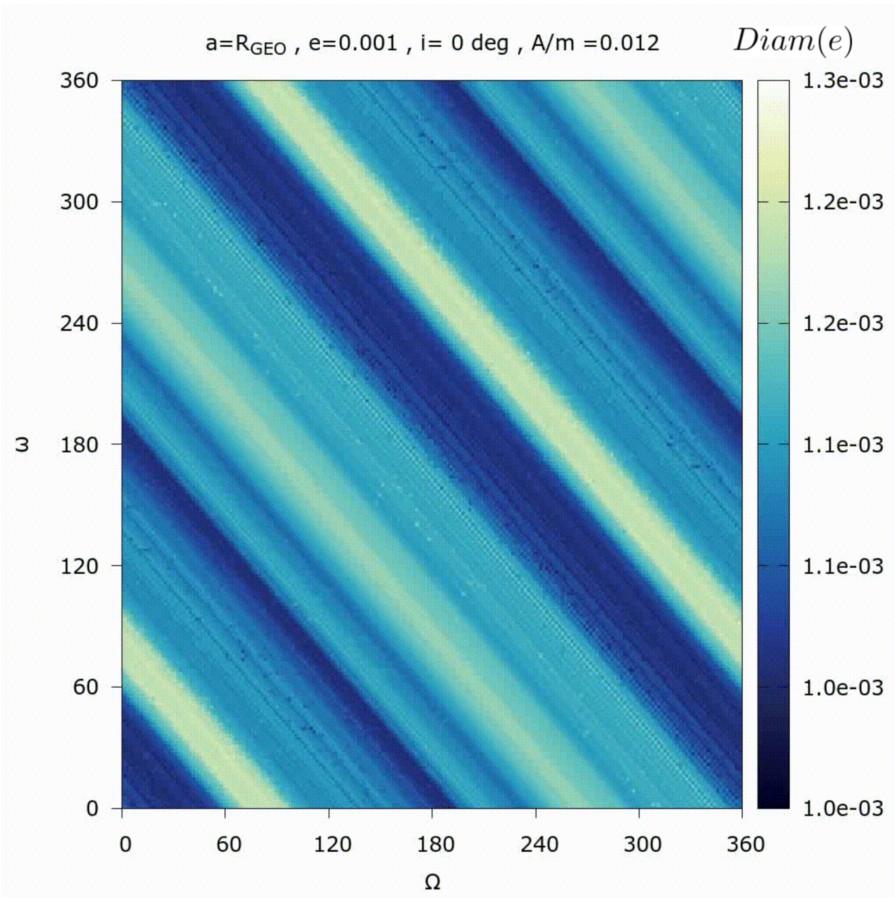


Disposal maps

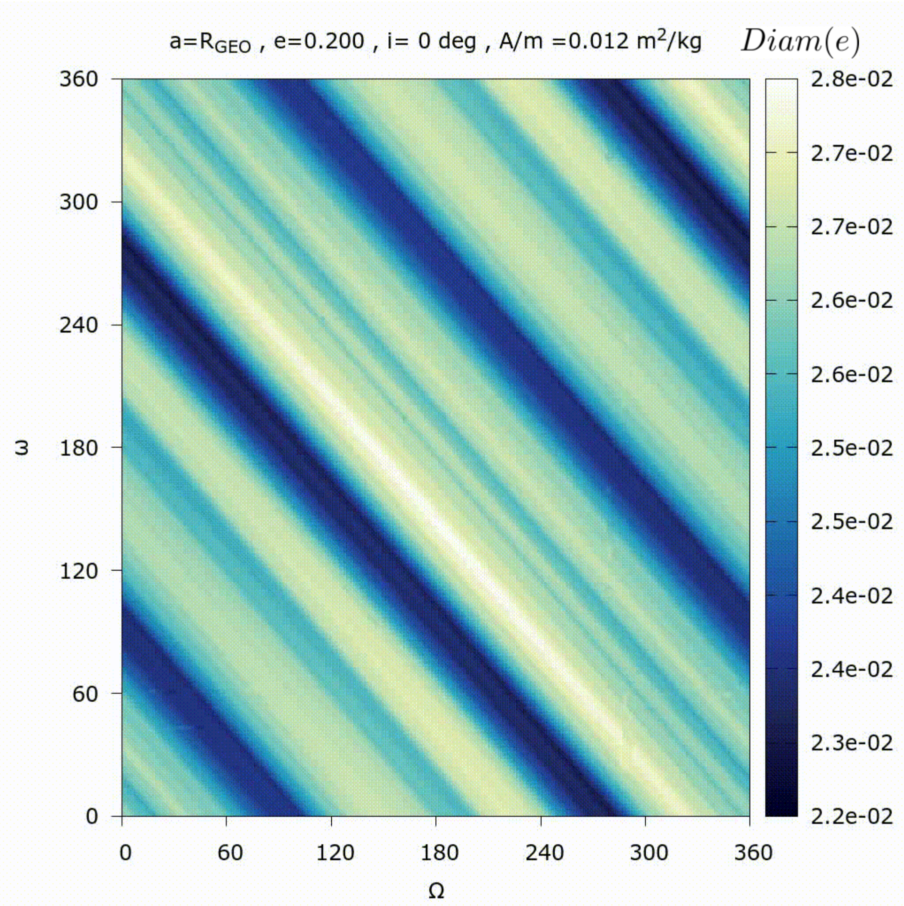
Standard s/c

$$A/m = 0.012\ m^2/kg$$

$$e_0 = 0.001$$

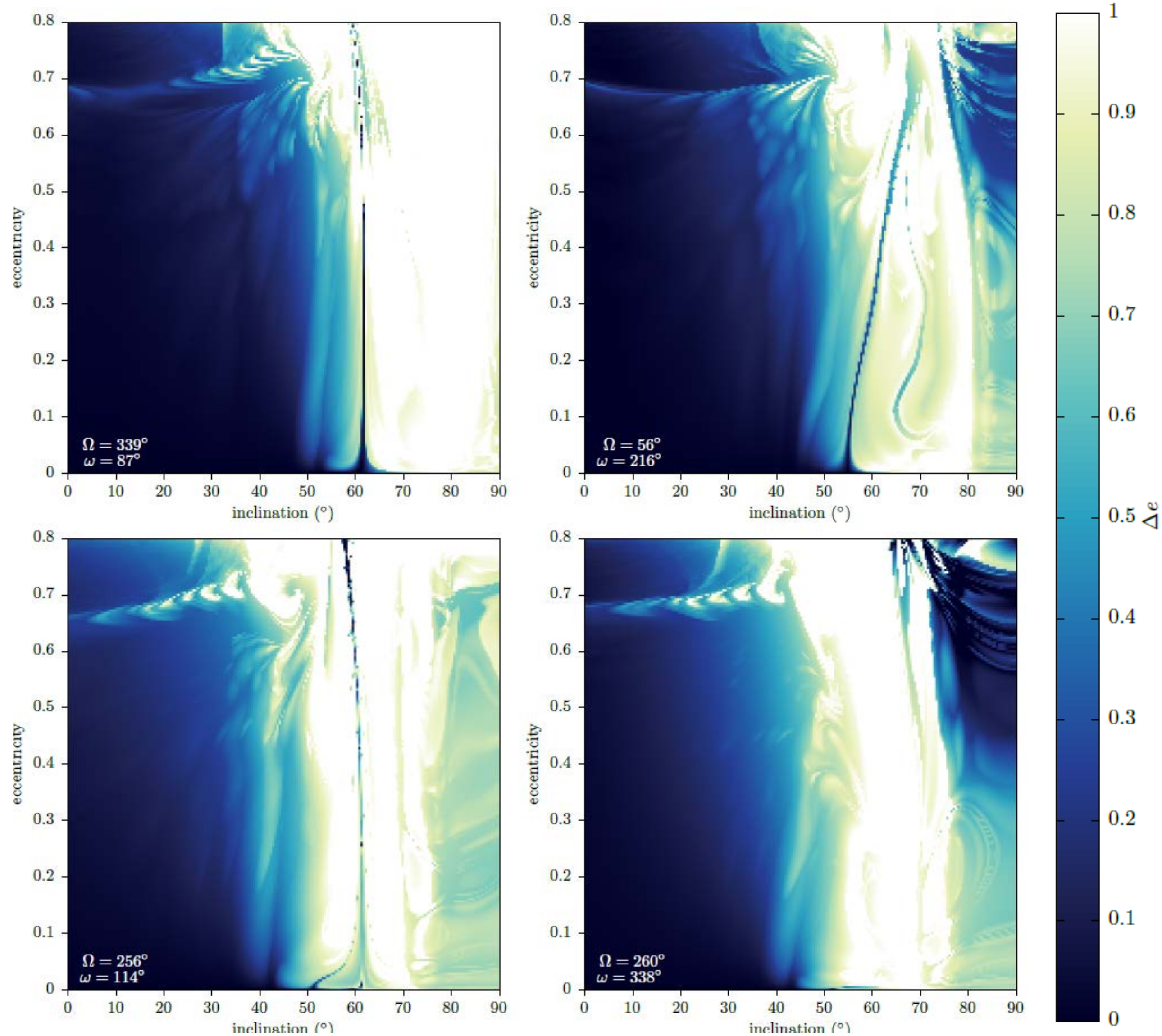


$$e_0 = 0.2$$



Eccentricity-inclination space

$a = R_{GEO}$



$\Delta e \to 1$

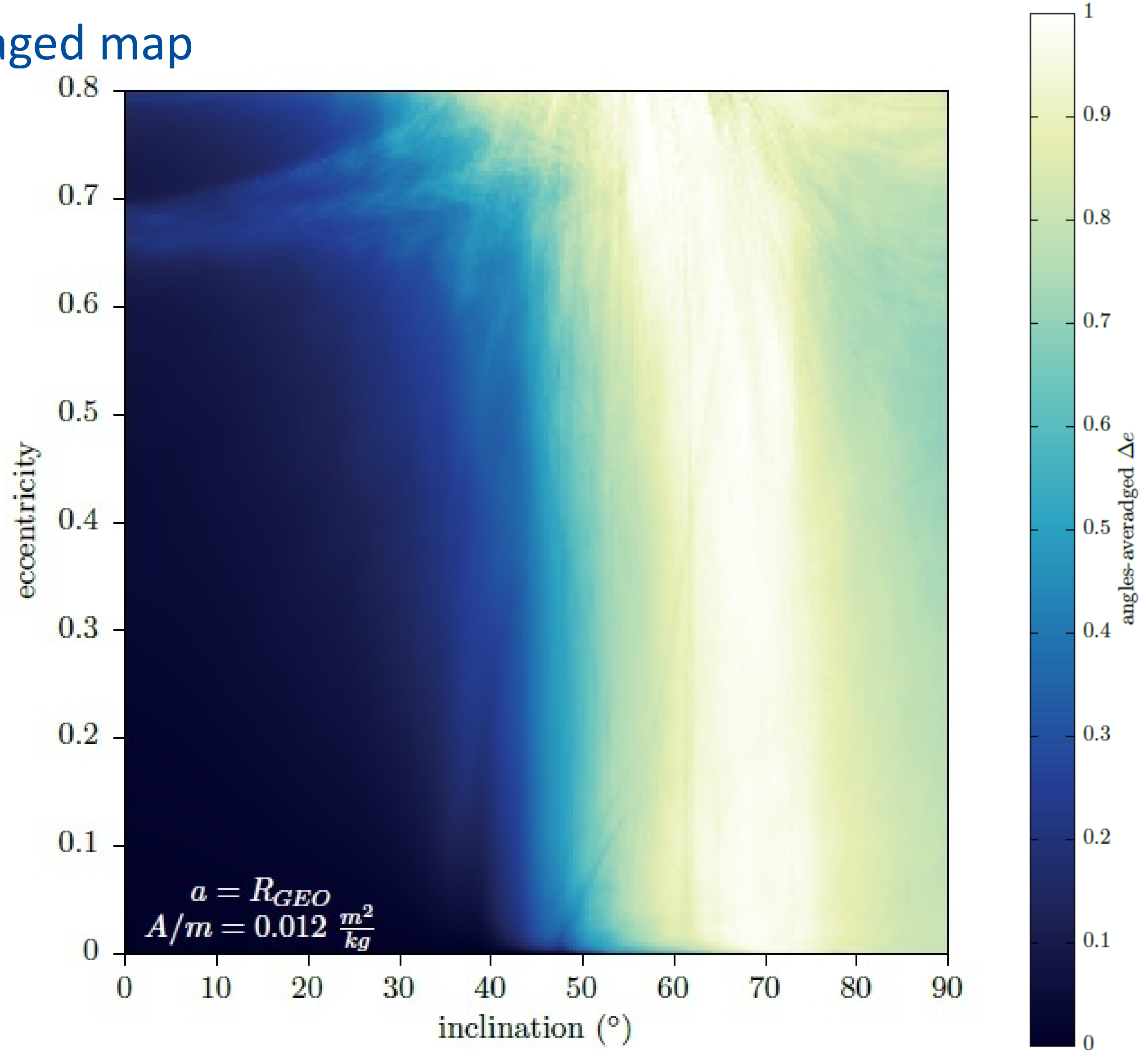
Re-entry

$\Delta e \to 0$

Bounded

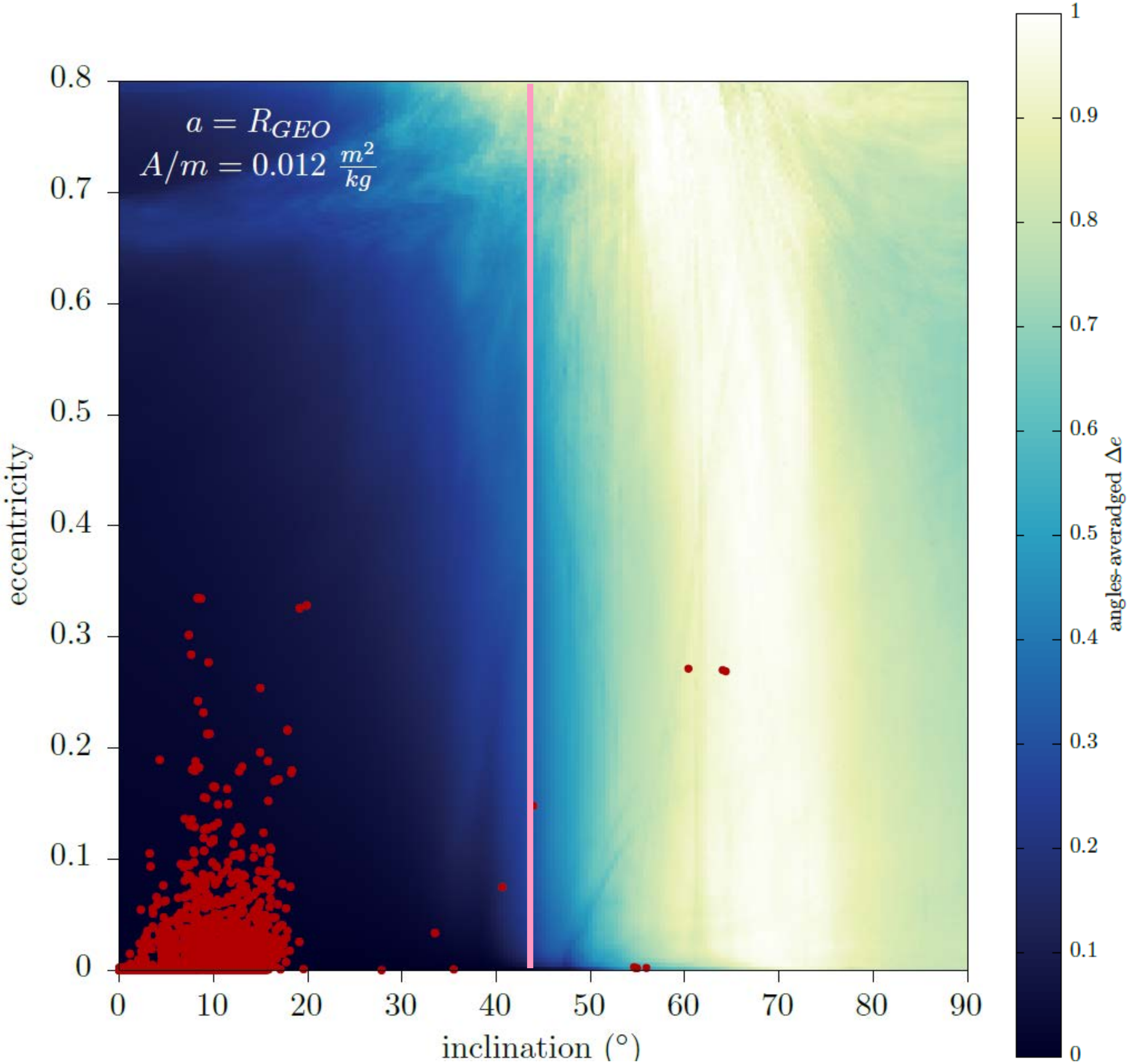
Action space

Angle-averaged map



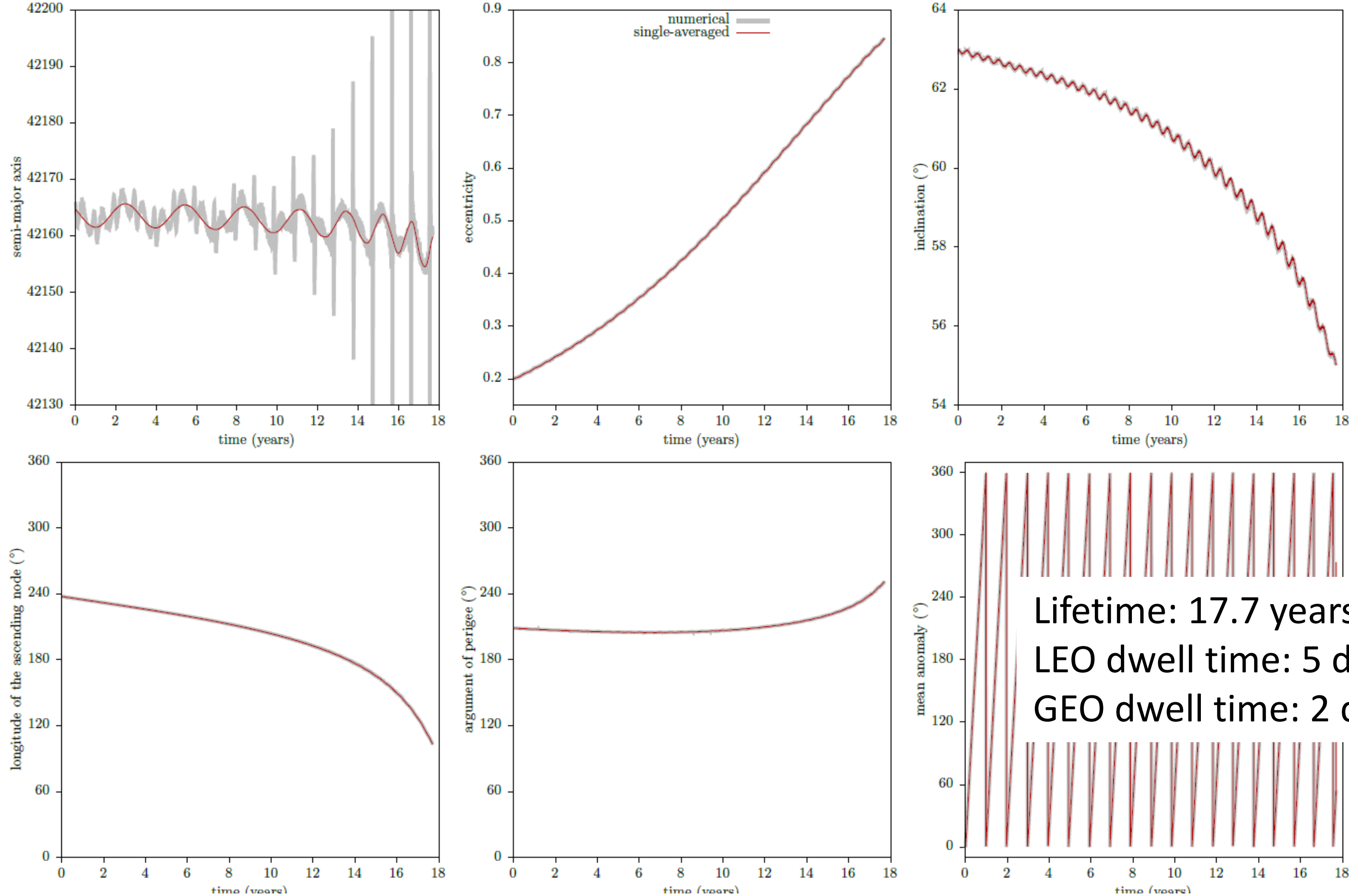
DISPOSAL ISSUES

Population and dynamics



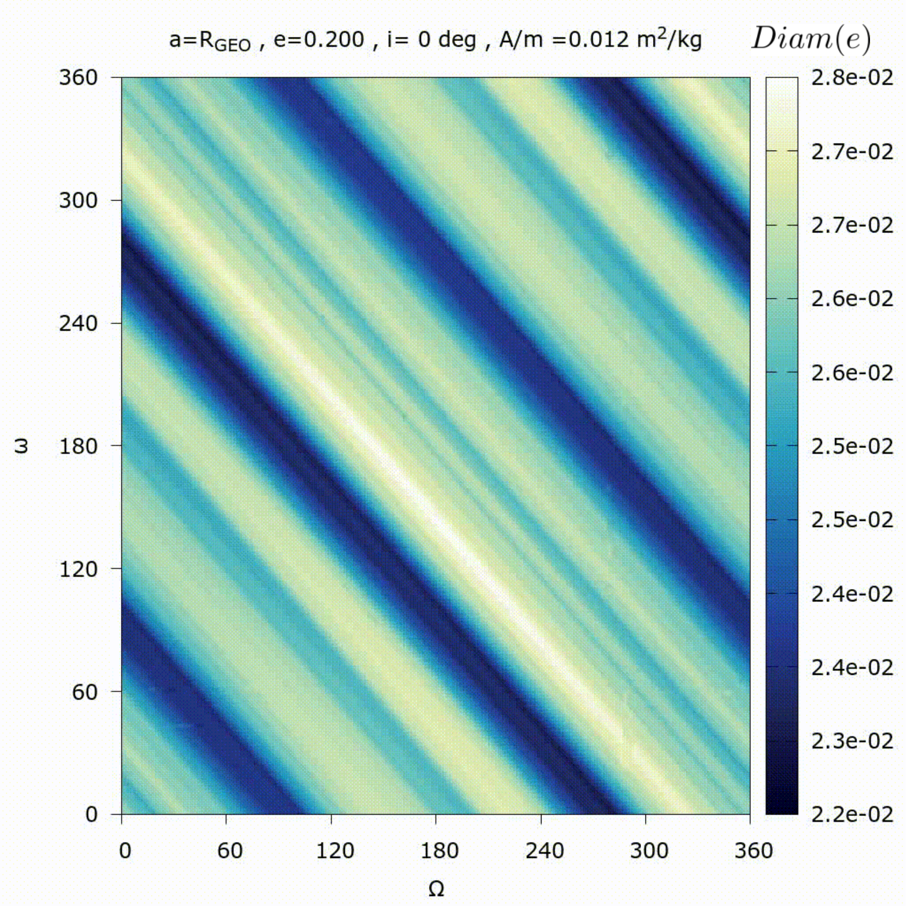
Effective cleansing mechanism

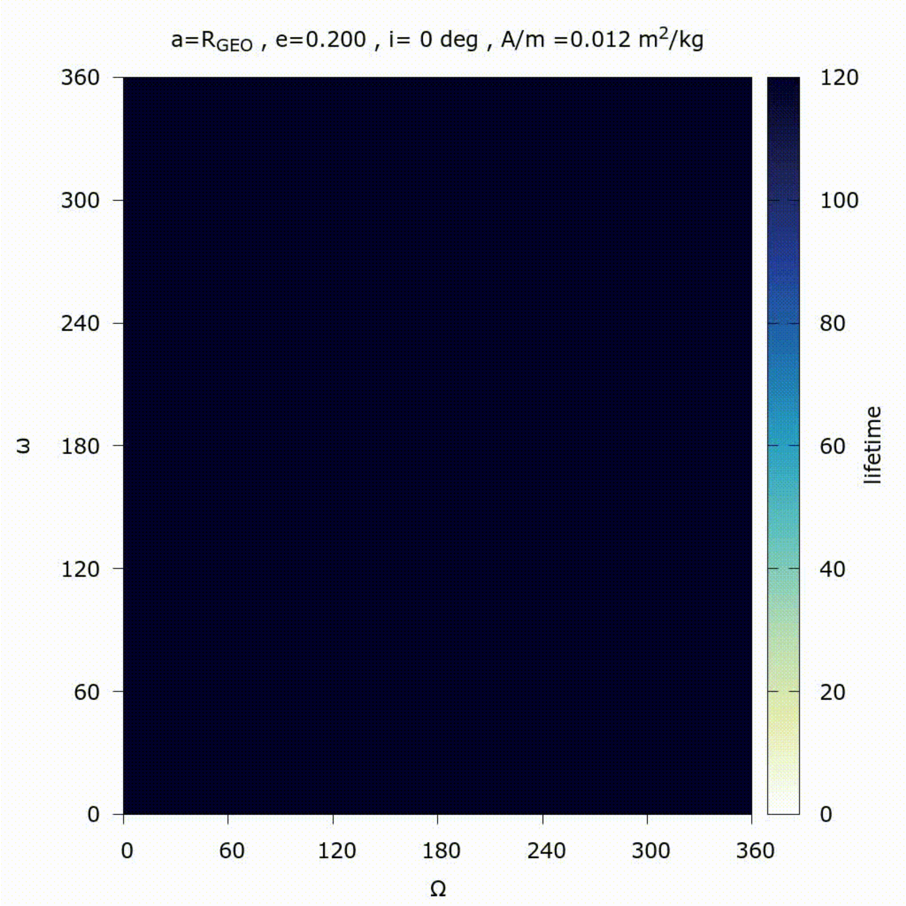
Fast re-entering orbits



Lifetime: 17.7 years!!!
LEO dwell time: 5 days
GEO dwell time: 2 days

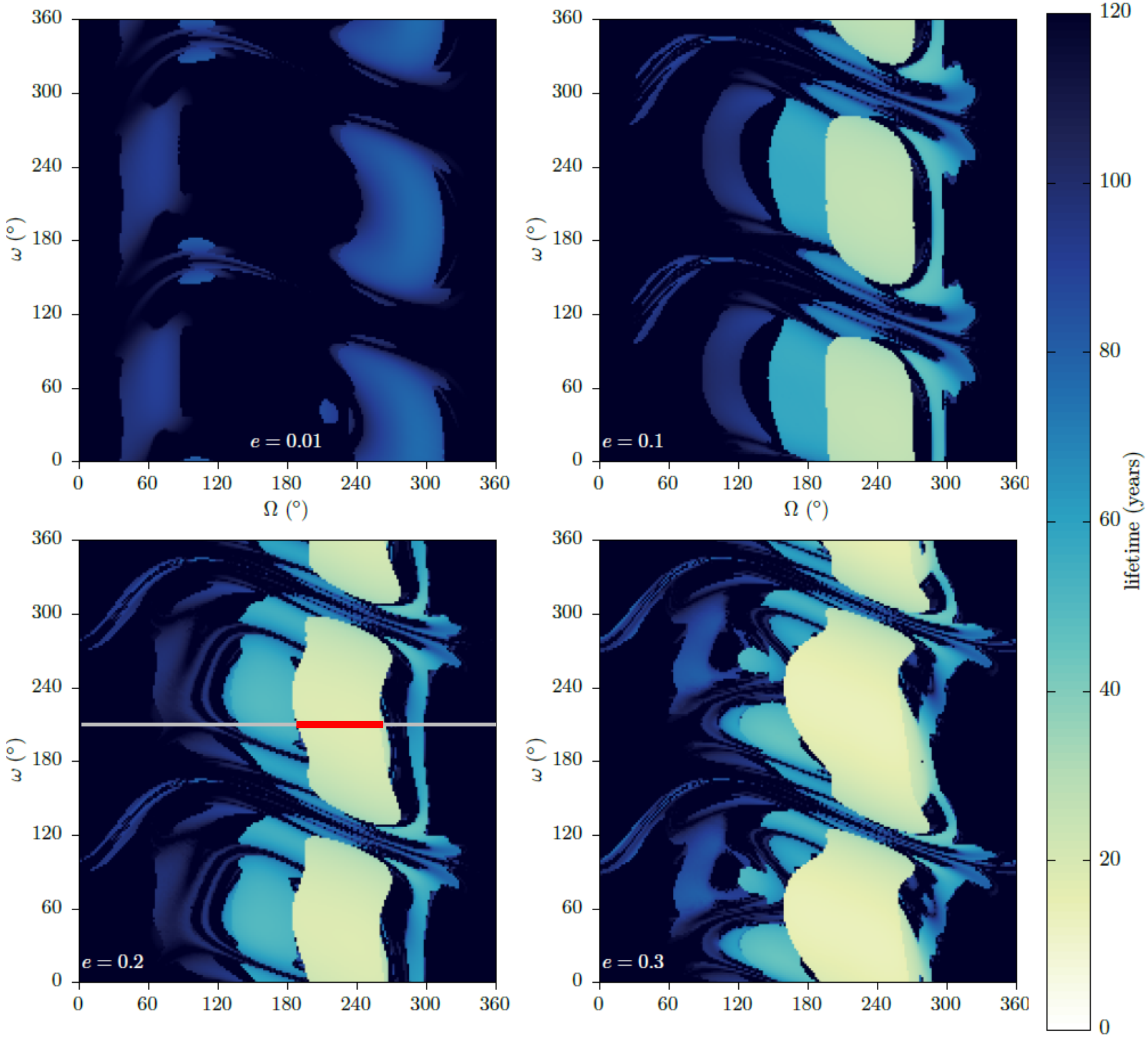
Effective cleansing mechanism



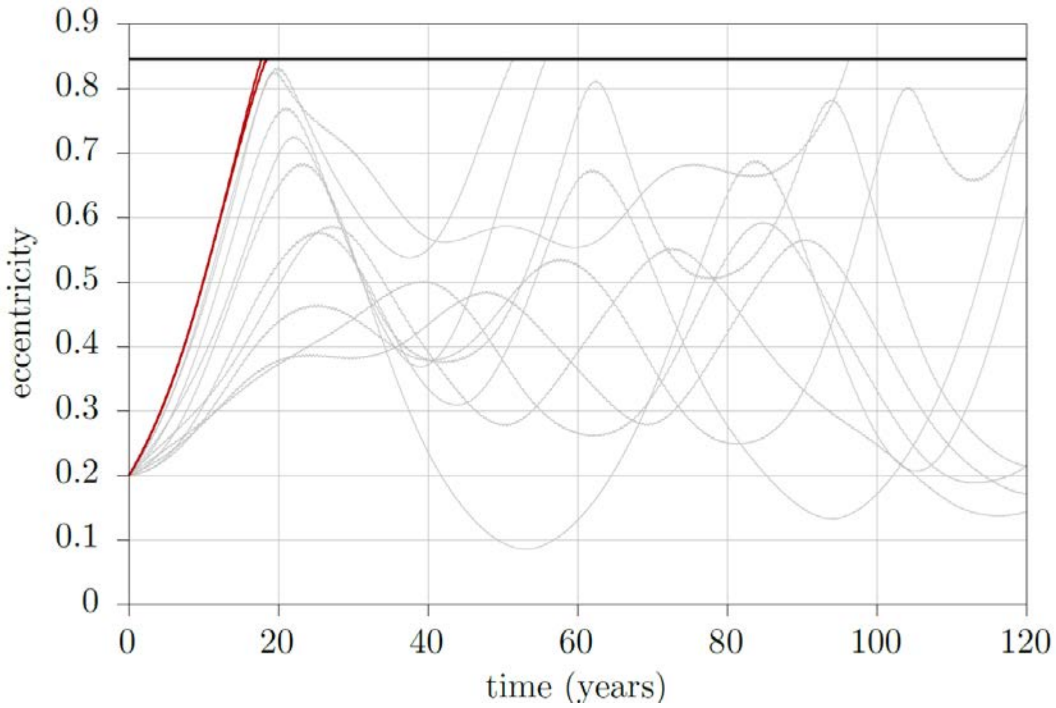


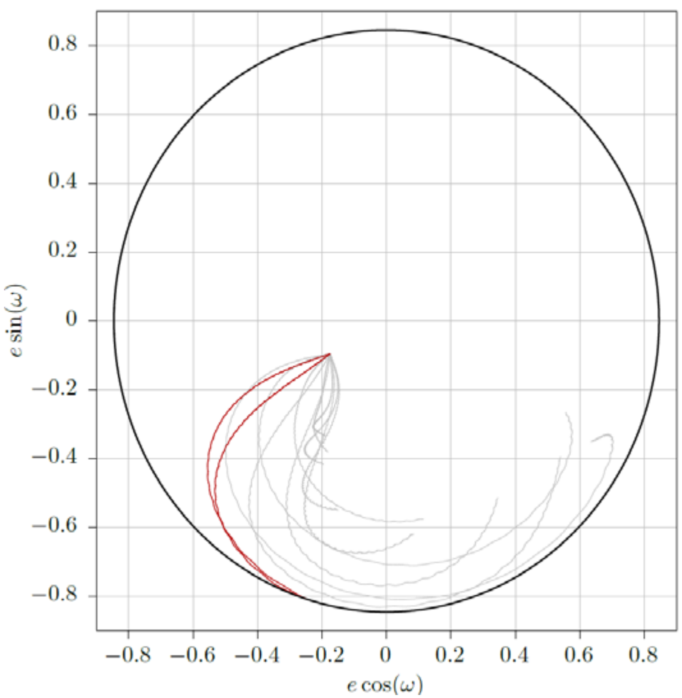
Effective cleansing mechanism

$a = R_{GEO}$
$i = 63^\circ$



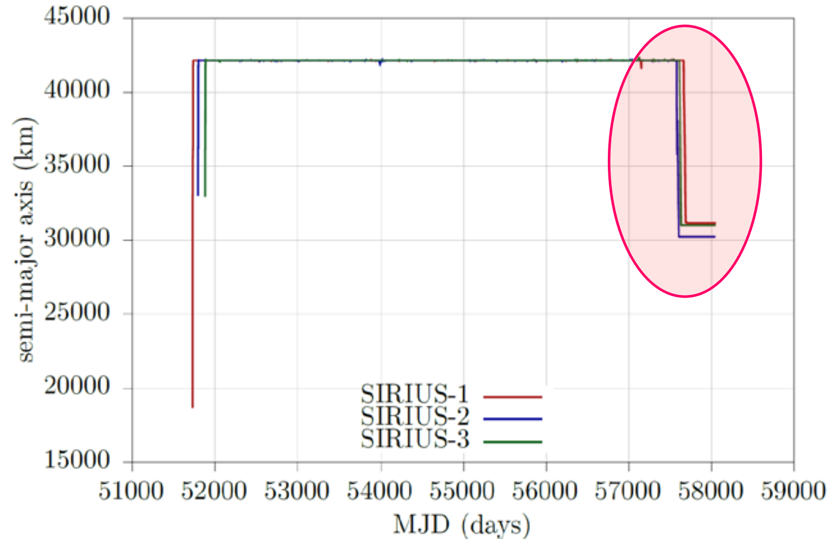
Effective cleansing mechanism

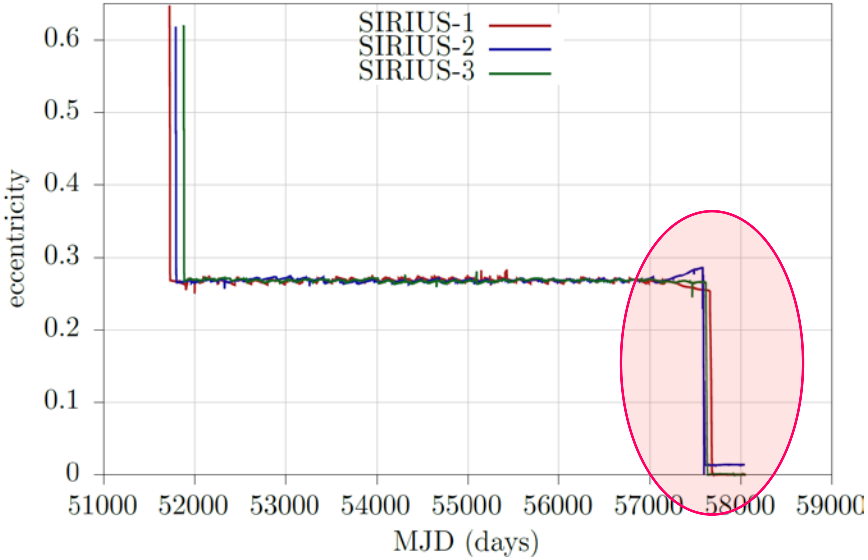


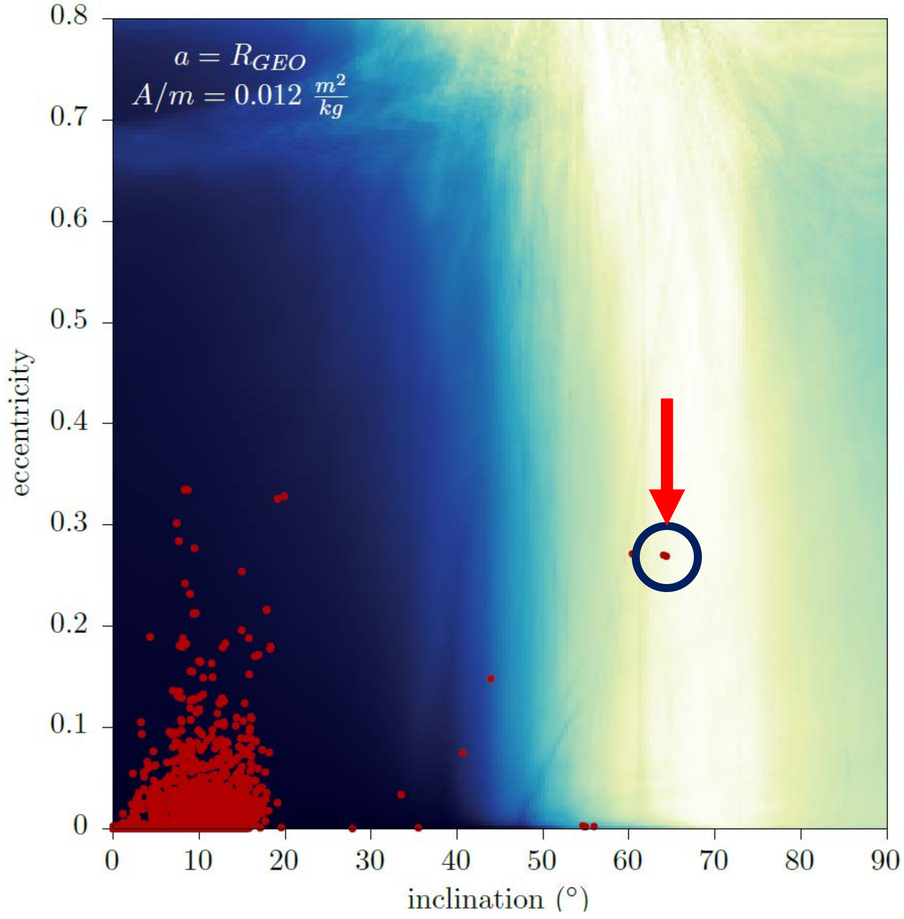


The Sirius constellation

“Missed” opportunity?







ANALYTICAL MODELING

Motivation

Hamiltonian reduction on the ecliptic

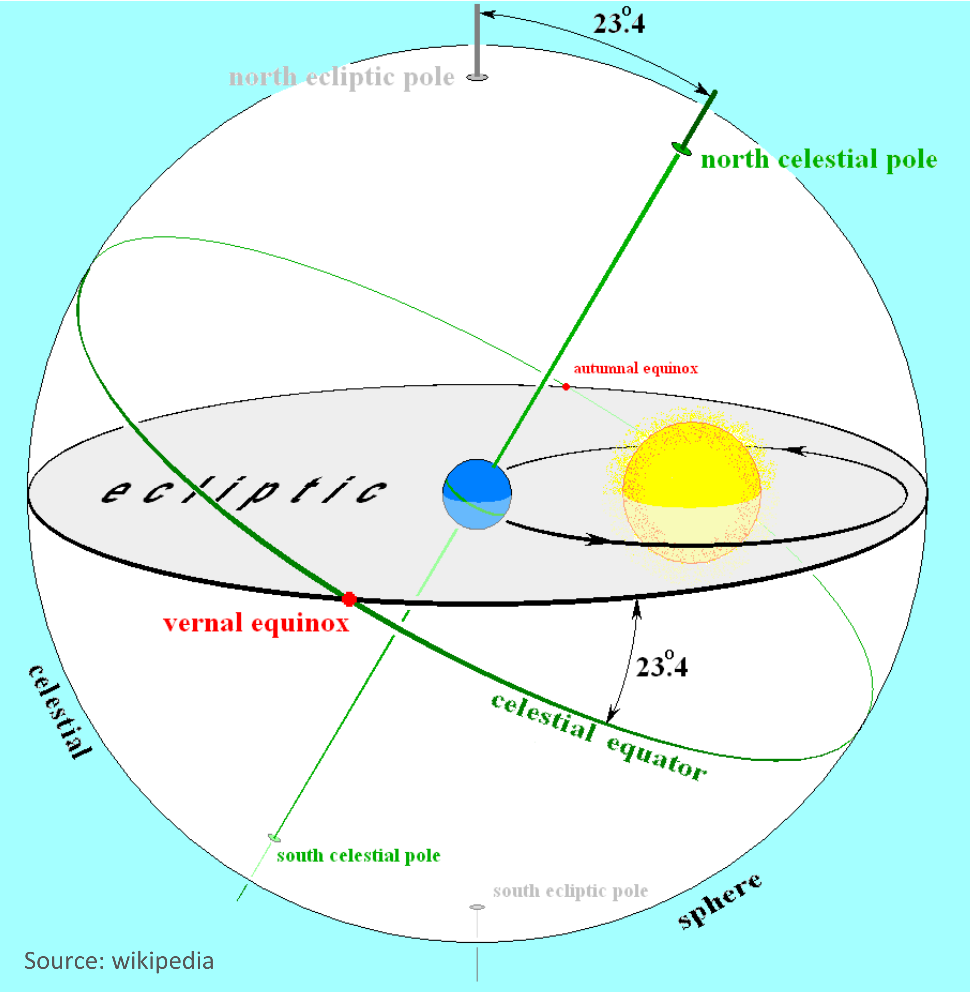
- Artificial satellite theories are developed in a coordinate frame that has the equator as the main plane.
- Geopotential is more conveniently expressed in this frame.
- Third body perturbations more conveniently expressed in the ecliptic.

Question

- Could an analytical theory developed on the ecliptic provide us with more insight for distant Earth satellite orbits?

Analytical modelling

Equatorial and Ecliptic frames



Source: wikipedia

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_1(-\epsilon) \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

Nonlinear relationship between ecliptic and equatorial inclinations

$$\cos I_Q = \cos\varepsilon \cos I - \sin\varepsilon \sin I \cos\Omega$$

$$\cos I = \cos\varepsilon \cos I_Q + \sin\varepsilon \sin I_Q \cos\Omega_Q$$

Analytical modelling

Body positions

equatorial frame

- Satellite's position:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} x_☾ \\ y_☾ \\ z_☾ \end{pmatrix} = R_1(-\epsilon)R_3(-\Omega_☾)R_1(-i_☾)R_3(-\theta_☾)\begin{pmatrix} r_☾ \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} x_\odot \\ y_\odot \\ z_\odot \end{pmatrix} = R_1(-\epsilon)R_3(-\theta_\odot)\begin{pmatrix} r_\odot \\ 0 \\ 0 \end{pmatrix}$$

ecliptic frame

- Satellite's position:

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} \xi_☾ \\ \eta_☾ \\ \zeta_☾ \end{pmatrix} = R_3(-\Omega_☾)R_1(-i_☾)R_3(-\theta_☾)\begin{pmatrix} r_☾ \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} \xi_\odot \\ \eta_\odot \\ \zeta_\odot \end{pmatrix} = R_3(-\theta_\odot)\begin{pmatrix} r_\odot \\ 0 \\ 0 \end{pmatrix}$$

Analytical modelling

Model formulation

The orbit of a massless Earth's satellite in high orbit (no drag) can be modelled as a perturbed Keplerian motion

$$\mathcal{H} = H_{\mathrm{kep}} + H_{\mathsf{zonal}} + H_{\mathrm{third\text{-}body}}$$

- Keplerian part:

$$H_{\mathrm{kep}} = -\frac{\mu}{2a}$$

- Zonal Harmonics:

$$H_{\mathsf{zonal}} = -\frac{\mu}{r}\sum_{j\geq 2}\left(\frac{R_{\oplus}}{r}\right)^{j} C_{j,0}P_{j,0}(\sin\phi)$$

- Third-body attraction (Sun and Moon):

$$H_{\mathrm{third\text{-}body}} = -\frac{\mu'}{r'}\left(\frac{r'}{||\mathbf{r}-\mathbf{r}'||} - \frac{\mathbf{r}\cdot\mathbf{r}'}{r'^2}\right)$$

Analytical modelling

Averaged potential in the quadrupolar approximation

Reduction of the J_2 part of the Hamiltonian $$H_{J_2} = \frac{\mu}{r}\left(\frac{R_\oplus}{r}\right)^2 J_2 P_2(\sin\phi)$$

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, -, -, -; \mu, J_2, R_\oplus)$$

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, \Omega, -, -; \mu, J_2, R_\oplus, \epsilon)$$

Reduction of the Sun's perturbing effect $$H_\odot = -\frac{n_\odot a_\odot^3}{r_\odot}\left(\frac{r}{r_\odot}\right)^2 P_2(cos\psi_\odot)$$

$$\bar{H}_\odot = \bar{H}_\odot(a, e, i, \Omega, \omega, -, \theta_\odot; n_\odot, a_\odot)$$

Reduction of the Moon's perturbing effect $$H_☾ = -\beta\frac{n_☾ a_☾^3}{r_☾}\left(\frac{r}{r_☾}\right)^2 P_2(cos\psi_☾)$$

$$\bar{\bar{H}}_☾ = \bar{\bar{H}}_☾(a, e, i, \Omega, \omega, -, \Omega_☾, -; \beta, n_☾, a_☾, i_☾, \epsilon, \eta_☾)$$

Analytical modelling

Advantage of the ecliptic frame

The full system is

$$\bar{\bar{H}} = \bar{H} + \bar{H}_{\odot} + \bar{\bar{H}}_{☾}$$

and is still of **2.5** degrees of freedom

$$\bar{\bar{H}} = \bar{\bar{H}}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{\odot}; \mu, J_2, R_{\oplus}, \epsilon, n_{\odot}, a_{\odot}, n_{☾}, a_{☾}, \eta_{☾})$$

HOWEVER

In the ecliptic representation time dependencies are always coupled with the ecliptic node of the satellite.

Analytical modelling

Further ecliptic reduction

Therefore, we can proceed with a further **elimination of the ecliptic node**. This is accomplished by working in a suitable **rotating frame** and is a valid operation when the perturbations are **of the same order**, i.e. for **distant** Earth's satellites.

$$\bar{\bar{H}}_{J_2} = \frac{J_2 R_\oplus^2 \mu (3\cos^2 i - 1)(3\sin^2\epsilon - 2)}{8 a^3 \eta^3}$$

$$\bar{\bar{H}}_\odot = a^2 n_\odot^2 \left(-\frac{15}{16} e^2 \cos 2\omega \sin^2 i + \frac{1}{16}(2+3e^2)(3\sin^2 i - 2) \right)$$

$$\bar{\bar{H}}_{☾} = -\frac{a^2 n_{☾}^2\, \beta (3\cos^2 i_{☾} - 1)((2+3e^2)(3\cos^2 i - 1) + 15e^2 \sin^2 i \cos 2\omega)}{32 \eta_{☾}^2}$$

Analytical modelling

Lidov-Kozai type Hamiltonian

The reduction on the ecliptic results in a **1 D.O.F** Lidov-Kozai type Hamiltonian

$$\bar{\bar{\bar{H}}} = \frac{A}{\eta^3}(2 - 3\sin^2 i) + B((2 + 3e^2)(2 - 3\sin^2 i) + 15e^2 \sin^2 i \cos 2\omega)$$

where

$$A = -\frac{J_2 R_\oplus^2 \mu}{8a^3}(2 - 3\sin^2 \epsilon)$$

and

$$B = -\frac{1}{16}\left(n_\odot^2 + \frac{n_{☾}^2}{\eta_{☾}}\beta\frac{3\cos^2 i_{☾} - 1}{2}\right)a^2$$

The system no longer depends on M and Ω, therefore the semi-major axis a is constant and

$$\sqrt{1 - e^2}\cos i = \text{constant}$$

Analytical modelling

Study of the reduced model

We introduce the non-singular elements

$$k = e\cos\omega, \ \ h = e\sin\omega$$

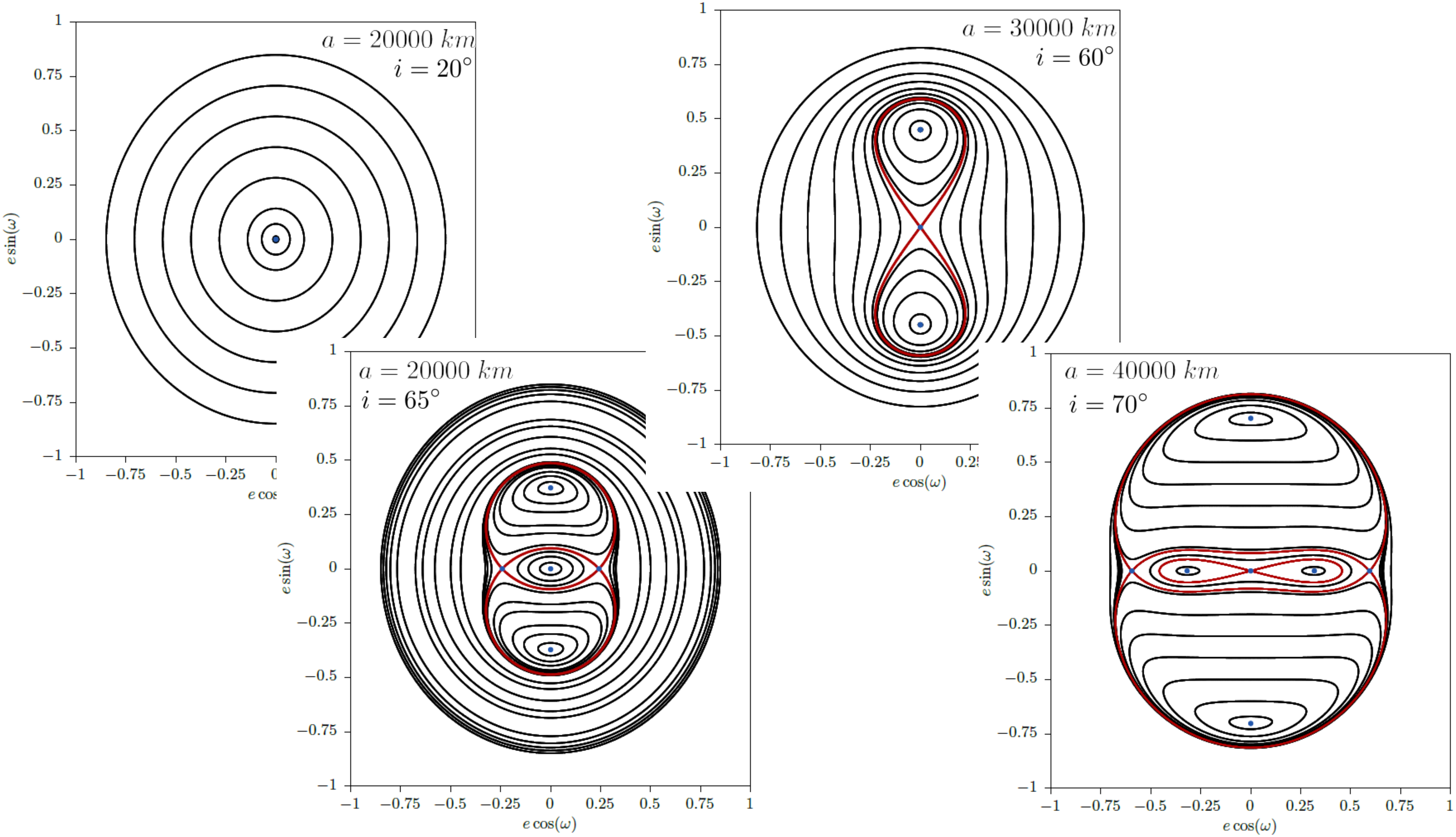
and the equations of motion are

$$\frac{dk}{dt} = -\frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dh}$$
$$\frac{dh}{dt} = \frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dk}$$

- Equilibrium points: $dk/dt = dh/dt = 0$
- Stability determined from the eigenvalues of the linearised system
- Parameter space of (a, i_{circ})

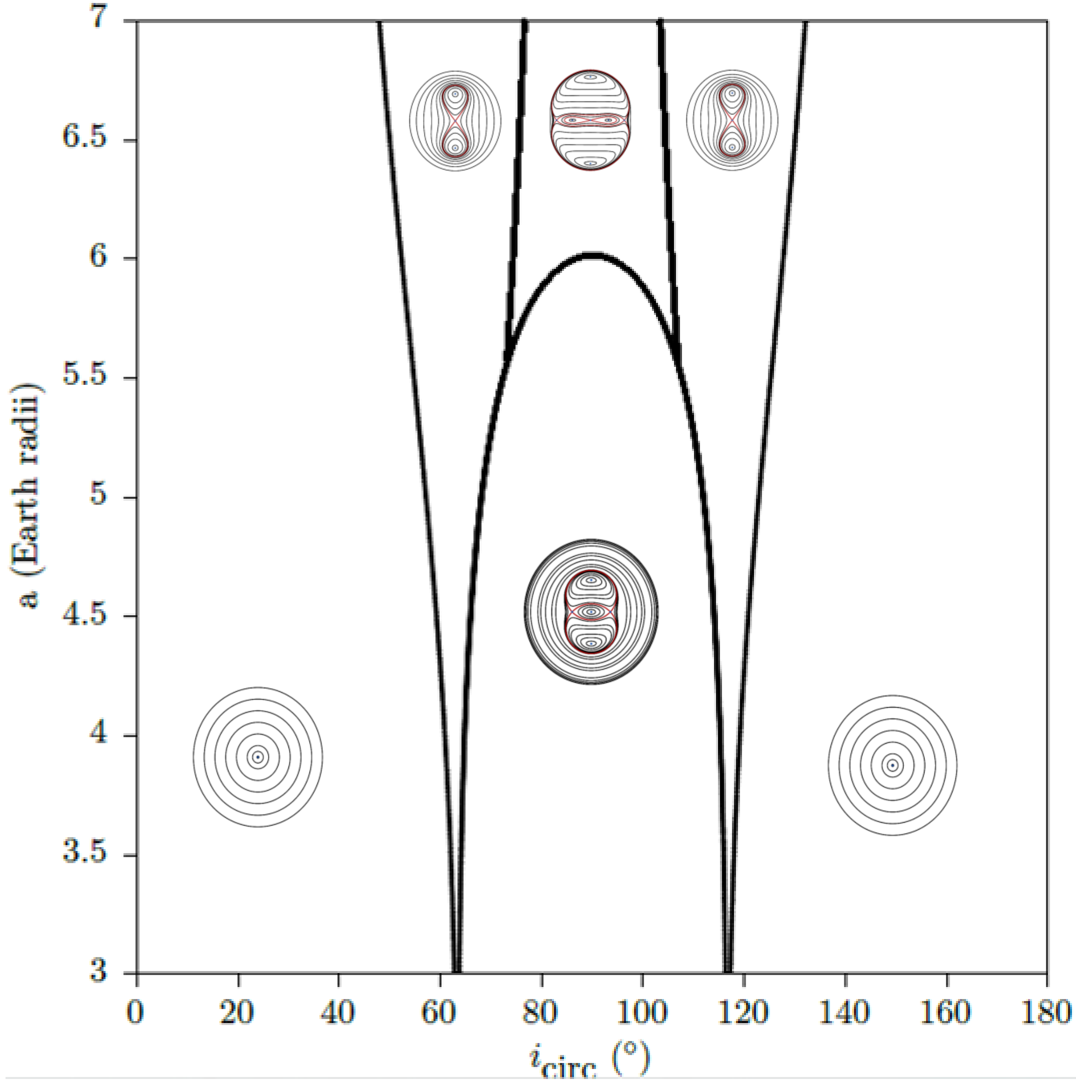
Analytical modelling

Study of the reduced model



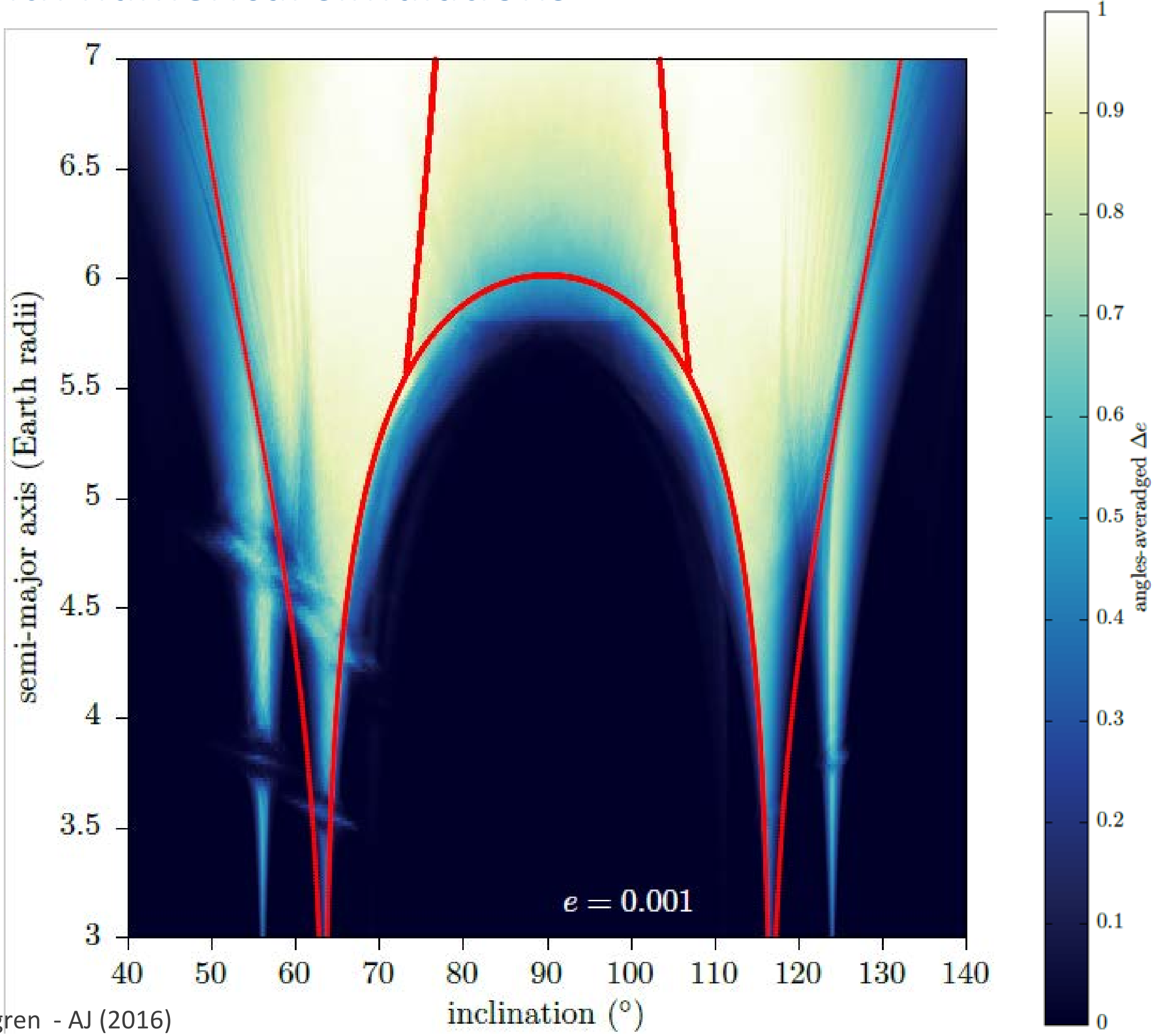
Analytical modelling

Bifurcation diagram



Analytical modelling

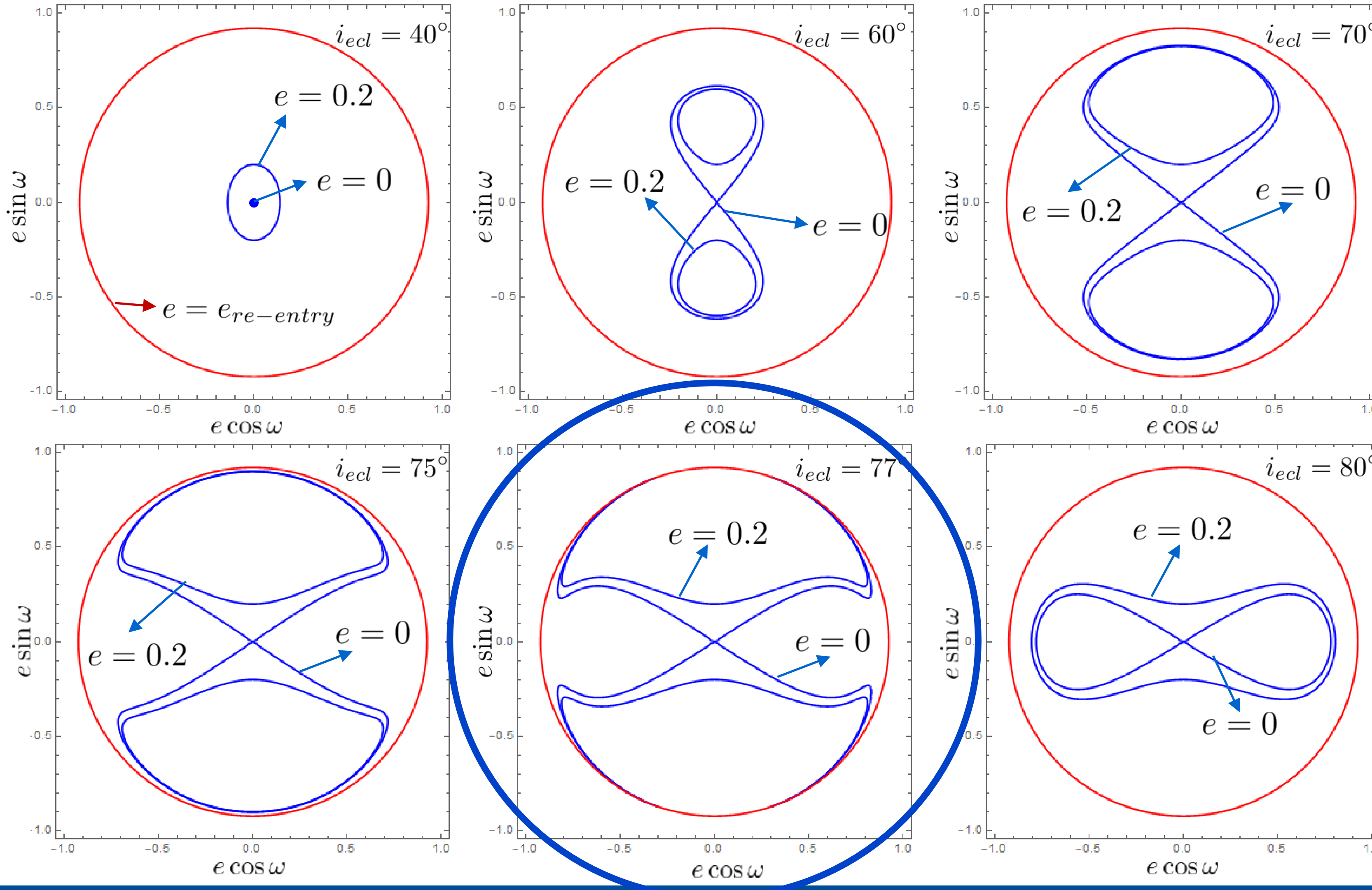
Comparison with numerical simulations



Gkolias, Daquin, Gachet, Rosengren - AJ (2016)

Analytical modelling

Disposal design



Conclusions

Numerical investigation

- For low initial inclinations, graveyard orbits with low variation of eccentricity are preferable.
- For inclined geosynchronous natural re-entry is possible.
- Optimise disposal manoeuvre for each particular end-of-life scenario.

- Is a single equation guideline for GEO enough?
- Could eccentric and inclined, small size constellations lead us to a sustainable exploitation of GEO?

- All maps calculated will be made public on the ReDSHIFT web site (http://redshift-h2020.eu).
- ReDSHIFT software tool for EOL disposal calculation will be available online

Conclusions

Analytical modelling

- We have reduced the problem of high Earth satellites using an analytical representation.
- The resulting 1 D.O.F. system describes the in plane stability.
- We studied the reduced phase-space by computing the equilibrium points and their stability.
- We have calculated the bifurcation diagram.

Further work:

- Recover the short-periodic terms.
- Add more perturbations, second order J_2 and up to P_4 for the Moon.
- Exploit the reduced dynamics for preliminary mission design.

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Source: ESA

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