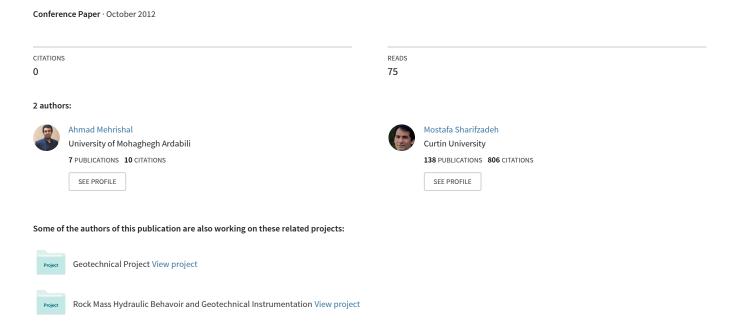
# Multi-scale joints roughness characterization using wavelet and shear modeling



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**ABSTRACT:** Mechanical behavior prediction of rock joints is very important in the rock mechanics. Many models have been proposed to predict the mechanical behavior of joints at which lack of correct evaluation of effective roughness coefficient has been the most important shortage. In this research, each of the upper and lower profiles of joint surfaces is considered as a 2-dimensional wave. Then, multi-scale decomposition based on wavelet theory has been applied studying on asperities. Upper and lower profiles have been combined to produce a composite surface having asperities characteristics of both joint surfaces. Each of the composed wave components (roughness and undulation) has been characterized with statistical quantity of arithmetic mean deviation ( $R_a$ ). This procedure of characterizing for 2-dimensional waves has been easily extended to 3-dimensional joint surfaces. Conformity in the results of shear and dilation modeling and laboratory tests satisfactorily verifies success of the proposed procedure.

**SUBJECT:** Modeling and numerical methods

**KEYWORDS:** Rock Joints

## 1 INTRODUCTION

Shear and dilatation behaviors of rock joints are both, of the most important design parameters in rock engineering. but the role of different parameters on joint behavior has not been characterized yet. For example, the lack of exact description of joint geometry comprises the main shortages of previously proposed models. Joint Roughness Coefficient (JRC) proposed by Barton, calculates one of the most known tools for joint roughness characterization. In this way, surface roughness variation in shearing is expressed by Mobilized JRC (JRC<sub>mob</sub>) (Barton 1973-1985). Several researchers such as Wu & Ali (1978), Krahn& Morgenstern (1979), Dight& Chiu (1981), Reeves (1999) and Maerz (1990) have used usual statistical parameters for characterization the joint surfaces asperities. Fractal geometry introduced by Mandelbort (1967), has encouraged researchers such as Miller (1990), Brown &Scholz (1985), Tullis& Power (1991), Huang et. al. (1992), Poon et. al. (1992), Odling (1994) and Den Outeret. al. (1995) to characterize joint roughness. Moreover, joint shear behavior has been modeled by Grasselli (2002-2003) making use of simulated 3-dimensional joint surface geometry. Also, Seok-Won Lee et al., (2006) have studied on the relates of joint shear strength and joint roughness coefficient (JRC). Although Barton's joint roughness coefficient (JRC) includes some shortages, it is still extensively used as joint roughness characteristics so far as lots of proposed fractal and statistical procedures are basically aimed to assess it.

In prior studies, some profiles that have been selected on only one of the joint surfaces have been applied to evaluate effective roughness coefficient. Although these procedures could have been unreliable, they have found comprehensive application. On the other hand, there are some method that consider 3-dimensional geometry of the surfaces and are more precise to estimate the joint mechanical behavior; however, because of high complexity of their algorithm, they could have not become commercially applicable.

Surface morphology greatly influences the joint mechanical behavior; therefore, effects of joint asperities have to be estimated by reliable and applicable methods. In rock mechanics joint asperities are divided into two scale: first order asperities (undulating) and second order asperities (roughness). The second order asperities affect the pre-peak behavior of joint shearing and the first order asperities manipulates the residual behavior of joint shearing (Yang & Chiang 2000). This finding has not been reflected in modeling of the previous studies. In the present research, two surface asperities of granitic tension joints have been studied using multi-scale wave decomposition method based on wavelet theory. Consequently, first height of the joint surface asperities has been decomposed to two groups of approximation scale (undulating) and detail

scale (roughness) by wavelet theory. Then, surface asperities have been characterized in two different scales by arithmetic mean deviation (R<sub>a</sub>). Finally, joint shear and dilation behavior have been modeled based on friction theory, considering effective joint surfaces roughness at each of the shearing steps.

#### 2 MULTI-SCALE WAVE DECOMPOSITION METHOD

wave is faded away (Fig. 1)(mirzaeian 2008).

Decomposition of a wave to several scales is known as multi-scale analysis method. Wavelet is a tool which allows for the simultaneous monitoring of both approximation and detail scale components of a wave. For example, profile of surface asperities is considered as a wave; so, undulating or first order asperities constitutes approximation scale component and roughness or second order asperities constitutes detail scale component of that wave (Muralikrishnan & Fu 2002). Using this transform, the approximated wave gets thicker and the detailed wave diminishes at each step. Finally, at the *n*-th stage of the transform, the approximated average wave can be considered as the numerical original wave where the detailed

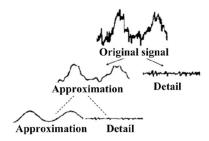


Figure 1. Decomposition of an arbitrary wave to constituent series of approximated and detailed waves.

According to the researchers' comments, three times (t=3) performing 2-dimensional surface decomposition produces multiscale roughness (Dashtizadeh & Biglari 2006). In addition, these three times decomposition on laser scanner data called the wavelet de-noising method (Khoshelham & Altundga 2010). Therefore, in this paper, we use this method in reliable characterization of roughness.

There are many wavelet transform. The 8db wavelet of the Daubechiez wavelet family has been allocated for this study. Also researchers have denoted that the higher orders of Daubechiez wavelets are more suitable for discovering the surface roughness rather than other wavelets in the wavelet bank (mirzaeian 2008). Then, surveyed profile of the joint surfaces is considered as a wave to separate the first order roughnesses from the second order ones perfectly. Figure 2. demonstrates the rock joint surfaces roughness and its decompositions on profile A-A.

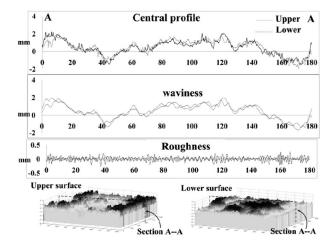


Figure 2. Joint surfaces asperities on profile A-A at pre-decomposition and post-decomposition (height of the asperities has been exaggerated)

#### 3 CHARACTERIZATIONS OF JOINT SURFACES ASPERITIES

Direct shear tests on granitic joints with compressive strength about 173MPa under low normal stress (1 and 3MPa) has been conducted using stress boundary conditions of constant normal load (CNL). Point samples have been surveyed by a laser scanner on a joint surfaces  $180 \times 100 \text{ m} \times \text{m}$  (length  $\times$  width) with 0.2 mm step at both directions of x and y. Heights of the cells (z) have been measured with precision of 0.001 mm (Sharifzadeh et al. 2006).

After decomposition, joint multi-scale roughness can be characterized at different scales. For the sake of familiarity, the terminology of this method is introduced as follow.

Composite surface: The composite surface is calculated using summation or subtraction of asperities heights of upper and lower joint surfaces. In order using of composite surface to calculate the influence of shear displacement on the effective roughness, the lower profile is fixed and the upper one moves toward the shear direction (figure 3). Therefore, new composite surface at the n-th displacement step is calculated by summation or subtraction of  $Z_n$  on the upper profile and  $Z_{n+1}$  on the lower profile.

Arithmetic mean deviation of profile (R<sub>a</sub>): arithmetic mean of absolute ordinate value during sampling on numerical profile is determined by Equation 1.

$$R_a = \frac{1}{n} \sum_{i=1}^{n} |Z_i - u| \tag{1}$$

Where, n = the number of the whole profile sample points, z = height of the sample point and u stands for height of the mean line.  $R_a =$  the most applicable parameter for surface structural decomposition which is known as centerline average (CLA).  $R_a$  has the unit of length (Dashtizadeh & Biglari 2006).

figure 3. demonstrates the variation of  $R_a$  during shearing. According to figure 3. if the  $R_a$  is calculated for two composite profiles, it will be observed that when the profile is completely matching, for the summation composite surface the minimum of  $R_a$  will be obtained (Fig. 3a) and for the subtraction composite surface the maximum of  $R_a$  will be attained (Fig. 3b).

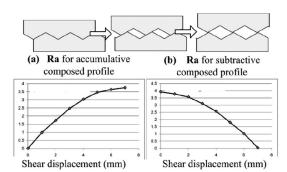


Figure 3. variation of Ra on composite surfaces during shear displacement.

The described analytical stages of profile could be simply expanded to joint surfaces using Equation 2 where the joint surfaces are constituted of many profiles. Therefore  $S_a$  is used for joint surfaces instead of  $R_a$  quantity.

$$S_a = \frac{1}{ij} \sum_{k=0}^{i-1} \sum_{l=0}^{j-1} |z(x_k, y_l) - u|$$
 (2)

Where  $z(x_k,y_l)$  = height of sample point surveyed on composite surface, i = number of sample points in direction of length and, j = represents the number of sample points in direction of width. In this study considering i=901 and j=499, u will be height of mean surface.

## 4 MODELING OF JOINT SHEAR BEHAVIOR

# 4.1 Peak Shear Displacement

At the beginning of direct shear tests, Joints under shear loading, tend to maximum interlocking. Joint interlocking characteristics differ in various directions because of the rock joint's surface heterogeneity and anisotropy (Grasselli 2002-2003).

Peak shear displacement is evaluated using roughness property  $(S_a)$ . As shown in figure 3. for the subtraction composite surface the maximum of  $S_a$  is coincided to the maximum interlocking of shear surfaces. Thus, both of small and large-scale asperities are simultaneously influences the pre-peak mechanical behavior of joint. Therefore, the maximum joint surfaces interlocking is determined by resulted values of  $S_a$  for each subtractive composite surface roughness.

The consequent value of peak shear displacement for sample (1) is 1.1 mm by using both Figure 4. and Figure 5; while, the peak shear displacement was measured 1.3mm in the experimental. Additionally, peak shear displacement which is evaluated to be 1.2 mm in sample (2) has been determined to be 1.4 mm in laboratory tests.

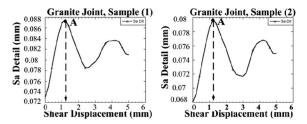


Figure 4. Variation of roughness property of composite surface (Sa) for small-scale roughness in two joint samples numbered (1) and (2).

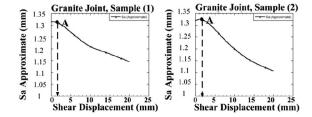


Figure 5. Variation of roughness property of composite surface (Sa) for large scale roughness in two joint samples numbered (1) and (2).

#### 4.2 Peak shear strength

In this research the influences of first order asperities (undulation) and second order asperities (roughness) has to be simultaneously considered to evaluate the pre-peak shear strength. Hence, pre-peak and peak shear strength can be evaluated by Equation 3.

$$\tau_i = \sigma_n \times tan[tan^{-1}(RF_i) + \varphi_b] \tag{3}$$

Where  $\tau_i$  = joint shear strength at *i*-th shear step (each step is equal to 0.2mm displacement),  $\sigma_n$  = applied normal stress (MPa) and,  $\varphi_b$  = basic friction angle which is dependent on rock material. *RF* (Friction of roughness) is the empirical roughness friction coefficient and can be calculated by Equation 4.

$$RF_{i} = \left[ \left( \frac{S_{a(i)}^{D}}{Z_{lmax}^{D}} \times ln(C.\sigma_{c}) \right) + \left( \frac{S_{a(i)}^{A}}{Z_{lmax}^{A}} \right) \right]$$
(4)

Where  $S^D_{a(i)}/Z^D_{|max|} = \text{small scale (detail)}$  roughness at subtractive composite surface at i-th shearing step normalized by maximum absolute value of composite surface asperities height.  $S^A_{a(i)}/Z^A_{|max|} = \text{large scale (approximation)}$  roughness at i-th shearing step normalized by maximum absolute value of composite surface asperities height.  $\sigma_c$  is the joint un-confined compressive strength and C represents a constant that varies with regard to rock type. In the present study, C is assumed to be 0.3.

#### 4.3 Post-peak shear strength

The increase of shear displacement causes the motion of joint surfaces thus resulting in the decrease of matched area of joint surfaces. So, many of the small scale asperities are miss-matched and joint behavior is controlled by the large scale asperities so called undulating, (Yang & Chiang 2000). Undulating  $S_a$  represents the amount of surface interlocking. Therefore, post-peak shear behavior is modeled on the basis of  $S_a$ . Then, Equation 3 is applied to predict the post-peak shear strength. The empirical Equation 5 is used to calculate the roughness coefficient ( $RF_i$ ) from joint surfaces large-scale roughness at the residual phase of joint shear behavior.

$$RF_i = \left[\frac{S_{\alpha(i)}^A}{Z_{|m\alpha x|}^A}\right] \tag{5}$$

Where  $S^{4}_{a(i)}/Z^{4}_{|max|}$  = the large scale roughness at *i*-th shear step normalized by maximum absolute value of asperities height.

The asperities damage will be occur in shear test even on hard rock samples. Asperities damage produces gauge material. Gauge material decreases joint surfaces friction coefficient which is considered by asperities damage degree (*DD*) in Equation 6.

$$DD_i = \left[1 - \left(\frac{u_i - u_p}{u_{max}}\right)\right] \tag{6}$$

At which  $u_i$  = the shear displacement at residual *i*-th step,  $u_p$  = the peak shear displacement evaluated in section 4 -1, and  $U_{max}$  = maximum shear displacement

#### 4.4 Modeling direct shear strength

A partial function is applied to complete modeling of joint shear behavior:

$$\tau_{i} = \begin{cases} \sigma_{n} \times tan \left[ tan^{-1} \left( \left[ \frac{S_{a(i)}^{D}}{Z_{|max|}^{D}} \times ln(C.\sigma_{c}) \right] + \left[ \frac{S_{a(i)}^{A}}{Z_{|max|}^{A}} \right] \right) + \varphi_{b} \right] u_{i} \leq u_{p} \\ \sigma_{n} \times tan \left[ tan^{-1} \left( \left[ \frac{S_{a(i)}^{A}}{Z_{|max|}^{A}} \right] \times DD_{i} \right) + \varphi_{b} \right] \\ u_{i} > u_{p} \end{cases}$$

(7)

The results of performed tests on granitic joints having basic friction angle of 30 degrees and unconfined compressive strength of 173MPa are used to verify the proposed model. Joint samples numbered (1) and (2) are tested under constant normal load of 1 MPa and 3MPa, respectively.

The shear stress—shear displacement resulted from simulation and laboratory tests are shown in figure 6. Which indicate that, the simulated results are in good agreement with laboratory tests. Also, peak shear displacement of simulation is in good correspondence with laboratory test.

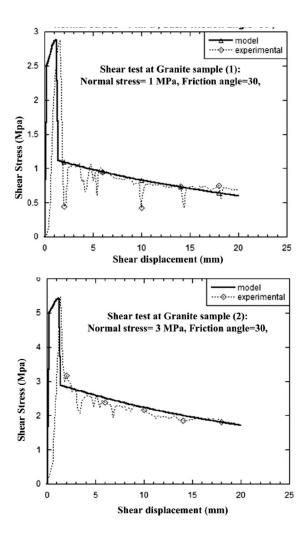


Figure 6. Comparison between the proposed shearing model and the laboratory test results from direct shear test on granitic joints numbered (1) and (2).

# 5 MODELING OF JOINT DILATION BEHAVIOR

Contrary to shear behavior, summation composite surface has been used for simulating dilation behavior. In this research, the maximum ordinate value of summation composite surface  $(Z_{max-com(i)})$  at each shear step is considered for evaluating the dilation.

The joint dilation is a function of joint asperities when normal stress is low (Huang et al. 2002).thus because of low normal stress, the damage of asperities which decreases the dilation is not considered in the present study. Also, large scale asperities is used for modeling of dilation behavior because the first order asperities generally controls the dilation behavior. Joint dilation behavior during shearing  $(D_i)$  can be determined by Equation 8.

$$D_{i} = \left(Z_{maxCom(i)} - Z_{maxCom(1)}\right) * \left(1 - \frac{\sigma_{n}}{\sigma_{c}}\right)$$
(8)

Where  $\sigma_n$  = the normal stress (MPa),  $\sigma_c$  = unconfined compressive strength (MPa) and,  $Z_{maxCom(i)}$  = maximum height of summation composite surface at each shearing step.

The dilation—shear displacement curve resulted from both simulation and laboratory tests are demonstrated in figure 7. Some of the differences between laboratory and simulation results are explained by the error of joint surfaces surveying and supposing a horizontal joint surfaces with the upper surface only moving upward without any rotation is in conflict with natural shear test condition. In addition, some other discrepancies may be resulted from damage of joint surfaces asperities.

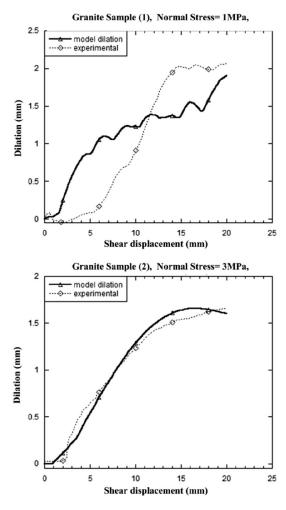


Figure 7. Comparison of dilation behavior resulted from proposed model and laboratory tests on granitic joint samples numbered (1) and (2).

## 6 DISCUSSION

The influence of surface roughness on joint mechanical behavior has been studied for both scales of roughness and undulation. Also, peak shear displacement, peak and residual shear strength and joint dilation have been evaluated using decomposition of surveyed morphology data on both upper and lower joint surfaces.

Comparisons results of modeling and laboratory tests have shown that the proposed algorithm is suitably capable to simulate shear and dilation behaviors of rock joints. Application of effective characteristics and considering to the main shear mechanism in hard rock joints accounts for the relative success of the presented approach.

As shown in the present study, friction is the main mechanism that controls the shear behavior in joint samples with high compressive strength under low normal stress. The friction coefficient is highly dependent on joint surfaces asperities. Therefore, joint geometry has a governing role on the shear strength that is in accordance with the most recent researchers' findings. Friction model presented by Barton (1973-1985) is also based on the joint surface roughness. Joint roughness coefficient (JRC) is one of the main effective parameters in the Barton model.

In the present study, degradation of joint asperities is not considered because the applied normal stress are negligible compared to samples compressive strength. So, joint mechanical behavior is under influence of joint surfaces morphology. Also, joint dilation is controlled by highest asperities at each shear step. These findings are in good agreement with the result of the laboratory tests performed by researchers such as Grasselli (2002), Yang & Chiang (2000), Seidel & Haberfield (2002), Hang et al. (2002).

At peak shear displacement both scales of joint surfaces asperities influence the shear behavior as synthesized in the present study. Then, as the shear displacement increases, shear behavior is completely controlled by joint undulation. Also, in order to evaluate the dilation behavior, only large-scale asperities (undulating) are used and small scale roughnesses do not affect the dilation. Barton (1985), Yang & Chiang (2000) and Patton (1996) also reported similar findings.

The findings of this research are as follow:

- Both upper and lower joint surfaces asperities, joint surfaces positioning and shear direction influence the friction coefficient.
- At the start of the shear displacement, both first and second order roughness influences the shearing. After the
  peak shear displacement and occurrence of dilation, second order asperities get inactive and residual shear
  behavior will be controlled only by the first order joint asperities. Therefore, multi-scale decomposition of
  surface asperities is necessary for evaluation of joint mechanical behavior.
- Peak shear displacement is divided into constant stiffness and plastic deformation. According to the results, peak plastic displacement is affected by joint surfaces geometry. Therefore, the primary plastic slip causes maximum joint interlocking at shear direction.
- Joint dilation behavior is especially function of surface asperities shape. In reality, joint surfaces undulating manipulates the joint dilation behavior.
- The agreement between the results of modeling and laboratory tests verifies the success of the proposed algorithm.

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