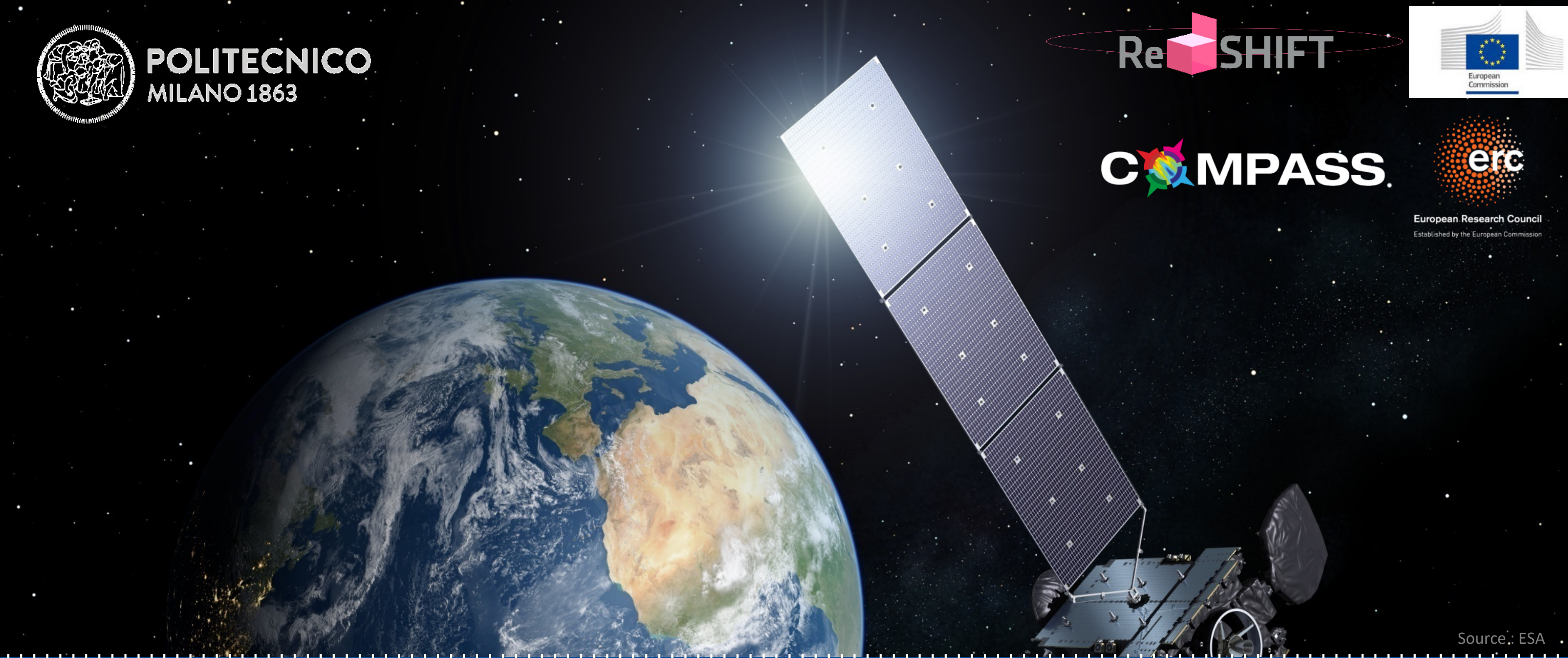


Source.: ESA

Towards a sustainable exploitation of the geosynchronous orbital region

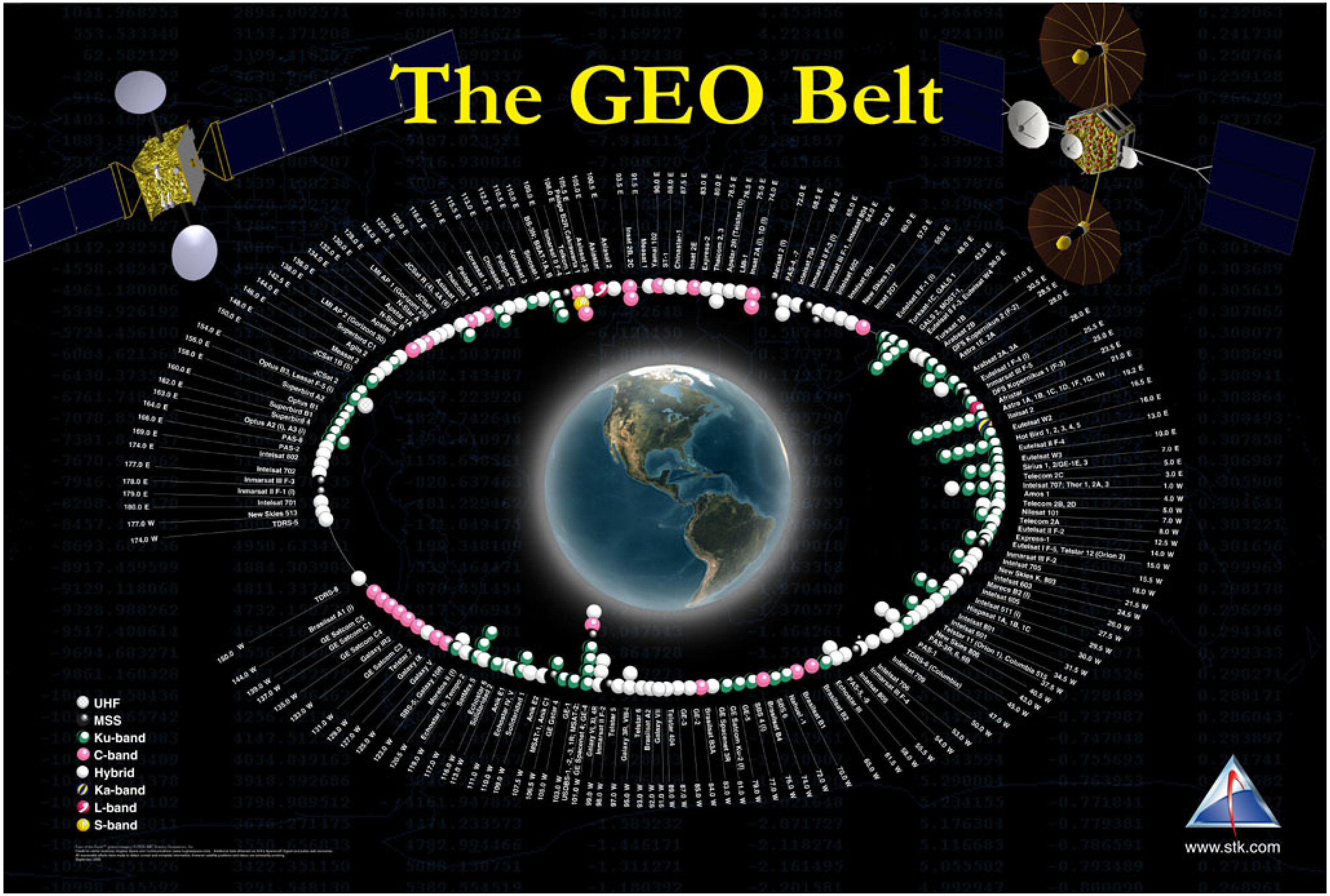
Ioannis Gkolias[1], Camilla Colombo[1], Martin Lara[2]

[1] *Politecnico di Milano,* [2] *University of La Rioja*

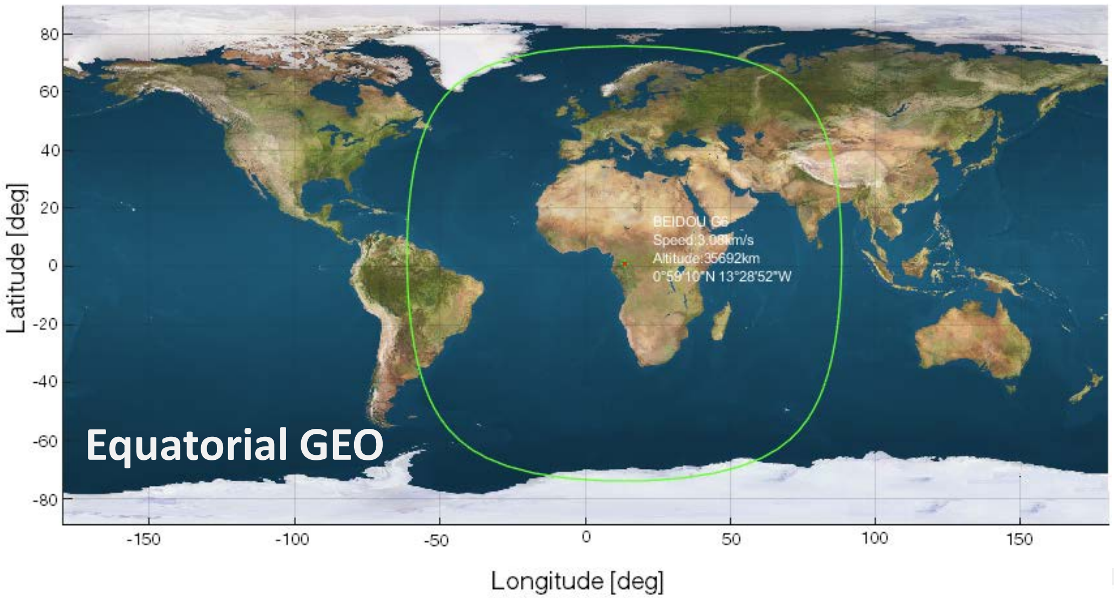
RCAAM, Academy of Athens, July 2018

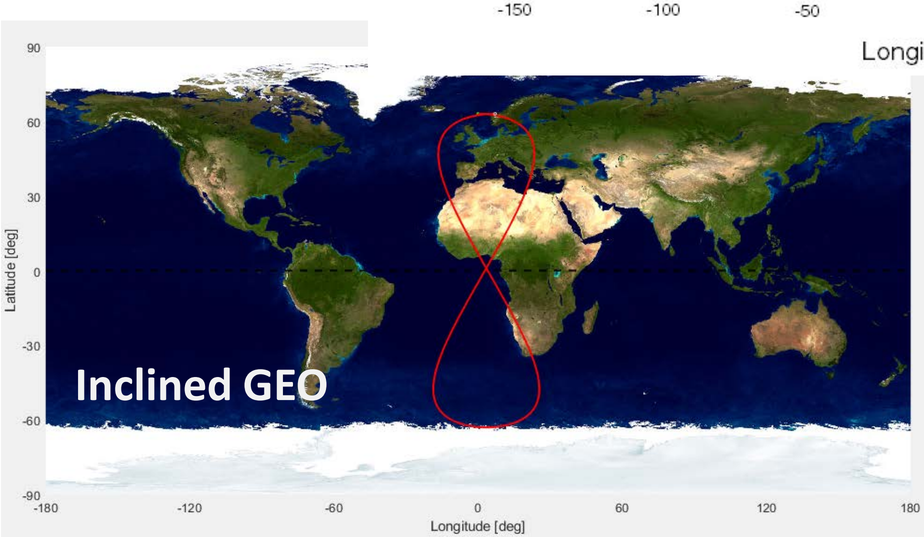
Introduction

Geostationary belt



Geosynchronous ground tracks



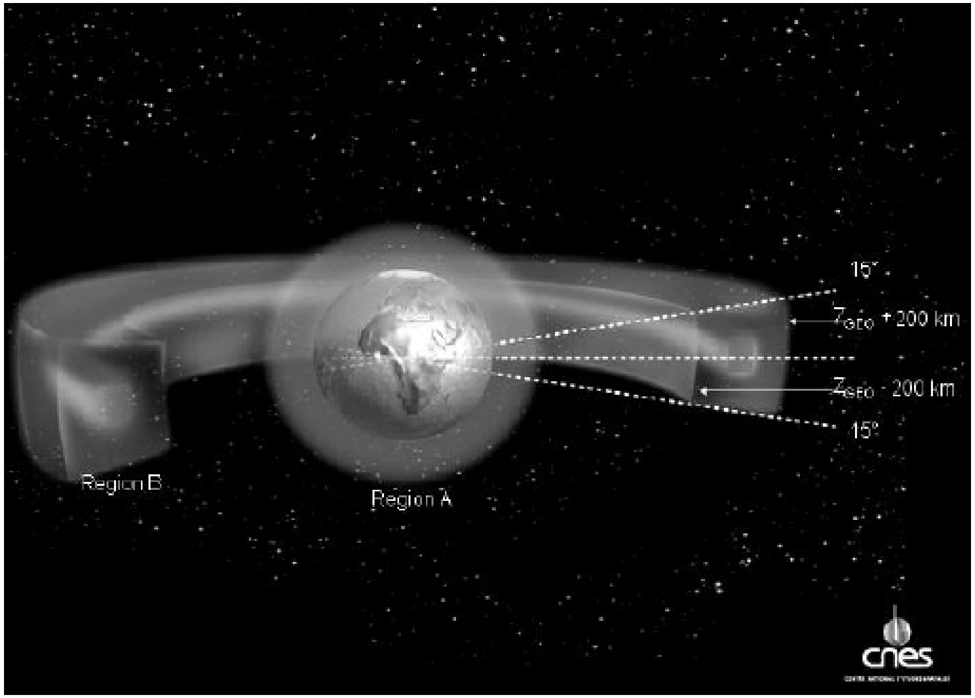


Introduction

Current ESA guidelines

Conformance with the GEO disposal requirement can be ensured by using a disposal orbit with the following characteristics:

- Eccentricity ≤ 0.005,
- Min perigee altitude above the GEO altitude $\Delta h_p \geq 235+1000\, c_R\, A/m$



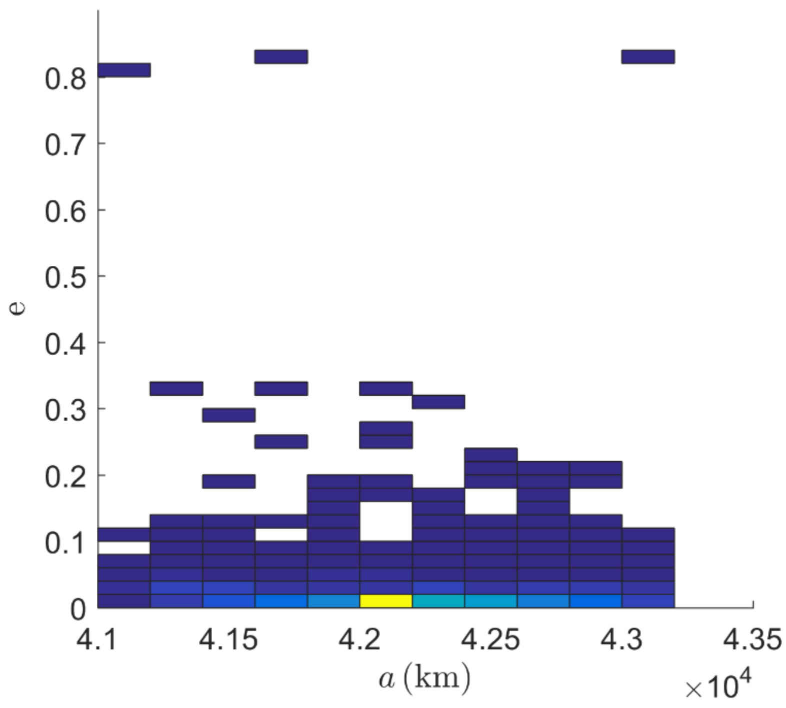
GEO protected region (GEO region): segment of spherical shell

- lower altitude boundary = geostationary altitude minus 200 km,
- upper altitude boundary = geostationary altitude plus 200 km,
- latitude sector: 15 degrees South ≤ latitude ≤ 15 degrees North

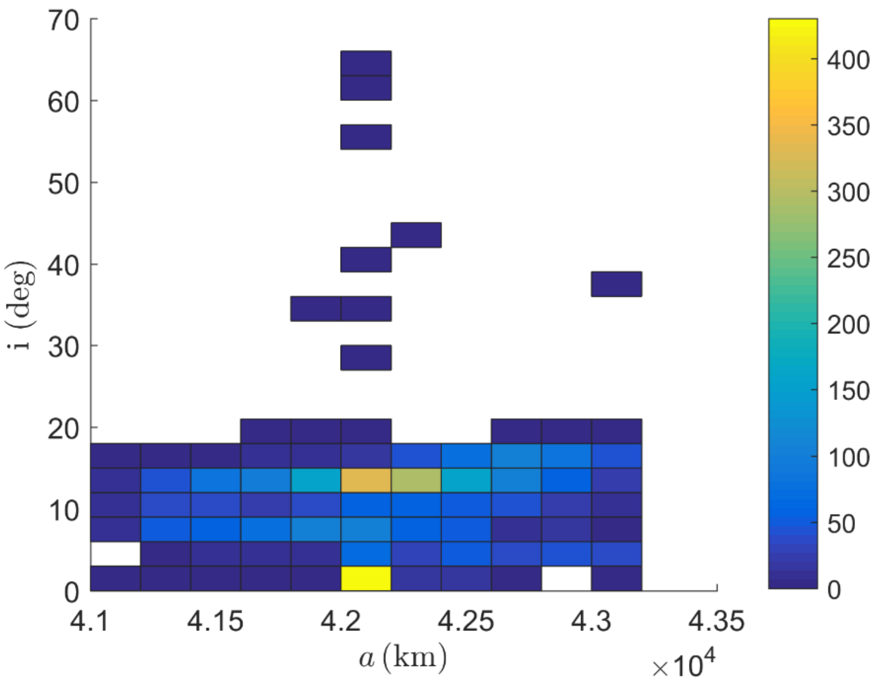
Introduction

GEO population

a-e distribution of all objects in GEO



a-i distribution of all objects in GEO



Introduction

Why revisit GEO disposal?

- Many people believe that the debris situation in GEO is shorted out, but is it really and in which timescale?
- Population models predict on average 1 GEO collision in the next 100 years.
- Satellites in graveyard orbits act as debris sources, even without collisions (e.g. HARM GEO population).
- From planetary defence point of view, if we keep the same rate of populating GEO, we will detectable by an equivalently advanced civilization by the year 2200.

Questions:

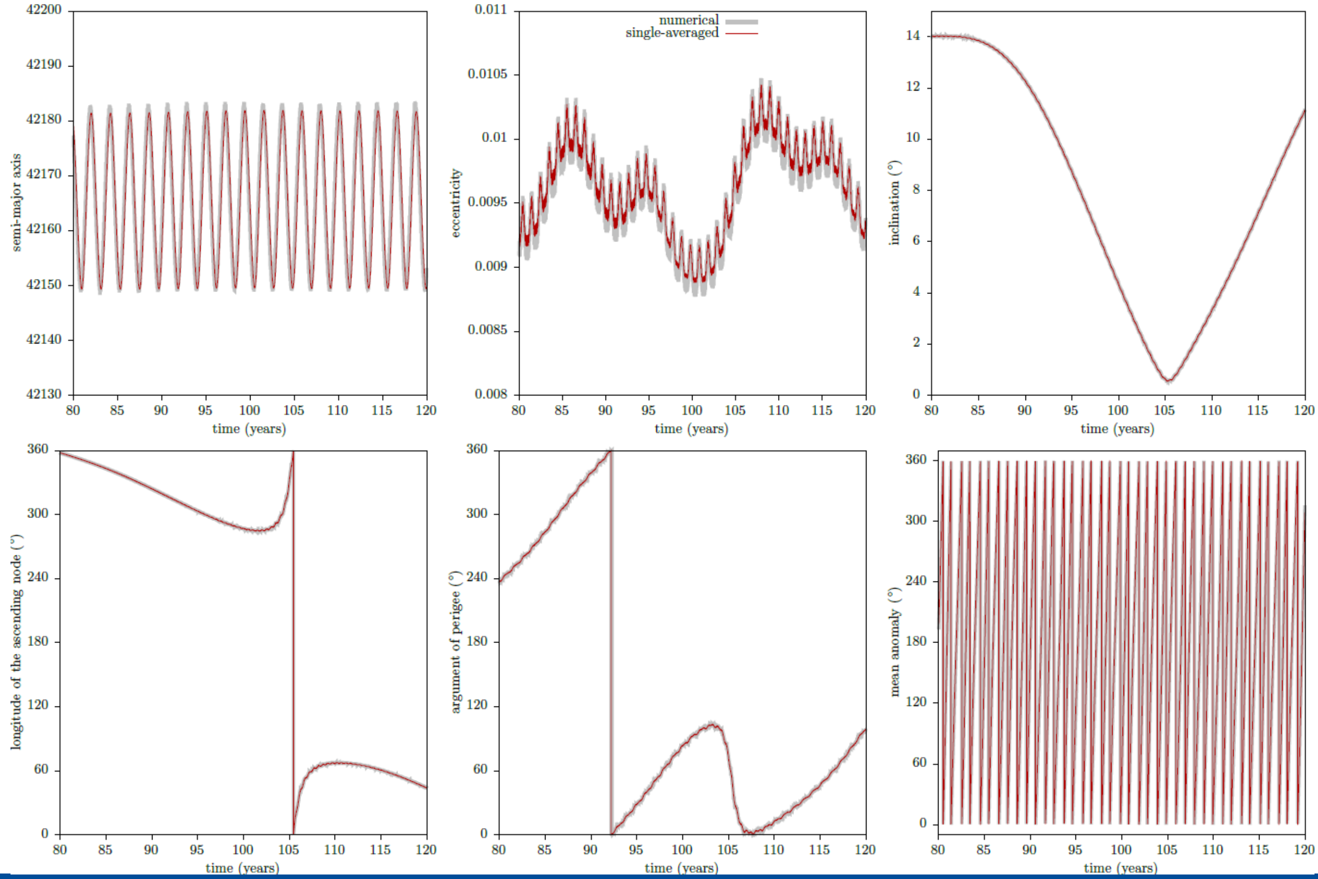
- Are current guidelines enough to ensure long-term GEO sustainability?
- Are there alternative ways to exploit the geosynchronous orbital region?

GEO DYNAMICAL MAPPING

Semi-analytical modelling

PlanODyn (semi-analytical orbit propagation)

Force model: 4x4 geopotential, 3rd body perturbations (up to 5th order in the parallax factor), solar-radiation pressure, Earth's precession



Grid definitions

Orbit propagation for 120 years

- Tesseral Maps
  - Main grid: *a* - *λ* (201x201) — > 4 Million
  - Parameters: *e*, *i*, *A*/m (5x11x2)
- Disposal Maps
  - Main grid: *ω* - *Ω* (201x201) — > 36 Million
  - Parameters: *e* , *i*, *A*/m (5x91x2)
- Action Maps
  - Main grid: *e* - *i* (201x201) — > 12 Million
  - Parameters : *a*, (*Ω*, *ω*), *A*/*m* (3x50x2)

Orbits propagated > 50 Million

Dynamical indicators:

$$Diam(e) = |e_{max} - e_{min}|$$

$$\Delta e = \frac{|e_{max} - e_0|}{|e_{re-entry} - e_0|}$$

$\Delta e \rightarrow 0$ Bounded

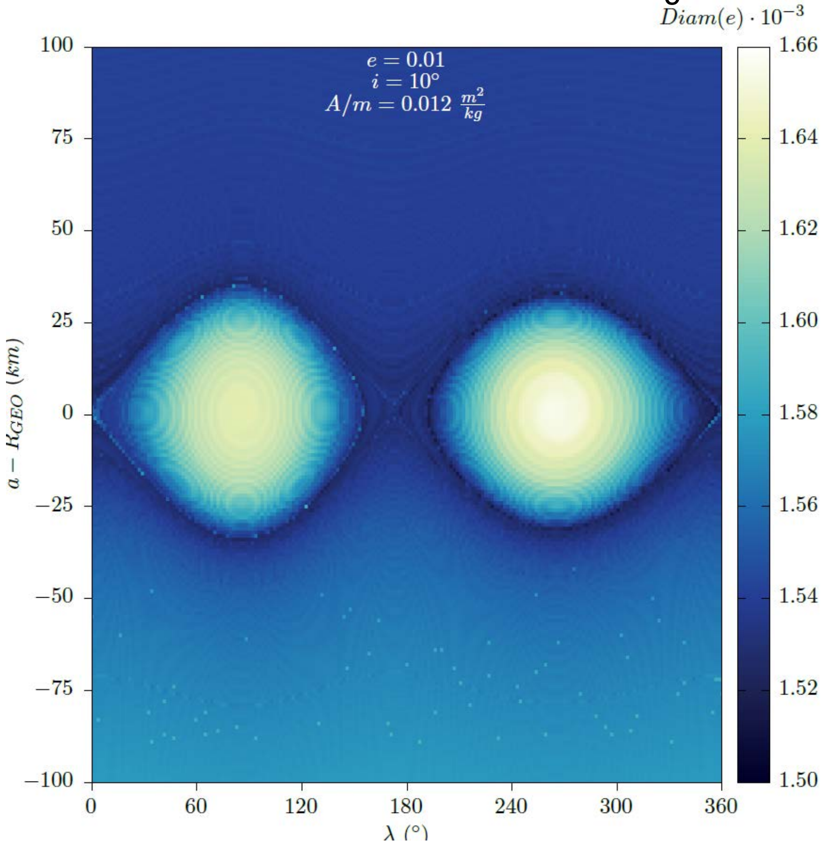
$\Delta e \rightarrow 1$ Re-entry

Tesseral maps

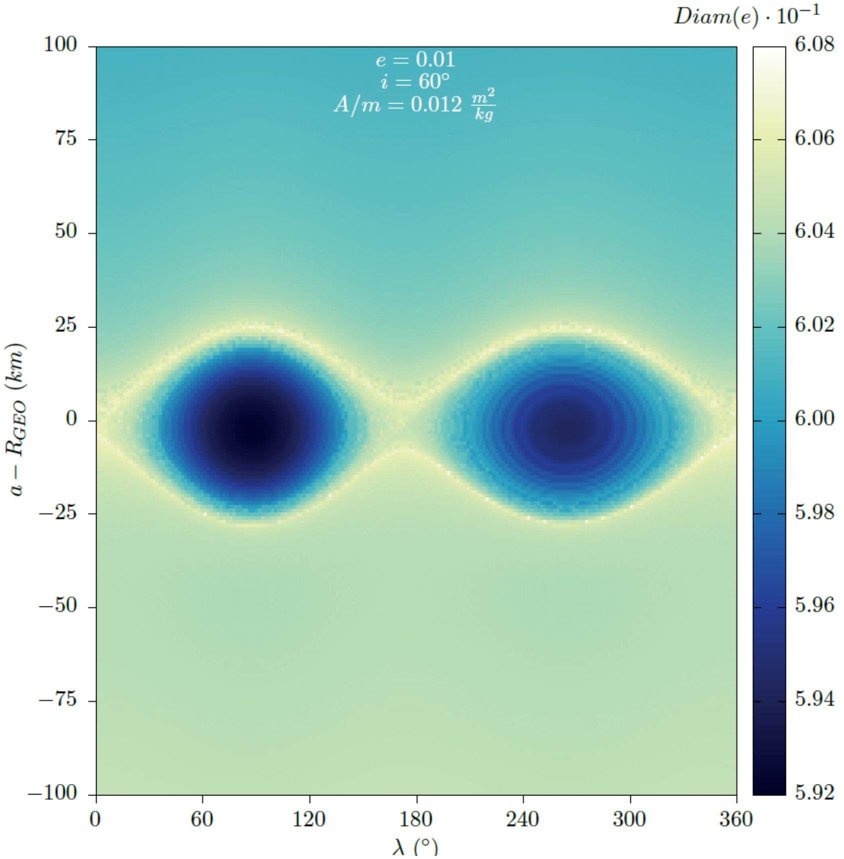
Standard s/c, initial circular orbit

A/m = 0.012 m^2/kg
e_0 = 0.01

low inclination i_0 = 10 deg



high inclination i_0 = 60 deg

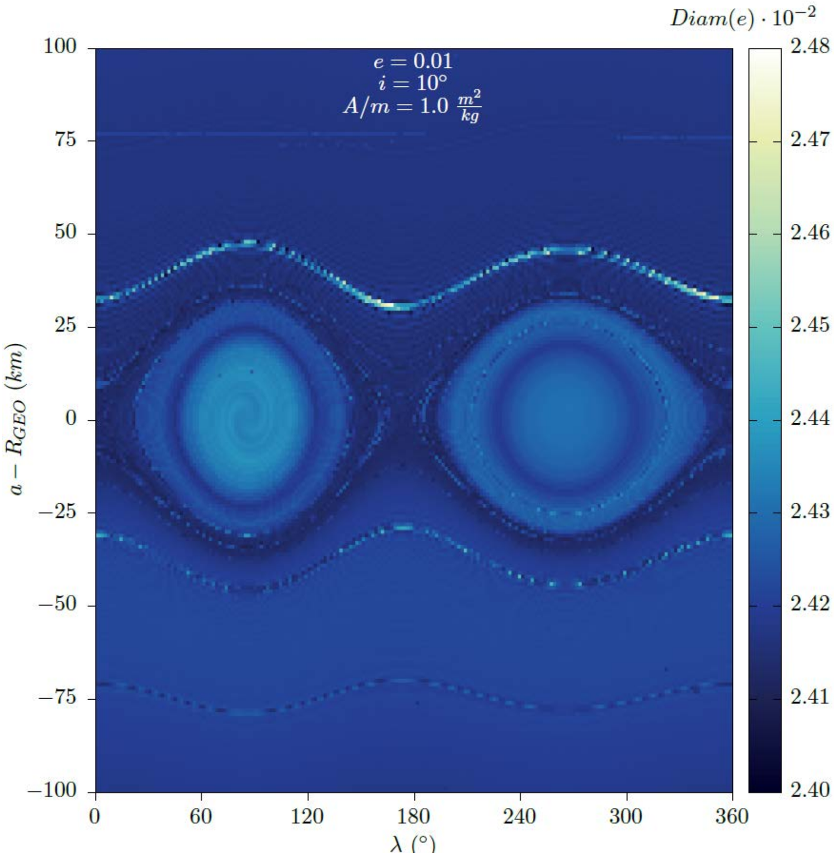


Tesseral maps

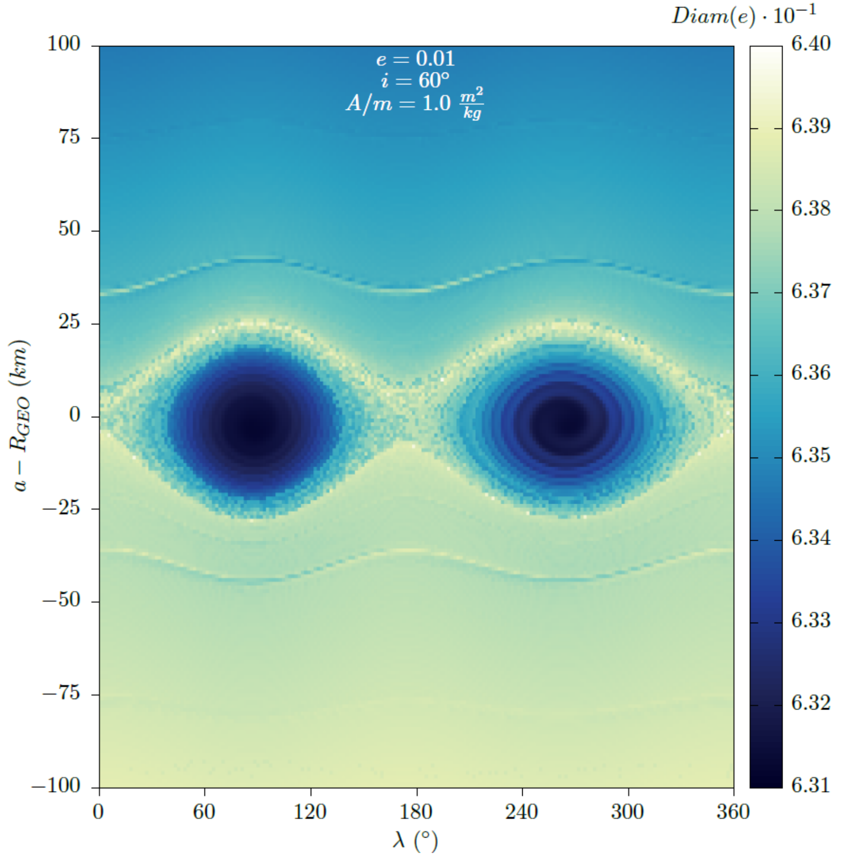
Enhanced-SRP s/c, initial circular orbit

A/m = 1 m^2/kg
e_0 = 0.01

low inclination i_0 = 10 deg



high inclination i_0 = 60 deg

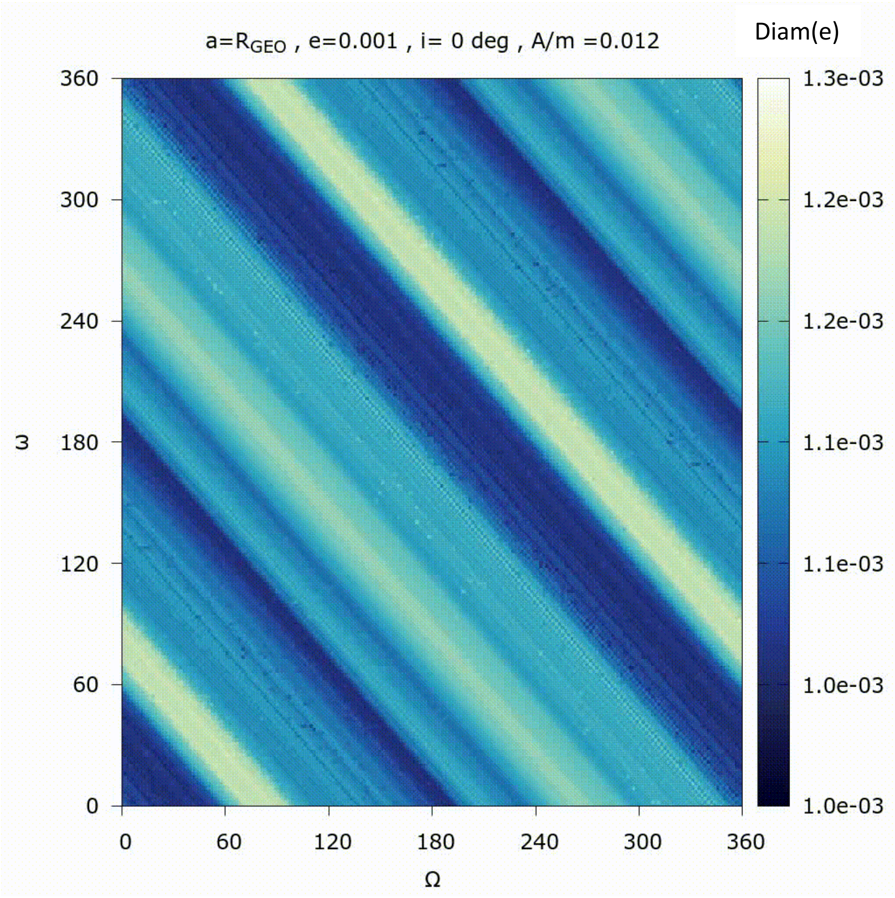


Disposal maps

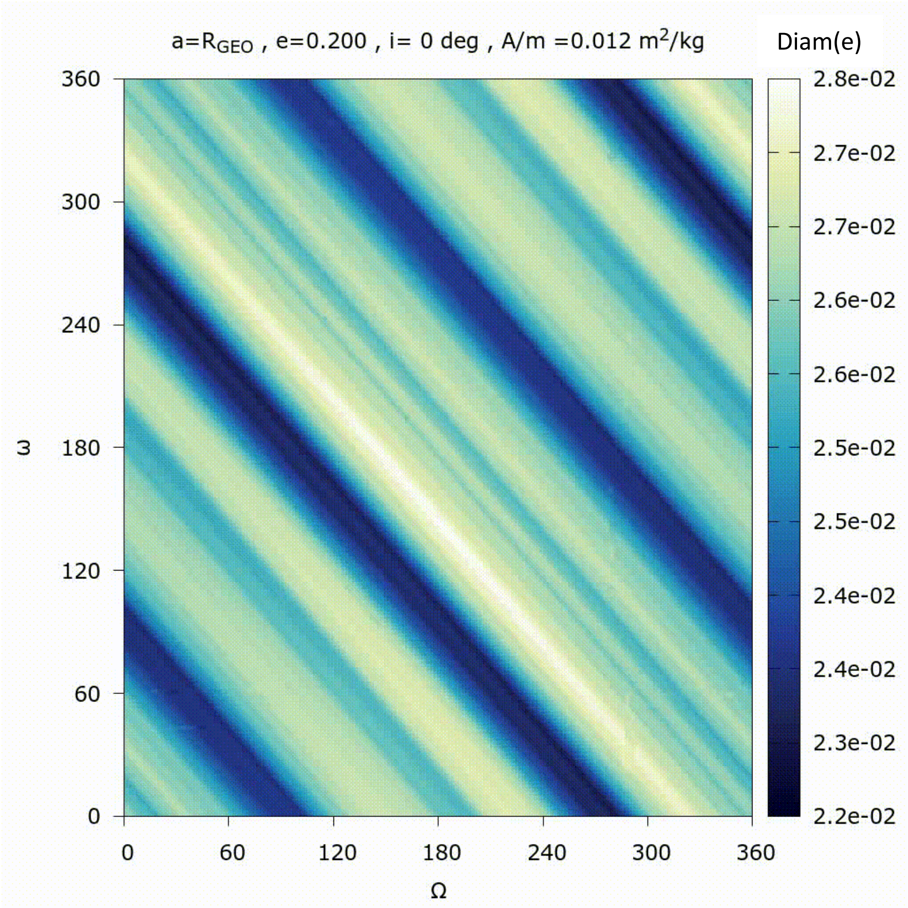
Standard s/c

$A/m = 0.012\ m^2/kg$

$e_0 = 0.001$



$e_0 = 0.2$

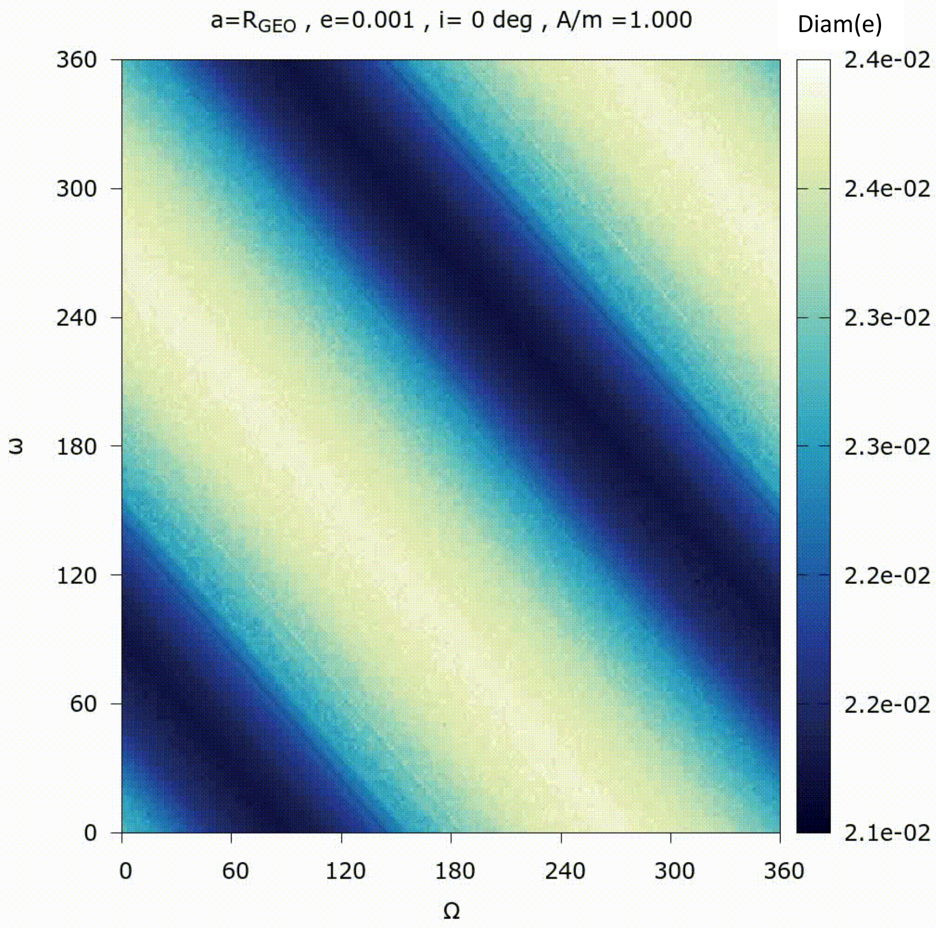


Disposal maps

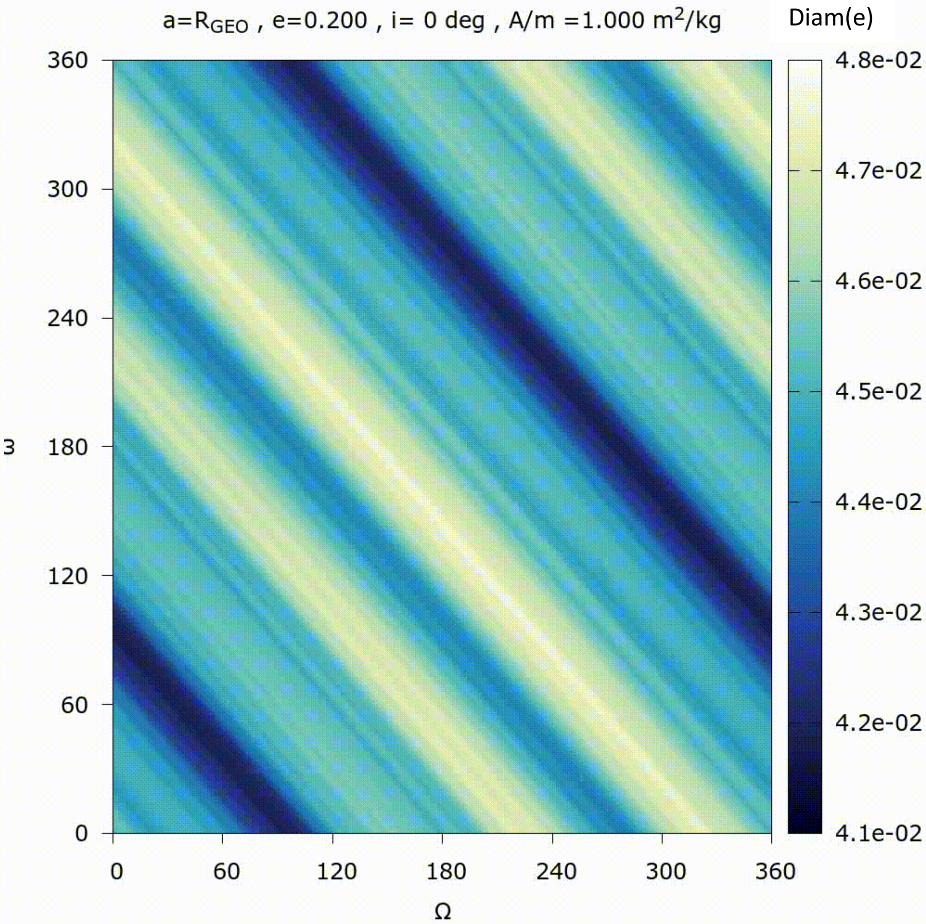
Enhanced-SRP

$A/m = 1\ m^2/kg$

$e_0 = 0.001$

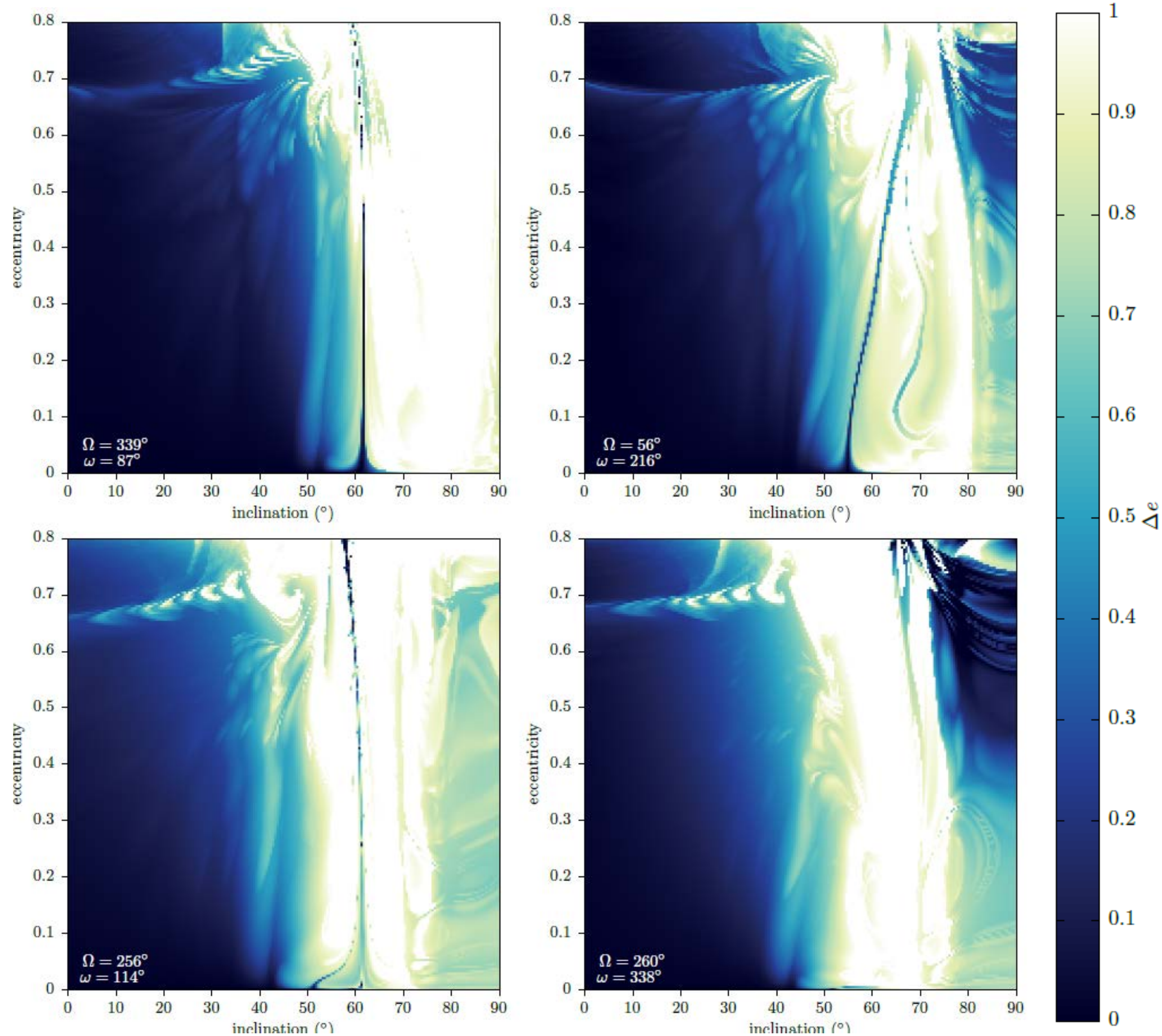


$e_0 = 0.2$



Eccentricity-inclination space

$a = R_{GEO}$



$\Delta e \to 1$

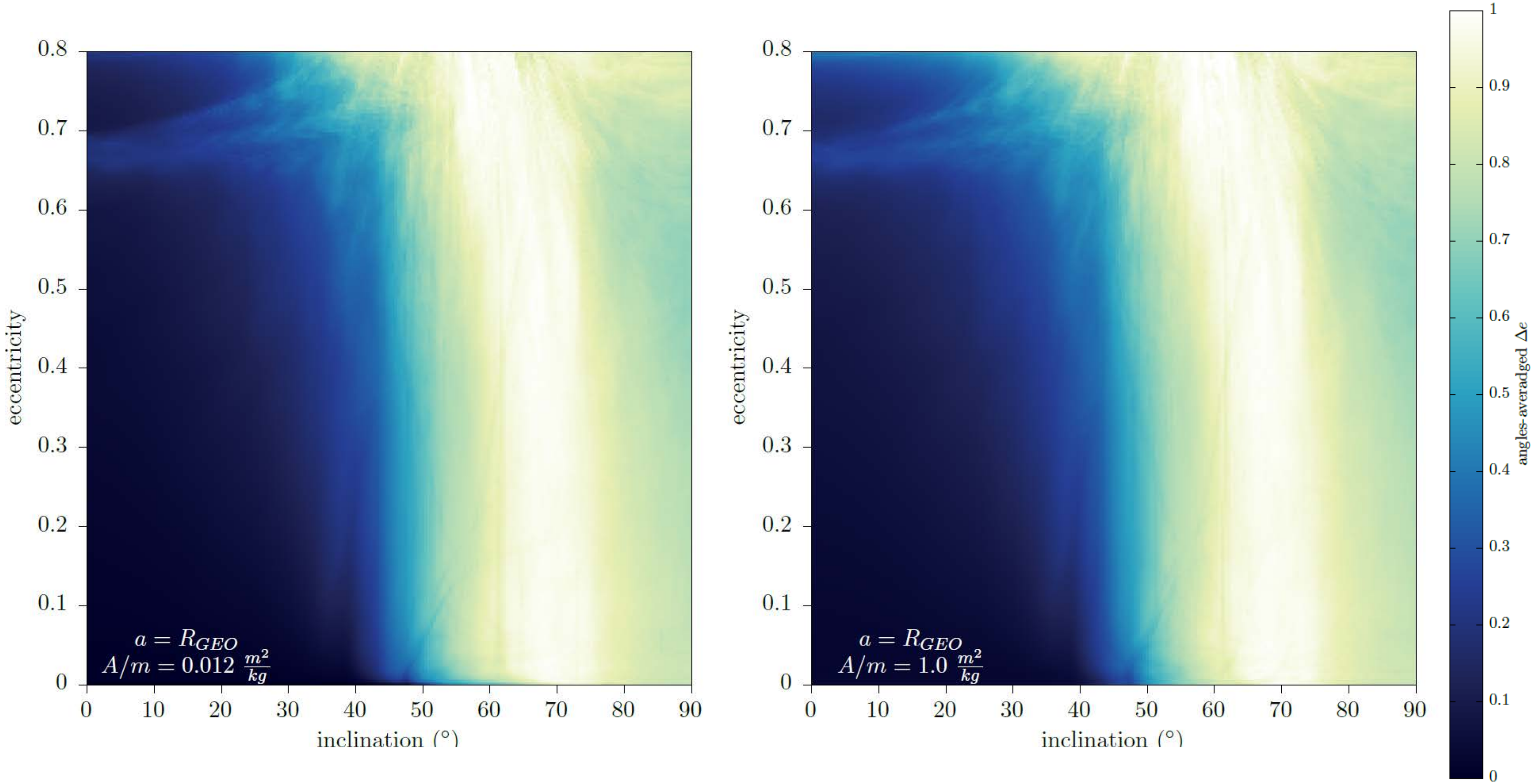
Re-entry

$\Delta e \to 0$

Bounded

Action space

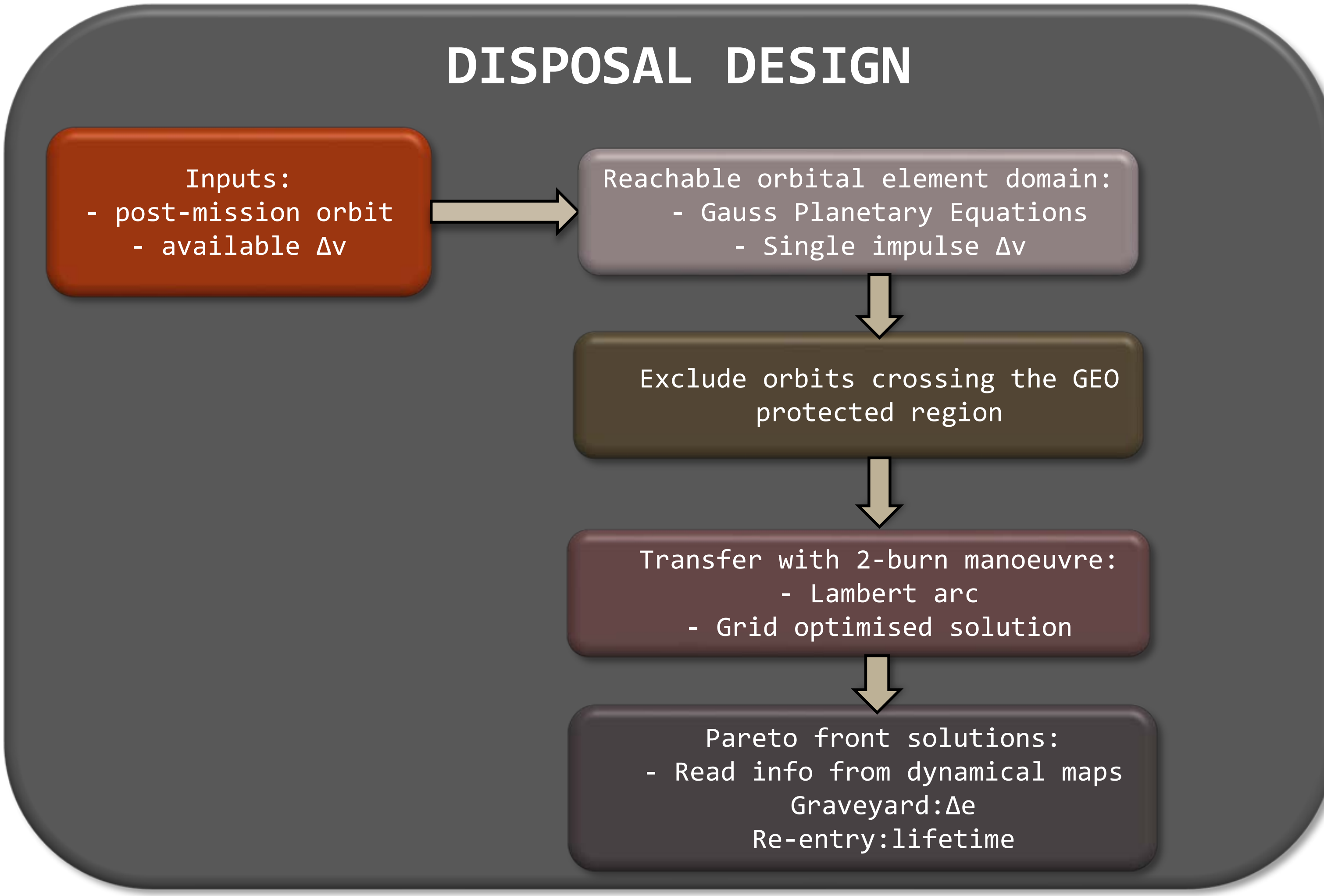
Angle-averaged maps



DISPOSAL MANOEUVRES

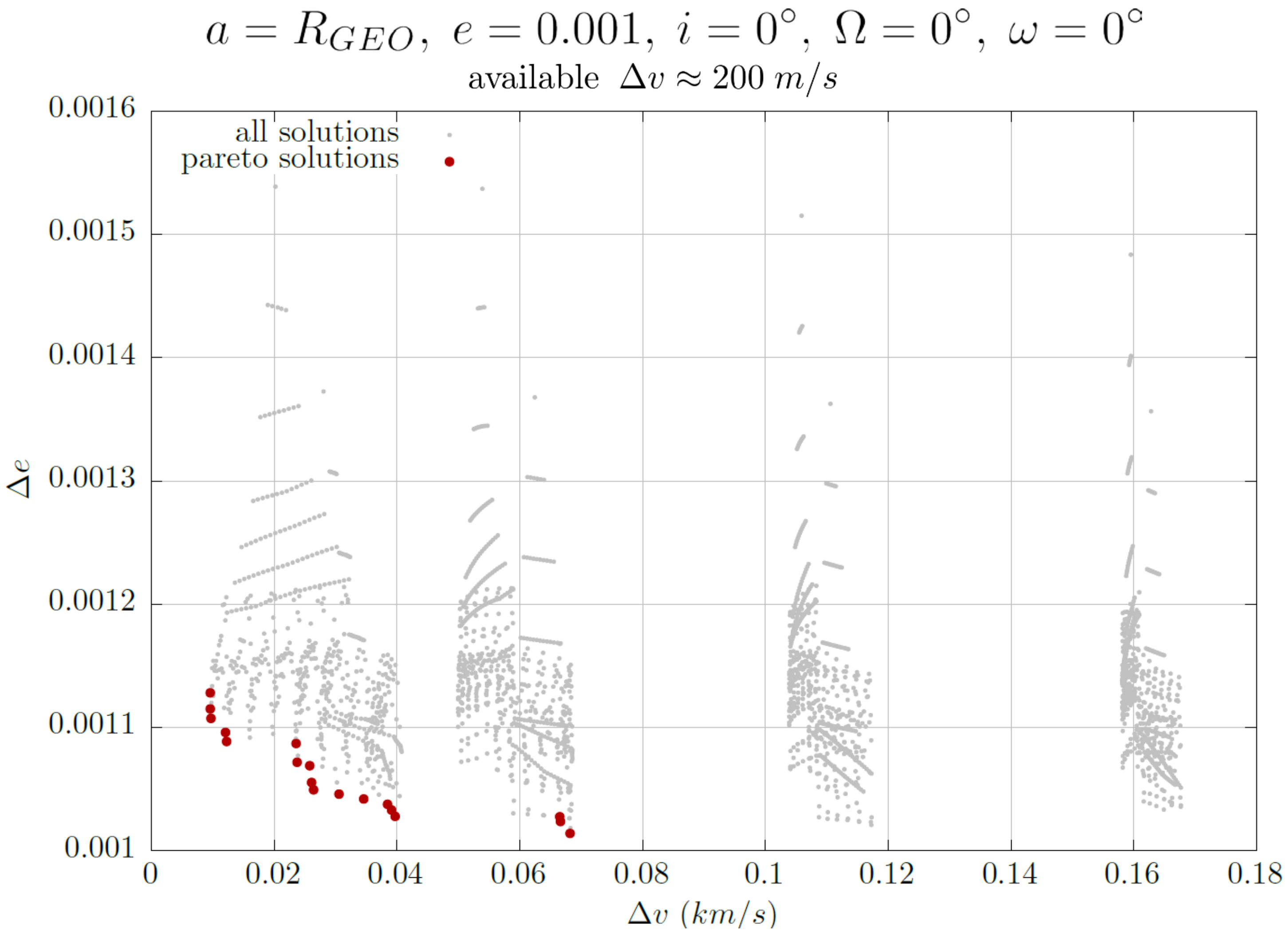
Disposal design

Process followed for each initial orbit



Graveyard design

Requirement for graveyard disposal

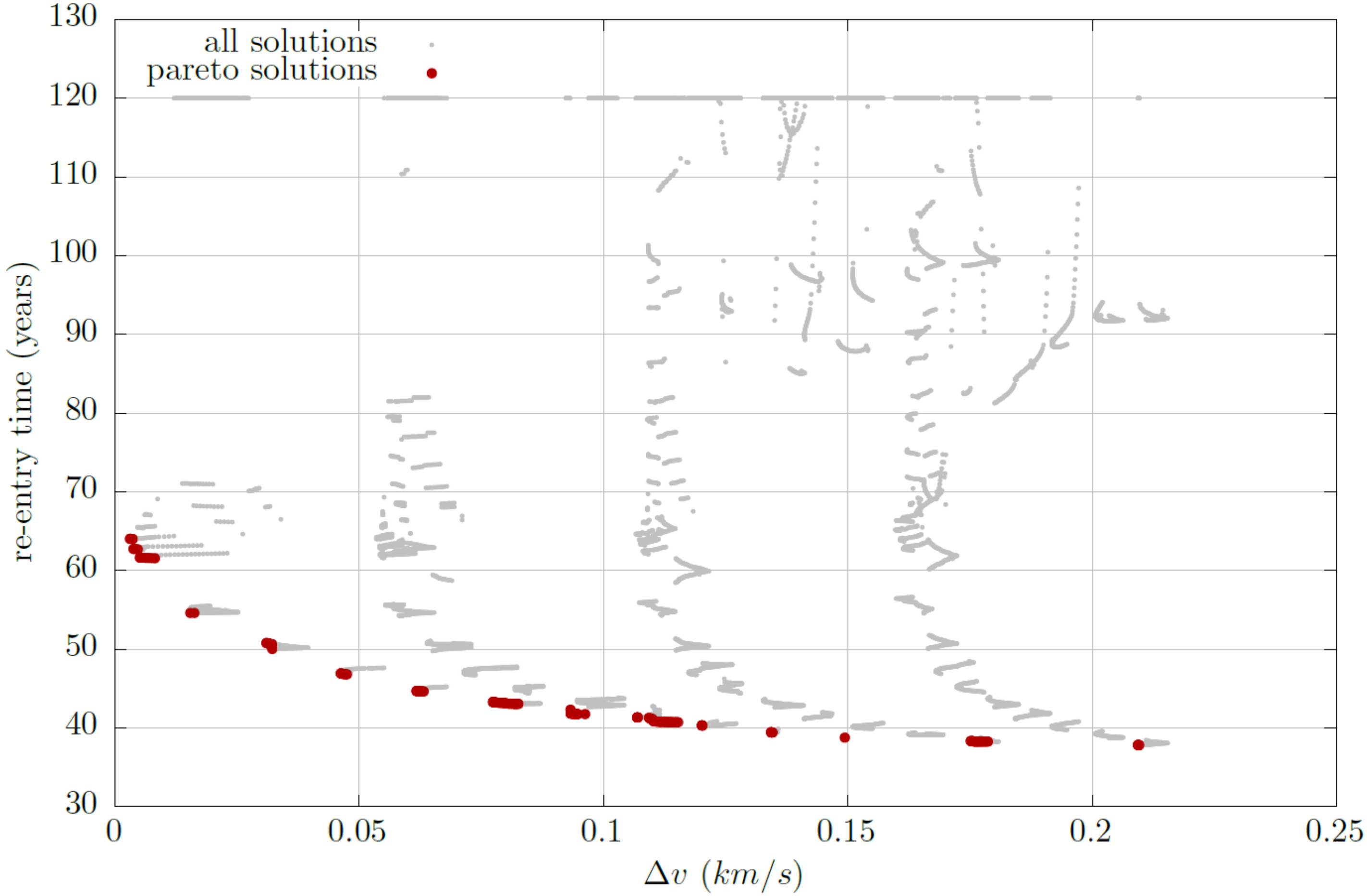


Re-entry design

Requirements for re-entry disposal

$$a = R_{GEO},\ e = 0.001,\ i = 70^\circ,\ \Omega = 0^\circ,\ \omega = 0^\circ$$

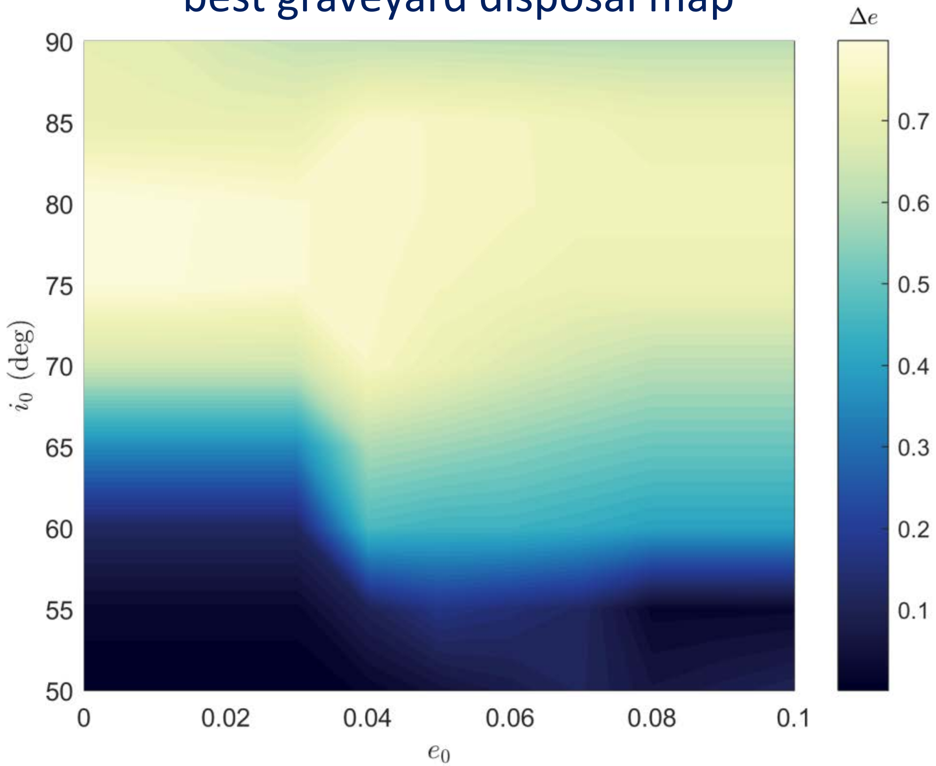
available $\Delta v \approx 200\ m/s$



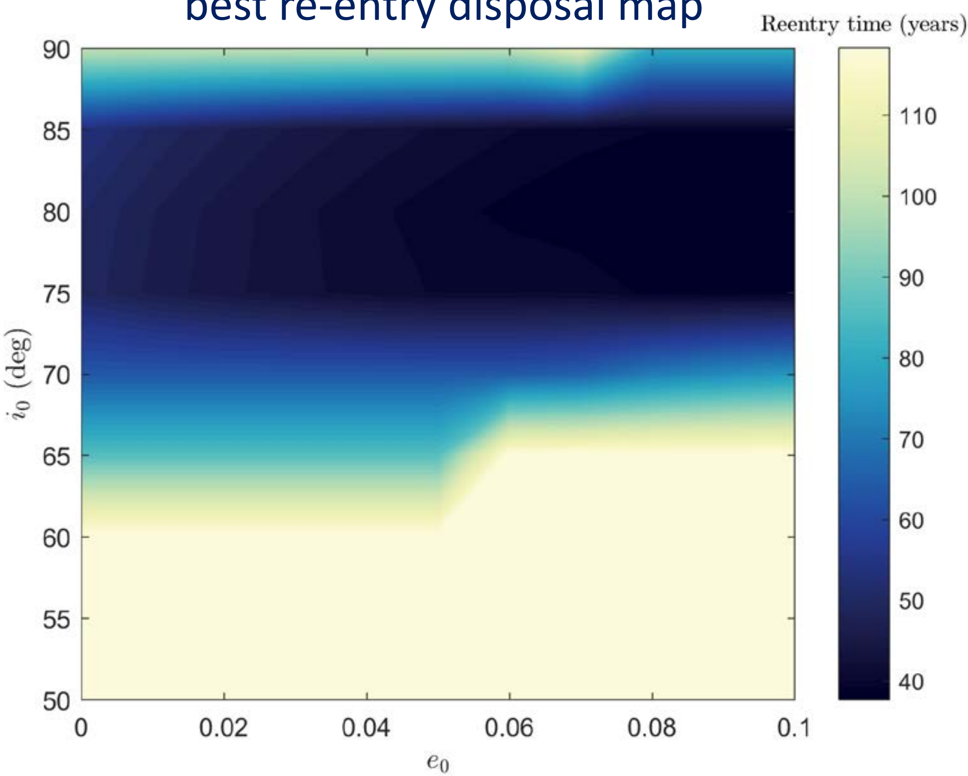
Best case scenario maps

Maximum available Δv = 50 m/s

best graveyard disposal map



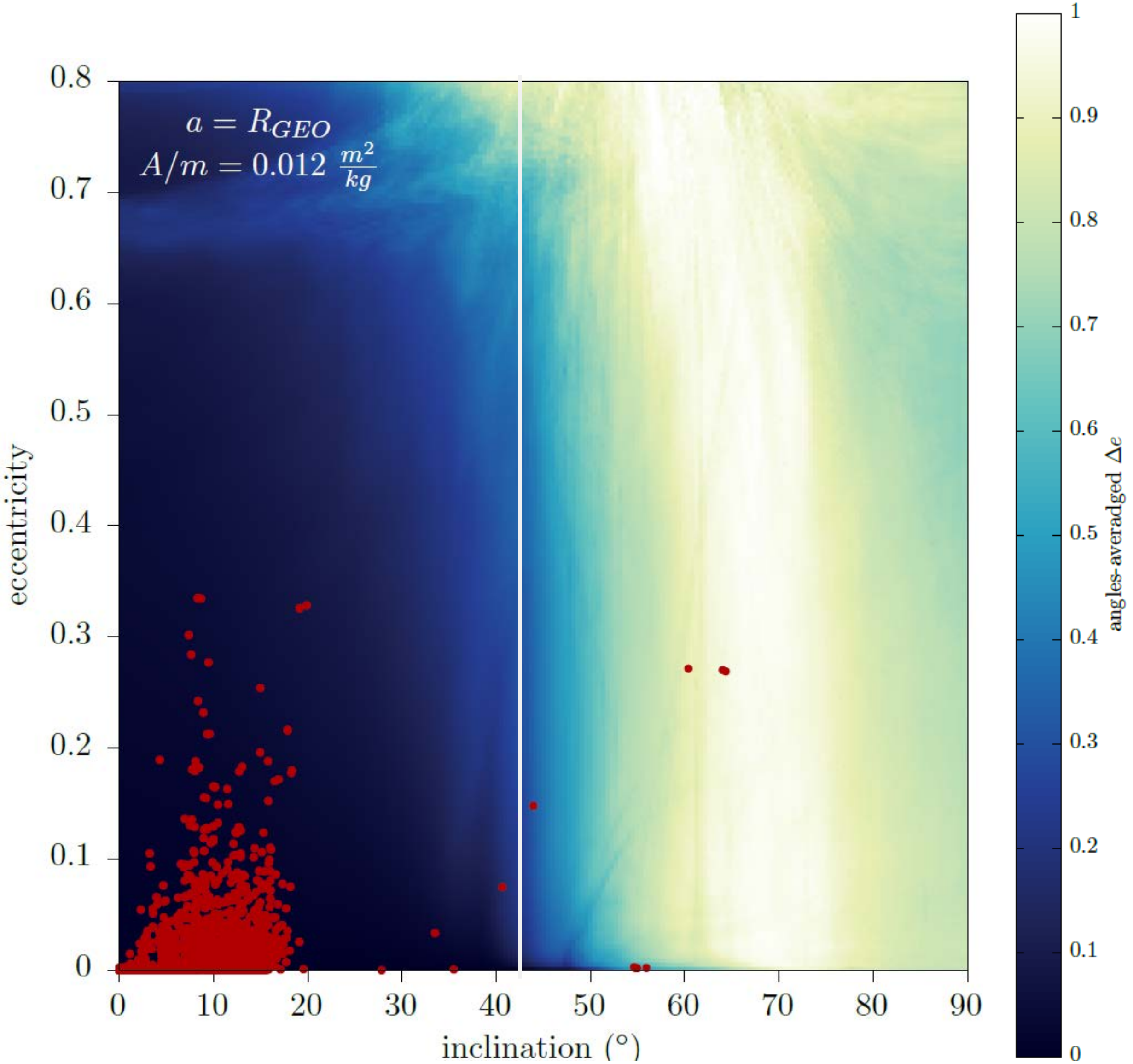
best re-entry disposal map



$\Omega_0 = 0, \omega_0 = 0$

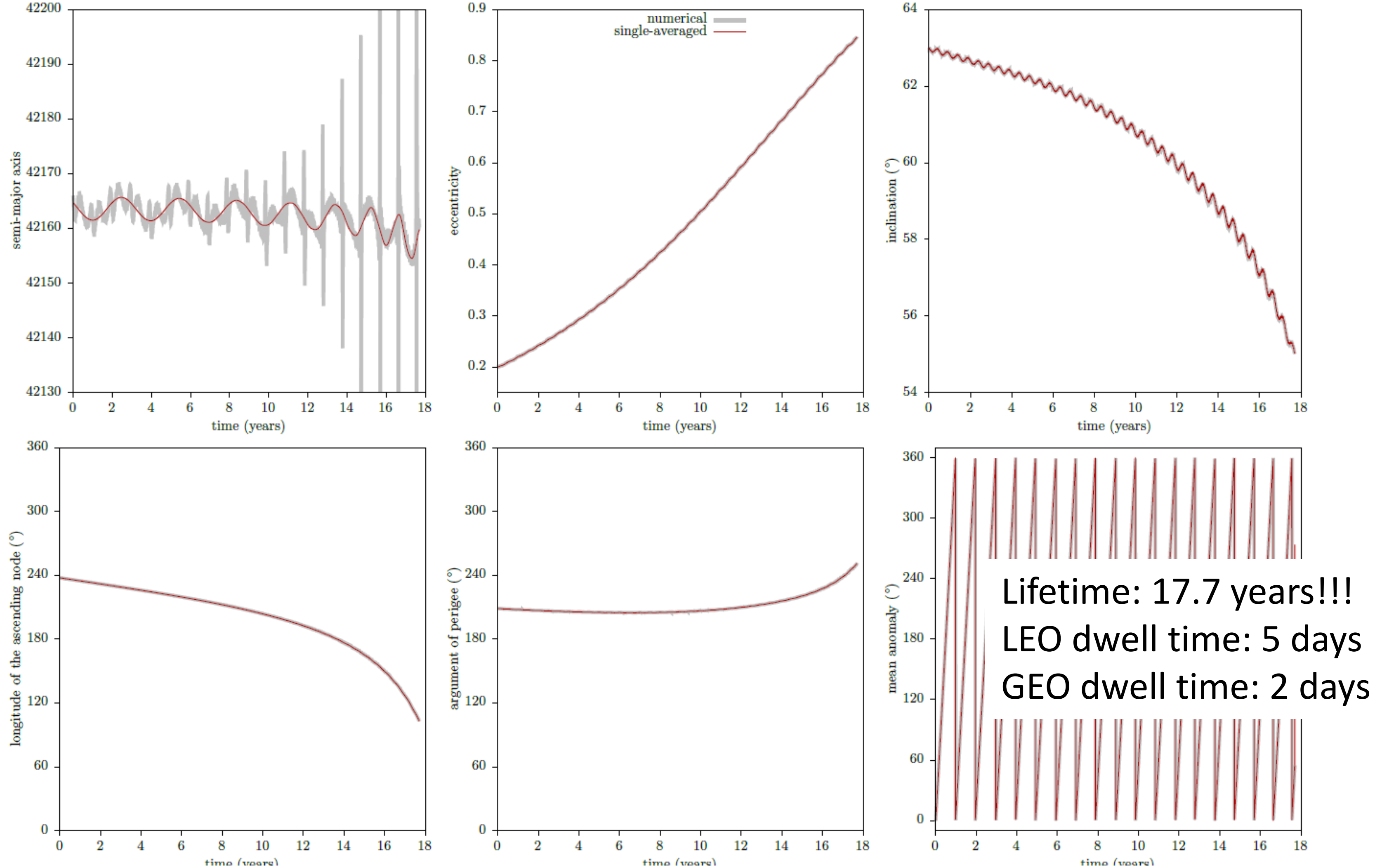
DISPOSAL ISSUES

Population and dynamics

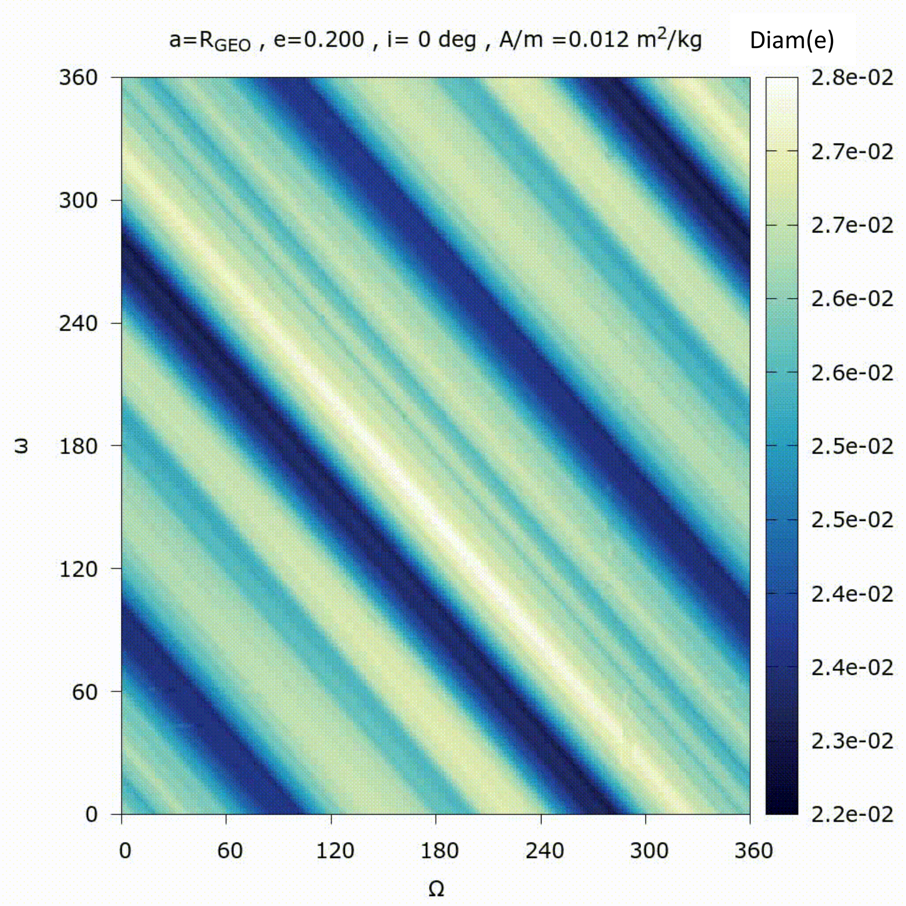


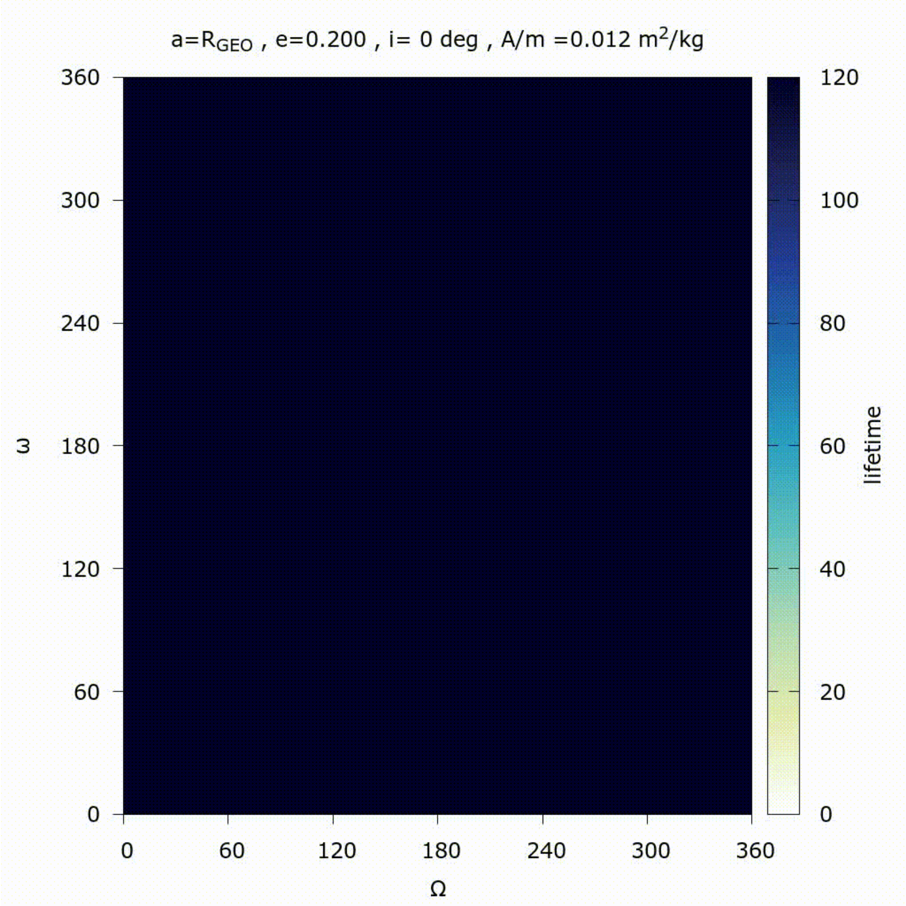
Effective cleansing mechanism

Fast re-entering orbits



Effective cleansing mechanism

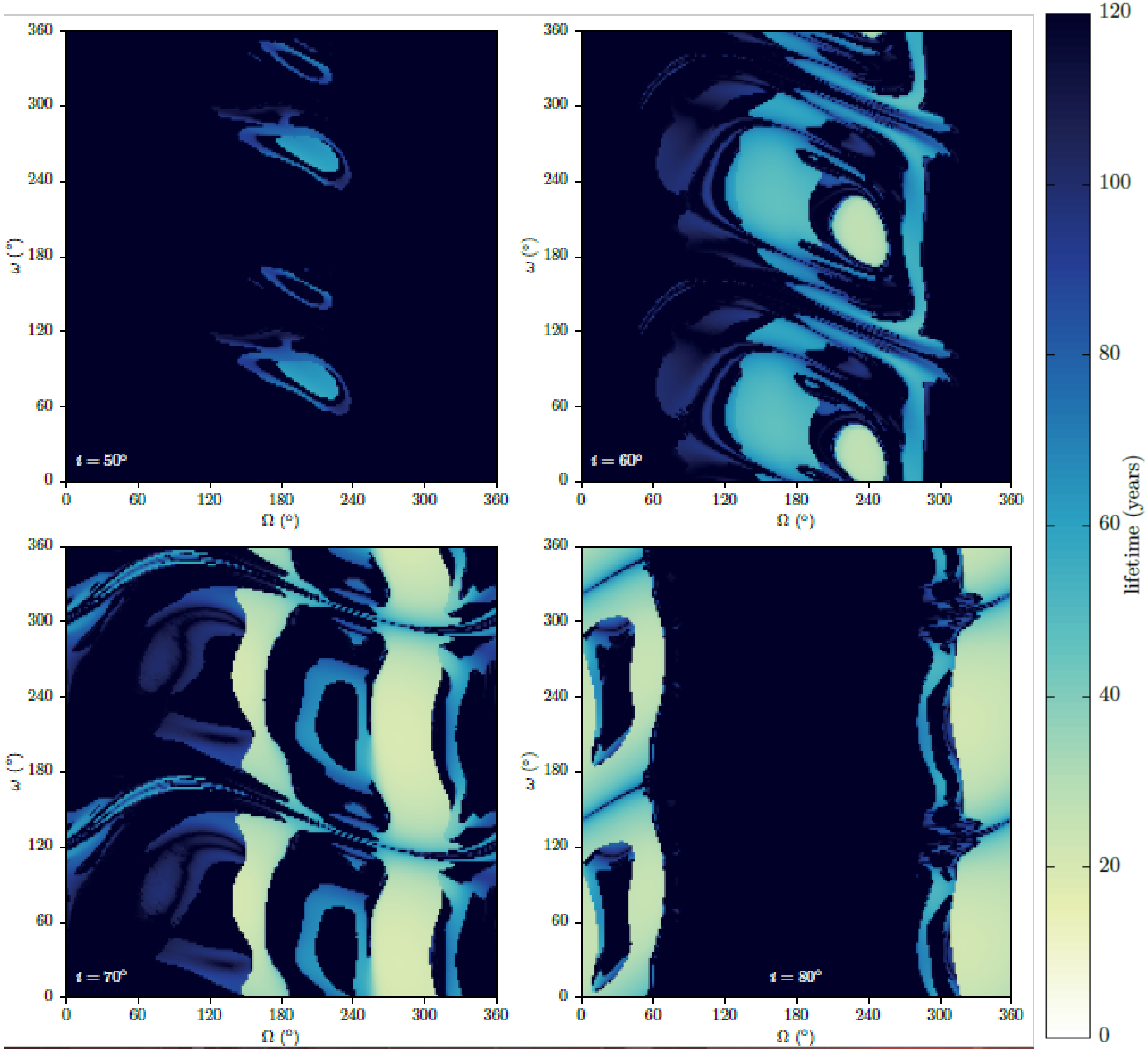




Effective cleansing mechanism

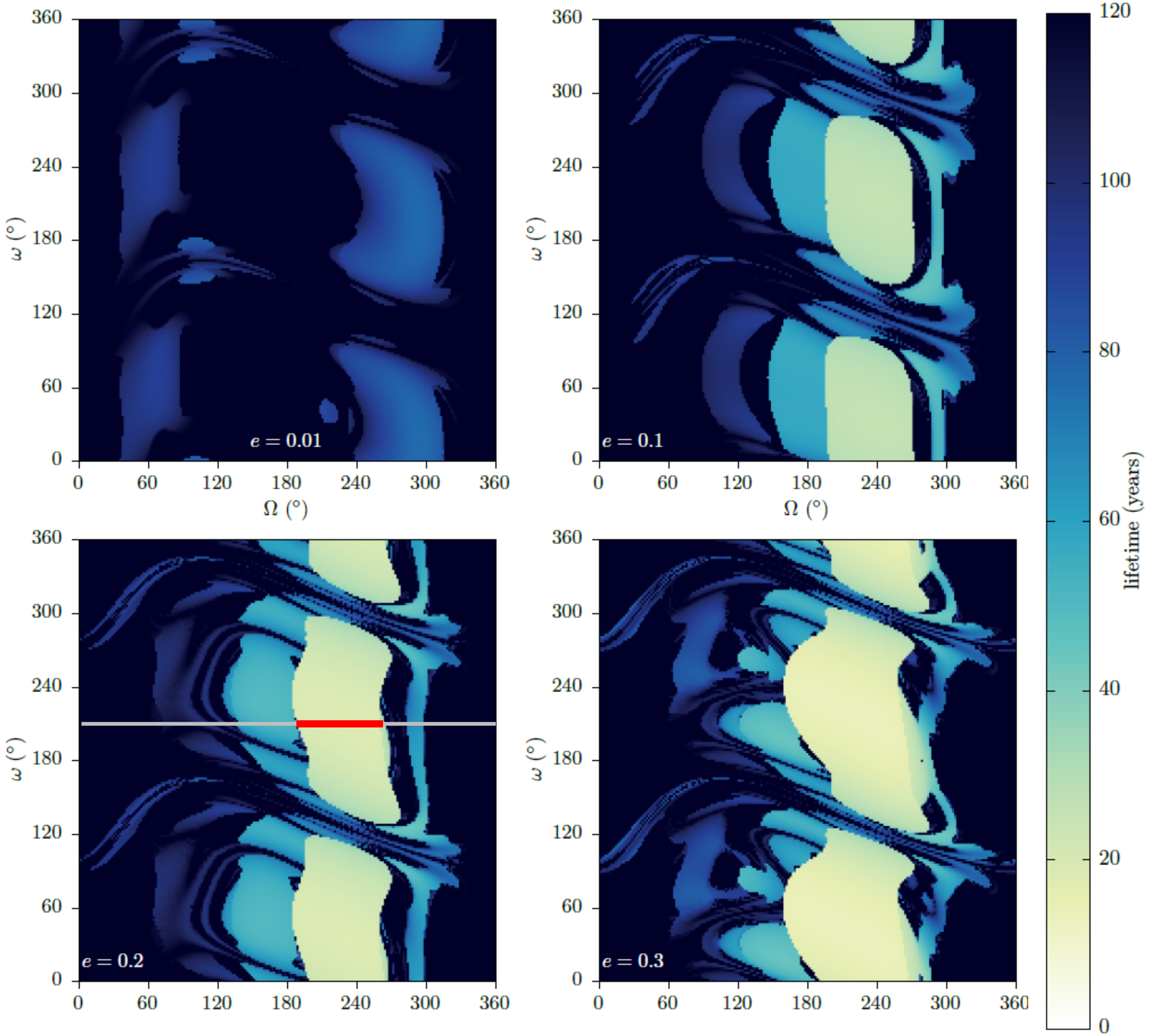
$a = R_{GEO}$

$e = 0.2$

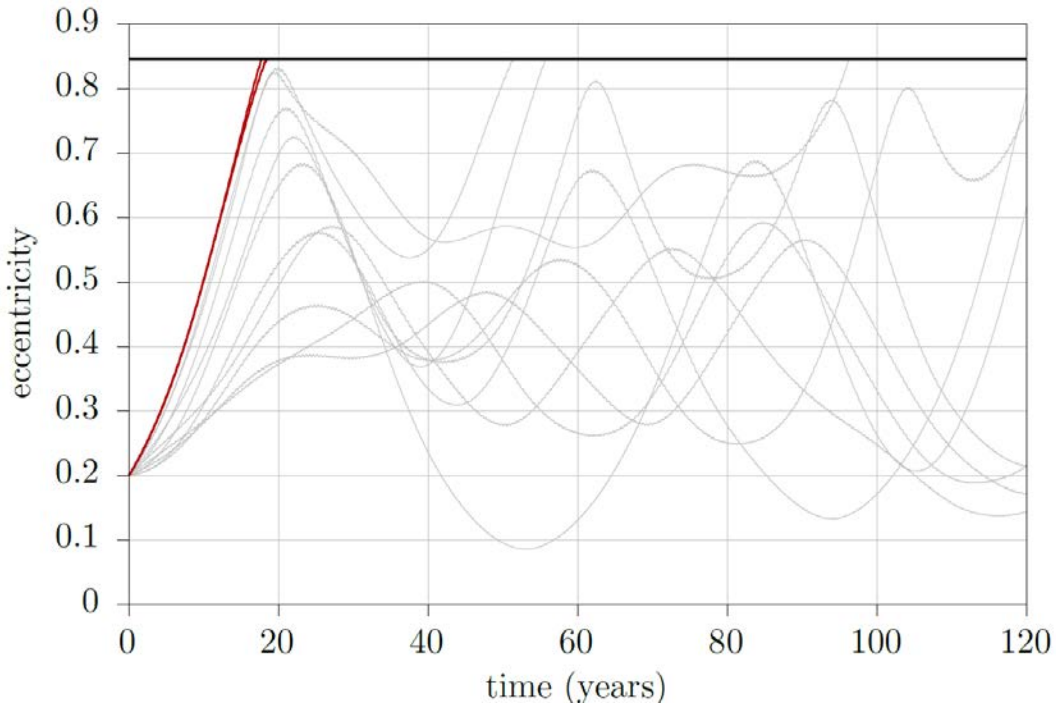


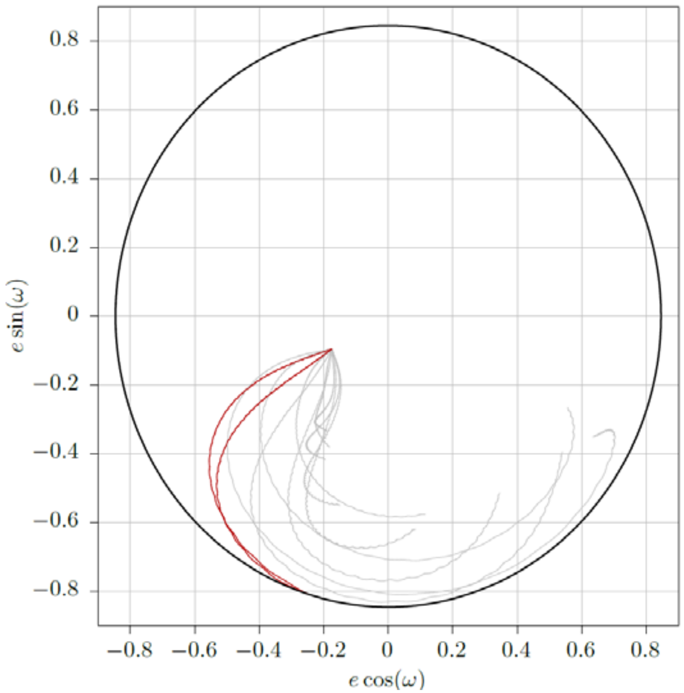
Effective cleansing mechanism

$a = R_{GEO}$
$i = 63^\circ$



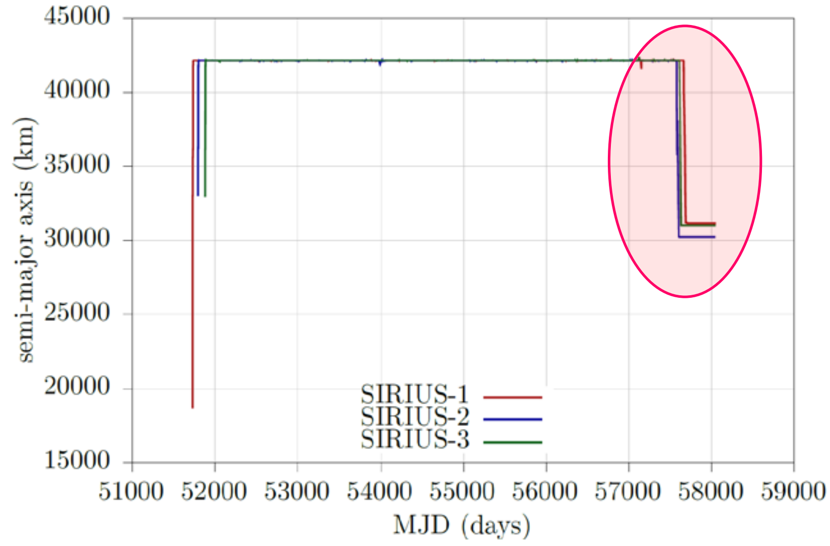
Effective cleansing mechanism

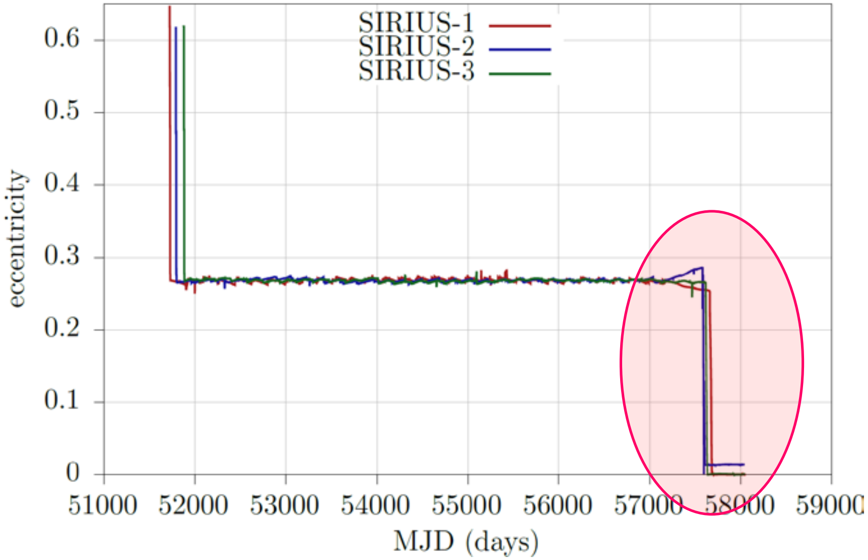


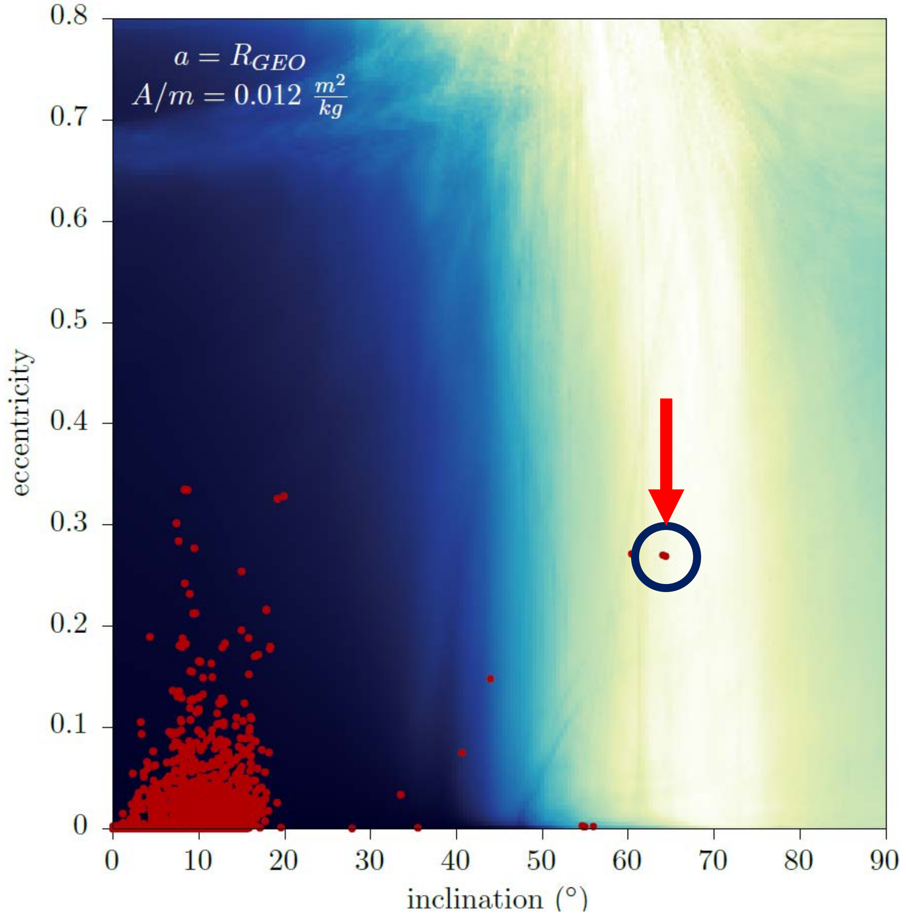


The Sirius constellation

“Missed” opportunity?

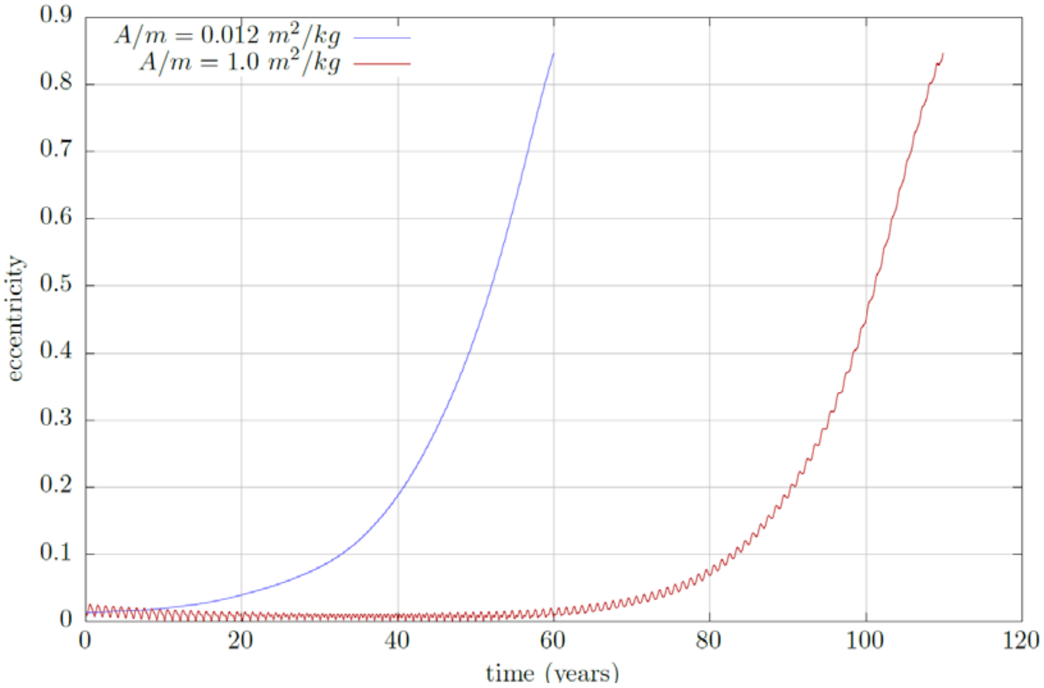


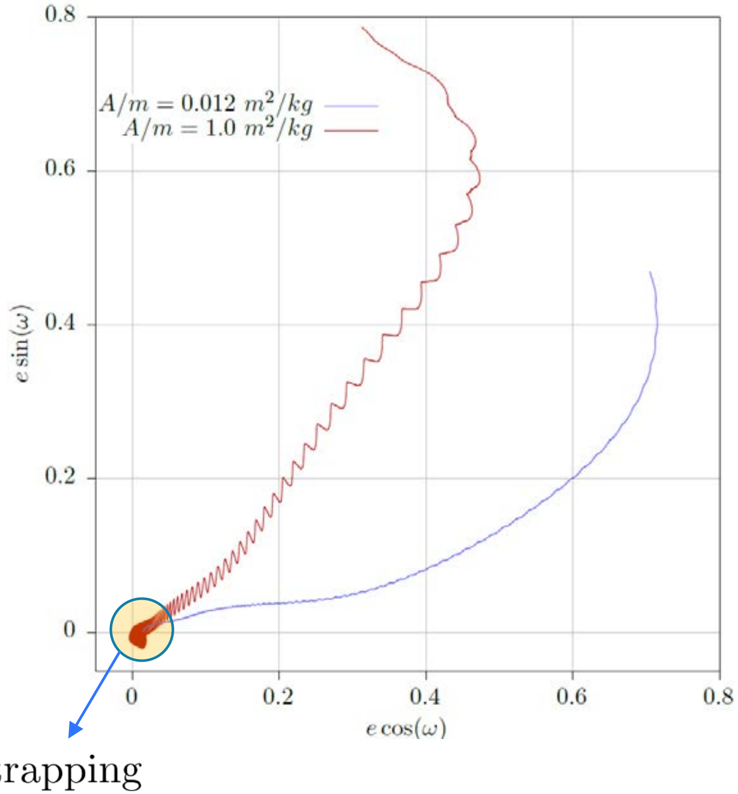




Does enhancing SRP always help?

Suppressing the Lidov-Kozai effect





ANALYTICAL MODELING

Motivation

Hamiltonian reduction on the ecliptic

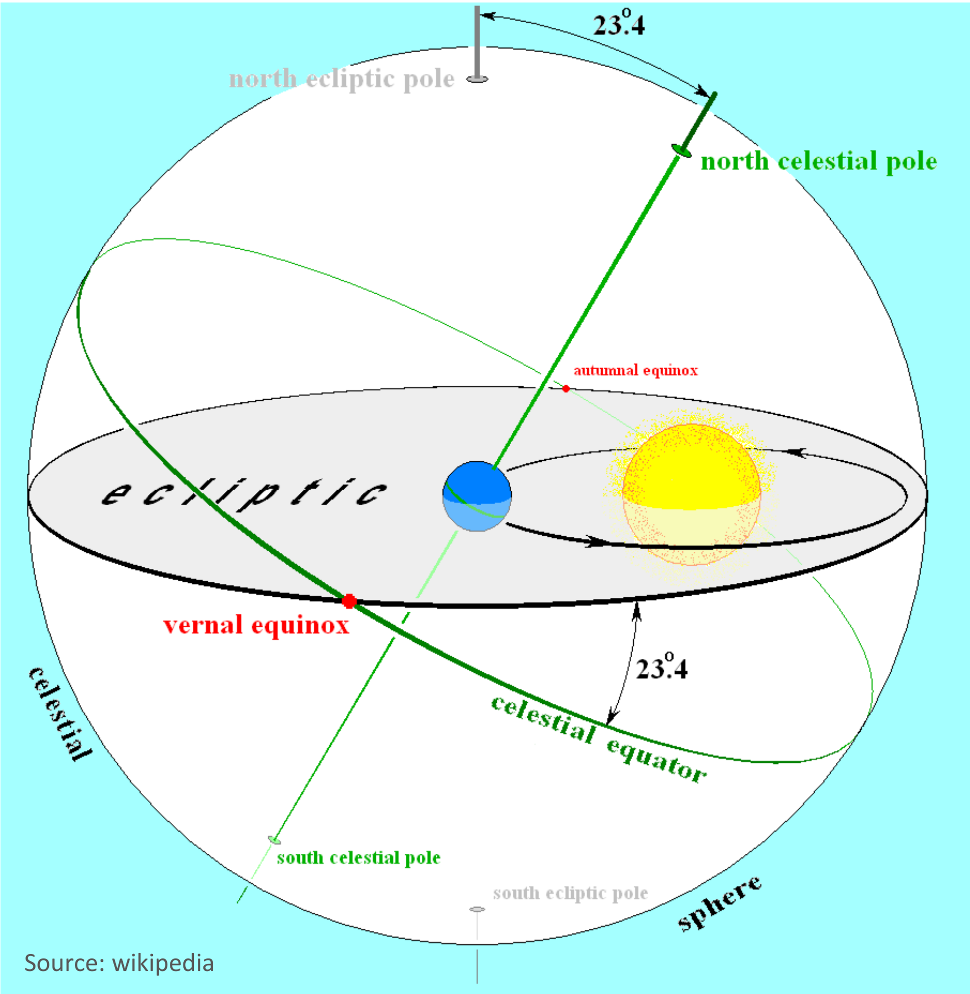
- Artificial satellite theories are developed in a coordinate frame that has the equator as the main plane.
- Geopotential is more conveniently expressed in this frame.
- Third body perturbations more conveniently expressed in the ecliptic.

Question

- Could an analytical theory developed on the ecliptic provide us with more insight for distant Earth satellite orbits?

Analytical modelling

Equatorial and Ecliptic frames



Source: wikipedia

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_1(-\epsilon) \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

Nonlinear relationship between ecliptic and equatorial inclinations

$$\cos I_Q = \cos\varepsilon \cos I - \sin\varepsilon \sin I \cos\Omega$$

$$\cos I = \cos\varepsilon \cos I_Q + \sin\varepsilon \sin I_Q \cos\Omega_Q$$

Analytical modelling

Body positions

equatorial frame

- Satellite's position:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} x_{☾} \\ y_{☾} \\ z_{☾} \end{pmatrix} = R_1(-\epsilon)R_3(-\Omega_{☾})R_1(-i_{☾})R_3(-\theta_{☾})\begin{pmatrix} r_{☾} \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} x_{\odot} \\ y_{\odot} \\ z_{\odot} \end{pmatrix} = R_1(-\epsilon)R_3(-\theta_{\odot})\begin{pmatrix} r_{\odot} \\ 0 \\ 0 \end{pmatrix}$$

ecliptic frame

- Satellite's position:

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} \xi_{☾} \\ \eta_{☾} \\ \zeta_{☾} \end{pmatrix} = R_3(-\Omega_{☾})R_1(-i_{☾})R_3(-\theta_{☾})\begin{pmatrix} r_{☾} \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} \xi_{\odot} \\ \eta_{\odot} \\ \zeta_{\odot} \end{pmatrix} = R_3(-\theta_{\odot})\begin{pmatrix} r_{\odot} \\ 0 \\ 0 \end{pmatrix}$$

Analytical modelling

Model formulation

The orbit of a massless Earth's satellite in high orbit (no drag) can be modelled as a perturbed Keplerian motion

$$\mathcal{H} = H_{\text{kep}} + H_{\text{zonal}} + H_{\text{third-body}}$$

- Keplerian part:

$$H_{\text{kep}} = -\frac{\mu}{2a}$$

- Zonal Harmonics:

$$H_{\text{zonal}} = -\frac{\mu}{r}\sum_{j\geq 2}\left(\frac{R_\oplus}{r}\right)^j C_{j,0}P_{j,0}(\sin\phi)$$

- Third-body attraction (Sun and Moon):

$$H_{\text{third-body}} = -\frac{\mu'}{r'}\left(\frac{r'}{||\mathbf{r}-\mathbf{r}'||} - \frac{\mathbf{r}\cdot\mathbf{r}'}{r'^2}\right)$$

Analytical modelling

Zonal part

Reduction of the J_2 part of the Hamiltonian:

$$H_{J_2} = \frac{\mu}{r}\left(\frac{R_\oplus}{r}\right)^2 J_2 P_2(\sin\phi)$$

$$\sin\phi = \frac{z}{r}$$

$$\sin\phi = \frac{z}{r} = \frac{\zeta\cos(\epsilon) + \eta\sin(\epsilon)}{r}$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, -, -, -; \mu, J_2, R_\oplus)$$

$$= \frac{J_2 R_\oplus \mu (3\sin^2 i - 2)}{4a^3\eta^3}$$

$$\eta = \sqrt{1 - e^2}$$

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, \Omega, -, -; \mu, J_2, R_\oplus, \epsilon)$$

Analytical modelling

Sun’s perturbing effect

Reduction of the Sun's perturbing effect

$$H_\odot = -\frac{n_\odot a_\odot^3}{r_\odot}\left(\frac{r}{r_\odot}\right)^2 P_2(cos\psi_\odot)$$

$$\cos(\psi_\odot) = \frac{xx_\odot + yy_\odot + zz_\odot}{rr_\odot}$$

$$\cos(\psi_\odot) = \frac{\xi\xi_\odot + \eta\eta_\odot + \zeta\zeta_\odot}{rr_\odot}$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_\odot = \bar{H}_\odot(a, e, i, \Omega, \omega, -, \theta_\odot; n_\odot, a_\odot)$$

Analytical modelling

Moon’s perturbing effect

Reduction of the Moon’s perturbing effect

$$H_{\leftmoon} = -\beta \frac{n_{\leftmoon} a_{\leftmoon}^3}{r_{\leftmoon}} \left(\frac{r}{r_{\leftmoon}}\right)^2 P_2(\cos\psi_{\leftmoon})$$

$$\cos(\psi_{\leftmoon}) = \frac{x x_{\leftmoon} + y y_{\leftmoon} + z z_{\leftmoon}}{r r_{\leftmoon}}$$

$$\cos(\psi_{\leftmoon}) = \frac{\xi \xi_{\leftmoon} + \eta \eta_{\leftmoon} + \zeta \zeta_{\leftmoon}}{r r_{\leftmoon}}$$

We average in closed form over the satellite’s mean anomaly

$$\bar{H}_{\leftmoon} = \bar{H}_{\leftmoon}(a, e, i, \Omega, \omega, -, \Omega_{\leftmoon}, \theta_{\leftmoon}; \beta, n_{\leftmoon}, a_{\leftmoon}, i_{\leftmoon}, \epsilon)$$

We average one more time again in closed form, over the Moon’s mean anomaly

$$\bar{\bar{H}}_{\leftmoon} = \bar{\bar{H}}_{\leftmoon}(a, e, i, \Omega, \omega, -, \Omega_{\leftmoon}, -; \beta, n_{\leftmoon}, a_{\leftmoon}, i_{\leftmoon}, \epsilon, \eta_{\leftmoon})$$

Analytical modelling

Advantage of the ecliptic frame

The full system is

$$\bar{\bar{H}} = \bar{H} + \bar{H}_{\odot} + \bar{\bar{H}}_{☾}$$

and is still of **2.5** degrees of freedom

$$\bar{\bar{H}} = \bar{\bar{H}}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{\odot}; \mu, J_2, R_{\oplus}, \epsilon, n_{\odot}, a_{\odot}, n_{☾}, a_{☾}, \eta_{☾})$$

HOWEVER

In the ecliptic representation time dependencies are always coupled with the ecliptic node of the satellite.

Analytical modelling

Further ecliptic reduction

Therefore, we can proceed with a further **elimination of the ecliptic node**. This is accomplished by working in a suitable **rotating frame** and is a valid operation when the perturbations are **of the same order**, i.e. for **distant** Earth's satellites.

$$\bar{\bar{H}}_{J_2} = \frac{J_2 R_\oplus^2 \mu (3\cos^2 i - 1)(3\sin^2\epsilon - 2)}{8a^3\eta^3}$$

$$\bar{\bar{H}}_\odot = a^2 n_\odot^2 \left(-\frac{15}{16} e^2 \cos 2\omega \sin^2 i + \frac{1}{16}(2+3e^2)(3\sin^2 i - 2) \right)$$

$$\bar{\bar{\bar{H}}}_{☾} = -\frac{a^2 n_{☾}^2 \beta (3\cos^2 i_{☾} - 1)((2+3e^2)(3\cos^2 i - 1) + 15e^2 \sin^2 i \cos 2\omega)}{32\eta_{☾}^2}$$

Analytical modelling

Lidov-Kozai type Hamiltonian

The reduction on the ecliptic results in a **1 D.O.F** Lidov-Kozai type Hamiltonian

$$\bar{\bar{\bar{H}}} = \frac{A}{\eta^3}(2 - 3\sin^2 i) + B((2 + 3e^2)(2 - 3\sin^2 i) + 15e^2 \sin^2 i \cos 2\omega)$$

where

$$A = -\frac{J_2 R_\oplus \mu}{8a^3}(2 - 3\sin^2 \epsilon)$$

and

$$B = -\frac{1}{16}\left(n_\odot^2 + \frac{n_{☾}^2}{\eta_{☾}}\beta\frac{3\cos^2 i_{☾} - 1}{2}\right) a^2$$

The system no longer depends on M and Ω, therefore the semi-major axis a is constant and

$$\sqrt{1 - e^2}\cos i = \text{constant}$$

Analytical modelling

Study of the reduced model

We introduce the non-singular elements

$$k = e\cos\omega,\ \ h = e\sin\omega$$

and the equations of motion are

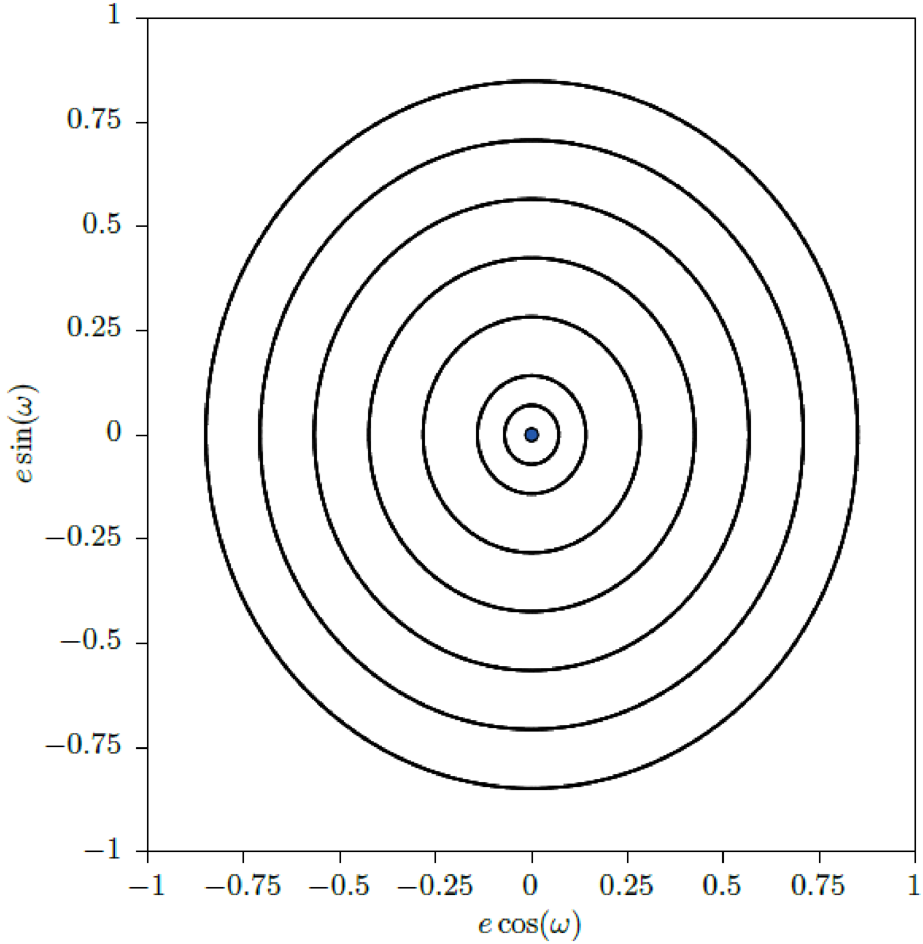
$$\frac{dk}{dt} = -\frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dh}$$
$$\frac{dh}{dt} = \frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dk}$$

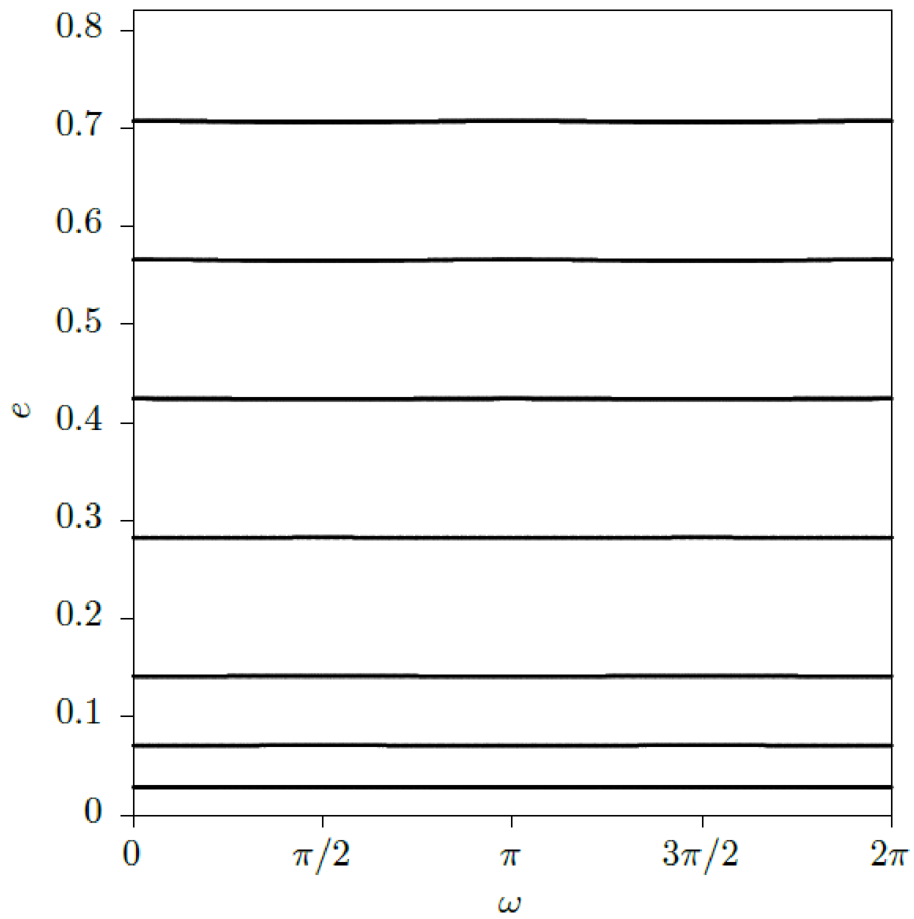
- Equilibrium points: $dk/dt = dh/dt = 0$
- Stability determined from the eigenvalues of the linearised system
- Parameter space of (a, i_{circ})

Analytical modelling

Study of the reduced model

Low inclinations at all altitudes

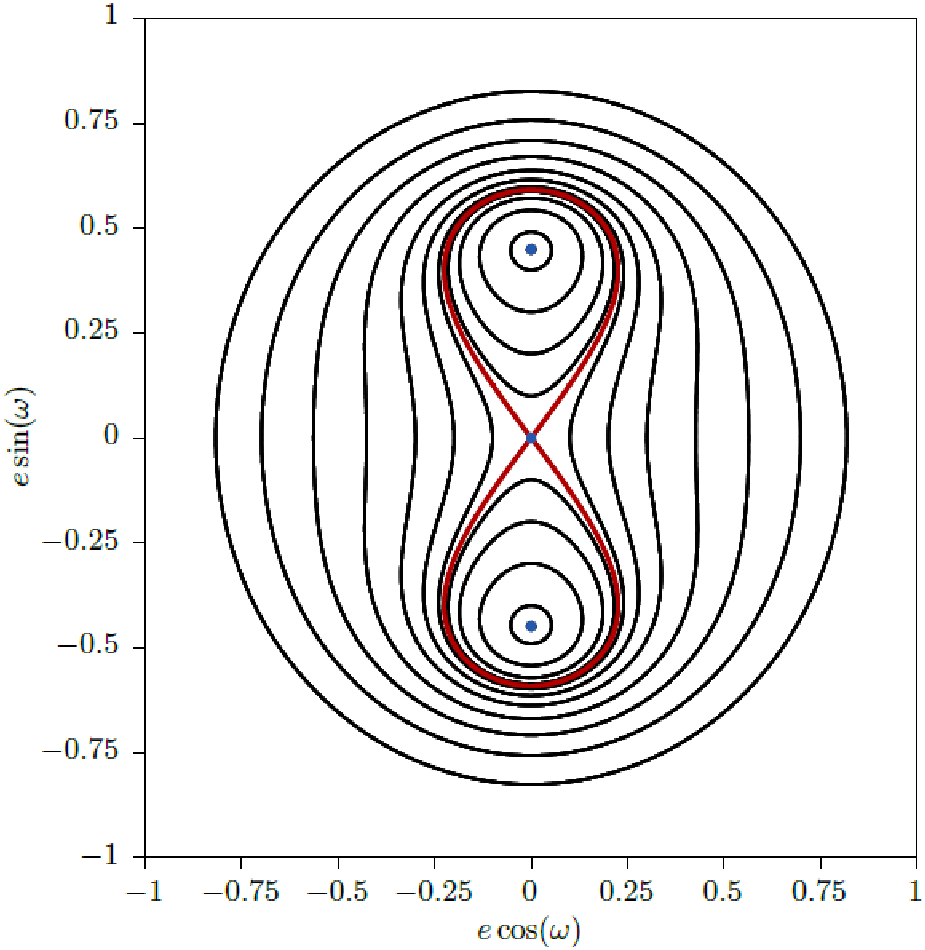


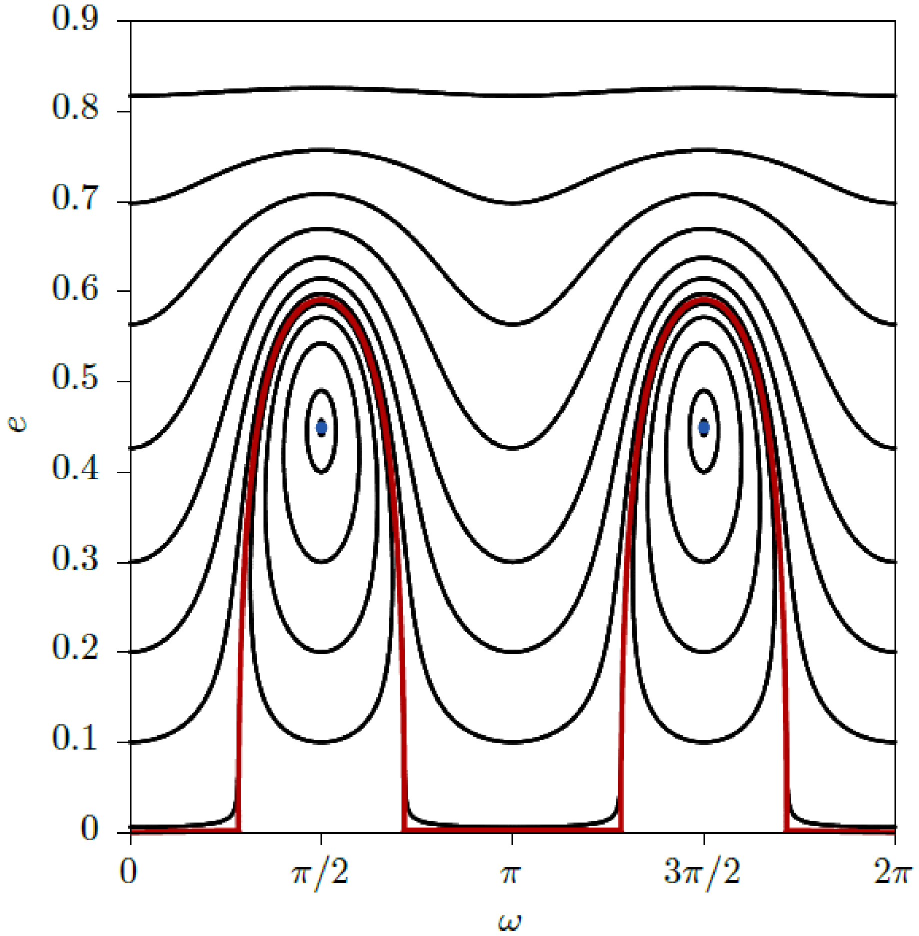


Analytical modelling

Study of the reduced model

Moderate inclinations at high altitudes

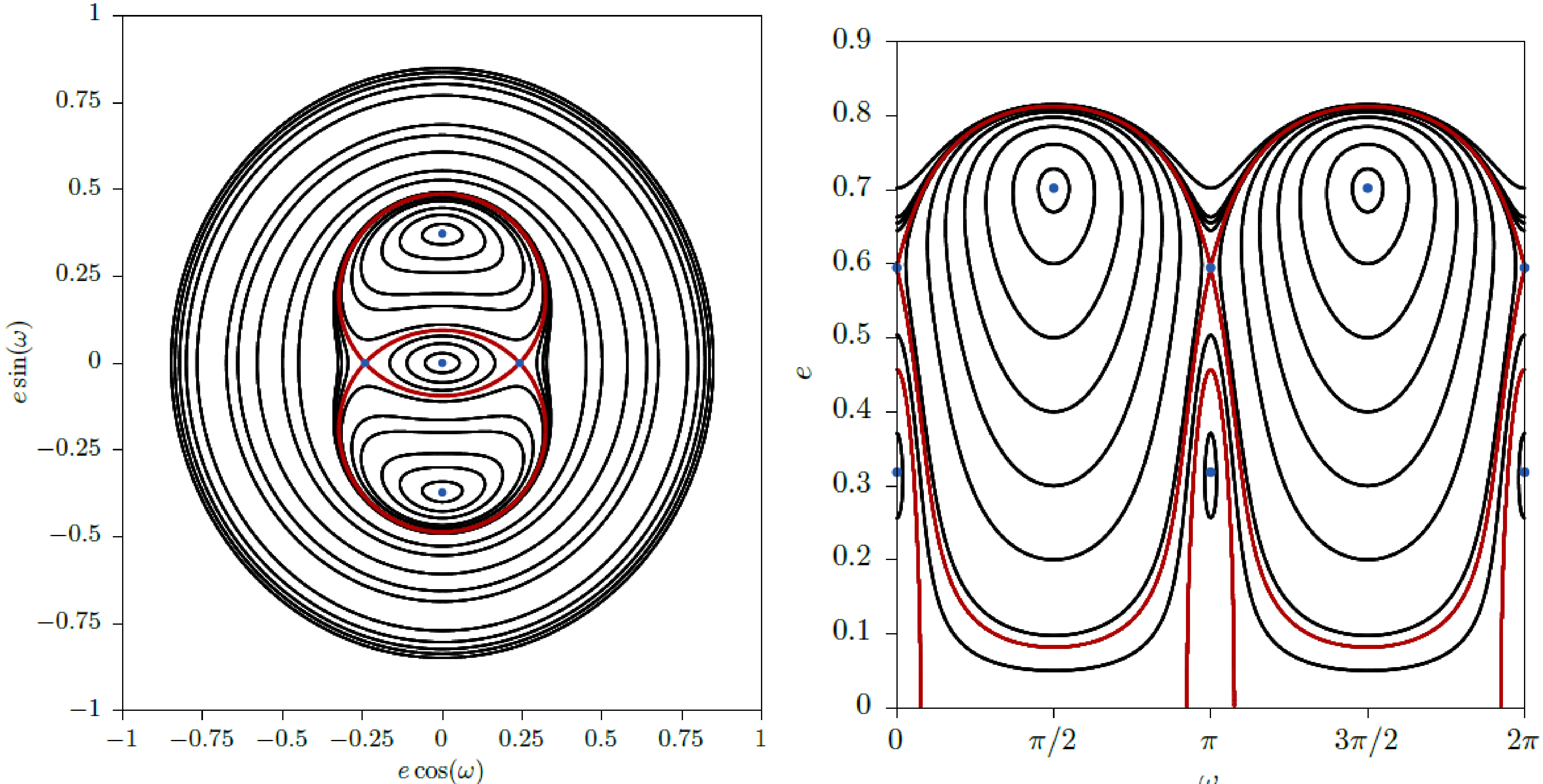




Analytical modelling

Study of the reduced model

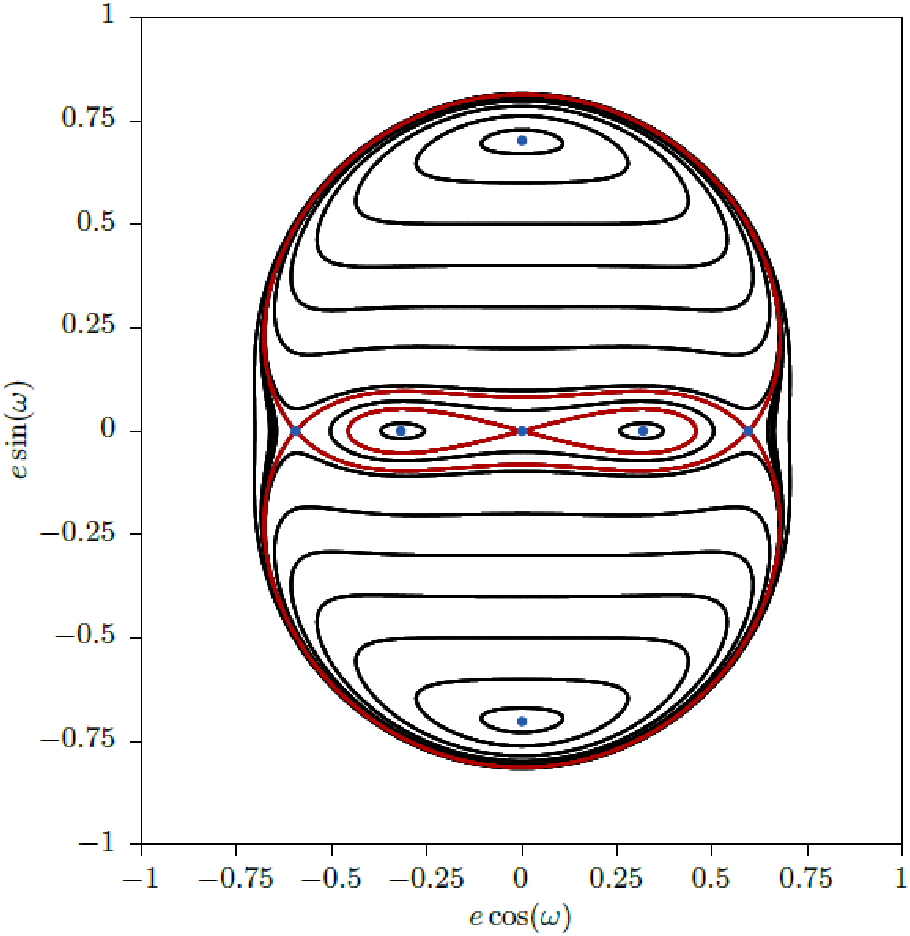
Polar inclinations at medium altitudes

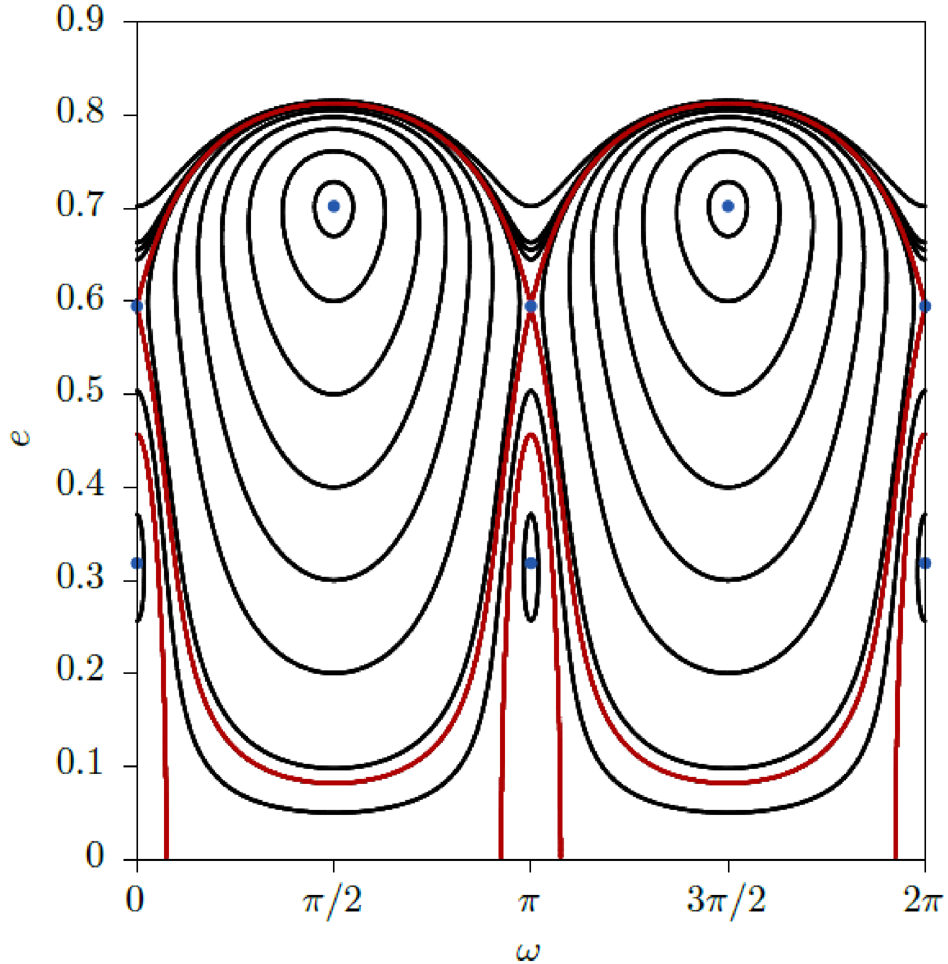


Analytical modelling

Study of the reduced model

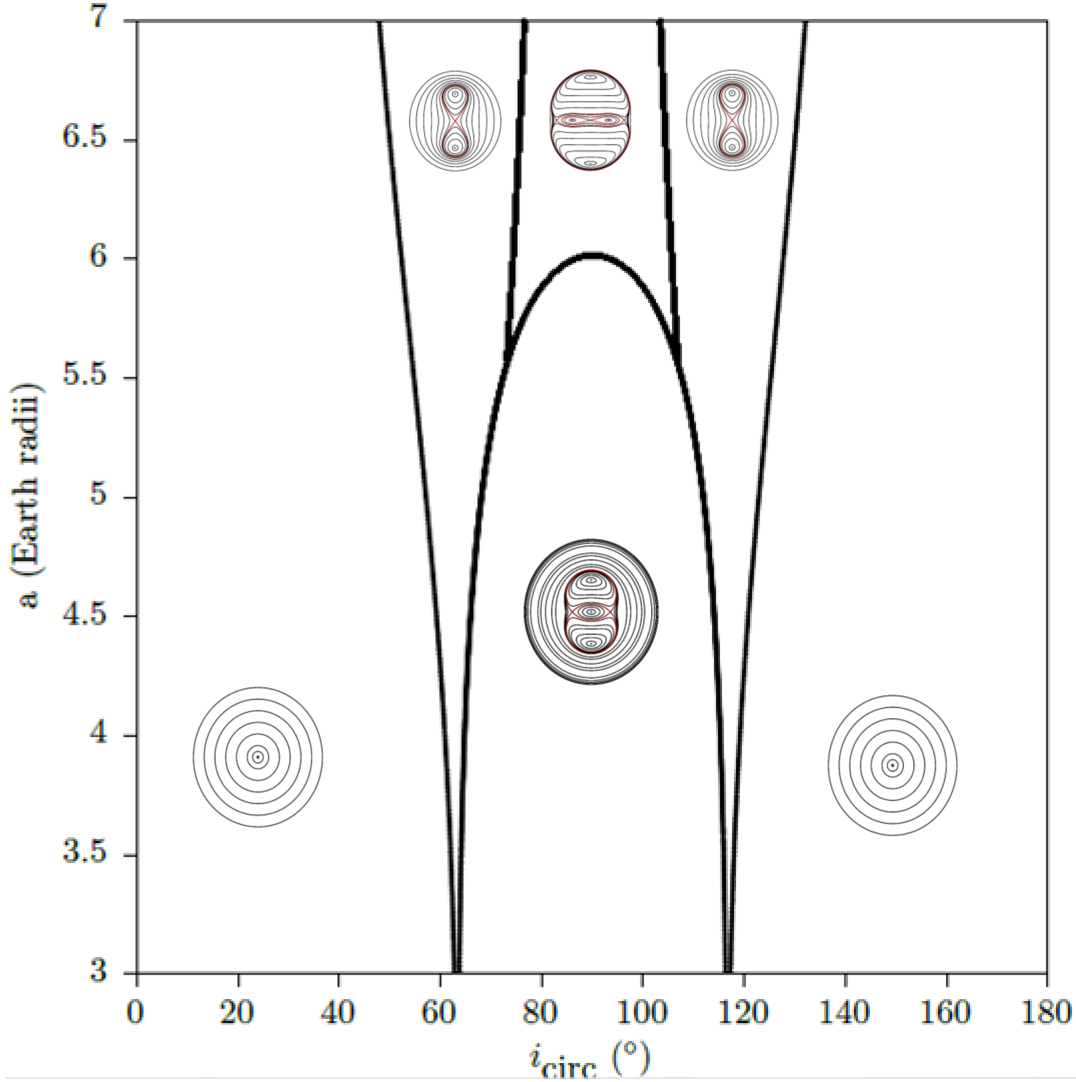
Polar inclinations at high altitudes





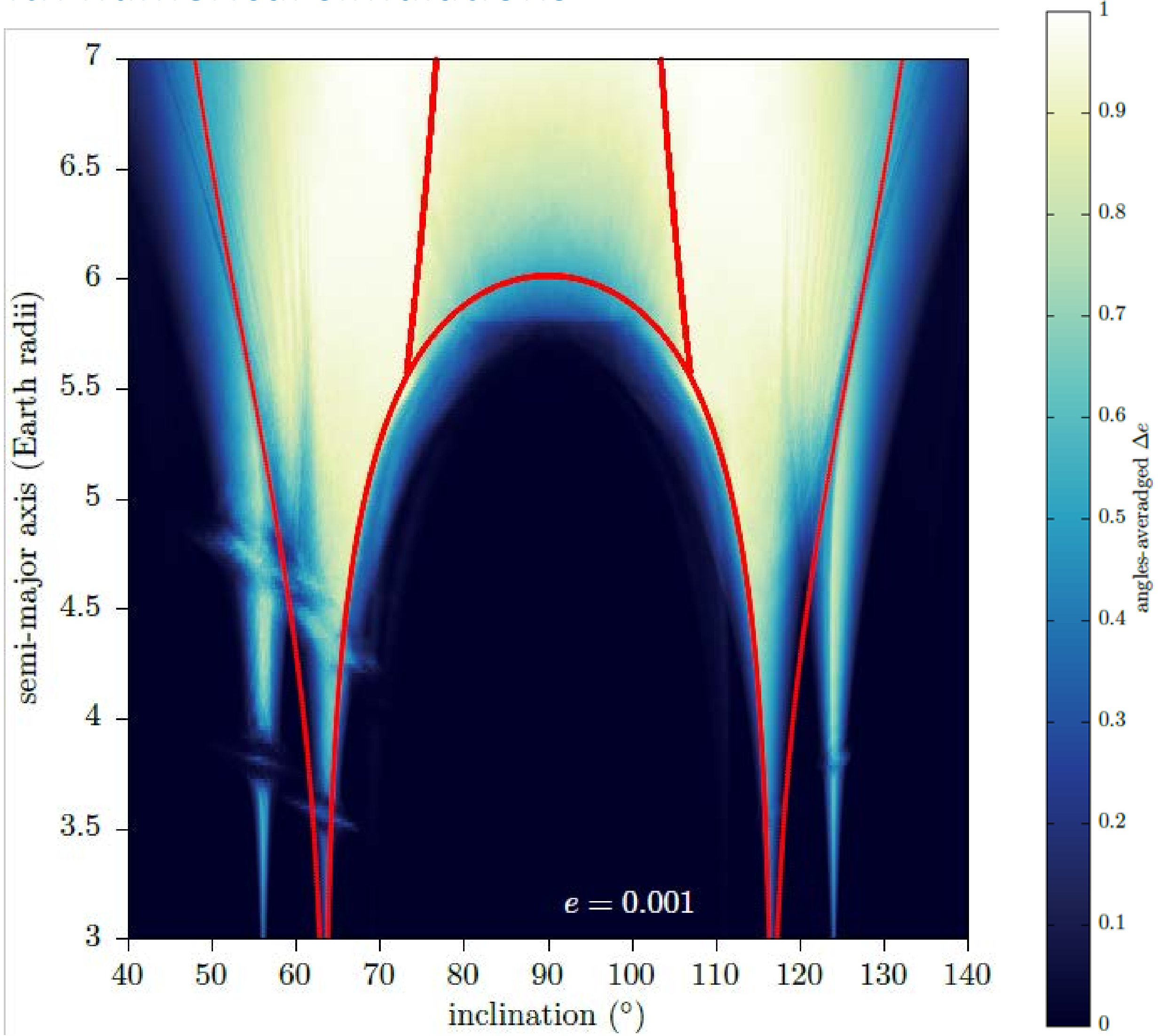
Analytical modelling

Bifurcation diagram



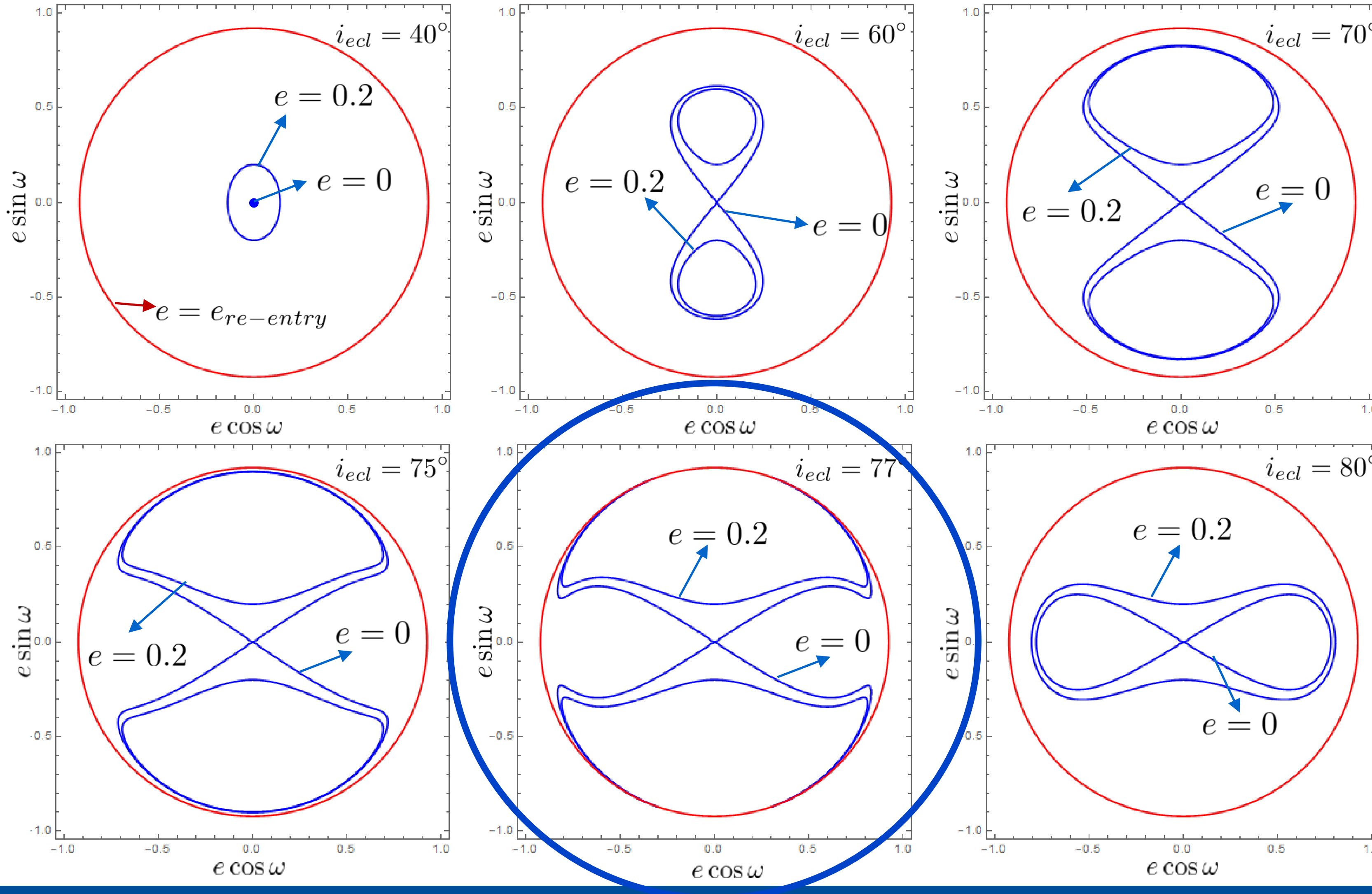
Analytical modelling

Comparison with numerical simulations



Analytical modelling

Disposal design



Conclusions

Numerical investigation

- For low initial inclinations: graveyard orbits with low variation of eccentricity.
- For inclined geosynchronous natural re-entry is possible.
- Optimise disposal manoeuvre for each particular end-of-life scenario.

- Is a single equation guideline for GEO enough?
- Could eccentric and inclined, small size constellations lead us to a sustainable exploitation of GEO?

- All maps and manoeuvres calculated will be made public on the ReDSHIFT web site (http://redshift-h2020.eu).
- ReDSHIFT software tool for EOL disposal calculation will be available online

Conclusions

Analytical modelling

- We have reduced the problem of high Earth satellites using an analytical representation.
- The resulting 1 D.O.F. system describes the in plane stability.
- We studied the reduced phase-space by computing the equilibrium points and their stability.
- We have calculated the bifurcation diagram.

Further work:

- Recover the short-periodic terms.
- Add more perturbations, second order J2 and up to P4 for the Moon.
- Study the equilibria and their bifurcation on a sphere.
- Exploit the reduced dynamics for preliminary mission design.

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Towards a sustainable exploitation of the geosynchronous orbital region

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