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EXPONENTIAL DECAY IN ONE-DIMENSIONAL TYPE III THERMOELASTICITY WITH VOIDS

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Abstract: In this paper we consider the one-dimensional type III thermoelastic theory with voids. We prove that generically we have exponential stability of the solutions. This is a striking fact if one compares it with the behavior in the case of the thermoelastic theory based on the classical Fourier law for which the decay is generically slower.

Keywords: Type III thermoelasticity, voids, exponential decay.

1. Introduction

In the last fifty years a big interest has been developed to propose alternative theories to the heat conduction, as the classical one violates the principle of causality. It is suitable to recall the theories of Green and Lindsay [12] or Lord and Shulman [20] which are based on the Cattaneo-Maxwell heat conduction equation [4] or the ones proposed by Gurtin and co-workers [5, 6, 7, 16]. In the 1990's Green and Nagdhi proposed three other thermoelastic theories in which the heat conduction proposes an innovation. They named them type I, II and III, respectively [13, 14, 15]. The linear version of the first one coincides with the classical theory based on the Fourier law. The second one is known as *thermoelasticity without energy dissipation* because the heat equation is not a dissipative process in that case. The third one is the most general and it contains the former two as limit cases. The main innovation of the types II and III theories consist in considering the *thermal displacement* between the independent variables. Therefore, these new theories suggest new systems of equations to study and understand. In fact these new theories have been also considered in the study of different problems [24, 25, 26].

We believe that an important aspect to study in continuum thermomechanics is to distinguish the different consequences of the several theories. Our note is addressed to this objective. In fact,

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we will see that the asymptotic behavior of the solutions for the type III theory is different from the one obtained for the classical theory of Fourier whenever we take into account the effects of the voids of the material. This is because in the type III theory new coupling terms are present which do not appear in the case of the classical theory. In this sense our work tries to show how new effects hold for the type II, III theories which are not present in the classical theory [18].

To clarify the differences, we want to point out in this paper that it is suitable to recall the theory of elasticity with voids proposed by Nunziato and Cowin [8, 9, 28]. In this theory the materials have a microstructure such that the mass in each point can be obtained as the product of the mass density by the volume fraction. The applications of these materials are relevant for solids with small distributed porous as rocks, soils, woods, ceramics, pressed powders or biological materials such as bones.

The study of the damping of the thermoelastic perturbations of materials with voids started in [32]. There, the author showed that generically the porous dissipation is not strong enough to guarantee the exponential decay of the solutions for a porous elastic structure. From this contribution a big quantity of contributions have been developed to clarify the decay of the thermomechanical perturbations for elastic solids with voids when different effects are taken into account [1, 3, 10, 11, 18, 21, 27, 29, 30, 33]. It is accepted that generically we would need two dissipative mechanisms to guarantee the exponential decay of solutions. To be more precise, it is needed that one of the mechanisms acts on the macrostructures of the material and the other on the microstructures. The time decay of the solutions of the problem corresponding to a thermoelastic material with voids based on the classical Fourier law was studied by Casas and Quintanilla [2]. There the authors proved the slow decay of solutions in the sense that the rate of decay cannot be controlled by an exponential. It was possible to obtain the exponential stability when some other dissipative mechanisms were also present as well as the microtemperatures [2] or the porous dissipation [3]. In fact Muñoz-Rivera and Quintanilla [27] proved the polynomial decay under suitable conditions on the coefficients and very recently new results for this problem have been obtained [33]. That is, in general the thermal effects (based on the classical Fourier law) are not strong enough to bring all the system to the exponential stability.

In this work we want to study the same problem, but in the context of the type III thermoelasticity and we will prove that generically the exponential stability is obtained. This is because in the context of the type III theory new and different couplings are present and in that case the heat conduction is strongly coupled with the macrostructures and the microstructures of the material. We want to emphasize that some new and different effects of this kind have been recently proved in the case of the thermoelasticity of type III with microtemperatures [22, 23] for the three-dimensional case. The present work tries to illustrate another aspect of these differences. We will consider the one-dimensioinal case, but we do not assume that the microtemperature effects are present.

2. Statement of the problem and well-posedness

In the context of the one-dimensional type III thermoelasticity with voids the system of field equations are determined by the evolution equations

$$\rho\ddot{u} = t_x, \tag{2.1}$$

$$J\ddot{\phi} = h_x + g, \tag{2.2}$$

$$\rho\dot{\eta} = q_x, \tag{2.3}$$

and the constitutive equations

$$t = \mu u_x + \gamma\phi - \beta\theta, \tag{2.4}$$

$$h = b\phi_x + m\psi_x, \tag{2.5}$$

$$g = -\gamma u_x + d\theta - \xi\phi, \tag{2.6}$$

$$\rho\eta = \beta u_x + a\theta + d\phi, \tag{2.7}$$

$$q = k\psi_x + m\phi_x + k^*\theta_x. \tag{2.8}$$

Here ρ is the mass density, J is the product of the mass density by the equilibrated inertia, t is the stress, h is the equilibrated stress, g is the equilibrated body force, q is the heat flux, η is the entropy and the variables u, ϕ, ψ and θ are the displacement, the volume fraction, the thermal displacement and the temperature, respectively. It is worth recalling that the thermal displacement is given by

$$\psi(x,t) = \int_0^t \theta(x,s)ds + \psi_0(x).$$

We also note that for the type I theory the thermal displacement is not present and therefore the parameters m and k vanish, meanwhile for the type II theory the parameter k^* vanishes. For the generic case of the type III thermoelasticity these three parameters are different from zero.

If we substitute the constitutive equations into the evolution equations, we obtain the field equations for the one-dimensional problem

$$\begin{cases} \rho\ddot{u} = \mu u_{xx} + \gamma\phi_x - \beta\dot{\psi}_x \\ J\ddot{\phi} = b\phi_{xx} + m\psi_{xx} - \xi\phi + d\dot{\psi} - \gamma u_x \\ a\ddot{\psi} = k\psi_{xx} + m\phi_{xx} - d\dot{\phi} - \beta\dot{u}_x + k^*\theta_{xx} \end{cases} \tag{2.9}$$

The parameters proposed in the system are related with the properties of the material. From now on, we assume that

$$b > 0,\ J > 0,\ \mu > 0,\ a > 0,\ \rho > 0,\ \mu\xi > \gamma^2,\ bk > m^2,\ k^* > 0. \tag{2.10}$$

Our assumptions agree with the thermomechanical axioms and empirical experiences. We want to emphasize that the condition on μ, ξ, b, k, m and γ can be interpreted with the help of the elastic stability. The condition on the thermal conductivity k^* is a consequence of the axioms of thermomechanics. The assumptions concerning mass density, the thermal capacity and the parameter J are also obvious.

We want to emphasize that the parameter β relates the displacement and the temperature. Furthermore m relates the thermal displacement with the volume fraction. These two parameters, jointly with γ, are responsible for the strong coupling between the variables. We will prove that if the coupling is strong enough, the thermal dissipation brings our system to the exponential stability.

To have the problem determined, we need to impose boundary and initial conditions. Thus, we assume that the solutions satisfy the boundary conditions

$$u(0,t) = u(\pi,t) = \phi_x(0,t) = \phi_x(\pi,t) = \psi_x(0,t) = \psi_x(\pi,t) = 0 \ \text{ for } \ t > 0, \tag{2.11}$$

and the initial conditions

$$
\begin{aligned}
&u(x,0)=u_0(x),\ \dot{u}(x,0)=v_0(x),\ \phi(x,0)=\phi_0(x),\ \dot{\phi}(x,0)=\varphi_0(x),\\
&\psi(x,0)=\psi_0(x),\ \dot{\psi}(x,0)=\theta_0(x)\ \text{ for }\ x\in[0,\pi].
\end{aligned}
\tag{2.12}
$$

The aim of this paper is to determine the asymptotic behavior in time of the solutions of the problem given by system (2.9), boundary conditions (2.11) and initial conditions (2.12).

First, we note that there are solutions (uniform in the variable x) that do not decay. To avoid these cases, we will also assume that

$$
\int_0^\pi \phi_0(x)\,dx=\int_0^\pi \varphi_0(x)\,dx=\int_0^\pi \psi_0(x)\,dx=\int_0^\pi \theta_0(x)\,dx=0.
\tag{2.13}
$$

We consider the Hilbert space

$$
\mathcal{H}=\big\{(u,v,\phi,\varphi,\psi,\theta)\in H_0^1\times L^2\times H_*^1\times L_*^2\times H_*^1\times L_*^2\big\},
\tag{2.14}
$$

where

$$
L_*^2=\{f\in L^2,\int_0^\pi f(x)dx=0\}\text{ and }H_*^1=L_*^1\cap H^1.
$$

Taking into account that $\dot{u}=v$, $\dot{\phi}=\varphi$ and $\dot{\psi}=\theta$ and writing $D=\frac{d}{dx}$, we can restate system (2.9) in the following way:

$$
\begin{cases}
\dot{u}=v\\
\dot{v}=\dfrac{1}{\rho}\big(\mu D^2u+\gamma D\phi-\beta D\theta\big)\\
\dot{\phi}=\varphi\\
\dot{\varphi}=\dfrac{1}{J}\big(bD^2\phi+mD^2\psi-\xi\phi+d\theta-\gamma Du\big)\\
\dot{\psi}=\theta\\
\dot{\theta}=\dfrac{1}{a}\big(kD^2\psi+mD^2\phi-d\varphi-\beta Dv+k^*\theta_{xx}\big)
\end{cases}
\tag{2.15}
$$

Moreover, if $U=(u,v,\phi,\varphi,\psi,\theta)$, then our initial-boundary value problem can be written as

$$
\frac{dU}{dt}=\mathcal{A}U,\ \ U_0=(u_0,v_0,\phi_0,\varphi_0,\psi_0,\theta_0),
$$

where $\mathcal{A}$ is the following 6×6-matrix

$$
\mathcal{A}=\begin{pmatrix}
0 & I & 0 & 0 & 0 & 0\\
\frac{\mu}{\rho}D^2 & 0 & \frac{\gamma}{\rho}D & 0 & 0 & -\frac{\beta}{\rho}D\\
0 & 0 & 0 & I & 0 & 0\\
-\frac{\gamma}{J}D & 0 & \frac{bD^2-\xi}{J} & 0 & \frac{m}{J}D^2 & \frac{d}{J}\\
0 & 0 & 0 & 0 & 0 & I\\
0 & -\frac{\beta}{a}D & \frac{m}{a}D^2 & -\frac{d}{a} & \frac{k}{a}D^2 & \frac{k^*}{a}D^2
\end{pmatrix}
\tag{2.16}
$$

and I is the identity operator. We note that $\mathcal{D}(\mathcal{A})$ is dense in $\mathcal{H}$.

If $U^* = (u^*, v^*, \phi^*, \varphi^*, \psi^*, \theta^*)$, then

$$\begin{aligned}\langle U, U^* \rangle_{\mathcal{H}} = \frac{1}{2} \int_0^\pi \Big(&\rho v \bar{v}^* + J \varphi \bar{\varphi}^* + a \theta \bar{\theta}^* + \mu u_x \bar{u}_x^* + b \phi_x \bar{\phi}_x^* + \xi \phi \bar{\phi}^* + \gamma (\phi \bar{u}_x^* + \bar{\phi}^* u_x) \\ &+ k \psi_x \bar{\psi}_x^* + m(\phi_x \bar{\psi}_x^* + \bar{\phi}_x^* \psi_x) \Big) dx. \end{aligned} \tag{2.17}$$

Here a superposed bar denotes the conjugate complex number. It is worth mentioning that this product is equivalent to the usual product in the Hilbert space $\mathcal{H}$.

Lemma 2.1. *For every $U \in \mathcal{D}(\mathcal{A})$, we have*

$$Re\langle \mathcal{A}U, U \rangle_{\mathcal{H}} \leq 0.$$

Proof. If we consider the inner product, we can see

$$\langle \mathcal{A}U, U \rangle_{\mathcal{H}} = -\frac{1}{2} \int_0^\pi k^* |\theta_x|^2 dx.$$

As we assume that k^* is positive the lemma is proved. □

Lemma 2.2. *0 belongs to the resolvent of $\mathcal{A}$ (in short, $0 \in \rho(\mathcal{A})$).*

Proof. The proof of this lemma is standard. It can be done (for instance) in a similar way as lemma 3.1 of [18] □

In view of these two lemmas and the fact that the domain of the operator is dense we can recall the Lumer-Phillips corollary to the Hille-Yosida theorem to conclude.

Theorem 2.3. *The operator given by matrix $\mathcal{A}$ generates a contraction C_0-semigroup $S(t) = \{e^{\mathcal{A}t}\}_{t \geq 0}$ in $\mathcal{H}$.*

3. Exponential decay of the solutions

In this section we will prove the exponential decay of the solutions of our problem. Apart from the assumptions proposed above on the constitutive coefficients, from now on we also impose that $m \neq 0$, $\beta \neq 0$ and $\gamma \neq 0$. These new assumptions say that the coupling between the three components of the problem is strong. In particular we note that for the classical theory the parameter m is not present.

Before proving the main result of this section, we recall the characterization stated in the book of Liu and Zheng that ensures the exponential decay (see [17], [19] or [31]).

Theorem 3.1. *Let $S(t) = \{e^{\mathcal{A}t}\}_{t \geq 0}$ be a C_0-semigroup of contractions on a Hilbert space. Then $S(t)$ is exponentially stable if and only if the following two conditions are satisfied:*

(i) $i\mathbb{R} \subset \rho(\mathcal{A})$,
(ii) $\overline{\lim}_{|\lambda| \to \infty} \|(i\lambda \mathcal{I} - \mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})} < \infty$.

Lemma 3.2. *The operator $\mathcal{A}$ defined in (2.16) satisfies $i\mathbb{R} \subset \rho(\mathcal{A})$.*

Proof. Following the arguments given by Liu and Zheng ([19], page 25), the proof consists of the following three steps:

(i) Since 0 is in the resolvent of $\mathcal{A}$, using the Neumann series argument, we see that for any real λ such that $|\lambda| < ||\mathcal{A}^{-1}||^{-1}$, the operator $i\lambda\mathcal{I} - \mathcal{A} = \mathcal{A}(i\lambda\mathcal{A}^{-1} - \mathcal{I})$ is invertible. Moreover, $||(i\lambda\mathcal{I} - \mathcal{A})^{-1}||$ is a continuous function of λ in the interval $(-||\mathcal{A}^{-1}||^{-1}, ||\mathcal{A}^{-1}||^{-1})$.

(ii) If $\sup\{||(i\lambda\mathcal{I} - \mathcal{A})^{-1}||, |\lambda| < ||\mathcal{A}^{-1}||^{-1}\} = M < \infty$, then by the contraction theorem, the operator

$$i\lambda\mathcal{I} - \mathcal{A} = (i\lambda_0\mathcal{I} - \mathcal{A})\Big(\mathcal{I} + i(\lambda - \lambda_0)(i\lambda_0\mathcal{I} - \mathcal{A})^{-1}\Big),$$

is invertible for $|\lambda - \lambda_0| < M^{-1}$. It turns out that, by choosing $\lambda_0 \in \rho(\mathcal{A})$ as close to $||\mathcal{A}^{-1}||^{-1}$ as we can, the set $\{\lambda, |\lambda| < ||\mathcal{A}^{-1}||^{-1} + M^{-1}\}$ is contained in the resolvent of $\mathcal{A}$ and $||(i\lambda\mathcal{I} - \mathcal{A})^{-1}||$ is a continuous function of λ in the interval $(-||\mathcal{A}^{-1}||^{-1} - M^{-1}, ||\mathcal{A}^{-1}||^{-1} + M^{-1})$.

(iii) Let us assume that the intersection of the imaginary axis and the spectrum is nonempty. Then there exists a real number ϖ with $||\mathcal{A}^{-1}||^{-1} \leq |\varpi| < \infty$ such that the set $\{i\lambda, |\lambda| < |\varpi|\}$ is in the resolvent of $\mathcal{A}$ and $\sup\{||(i\lambda\mathcal{I} - \mathcal{A})^{-1}||, |\lambda| < |\varpi|\} = \infty$. Therefore, there exist a sequence of real numbers λ_n with $\lambda_n \to \varpi$, $|\lambda_n| < |\varpi|$ and a sequence of vectors $U_n = (u_n, v_n, \varphi_n, \phi_n, \psi_n, \theta_n)$ in the domain of the operator $\mathcal{A}$ and with unit norm such that

$$\|(i\lambda_n\mathcal{I} - \mathcal{A})U_n\| \to 0. \tag{3.1}$$

Writing this condition term by term we get

$$i\lambda_n u_n - v_n \to 0 \text{ in } H^1, \tag{3.2}$$

$$i\lambda_n v_n - \frac{1}{\rho}\left(\mu D^2 u_n + \gamma D\phi_n - \beta D\theta_n\right) \to 0 \text{ in } L^2, \tag{3.3}$$

$$i\lambda_n \phi_n - \varphi_n \to 0 \text{ in } H^1, \tag{3.4}$$

$$i\lambda_n \varphi_n - \frac{1}{J}\big(- \gamma D u_n + bD^2\phi_n - \xi\phi_n + mD^2\psi_n + d\theta_n\big) \to 0 \text{ in } L^2, \tag{3.5}$$

$$i\lambda_n \psi_n - \theta_n \to 0 \text{ in } H^1, \tag{3.6}$$

$$i\lambda_n \theta_n - \frac{1}{a}\left(-\beta D v_n + mD^2\phi_n - d\varphi_n + kD^2\psi_n + k^* D^2\theta_n\right) \to 0 \text{ in } L^2. \tag{3.7}$$

In view of the dissipative term for the operator, we see that

$$\theta_n \to 0 \text{ in } H^1. \tag{3.8}$$

Then $\lambda_n\psi_n$ also tends to zero in H^1. Now, we multiply (3.7) by ϕ_n. We obtain that

$$\langle ia\lambda_n\theta_n, \phi_n\rangle + \beta\langle Dv_n, \phi_n\rangle + m||D\phi_n||^2 + d\langle\varphi_n, \phi_n\rangle \to 0.$$

We note that

$$\langle ia\lambda_n\theta_n, \phi_n\rangle = \langle ia\theta_n, \lambda_n\phi_n\rangle \to 0,$$

because $\lambda_n\phi_n$ is bounded. Next,

$$i\beta\lambda_n\langle Du_n, \phi_n\rangle + m||D\phi_n||^2 + d\langle\varphi_n, \phi_n\rangle \to 0.$$

Therefore

$$i\beta\lambda_n\langle Du_n, \phi_n\rangle + m||D\phi_n||^2 + i\lambda_n d||\phi_n||^2 \to 0.$$

We now want to prove that $D\phi_n$ tends to zero. It will be sufficient to show that $\langle Du_n, \phi_n\rangle$ tends to a real number. From (3.5) and after a multiplication by ϕ_n we see that

$$\langle iJ\lambda_n\varphi_n, \phi_n\rangle + \gamma\langle Du_n, \phi_n\rangle + b||D\phi_n||^2 + \xi||\phi_n||^2 \to 0.$$

But

$$\langle iJ\lambda_n\varphi_n, \phi_n\rangle \to -J||\varphi_n||^2$$

which is a real number. Thus, we have proved that $\langle Du_n, \phi_n\rangle$ tends to a real number and $||D\phi_n|| \to 0$.

We then consider (3.7) after a multiplication by $\lambda_n^{-1}Du_n$. We find

$$\beta||Du_n||^2 - m\langle D\phi_n, \lambda_n^{-1}D^2u_n\rangle - k\langle D\psi_n, \lambda_n^{-1}D^2u_n\rangle - k^*\langle D\theta_n, \lambda_n^{-1}D^2u_n\rangle \to 0.$$

But $\lambda_n^{-1}D^2u_n$ is bounded in view of (3.3). Thus we also see that $||Du_n|| \to 0$. If we multiply (3.3) by u_n and (3.5) by ϕ_n we also conclude that v_n and φ_n tend to zero. We have thus obtained a contradiction and the lemma is proved.

□

Lemma 3.3. *The operator $\mathcal{A}$ satisfies*

$$\overline{\lim_{|\lambda|\to\infty}} \|(i\lambda\mathcal{I} - \mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})} < \infty.$$

Proof. We can also prove this lemma by a contradiction argument. Suppose that the thesis of the lemma is not true. Then, there exists a sequence λ_n with $|\lambda_n| \to \infty$ and a sequence of vectors U_n in the domain of the operator $\mathcal{A}$ with unit norm and such that (3.1) holds. We obtain again (3.2)-(3.7) and repeat the arguments proposed in the proof of the previous lemma since the key point in the proof is that λ_n does not converge to zero. □

Theorem 3.4. *The C_0-semigroup $S(t) = \{e^{\mathcal{A}t}\}_{t\geq 0}$ is exponentially stable. That is, there exist two positive constants M and α such that $\|S(t)\| \leq M\|S(0)\|e^{-\alpha t}$.*

Proof. The proof is a direct consequence of Lemma 3.2, Lemma 3.3 and Theorem 3.1. □

It is worth noting that the behavior of the solutions for this model completely differs from the behavior in the one-dimensional classical thermoelasticity with voids, where slow decay is observed. The exponential stability obtained in our case is a consequence of the strong coupling between the porosity and the temperature. This coupling is not present in the classical model. This behavior is another striking effect of the type III thermoelasticity.

4. Conclusion

In this paper we have proved that, under suitable hypotheses on the different constitutive parameters, the solutions of the system of equations that models the type III thermoelasticity with voids decay exponentially. This behavior differs significantly from the one obtained for the classical theory.

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