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Design of disposal orbits for high altitude spacecraft with a semi-analytical model

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Abstract

This work presents the design of optimal strategies for the post-mission disposal of satellites in high-altitude orbits by exploiting the luni-solar effects. The dynamics of distant Earth satellites is mainly influenced by the third body perturbations of the Moon and the Sun, coupled with the Earth's oblateness. In this paper, an analytical model is proposed for designing the spacecraft's disposal strategy to achieve natural re-entry by exploiting the long-term effect of the natural perturbations, enhanced also by impulsive manoeuvres. The design of the disposal manoeuvre is fully done on the reduced phase space and the results are tested against an optimisation procedure using a semi-analytical propagation. A hypothetical Venus mission and the ESA's INTEGRAL mission are chosen as test cases to demonstrate the efficiency of the fully-analytical procedure. The proposed method can be used for the preliminary design of re-entry or graveyard disposal orbits for satellites operating in high-altitude orbits.

Keywords: end-of-life disposal, optimal manoeuvres, orbital perturbations, averaged models.

Introduction

The increasing number of satellites orbiting the Earth gives rise to the need for investigating disposal strategies for space vehicles, to keep operative orbits safe for future space missions. In the last years, several studies have been conducted focused on designing end-of-life trajectories. This work aims to define optimal disposal options for the end-of-life manoeuvre of spacecraft in Highly Elliptical Orbits (HEOs), employing a completely analytical model for the underlying dynamics. An adequate approximation of the orbit evolution in time requires a model including at least the J_2 and the third body disturbing function, the latter expanded up to the fourth order in the parallax factor [1]. Following the classical theory, the analytical expressions of the disturbing potential due to the external perturbations are derived in the planetocentric equatorial frame. To model the end-of-life disposal, the short-period effects are negligible, and they are removed by using a double averaging procedure on the potential function. This procedure results in secular and long-term variation in all the orbital parameters, except for the semi-major axis, which becomes a constant of the motion.
From the latest report provided by ESA's Space Debris Office [2], the number of debris objects regularly tracked by Space Surveillance Networks is about 21,000. Since its foundation in 1993, the Inter-Agency Space Debris Coordination Committee (IADC) defines the recommended guidelines for the mitigation of space debris. The removal of any object in Low Earth Orbits (LEOs) is required within 25 years after the end-of-mission, while for Geostationary Orbits

(GEOs) the guideline is to move to a graveyard orbit 250 km above. For the HEOs there is no regulation yet, but, since many current and future missions target that region (e.g. Proba-3, INTEGRAL, XMM-Newton, Cluster II, Image, Themis, Chandra, IBEX), the implementation of a strategy is highly recommended. In this work, we consider the exploitation of luni-solar perturbations for the post-mission disposal of satellites in high-altitude orbits about the Earth. The dynamics of this region is mainly influenced by the effects of the third body perturbations due to the gravitational attraction of the Moon and the Sun, coupled with the Earth's oblateness [3]. The orbital evolution can be described through the variation of Keplerian elements double averaged over the orbital periods of the spacecraft and the perturbing bodies [4,5]. The luni-solar attraction induces secular and long-term variation to all orbital elements except the semi-major axis. A representation of the system with respect to the plane of the perturbing body yields a one degree of freedom Hamiltonian (system) [1].

In this paper, an analytical model is proposed for designing the disposal manoeuvre to be given to a spacecraft in these orbits with the goal to achieve natural re-entry by exploiting the long-term effect of the natural perturbations, enhanced also by impulsive manoeuvres. The optimal initial conditions during the natural evolution of the argument of perigee and the orbit eccentricity are selected such that, through an impulsive manoeuvre, the new orbit conditions will lead to a natural increase of the orbit eccentricity until re-entry is reached. The design of the disposal manoeuvre is fully done on the averaged phase space and then the results of the fully and semi-analytical model are tested against numerical optimisation, implemented in previous works [4]. Two situations are chosen as test case scenarios. A first simple system involving the effect of only one perturbing body: an orbiter on an HEO trajectory in the Venus-Sun system. A second case scenario, involving a more complex model under the influence of two different third disturbing bodies: an HEO satellite for the Earth-Moon-Sun system. For this situation, the design of the end-of-life disposal of the INTEGRAL mission is chosen as a test case. The optimisation procedure is implemented to compute the optimal manoeuvre magnitude and direction, as well as the true anomaly along the orbit where the disposal is performed. Global optimisation is imposed through the genetic algorithm together with a multi start method to validate the results. The disposal options for the real case scenario, INTEGRAL mission, are designed. The re-entry is imposed by targeting a perigee altitude after the delta-v manoeuvre below the atmospheric interface: for this mission, it was already studied that an altitude equal or lower than 50 km is necessary to correctly assess the disposal [4]. As demonstrated in previous works [1,6], numerical optimisation is necessary for a very accurate design of the disposal manoeuvre in the Earth case scenario, while the purpose of this work was to produce a method to preliminary estimate the manoeuvre effort. The proposed method can be used for the preliminary design for the disposal of satellites in high altitude orbits with the operational orbit in Medium Earth orbit or above.

Model Definition

The dynamics of satellites on HEOs is here discussed, considering the most relevant external perturbations, starting from literature analysis [3]. For the secular and long-term analysis, first a double-averaged model is implemented, then, to produce a two-dimensional Hamiltonian representation of the system, the node elimination is applied to drop the dependence on the satellite node. The overall model is implemented in the planet equatorial frame considering the following disturbing effects:

- second order of the zonal harmonics of the planet's gravity potential J_2;
- third body perturbation up to the 4th order, as in [4], using the Legendre polynomials:
  - Sun perturbation for the Venus case,
  - Sun and Moon perturbation for the Earth case.

The dynamic of a satellite is then represented in the Hamiltonian formulation for a massless orbiting body [7]:

$$H = H_{kep} - R = -\frac{\mu}{2\,a} - R_{zonal} - R_{3b}, \tag{1}$$

where the first term represents the Keplerian contribution, with μ the planet's gravitational parameter, R_{zonal} represents the zonal harmonic contribution and R_{3b} the third body perturbation effect [6,8,9]:

$$R_{zonal} = -\frac{\mu}{r} J_2 \left(\frac{R_\alpha}{r}\right)^2 (3\sin^2\delta - 1), \tag{2}$$

where J_2 is the zonal coefficient, R_α is the planet mean equatorial radius, r is the satellite position vector and δ is the geocentric latitude.

$$R_{3b} = -\frac{\mu_{3b}}{r_{3b}} \sum_{l=2}^{4} \delta^l \left(\frac{r}{a}\right)^l P_l[\cos S], \tag{3}$$

where μ_{3b} is the third body gravitational parameter, r_{3b} is the third body's position vector, S is the angle between the satellite and the third body position vector measured from the central planet and δ is the ratio between the satellite semi-major axis and the third body position vector, where $\delta = a/r_{3b}$. The cos S term can be expressed in terms of the satellite and the third body position vector and, collecting the dependence on the satellite true anomaly, it becomes [8]:

$$\cos S = \hat{P}\cos f + \hat{Q}\cos f. \tag{4}$$

Averaging Procedure

To study the secular and long-term dynamics of the satellites, the first step is to cancel out the short-term effects due to the true anomaly variation along the orbit [6,8]:

$$\bar{\bar{R}} = \int_0^{2\pi}\int_0^{2\pi} R\, dM\, dM_{3b}. \tag{5}$$

Averaging out the short-term effect is beneficial from a computation point of view to study the secular motion of a satellite. The analytical expression of the double-averaged of the potential was cross-checked with literature results [6,8].

The level of accuracy of the double-averaged model was verified by comparing the time propagation of the full, the single and the double averaged model for 25 years of an INTEGRAL-like orbit: the reference condition under study is taken from NASA JPL Horizon Web Interface for INTEGRAL ephemeris at 22/03/2013. The results are comparable with the analysis done for XMM-Newton mission in [4], and they were presented in detail for both Venus' and Earth's missions in [10]. It was found that for the Earth case, the coupling term between the satellite and the Moon node is important for secular and long-term propagation.

Venus-Sun system: double-averaged Hamiltonian

Considering a Venus' orbiter, the long-term evolution is governed by the J_2 zonal contribution and the Sun third body disturbance. After the averaging procedure, the Hamiltonian expression does not depend anymore on the satellite and Sun true anomaly, in addition, Venus is assumed on a circular orbit about the Sun, resulting in a constant r_{3b}, and the J_2 coefficient is constant in time as well. Therefore, it is a function of satellite's Keplerian elements only:

$$\bar{\bar{H}} = \bar{\bar{H}}(e, i, \omega, \Omega) \tag{6}$$

Earth-Moon-Sun: system double-averaged Hamiltonian

Similarly, as in the Venus system, the satellite dynamics for an HEO orbit is mainly influenced by the J_2 zonal contribution coupled with the Moon and Sun third body attraction. In this case, Earth orbit around the Sun is assumed circular, but differently from previous analyses in [6], here the eccentricity of the Moon's orbit about the Earth is retained. If the semi-major axis, the eccentricity, the inclination and argument of perigee of the Moon's orbit are considered constant in time, as well as the Sun distance and the J_2 coefficient, the resulting Hamiltonian expression is a function of satellite's Keplerian elements, Moon's node only:

$$\bar{\bar{H}} = \bar{\bar{H}}\left(e, i, \omega, \Omega - \Omega_{☾}\right) \tag{7}$$

Note the coupling effect between the satellite and the Moon node: $\Omega - \Omega_{☾}$. This term is important for the correct dynamic evolution of satellite orbit about the Earth.

Reduced Hamiltonian Formulation

To obtain a two-dimensional phase space representation it is necessary to reduce the Hamiltonian expression for orbit propagation to a one-degree-of-freedom formulation. The node elimination procedure, already used in [10], consists of averaging out the expressions in Eq. 6 and 7 over the satellite's node Ω. After this procedure, the Hamiltonian formulation is a time-invariant expression and, since the semi-major axis is a constant of motion in the secular evolution, it is a function of eccentricity inclination and argument of perigee. At this point, the Kozai parameter is introduced to relate the eccentricity and inclination to the initial condition of the orbit. It is a constant of motion and it is defined from [11]:

$$\Theta = (1 - e^2)\cos^2 i = \Theta_0 \tag{8}$$

By expressing the satellite orbit's inclination as a function of eccentricity and initial conditions only, the resulting Hamiltonian is a one-degrees-of-freedom expression, related to the initial condition of the orbit:

$$H = H(e, \omega, a_0, e_0, i_0) \tag{9}$$

where the subscript zero refers to an initial condition in semi-major axis, eccentricity and inclination. The Hamiltonian expresses the time evolution of eccentricity and perigee anomaly of the satellite orbit for a specific initial condition (e_0, ω_0) [10]:

$$\mathcal{F} = H(e, \omega) - H(e_0, \omega_0) \tag{9}$$

From this relation, the time evolution of satellite eccentricity can be computed from the satellite argument of perigee and initial conditions only: it is a one-degree-of-freedom expression. The phase space maps are now presented for the Venus orbiter and the INTEGRAL-like satellite in Fig. 1. This representation allows computing the maximum eccentricity condition reached by the satellite's orbit in time analytically from Eq. 9, just by computing the stationary points of that function.

End-of-Life Atmospheric Re-Entry

The end-of-life disposal manoeuvre gained importance due to the necessity of reducing the amount of space debris around the Earth for future space missions. For Earth's satellites, the IADC set some guidelines for space debris mitigation in GEO and LEO. For HEOs the disposal is highly recommended, but no guidelines currently exist. For HEOs, two possible strategies can be used, as described in [12]: target a re-entry trajectory or a graveyard orbit. During the disposal, no interaction at all with other orbiting objects, and some very stringent regulation exists in case of a passage in the protected region (GEO and LEO). In this work, the first strategy is implemented. The disposal through an atmospheric re-entry uses the natural decay due to atmospheric drag for the satellite disintegration at end-of-life.

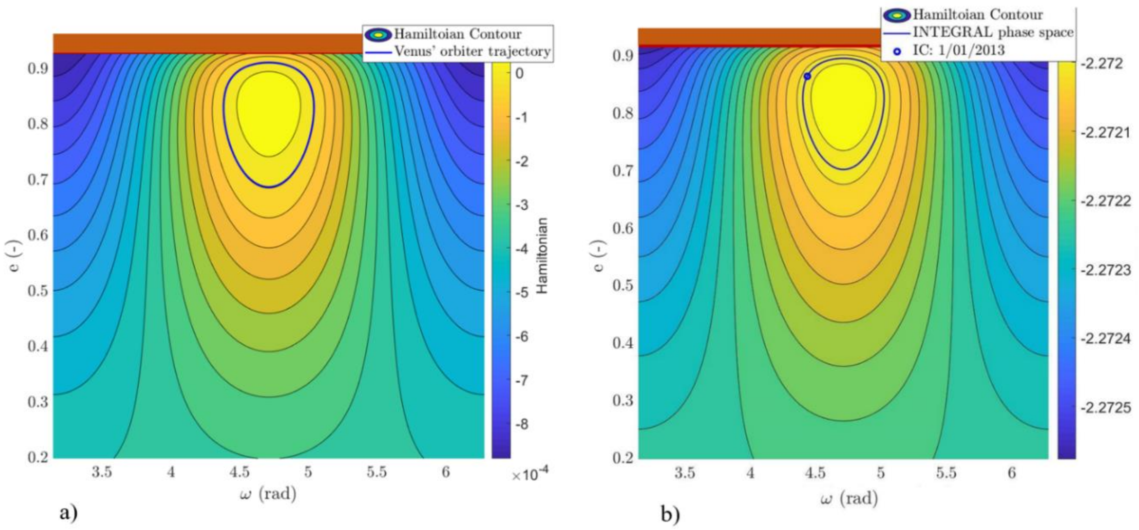

Fig. 1: a) Venus' phase space for an orbiter on an HEO trajectory with a_0=87000 km, e_0=0.87, i_0=60°. The blue trajectory is the orbital time evolution in terms of eccentricity and argument of perigee. b) INTEGRAL phase space with a_0=87839 km, e_0=0.87, i_0=61.5°. With orbital time evolution described by the blue line.

A satellite, orbiting on a trajectory with at least the perigee below the atmospheric interface, suffers the drag effect and tends to naturally decay towards the Earth surface. The atmospheric interface for atmospheric re-entry is typically set at 120 km for the Earth as in [13], and at 250 km for Venus as in [14], where above that altitude, the drag effect is not significant for satellite disposal, even if produce a variation in orbital elements. For the Earth atmospheric re-entry, a stringent procedure exists. The re-entry results in the space vehicle breakup. The major breakup shall result in a minimal amount of survival mass at the Earth's surface. The secondary risk is evaluated for each re-entry vehicle: the probability of impact with the ground must be minimised since it could lead also to human casualties. The risk for aviation or other operative satellites impact is considered as well, but it's very difficult to produce a precise model during the uncontrolled re-entry. A better approach is to develop at least a semi-controlled re-entry, with known orbital parameters at the atmospheric interface [15,16,17]. In this way, the risk assessment is more accurate since the satellite is inserted in a specific trajectory.

Venus atmospheric entry condition

The target altitude for atmospheric entry of a Venus' orbiter is selected starting from the study done for Venus Express mission, which ends its operative life with a de-orbit trajectory [18]. The de-orbit happens when the perigee altitude is below the atmospheric interface. For Venus express, the drag effect starts being significant below 200 km. To maintain the operative orbit of Venus Express during the mission extension several perigee risings were performed to gain an altitude above 200 km. To assure a correct disposal design for the preliminary analysis, the atmospheric entry condition was set at 130 km for Venus' orbiter:

$$h_{p,min} \leq 130 \text{ km} \tag{10}$$

Earth atmospheric re-entry condition

For the Earth's re-entry condition, several studies were already developed [4,19]. For INTEGRAL mission the target perigee is selected around 50 km, as in [4], to achieve a correct re-entry. This is due to the higher velocity at the atmospheric interface, which could result in a partial fragmentation if the target perigee is above 50 km.

$$h_{p,min} \leq 50 \text{ km} \tag{11}$$

Atmospheric Re-Entry Disposal Design

The disposal strategy for atmospheric re-entry is designed performing a single impulsive manoeuvre to enhance the effect of natural orbit evolution of the spacecraft under the influence of external perturbations of the third body and the zonal harmonic. This strategy was already adopted in previous analyses in [4] and in [6] to produce a change in the orbital parameter so that the resulting trajectory evolution in time produces a perigee altitude below the target perigee. The new set of the orbital elements, after the delta-v is produced, is propagated in time, considering the effect of external source of perturbation through two different approaches: the numerical propagation of the double-averaged model and the time propagation through the one-degrees-of-freedom Hamiltonian formulation in Eq. 9.

Disposal Manoeuvre Design

The single impulsive manoeuvre is modelled by following the approach described in [4] and in [6]. The impulsive manoeuvre in terms of delta-v is described in the {***t, n, h***} frame, with ***t*** the unit vector tangent to the velocity vector, ***h*** the unit vector in the direction of the orbital angular momentum and ***n*** completes the orthogonal frame, by:

- the magnitude of the impulse, Δv,
- the in-plane angle, α,
- the out-of-plane angle, β.

The geometry of the delta-v is represented in Fig. 2. The impulse is characterised by three components: tangential, normal and out-of-plane. The mathematical description is provided in terms of the angles α and β:

$$\mathbf{\Delta v} = \Delta \mathrm{v} \begin{bmatrix} \cos\alpha\cos\beta \\ \sin\alpha\cos\beta \\ \sin\beta \end{bmatrix} \tag{12}$$

This delta-v provides a finite variation of the orbital elements through the Gauss planetary equations. In case an impulsive manoeuvre is given, the Gauss planetary equations can be written, in first approximation, in terms of impulsive variation of the velocity vector [20], as function of the radius and the velocity at the point where the instantaneous change is provided. The Gauss planetary equations are the following:

$$\begin{aligned}
\delta a &= \frac{2a^2 v_d}{\mu}\delta v_t \\
\delta e &= \frac{1}{v_d}\left[2(e+\cos f)\delta v_t - \frac{r_d}{a}\sin f\ \ \delta v_n\right] \\
\delta i &= \frac{r_d \cos u_d}{h}\delta v_h \\
\delta\omega &= \frac{1}{e v_d}\left[2\sin f\ \ \delta v_d + \left(2e + \frac{r_d}{a}\cos f\right)\delta v_n\right] - \frac{r_d \sin u_d \cos i}{h\sin i}\delta v_h \\
\delta\Omega &= \frac{r_d \sin u_d}{h\sin i}\delta v_h \\
\delta M &= -\frac{b}{e\ a\ v_d}\left[2\left(1+\frac{e^2 r_d}{p}\right)\sin f\ \delta v_t + \frac{r_d}{a}\cos f\ \delta v_n\right]
\end{aligned} \tag{13}$$

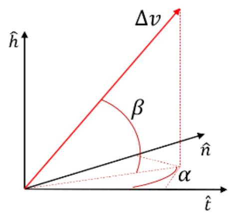

Fig. 2: Impulsive delta-v representation in the t,n,h frame depending on the in-plane angle α and out-of-plane angle β.

where h is the angular momentum, p is the semi-latus rectum, $u_d = \omega + f$ is the argument of latitude, b is the semi-minor axis, dM considers only the instantaneous change in the mean anomaly, r_d and v_d are respectively the radius and the velocity at the point where the instantaneous change is provided. This results in a finite variation of orbital elements of the satellite: $\Delta\mathrm{kep} = \mathrm{Gauss}(\mathrm{kep}(t_m), f_m, \boldsymbol{\Delta v})$. Once the variation of the orbital elements is computed from the manoeuvre, the new orbital parameters are computed as:

$$\mathrm{kep_{new}} = \mathrm{kep(t_m)} + \Delta\mathrm{kep} \tag{14}$$

The new set of orbital elements after the manoeuvre is then used to check for the atmospheric re-entry condition assessment in two different ways:

- semi-analytical method, with time propagation of the double-averaged model,
- fully-analytical method, with an analytical computation of the minimum perigee altitude in time.

Semi-Analytical Method

After the application of the delta-v, the new Keplerian elements are propagated in time using the double-averaged model. This procedure is more efficient than the full numerical propagation but requires some computational time: for each new condition after the delta-v, a time propagation is required to check for the maximum eccentricity value, as in [4,6]. The natural evolution in terms of perigee altitude is computed for the available time interval, resulting in a time history $h_p(t)$. The minimum perigee altitude is then computed as:

$$h_{p,min} = \min\left(h_p(t)\right) \tag{15}$$

where $h_{p,min}$ represents the minimum perigee reached by the satellite orbits after the delta-v manoeuvre is applied. Note that for the manoeuvre, the actual true anomaly of the satellite can be optimised, since in the double-averaged model no dependence on f is retained. For this reason, fixing the other orbital parameters of the satellite a, e, i, ω, Ω the manoeuvre can be optimised in terms of delta-v impulse and satellite true anomaly. Hence, disposal condition is computed with the numerical propagation of each condition after the manoeuvre is applied.

Fully-Analytical Method

This new procedure is implemented in this work starting from the triple-averaged model resulting from the node elimination procedure. The minimum perigee altitude depends on the orbital elements: $r_p = a(1-e)$ and $h_p = r_p - R_\alpha$, with R_α the equatorial mean radius of the central planet. Hence, the target perigee is related to critical eccentricity, once the semi-major axis is defined:

$$e_{cr} = 1 - \frac{h_{p,target} + R_\alpha}{a} \tag{16}$$

This means that the maximum satellite eccentricity e_{max}, corresponding to the minimum perigee altitude $h_{p,min}$, should reach the critical value to assess the atmospheric re-entry. From the Hamiltonian expression of the time evolution of the satellite orbit in Eq. 9, the maximum eccentricity condition can be simply computed from the stationary points of the function:

$$e_{max} = \max(\mathcal{F}(e, \omega)) \tag{17}$$

This procedure has several advantages: computing analytically the stationary point of a function is a much more computationally efficient method than the semi-analytical one. In addition, this means that Eq. 9 can be used to compute the orbital evolution of the satellite in time without any numerical integration for orbit propagation at all. In this case, to understand how the delta-v changes the orbit, the phase space can be used to compute the eccentricity evolution of the orbit, as proposed in [1]. In fact, the phase space representation is very intuitive and allows the visualization of the manoeuvre effect. The delta-v will change the condition of the orbital parameter so that the final trajectory in the phase space would target the critical eccentricity. This means that the final trajectory is tangent to that value, indicating that in time the re-entry condition is achieved. A first simple approach, represented in Fig. 3a, is to target another trajectory in the same phase space, for which the maximum eccentricity is the critical one. The impulsive manoeuvre shall provide a variation in orbital element to maintain the Kozai parameter constant. This is the case of an impulse with null tangential component, hence, the in-plane contribution is all in the normal direction: the in-plane angle is set to 90°. From the analytical expression of the semi-major axis variation in the Gauss equation, a remain constant after the delta-v if it is performed in the normal direction only:

$$\delta a = \frac{2a^2 v_d}{\mu} \delta v_t = 0 \qquad \text{if } \delta v_t = 0 \tag{18}$$

The manoeuvre changes the parameters of the satellites to remain in the same phase space. The disposal is obtained due to an increase of the amplitude of eccentricity oscillations in the phase space. A second approach, represented in Fig. 3b, is to provide an impulsive manoeuvre in a generic direction: both angles α and β can assume a generic value, changing the Kozai parameter. This provides a variation of the phase space representation. Depending on the new value of the semi-major axis, the Hamiltonian contour line could translate up or down. For a reduction of a, the phase space translates towards higher values of the eccentricity, enhancing the disposal condition. The variation in the orbital parameters shall provide a trajectory with the maximum eccentricity equal to the critical one, hence the phase space contours translate up through higher eccentricity values.

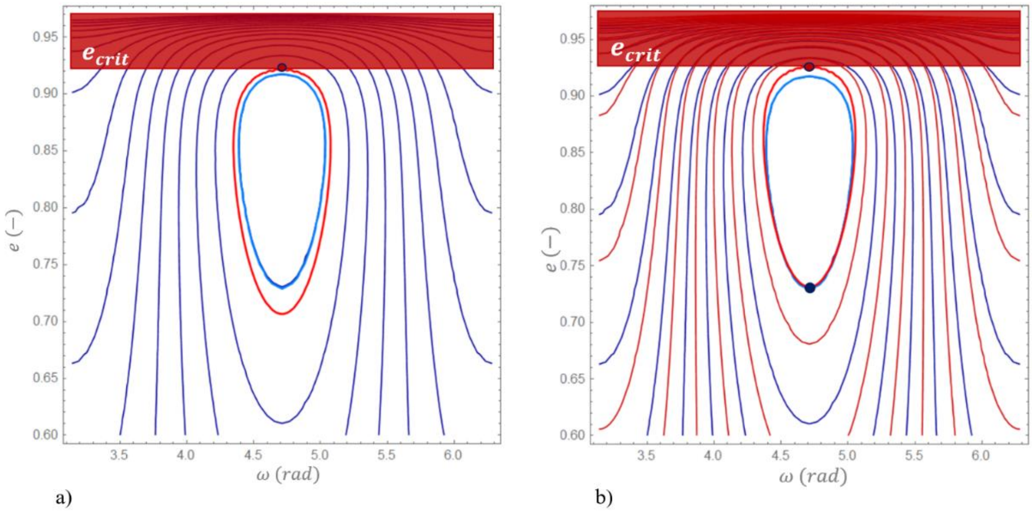

Fig. 3 The light blue trajectory is the evolution from the initial condition, the red one is the evolution of the orbital condition after the manoeuvre. a) Manoeuvre in the same phase space (a_0 = 87000 km). b) Manoeuvre among two phase space (form the blue one to the red one).

Optimisation procedure

An optimisation procedure is necessary to evaluate which is the optimal impulse that provides the desired solution, not only the magnitude of the impulse is optimised, but also its direction. For each initial condition, the re-entry manoeuvre is assessed through an optimisation procedure. This aims to determine the optimal parameters for the definition of the delta-v impulse and the optimal true anomaly for the manoeuvre f_m. An optimal set of parameters is defined: $\boldsymbol{x} = [\alpha, \beta, \Delta v, f_m]$. The optimisation procedure is used to determine the optimal solution for the target eccentricity and the minimum $\boldsymbol{\Delta v}$. Hence, it is a multi-objective optimisation, but the former condition related to the minimum perigee has a higher relative importance than the delta-v optimisation. A multi-objective optimisation aims to optimise more than one function of merit [21]. The cost function for the optimal control problem is select as a quadratic function, that shall provide the solution with the desired accuracy. The general expression of a multi-objective cost function is:

$$J = \frac{1}{2} \sum_i \lambda_i J_i \tag{19}$$

where J_i is a quadratic function and λ_i is the weighting function. The aim of the optimisation is to target the re-entry altitude, set below the atmospheric interface. The disposal manoeuvre shall target the critical eccentricity, corresponding to 50 km of altitude for the Earth's case and to 130 km for Venus. The maximum eccentricity value reached during the long-term propagation shall be compared with the critical one e_{cr}, so that the goal of the optimisation is that in time the maximum eccentricity, corresponding to the minimum perigee altitude, reaches the critical one. The cost function for the optimal altitude is defined as:

$$J_{h_p} = \max\left(\frac{h_{p,min} - h_{p,target}}{h_{p,target}}, 0\right)^2 \tag{20}$$

Differently from the cost function used in [4], the variation in the perigee altitude is divided by the target altitude since it represents the accuracy coefficient for solution determination and acts as a weighting coefficient for the objective function, resulting in an a-dimensional cost function. The second objective of the optimisation is to maintain the delta-v cost the smallest as possible. The onboard fuel at the end of mission is typically very low, and the aim of the strategy shall be to use as much as possible the natural evolution and reducing the propellant consumption. The cost function for the optimal delta-v impulse is defined as:

$$J_{\Delta v} = \left(\frac{\Delta v}{\sigma_v}\right)^2 \tag{21}$$

where σ_v is set equal to 1 km/s to have an a-dimensional representation of the cost function. The main difference with the cost function used in previous works, [4] and [6], is the insertion of the weighting factors $(h_{p,target}, \sigma_v)$ to refer the cost function to the target condition. The goals for the cost function are identified by the performance indices previously defined by J_{h_p} and $J_{\Delta v}$:

$$J = \frac{1}{2}\left(K\, J_{h_p} + W\, J_{\Delta v}\right), \tag{22}$$

where K, W are the weighting constants for the optimisation, as: $K = 1$ and $W = 1 \times 10^{-2}$. The weights act as accuracy coefficient and make the cost function a-dimensional. The weighting constant K and W have been selected to grant the convergence in terms of target perigee for the re-entry condition and minimum variation of the eccentricity for the graveyard case. The delta-v is optimised only after the target condition has been reached. The procedure is performed with a multi-start method, for the search of the best local minima. The

MultiStart.m algorithm in MATLAB® generates multiple local solutions starting from various initial points. The solution is generated in the GlobalOptimSolution.m. This is a MATLAB® object containing information on the local minima: the value of the local minimum, the objective function value, the start and the point that leads to the minimum. On the other hand, in [4], the optimisation was done with the genetic algorithm only. In that work, the optimisation was performed introducing the tournament selection and mutation to maintain the genetic diversity and enhance the algorithm convergence.

Disposal Constraint

The optimisation problem is not an un-constrain problem but requires the setting of bounds and constraints. Linear and non-linear constraints can be imposed on the procedure, based on the set of parameters $\boldsymbol{x} = [\alpha, \beta, \Delta v, f_m]$, that, during the optimisation process, can vary in a certain interval:

$$\begin{array}{ll} \alpha \in (-\pi, \pi) & \beta \in (-\pi/2, \pi/2) \\ \Delta v \in (\Delta v_{min}, \Delta v_{max}) & f_m \in (0, 2\pi), \end{array} \tag{23}$$

where the bounds in Δv are mission dependent. On the other hand, some nonlinear constraints are imposed on the minimisation.

- The perigee radius shall be higher than the Earth radius: $h_p > 0$.
- The new target orbit shall be elliptical: $e_{new} \in (0,1)$

The strategy adopted in this work consists of exploiting one single manoeuvre to assess the final orbit. In a further study, a multi-manoeuvres design can be developed.

Disposal Algorithm

The logic behind the disposal algorithm is now explained. Starting from the initial orbital parameters of the satellite, the algorithm requires as input only three initial conditions:

- initial data of observation at *T*0 = [*yy, mm, dd, h, min, sec*],
- Satellite ephemeris at *T*0: kep0 = $\{a_0, e_0, i_0, \omega_0, \Omega_0, M_0\}$,
- maximum Δv available onboard.

Then, two different solvers are used to validate the procedure. At first the ga.m solver was used, as in [6]. The Genetic Algorithm (ga.m) is a heuristic method based on the natural evolution theory. It is based on the natural selection process to eliminates the bad conditions from one generation to another: during the iterations only, the best solution passes at the successive generation yielding to the best fitness selection. In addition, during the process the mutation was introduced to maintain the diversity within the population: this prevents a premature convergence and ensures that the algorithm terminates once there is no significant difference between two consecutive generations.

On the other hand, the same solution was computed with MultiStart.m. It does not rely on a heuristic method, but the solver searches the best solution by running multiple local solvers starting from various points. It uses a non-linear programming solver (fmincon.m) to find the minimum of a constrained multi-variable function. The assessment of the convergence of this algorithm is more difficult than for the Genetic Algorithm. It is very important to correctly impose the initial conditions and the solver options: the settings of the number of initial points to run is essential to achieve the convergence: in this analysis it was set to 20 initial points. In addition, it must be specified that the initial points should be within the bounds of the inequality's constraints.

The initial conditions for the optimisation are defined by the delta-v parameters: angles (α, β) and magnitude Δv. In addition, the solver can select the best true anomaly for the manoeuvre, since in the single and double-averaged approach the dependence on it was cancelled out: each solution is valid for any value of the true anomaly along the orbit. This should be in any case checked at the end of the optimisation, verifying that the target condition is reached. The value of the objective function defines the accuracy of the solution: one solution is better than another if the objective function is smaller. MultiStart.m and ga.m take the same function evaluations for the computation of the optimal solution, providing two equivalent optimum points. Nevertheless, MultiStart takes about half of the time to find the minima, hence it results more efficient for the case under study. For both the solvers, the same objective function was considered.
In Table 1 the solutions obtained by using the two different solvers are compared. The difference in the final value of the cost function is negligible, while the main aspect to point out is the difference in the computational time. The MultiStart solver is faster in reaching the solution and can be used to generate a family of results for many initial conditions in time for the original orbit. In this way, the best solution in terms of the propellant consumption can be identified: each of them is connected to an optimal epoch for the disposal. These performances are for the fully-analytical methods, where the solver must solve the Hamiltonian for the critical eccentricity condition. Similar results are obtained with the semi-analytical propagation method. In the latter case, the computational time is significantly longer than in the fully analytical computation (2-4 h).

Table 1: Results obtained for the Multistart and the Genetic Algorithm methods for an INTEGRAL like orbit, using a fully-analytical method. Note the difference in the computational time.

Parameters	MultiStart	Genetic Algorithm
$\mathrm{kep_{in}}$	$\{8.7709 \times 10^4\ km, 0.8975, 0.9841\ rad, 4.7123\ rad, 3.0596\ rad, 3.141\ rad\}$	
α_{opt}	-3.137 rad	-3.140 rad
β_{opt}	7.05×10^{-5} rad	7.06×10^{-5} rad
Δv_{opt}	67.9 m/s	67.9 m/s
f_{opt}	3.1411 rad	3.1415 rad
Cost fun J_{opt}	0.023	0.0052
$h_{p,min}$	50.0048 km	50.00 km
$\mathrm{kep_{new}}$	$\{8.6412 \times 10^4\ km, 09256, 0.9841\ rad, 4.7123\ rad, 3.0596\ rad, 3.141\ rad\}$	
Computational time	2.55 min	11.21 min

Results

Two different application are here presented with the application of both the semi and the fully analytical method, to verify the potential efficiency of this new approach. The results from those methods are compared to validate the solution.

- Venus' orbiter: the disposal trajectory is designed as an atmospheric entry with the fully-analytical method only. An altitude of 130 km is set as the target condition.
- Earth's satellite atmospheric re-entry. The case of the INTEGRAL satellite is considered for the design of the disposal manoeuvre, both the fully-analytical and semi-analytical optimisations are considered. In addition, the results of both codes are compared with the manoeuvre options computed in literature.

Venus' Orbiter Disposal Design

In the past years, many space probes visited Venus for scientific purposes. Since the 1960s, NASA starts planning future scientific missions to Venus [22]. In particular, the opportunity to study the atmosphere and clouds of Venus was of great interest: Pioneer Venus probes, Venera missions, Magellan and Venus Express are just some example of missions that visits the planet [23]. Nevertheless, none of them is a suitable case of study to see how the third body perturbation can be used for orbital navigation. In fact, the inclination, the eccentricity or the semi-major axis were not suitable for the analysis, or their trajectory was much affected by the atmospheric drag. For this reason, a fictitious orbiter was considered. It is the equivalent to an HEO trajectory in the Earth-Moon system. Since the dimension and the gravitational attraction of Venus and Earth are quite similar, the parameters for the HEO are considered like the INTEGRAL mission. For the optimisation procedure, the following constraints were identified:

- disposal window of 15 years,
- delta-v interval for the optimisation: 0 - 1.20 km/s,
- target perigee 130 km.

Venus' Orbiter: Mission Scenario

A generic initial condition around Venus is propagated in the time-space 22/03/2013 for 25 years, to study the time evolution of an HEO trajectory. The initial Keplerian elements are the following: $a = 87000$ km, $e = 0.87$, $i = 60°$, $\omega = 4.42$ rad, $\Omega = 4.64$ rad, $M = 2.25$ rad. The orbit evolution is computed in the Venus equatorial frame, considering a satellite with a small area-to-mass ratio (namely $\ll 1$), so that the solar radiation pressure can be neglected. Fig. 4 show the time propagation of the initial Keplerian elements using the single, double and triple averaged model. This is used to verify the accuracy level of the fully-analytical model for the disposal manoeuvre design since it uses the reduced Hamiltonian formulation (called also triple-averaged model).

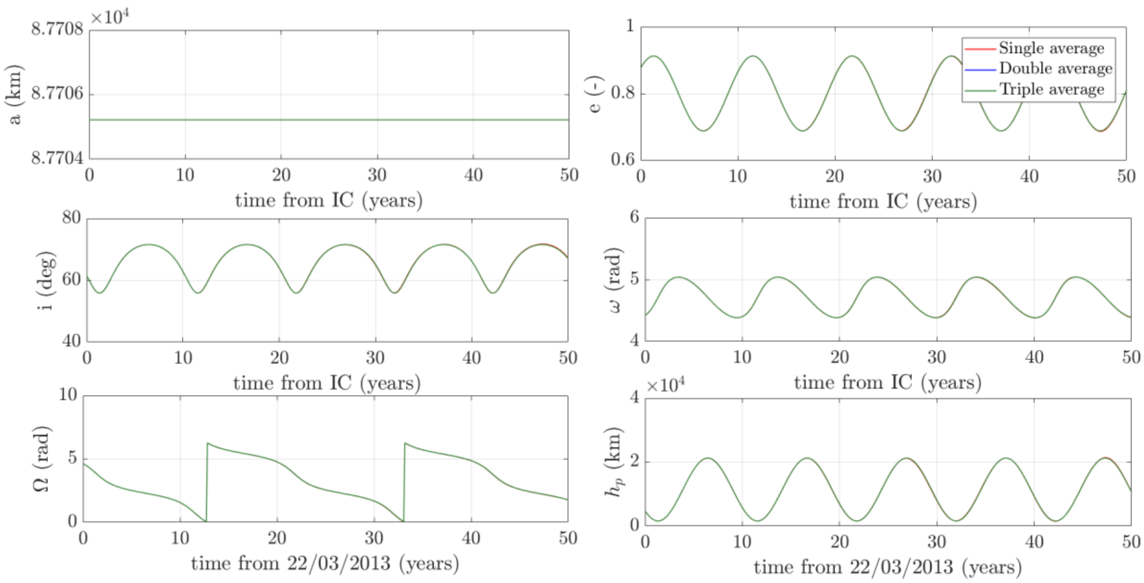

Fig. 4: Time evolution using different models (single, double and triple-averaged model) for a Venus' orbiter on an HEO trajectory, under the effect of J_2 and Sun perturbations.

Model validation

The first optimisation was done for the maximum and the minimum eccentricity point to define the interval of the manoeuvre cost. The manoeuvre was modelled both with the fully-analytical and semi-analytical approach, and the results were already reported in [10]. Those analyses prove the accuracy of the reduced Hamiltonian model for the computation of the target eccentricity condition, corresponding to the minimum altitude of perigee. The results from the semi and the fully analytical model are completely comparable in terms of optimal parameters $\alpha, \beta, \Delta v$ and f. On the other hand, the most important consideration concerns the computational time for the optimisation in the two different approach: even if the semi-analytical model is more efficient than the full numerical propagation, the fully analytical model requires about 10 times less computational time to optimise the manoeuvre for one initial condition:

- semi-analytical method (numerical propagation of double-averaged model): $\sim$1 h,
- fully-analytical method (analytical computation with reduced Hamiltonian): $\sim$5 min.

Atmospheric entry options

The possible manoeuvres for the atmospheric entry are computed in a time span of 25 years from 22/03/2013 for the Venus' orbiter. Within the time window considered, an atmospheric entry below 130 km is possible with different level of delta-v effort. Ten initial conditions are investigated for the period of 2013-2022. The minimum perigee altitude is reached by each disposal options. Nevertheless, the cost of the manoeuvre depends on the orbital elements of the satellite and on the couple (e, ω) depending on the position in the phase space. It results in different delta-v values, varying from a minimum of 55.3 m/s to a maximum of 70.2 m/s, as reported in Table 2. The minimum perigee altitude below 130 km is assured for each initial condition. The solution is shown in terms of the optimal angle of the delta-v firings and the optimal true anomaly for the manoeuvre. The computational time for the optimisation is about 45-60 minutes, so it's a very efficient method for manoeuvre design. The solution is reported graphically in Fig. 5, where the in and out of plane angles are reported, α and β, together with the magnitude of the Δv. Finally, the optimal true anomaly for the manoeuvre is reported.

Table 2: Venus' probe optimal atmospheric entry options with the fully-analytical approach (using the triple averaged Hamiltonian model)

	Fully-analytical approach			
Manoeuvre date (dd/mm/yy)	$h_{p,min}$ (km)	Δv (m/s)	α (rad)	β (rad)
22/03/2013	130.60	56.6	-2.51	0.60
25/03/2014	130.53	55.3	-2.90	0.17
29/03/2015	130.56	56.1	-3.14	-0.31
01/04/2016	130.62	57.2	2.49	-0.73
04/04/2017	130.69	59.2	2.59	-0.97
08/04/2018	130.83	63.0	2.76	-1.11
12/04/2019	131.13	70.2	2.98	-1.20
14/04/2020	131.04	68.1	-2.92	1.18
18/04/2021	130.79	61.9	-2.72	1.08
22/04/2022	130.67	58.6	-2.55	0.92
Computational time	$\sim$1 h			

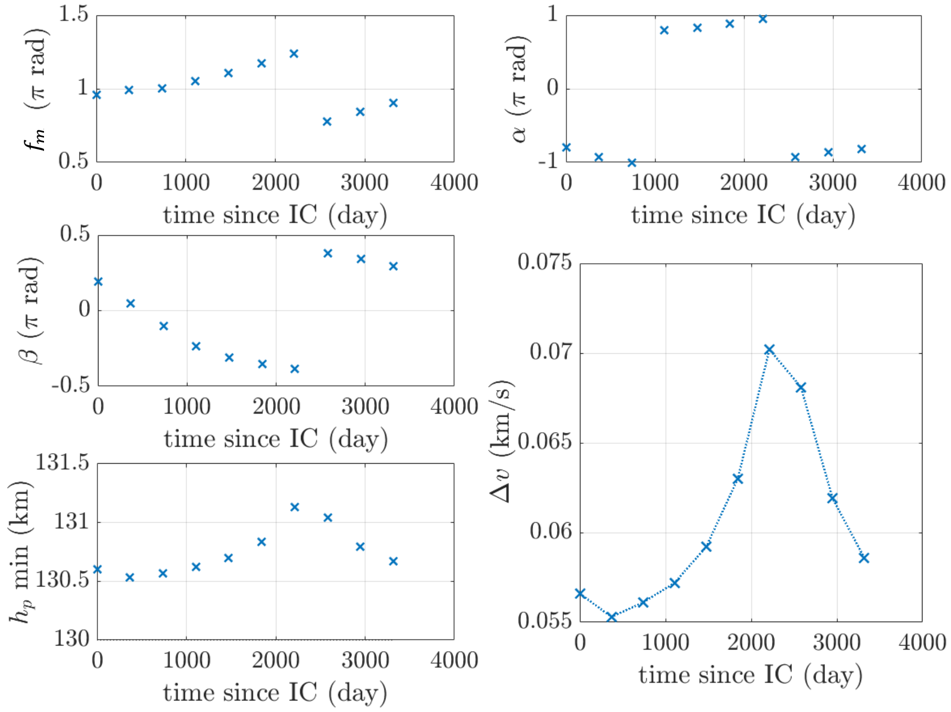

Fig. 5: Optimal manoeuvre parameters for Venus' atmospheric entry case.

INTEGRAL Mission Disposal Design

The INTErnational Gamma-Ray Astrophysics Laboratory (INTEGRAL) was a European mission of ESA for the "Horizon 2000" program. It was dedicated to spectroscopy and imaging of gamma-ray sources [24]. It was designed to have a nominal mission of two years, but its lifetime was then extended from 2004 to 31 December 2021. The main characteristics of INTEGRAL mission are described in Table 3, where the initial parameters of the spacecraft were computed on January 2013, as in [4].

Table 3: INTEGRAL condition at 01/01/2013 [4]

	Condition in Jan 2013 [4]
Operational orbit	$\{8.7705 \times 10^4, 0.8766, 61.5°, 4.42\ \text{rad}, 4.64\ \text{rad}, 2.25\ \text{rad}\}$
Fuel mass	61.5 kg
Equivalent delta-v	61.9 m/s

The end-of-life strategy of INTEGRAL mission was already studied in many works, see for example analysis in [4] and [19]. In both, the authors exploit a semi-analytical orbit propagator to describe the orbital motion of the satellite with a high-fidelity model, using a double-averaged model to speed up the computations. The former works investigate the Earth's atmospheric re-entry possibility for the satellite in the time window 2013-2029, the latter focused his works on the design of a graveyard trajectory. The aim of the present work is to design an Earth's re-entry trajectory, using both the semi-analytical propagation of the orbit and the fully-analytical recovery of the critical condition. For the optimisation procedure, the following constraints were identified:

- disposal time window: 2013-2029,
- delta-v interval for the optimisation: 0-1.20 km/s,
- target perigee altitude after the manoeuvre 50 km.

INTEGRAL Mission Scenario

The INTEGRAL orbit was propagated in time starting from the Keplerian elements in Table 3. The future evolution of the orbital elements computed in the Earth-centred equatorial inertial frame using the full, the single, double and triple-averaged model, as reported in Fig. 6. The non-accurate approximation with the reduced Hamiltonian model is caused by the approximation done in the model: the assumption of constant Moon orbital elements and the drop of the coupling between the satellite and the Moon node. The latter is related to the elimination of the node and will be addressed in the "Problem of the Node Elimination for the Earth system" section.

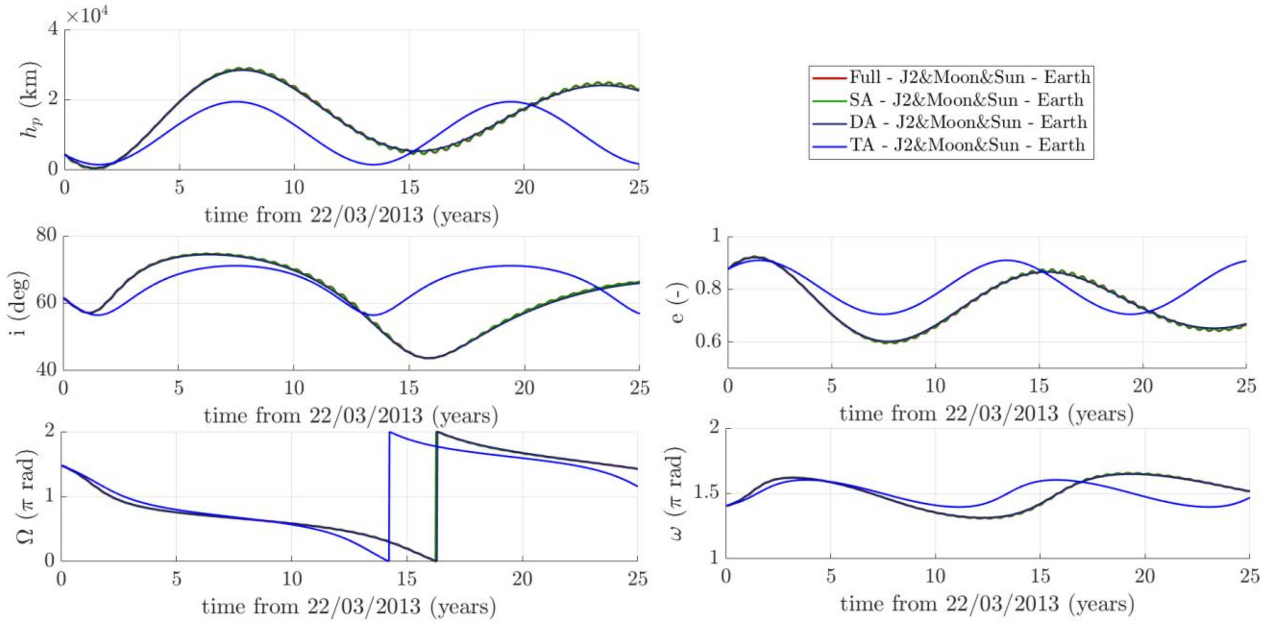

Fig. 6: Time evolution of satellite orbital elements using the full, single, double and triple (reduced Hamiltonian) averaged model. The triple averaged is not able to represent the actual time evolution of the orbit.

INTEGRAL Re-entry Options

The possible disposal options are computed starting from 2013 for 25 years, with both semi and fully analytical methods. Those results were compared with the ones in [4]. As already demonstrated in [10], the reduced Hamiltonian representation for the Earth case suffers from the elimination of the node problem, due to the drop of the coupling effect of the satellite and the Moon node, and of the assumptions that the Moon elements are constant in time. For this reason, the fully analytical method can be used just as a very preliminary approach to estimate the order of magnitude of the manoeuvre effort. On the other hand, it results that the computations performed with the semi-analytical method are much more accurate than the one from the fully-analytical model. The numerical results are reported in Table 4. The Δv value changes significantly in the two different approaches. In particular, the semi-analytical model has a behaviour like the results obtained in [4]. In the table, the best solutions are coloured in blue. There are three best options: the first in 2014, which would not be considered, it is already passed. The second in 2023, it is a very good option, since in this analysis it requires the lowest value of delta-v. The third option is on 2032, with a delta-v of 25.6 m/s. The best period to perform the disposal appears to be around 2023, when the delta-v necessary for the re-entry is only 17.2 m/s. This is a good solution since it is in the same period ESA wants to dismiss it.

Table 4: INTEGRAL disposal options with the fully-analytical method using the reduced Hamiltonian model and the semi-analytical method propagating the satellite orbit with the double-averaged potential. The results from the semi-analytical approach are comparable with literature values in [4].

	Fully-analytical model		Semi-analytical model	
Manoeuvre date (dd/mm/yy)	Δv (m/s)	$h_{p,min}$ (km)	Δv (m/s)	$h_{p,min}$ (km)
01/06/2013	73.3	34.69	75.2	50.02
04/06/2014	67.7	33.18	35.5	49.5
08/06/2015	67.4	43.45	36.5	50.3
11/06/2016	73.1	43.17	48.8	49.8
14/06/2017	74.9	53.23	50.6	49.7
18/06/2018	77.7	38.76	100.1	49.1
22/06/2019	83.7	44.37	112.8	50.2
24/06/2020	94.0	32.37	100.3	49.4
28/06/2021	97.8	35.37	118.5	49.9
02/07/2022	85.9	44.38	50.1	50.0
05/07/2023	78.9	37.43	17.2	47.8
08/07/2024	75.5	37.21	47.9	45.6
12/07/2025	73.8	37.63	52.6	48.5
15/07/2026	68.8	33.12	80.4	49.7
19/07/2027	66.8	52.19	96.3	50.8
22/07/2028	74.2	40.47	83.2	47.3
25/07/2029	74.8	43.05	70.6	50.3
29/07/2030	77.3	45.10	55.3	46.2
02/08/2031	83.0	40.85	37.2	48.8
05/08/2032	92.2	42.49	25.6	49.3
Computational time	~ 1 h		~8 h	

Studying the results in Fig. 7, the solution obtained in previous works is more accurate since a bigger number of initial conditions were studied. Nevertheless, the behaviour of the semi-analytical method is very similar to those results, indicating an equal trend in time. In fact, the red and the green lines have the maximum and the minimum in the same time period. On the other hand, the results from the fully-analytical model are not accurate, in the first part seems to follow at least the average trend, but then it behaves just the opposite as the real delta-v required. Moreover, the big difference in the computational time justifies the need of finding a better approximation with the fully-analytical model, as demonstrated for the Venus system, where the model correctly works.

Further analyses including the atmospheric drag and the SRP are necessary, to understand how those components can change the behaviour of the satellite in time. Hence, a new model shall be developed, for a high-fidelity propagation. The atmospheric leg design is necessary for analysing the casualty risk and the probability of impact on the ground. Moreover, since with this method the Keplerian elements at the entry altitude are known, the Earth's re-entry happens as semi-controlled disposal. This is very good for the risk statistics analysis.

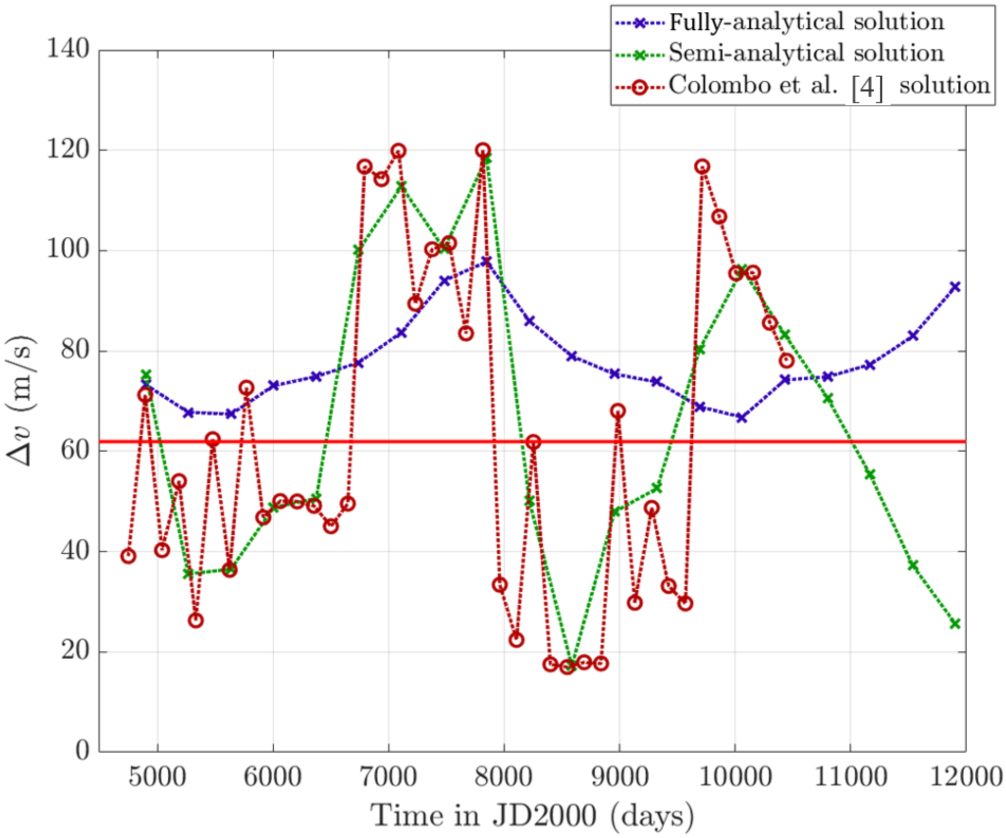

Fig. 7: INTEGRAL optimal disposal options. The results from the fully and semi-analytical model are compared with literature results in [4].

Semi-analytical and Fully-analytical methods performances

The comparison between the computational time for disposal options demonstrates that using a semi-analytical propagation for the manoeuvre optimisation is more expensive than a full-analytical method based on the solution of the Hamiltonian, even if it is more time efficient than a full dynamics' integration. It could take several hours to produce the optimal results. For this reason, the optimal solution shall be computed on ground and then the instructions are sent to the onboard system. Instead, the approach presented in this work aims to reduce significantly the computational time for the optimisation design. The power of the fully-analytical approach is based on the computational time to find the stationary points condition. The performances are referred to the following processor: 2.60 GHz and 16.0GB of RAM. It is evident that the computational time is reduced significantly, yielding to the necessity of developing a much more accurate analytical model. The performances are reported in Table 5.

Table 5: Difference in computational time between a numerical and a semi-analytical approach.

Semi-Analytical Method	
Orbit propagation for 25 years	3.63 s
Optimisation of 1 initial condition	$>$ 1 h
Fully-Analytical Method	
Stationary point computation for e_{max}	0.02163 s
Optimisation of 1 initial condition	$<$ 10 min*

* averaged value for the optimisation of different initial conditions

Problem of the Node Elimination for the Earth system

The results, given by the Earth-Moon-Sun model, highlight the limitations of the triple-averaged model in some cases. In particular, the model developed correctly works for a system where the satellite node is not coupled with the third body node and the orbital elements of the third body does not change in time. As a result, approximating the Sun and the Moon on the equatorial plane provides good results as well as the Venus' case. Nevertheless, the results for the equatorial case reveal how the Earth-Moon-Sun system has a complex behaviour. It is not as simple as the Venus' one, for which the reduced Hamiltonian approximation correctly works. This suggests that different approaches for the elimination of the node should be used. In fact, the idea is very promising since allows the determination of the critical eccentricity (maximum eccentricity value in time) without propagating the dynamics, but simply by solving the reduced one-degree-of-freedom Hamiltonian equation. The limitation of the present model is the non-correct elimination of the node procedure in case of an inclined perturbing body, with orbital elements varying in time (as for the Moon). This is a very complex problem and should be addressed in future works. The low accuracy is therefore caused by the elimination of the dependence in the Hamiltonian expression from the coupling effect $\Omega - \Omega_{☾}$ and by the assumption of a constant inclination in time of the Moon's orbital plane. In fact, the Moon node has a non-linear variation on the equatorial frame and its coupling effect with the satellite node causes complex dynamical behaviour of the secular evolution of the satellite orbit. Therefore, the node elimination and the other assumptions in the model drop some important contribution for the determination of the secular and long-term satellite evolution.

Conclusions

This paper presents an efficient method for designing the end-of-life disposal of spacecraft in Highly Elliptical Orbits around the Venus and the Earth. As a case of study, a Venus' orbiter is considered, and the design of possible atmospheric entry trajectories is presented. Moreover, the disposal options for INTEGRAL mission are designed, comparing the accurate results from the semi-analytical method with the fully analytical one. Depending on the initial condition selected, and therefore on the date at which the manoeuvre is performed, the delta-v cost varies, and it could increase or decrease the eccentricity value. The manoeuvre is used to navigate in the luni-solar, coupled with the Earth oblateness, perturbed environment of the Earth and in the solar and J_2 perturbation space for Venus. By computing the orbital evolution after the manoeuvre in such environment, it is checked that the critical eccentricity is reached, i.e. the perigee altitude goes below the re-entry altitude.

Two different application are here presented to model the disposal manoeuvre. The proposed analytical model based on the reduced Hamiltonian allows the computation of the orbital evolution solving one equation for some peculiar cases: the orbital elements of the third body can be considered constant in time, as a first approximation, and the elimination of the node does not cancel out complex dynamical behaviour of the system. This can be used to surf among the phase space to find the optimal manoeuvre to reach the target condition, as the disposal atmospheric re-entry for a Venus' orbiter. In fact, the Venus-Sun system is a proper system to apply the model, where the satellite moves in the phase space under the coupling effect of the Sun and J2.

On the other hand, the application of the fully-analytical method to the INTEGRAL case shows the problem of the elimination of the node and of the assumptions done to recover the model itself. This cancels out complex dynamics, and the Hamiltonian reduction is based on too stringent hypotheses to describe in an accurate way the real satellite dynamic. The problematic arises from the node elimination. The right ascension of the satellite is coupled with the Moon node: it has a non-linear time behaviour in the equatorial frame. The fully-analytical model is not accurate for the Earth's system, and therefore can only be used for a very preliminary

estimation of the order of magnitude of the manoeuvre effort. On the other hand, the semi-analytical model, considering the orbit propagation in time produces suitable results for the disposal strategy of the INTEGRAL mission.
However, further analyses are required to fully characterise the final leg of the atmospheric re-entry, considering the drag effect on the disposal trajectory, but more important an analysis concerning the problem of the node elimination should be proposed to produce a more reliable fully-analytical method for the Earth case. A first study could investigate the feasibility of finding a solution to the 2.5 degrees of freedom Hamiltonian, that is produced after the double averaging. Some works were already implemented in this direction. Moreover, the need to reduce the computational time is related to the possibility of generating software that can compute the optimal disposal trajectory from the current ephemeris of the satellite. This is fundamental during the design of the mission since it allows the correct dimension of each satellite's subsystem. This idea is very promising and could be achieved only by solving the satellite dynamics in a fully-analytical way, as it was done in this work.

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References

1. C. Colombo, E. M. Alessi and M. Landgraf. 'End-of-life Disposal of spacecraft in Highly elliptical Orbits by Means of Luni-Solar Perturbations and Moon Resonances'. In: *6th European Conference on Space Debris. Darmstadt, Germany.* Vol. 723. 22 - 25 April 2013.

2. ESA website, Space Debris by the numbers: https://www.esa.int/Our_Activities/Operations/Space_Debris/Space_debris_by_the_numbers, last visited on 24/10/2018

3. L. Blitzer. Handbook of Orbital Perturbations, Astronautics 453, Univ. of Arizona, 1970.

4. C. Colombo, F. Letizia, E. M. Alessi, M. Landgraf et al. 'End-of-life Earth re-entry for highly elliptical orbits: the INTEGRAL mission'. In: *The 24th AAS/AIAA space flight mechanics meeting.* Santa Fe, New Mexico, U.S.A. 26-30 January 2014, pp. 26–30.

5. C. Colombo. 'Planetary Orbital Dynamics (PlanODyn) suite for long term propagation in perturbed environment'. In: *6th International Conference on Astrodynamics Tools and Techniques (ICATT).* Darmstadt, Germany. 14-17 March 2016, pp. 14–17.

6. C. Colombo. 'Long-term evolution of highly-elliptical orbits: luni-solar perturbation effects for stability and re-entry'. In: *25th AAS/AIAA Space Flight Mechanics Meeting.* Williamsburg, Virginia, U.S.A. 11-15 January 2015.

7. M. Valtonen and H. Karttunen. The three-body problem. Cambridge University Press, 2006.

8. B. Kaufman. "Variation of parameters and the long-term behaviour of planetary orbiters," *Astrodynamics Conference*. Santa Barbara, California, U.S.A., 19-21 August 1970, p. 1055.

9. E. Tresaco, J. P. S. Carvalho, A. FBA Prado, A. Elipe and R. Vilhena de Moraes. ‘Averaged model to study long-term dynamics of a probe about Mercury’. In: *Celestial Mechanics and Dynamical Astronomy* 130.2 (2018), p. 9.

10. C. Colombo, F. Scala and I. Gkolias. ‘Surfing in the phase space of Earth’s oblateness and third body perturbations’. In: *29th AAS/AIAA Space Flight Mechanics Meeting.* Ka’anapali, Hawaii, U.S.A. 13-17 January 2019.

11. Y. Kozai, ‘Secular perturbations of asteroids with high inclination and eccentricity’. In: *The Astronomical Journal*, Vol. 67, 1962, p. 591.

12. C. Colombo, E. M. Alessi, W. van der Weg, S. Soldini, F. Letizia, M. Vetrisano, M. Vasile, A. Rossi and M. Landgraf. ‘End-of-life disposal concepts for Libration Point Orbit and Highly Elliptical Orbit missions’. In: *Acta Astronautica* 110 (2015), pp. 298–312.

13. T. Sgobba. ‘Space debris re-entries and aviation safety’. In: *International Association for the Advancement of Space Safety’s 2013 conference.* Montreal, Canada. 21-23 May 2013.

14. C. G Justus and R. D Braun. ‘Atmospheric Environments for Entry, Descent and Landing (EDL)’. In: *5th International Planetary Probes Workshop and Short Course*, Bordeaux, France. 23-29 June 2007.

15. A. Gaudel, C. Hourtolle, J.-F. Goester, N Fuentes and M Ottaviani. ‘De-orbit strategies with low-thrust propulsion’. In: *Space Safety is No Accident*. Springer, 2015, pp. 59–68.

16. A. E. Comellini, B. Dellandrea, G. D. and P. Di Lizia. ‘The semi-controlled reentry: development of a simulator and feasibility study’. In: *14th Pegasus-AIAA Aerospace Student Conference*. Madrid, Spain. 18-20 April 2018. p. 1-11.

17. ESA. *Basics about controlled and semi-controlled re-entry*. url: http://blogs.esa.int/cleanspace/2018/11/16/basics-about-controlled-and-semi-controlledreentry/.

18. H. Svedhem, D. Titov, F Taylor and O Witasse. ‘Venus express mission’. In: *Journal of Geophysical Research: Planets* 114.E5 (2009).

19. R. Armellin, J. F. San-Juan and M. Lara. ‘End-of-life disposal of high elliptical orbit missions: The case of INTEGRAL’. In: *Advances in Space Research* 56.3 (2015), pp. 479–493.

20. C. Colombo. ‘Optimal trajectory design for interception and deflection of Near Earth Objects, Ph.D. Thesis’. PhD thesis. University of Glasgow, 2010.

21. A. Shirazi. ‘Multi-objective optimisation of orbit transfer trajectory using imperialist competitive algorithm’. In: *Aerospace Conference, 2017 IEEE. Big Sky, Montana, U.S.A.* IEEE. 2017, pp. 1–14.

22. B. J Bienstock. ‘Pioneer Venus and Galileo entry probe heritage’. In: *Planetary Probe Atmospheric Entry and Descent Trajectory Analysis and Science*. Vol. 544. 2004, pp. 37–45.

23. DV Titov, KH Baines, AT Basilevsky, E Chassefiere, G Chin, D Crisp, LW Esposito, J-P Lebreton, E Lellouch, VI Moroz et al. *Missions to Venus*. 2002.

24. C. Winkler, G Di Cocco, N Gehrels, A Giménez, S Grebenev, W Hermsen, JM Mas-Hesse, F Lebrun, N Lund, GGC Palumbo et al. ‘The INTEGRAL mission’. In: *Astronomy & Astrophysics* 411.1 (2003), pp. L1–L6.