

1 **Design of ballasted railway track foundations using numerical modelling**

2 **Part II: Applications**

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## 25     **Design of ballasted railway track foundations using numerical modelling**

### 26                             **Part II: Applications**

27

28     **Abstract:** This paper is the second of two companion papers in relation to a new design  
29     method for ballasted railway track foundations. The development of the new design method  
30     has been explained in the first paper (i.e., Part I: Development), and the procedures for using  
31     the method and its practical application on some field case studies are presented in this paper.  
32     Special feature of the proposed design method is that it considers the true impact of train  
33     dynamic moving loads and number of repeated applications of the traffic tonnage. The  
34     proposed method is then applied to four case studies of actual tracks and the results are  
35     compared with field measurements and found to be in good agreement. It should be noted  
36     that, although the proposed design method is able to overcome most shortcomings of the  
37     existing methods and found to provide excellent outcomes, further verification for more field  
38     case studies is highly desirable.

39     Keywords: Finite elements, numerical modelling, ballasted railway track foundations,  
40     dynamic amplification factor, high-speed trains.

41

## 42 **Introduction**

43 A new method is developed for design of ballasted railway track foundations for determining  
44 the granular ballast layer thickness required to prevent the railway track failures induced by  
45 the repeated train (dynamic) moving loads. Two common track failure criteria are considered  
46 to govern the new design method, namely the subgrade progressive shear failure and  
47 excessive plastic deformation of track substructure. The process leading to development of  
48 the new design method, including all affecting design parameters, are studied in detail and  
49 presented in a separate companion paper, i.e., Part I: Development (Sayeed and Shahin 2017).  
50 In this paper, the design procedures that need to be followed for using the new design method  
51 is described and the applicability of the method is verified by conducting a comparison  
52 between the method outcomes and field measurements, for some well-documented case  
53 studies. The results obtained from the new design method are found to be in good agreement  
54 with the field measurements, thus, the method can be used with confident in routine design  
55 by practitioners.

## 56 **Description of design procedures of new proposed method**

57 This section presents detailed procedures for using the new design method of selecting a  
58 granular layer thickness with the aid of the design charts developed in the companion paper  
59 (i.e., Part I: Development). The method has two design procedures corresponding to two  
60 different criteria of preventing railway track failures. One procedure is meant for preventing  
61 the progressive shear failure at the top subgrade surface, while the other focusses on  
62 preventing the excessive plastic deformation of the track. The thickness of the granular layer  
63 that should be used for design should be the maximum thickness obtained from applying the  
64 two procedures. It should be noted that if the subgrade is very stiff and dynamic wheel load is  
65 low, the obtained design thickness might be very small and in such a case it is suggested to

66 use a standard minimum thickness of granular layer equal to 0.45 m, including 0.30 m of  
67 ballast plus 0.15 m of sub-ballast, as suggested by Li et al. (2016). However, if the subgrade  
68 is soft (e.g.,  $E_s = 15$  MPa, i.e., shear wave speed  $\approx 54$  m/s), before proceeding to calculate the  
69 granular layer thickness using the design charts, the practitioner needs to double check  
70 whether the design speed is higher than the critical speed of the train-track-ground condition  
71 at hand. To quantify the critical speed of the train-track-ground condition, readers are referred  
72 to Sayeed and Shahin (2016). If the design train speed is higher than the critical speed, the  
73 soft subgrade will be susceptible to failure and it is thus recommended to improve the  
74 subgrade (e.g., by chemical additives) so that the subgrade modulus can be increased and in  
75 turn the critical speed becomes higher than the train design speed.

#### 76 **Design procedure for preventing progressive shear failure**

77 The design procedure for preventing the progressive shear failure is based on limiting the  
78 cumulative plastic strain at the subgrade surface below a threshold value. As discussed  
79 earlier, limiting the cumulative plastic strain is achieved automatically by limiting the  
80 deviatoric stress induced by the dynamic train moving loads. Li and Selig (1998a, b)  
81 developed a design procedure for preventing this mode of track failure; however, their  
82 method has several limitations discussed in the companion paper (i.e., Part I: Development).  
83 The intention of the proposed new design method is to overcome most of the current  
84 limitations of the available design methods including Li-Selig's method, by providing a  
85 methodology that suits the modern railway traffics.

86 Fig. 1 shows a flowchart that can be used for calculating the granular layer thickness needed  
87 to prevent the progressive shear failure. The flowchart has four main steps: (1) data collection  
88 and preparation; (2) determination of allowable deviatoric stress; (3) determination of

89 allowable strain influence factor; and (4) selection of the granular layer thickness using the  
90 developed design charts. The above steps are described in some detail below.

91 Step 1: The designer should collect and prepare the following information:

- 92 • *Loading conditions*: this requires calculation of the design dynamic wheel load,  $P_d$ ,  
93 and number of equivalent repeated application of wheel load in the subgrade layer,  $N_s$ ,  
94 for a given design traffic tonnage. In order to establish the dynamic wheel load,  $P_d$ , it  
95 is required to determine the wheel spacing factor (WSF) corresponding to the wheel  
96 spacing, which can be obtained from Fig. 12(b) of the companion paper (i.e., Part I:  
97 Development). It is also required to determine the dynamic amplification factor  
98 (DAF) corresponding to the train speed, which can be obtained from Fig. 13 of the  
99 companion paper (i.e., Part I: Development) and best corresponds to the track-ground  
100 condition under consideration. The dynamic wheel load,  $P_d$ , can then be estimated  
101 using Equation (7) of the companion paper (Part I: Development), and the number of  
102 load repetitions in the subgrade layer can be calculated using Equation (9) of the  
103 companion paper (i.e., Part I: Development). If there are some major groups of wheel  
104 loads, the corresponding groups of dynamic wheel loads and number of repeated loads  
105 should be determined separately. Equations (10) and (13) of the companion paper  
106 (i.e., Part I: Development) have then to be employed to determine the total number of  
107 equivalent load applications in the subgrade,  $N_s$ , of the wheel load,  $P_s$ .
  
- 108 • *Design criterion*: the design proceeds by selecting an acceptable level of the  
109 cumulative plastic strain at the subgrade surface,  $\varepsilon_{(p_s)a}$ , for certain number of  
110 repeated loads (i.e., for the design traffic tonnage).

- 111 • *Subgrade characteristics*: this design item requires selection of the subgrade soil type  
 112 and determination of the soil monotonic strength,  $\sigma_{s\_s}$ , from the unconfined  
 113 compressive strength (UCS) test and soil modulus,  $E_s$ , obtained from the cyclic  
 114 triaxial compression test under a confining pressure equal to 100 kPa.
- 115 • *Granular material characteristics*: the mechanical properties of the granular materials  
 116 in the form of the ballast modulus,  $E_b$ , need to be determined from the cyclic triaxial  
 117 compression test under a confining pressure equal to 100 kPa.

118 Step 2: The allowable deviatoric stress at the subgrade surface is determined using the  
 119 following equation developed in the companion paper (i.e., Part I: Development):

120

$$121 \quad \sigma_{(d\_s)a} = \left( \frac{\varepsilon_{(p\_s)a}}{aN_s^b} \right)^{\frac{1}{m}} \sigma_{s\_s} \times 100 \quad (1)$$

122

123 where,  $\sigma_{(d\_s)a}$  is the allowable deviatoric stress at the subgrade surface;  $\varepsilon_{(p\_s)a}$  is the  
 124 allowable cumulative plastic strain at the subgrade surface needed to prevent the progressive  
 125 shear failure;  $\sigma_{s\_s}$  is the soil unconfined compressive strength;  $a$ ,  $b$  and  $m$  are material  
 126 parameters pertinent to the subgrade soil type (see Table 2 of the companion paper, i.e., Part  
 127 I: Development);  $N_s$  is the total equivalent number of repeated applications of the design load  
 128 obtained from Step 1.

129 Step 3: The allowable strain influence factor at the subgrade surface is determined, using the  
 130 following equation derived in the companion paper (i.e., Part I: Development):

131

$$132 \quad I_{(\varepsilon\_s)a} = \frac{\sigma_{(d\_s)a} \times A}{P_d} \quad (2)$$

133 where,  $I_{(\varepsilon_s)a}$  is the allowable strain influence factor based on the allowable deviatoric stress,  
134  $\sigma_{(d_s)a}$ , obtained from Step 2;  $P_d$  is the design dynamic wheel load obtained from Step 1;  
135 and the area coefficient,  $A = 1 \text{ m}^2$ .

136 Step 4: The required granular layer thickness needed to prevent the progressive shear failure  
137 at the subgrade surface is determined, as follows:

- 138 • Select a design chart from *Appendix A* (e.g., Fig. 2) that best corresponds to the ballast  
139 modulus; and
- 140 • Using the design chart, calculate the granular layer thickness corresponding to the  
141 modulus of subgrade soil,  $E_s$ , and allowable strain influence factor,  $I_{(\varepsilon_s)a}$ , obtained  
142 from Step 3.

#### 143 **Design procedure for preventing excessive plastic deformation**

144 The design procedure for preventing the excessive plastic deformation of ballast layer is  
145 developed in this section. It should be noted that most existing methods are limited to  
146 determination of the subgrade deformation only, although about 40% of the total track  
147 deformation may occur from the granular layer (Li et al. 2016; Stewart 1982). The key  
148 advantage of the current proposed design method is that the design procedure for preventing  
149 the excessive plastic deformation is based on limiting the total plastic deformation including  
150 both the ballast and subgrade layers. According to this design criterion and the above  
151 procedure, a flowchart for calculating the granular layer thickness is presented in Fig. 3. As it  
152 is difficult to assume the exact value of the granular layer thickness initially, this procedure  
153 provides an optimum granular layer thickness after several repetitions following Steps 2-4, as  
154 follows:

155 Step 1: Initially, the designer should collect and prepare the required design information, as  
 156 presented in the previous section, and some other information such as the thickness of the  
 157 deformable subgrade layer,  $H_s$ , ballast type, compressive strength of ballast at 50 kPa  
 158 confining pressure,  $\sigma_{s_b}$ , and number of load repetitions in the ballast layer,  $N_b$ . The number  
 159 of load repetitions in the ballast layer can be calculated using Equation (8) of the companion  
 160 paper (i.e., Part I: Development). Similar to the load repetitions in the subgrade soil, if there  
 161 are some major groups of wheel loads, the corresponding groups of the dynamic wheel loads  
 162 and number of repeated loads should be determined separately. Afterwards, Equations (11)  
 163 and (12) of the companion paper (i.e., Part I: Development) can be employed to determine the  
 164 total number of equivalent repeated load applications of the wheel load on the ballast layer.  
 165 The design criterion for preventing the progressive shear failure (i.e., allowable plastic strain  
 166 at the subgrade surface,  $\varepsilon_{(p_s)a}$ ) is thus substituted by enforcing the allowable total plastic  
 167 deformation of the track substructure layers,  $\rho_{ta}$ .

168 Step 2: This step is to determine the deformation of granular ballast layer, as follows:

- 169 • Assume a granular layer thickness,  $H_b$ , equal to the granular layer thickness obtained  
 170 from the design procedure used earlier for preventing the progressive shear failure.
- 171 • Select a suitable chart from *Appendix B* for estimating the distribution of the  
 172 dimensionless strain influence factor,  $I_{\varepsilon_b}$ , with depth for the granular the ballast  
 173 layer (e.g., Fig. 4) that best corresponds to the elastic modulus of the ballast and  
 174 subgrade, and the granular layer thickness.
- 175 • Determine the deformation of the granular ballast layer,  $\rho_b$ , using the following  
 176 equation developed in the companion paper (i.e., Part I: Development):



177

$$178 \quad \rho_b = \frac{x[1 + \ln(N_b)]^z}{100} \left( \frac{P_d}{A\sigma_{s_b}} \right)^y \int_0^{H_b} (I_{\varepsilon_b})^y dh \quad (3)$$

179

180 where,  $P_d$  is the design dynamic wheel load;  $\sigma_{s_b}$  is the static strength of ballast;  $N_b$   
 181 is the total number of equivalent repeated load applications of the wheel load for the  
 182 ballast layer;  $x$ ,  $y$  and  $z$  are material parameters for a particular ballast type (see Table  
 183 1 of the companion paper, i.e., Part I: Development);  $H_b$  is the granular ballast  
 184 thickness;  $I_{\varepsilon_b}$  is the distribution of strain influence factor with ballast depth; and  $A$  is  
 185 the area coefficient ( $= 1 \text{ m}^2$ ). All corresponding information are obtained from Step 1.

186 Step 3: This step is to determine the allowable subgrade deformation influence factor,  $I_{(\rho_s)a}$ ,  
 187 using the information obtained from Steps 1 and 2 and applying the following equation  
 188 developed in the companion paper (i.e., Part I: Development):

189

$$190 \quad I_{(\rho_s)a} = \frac{\rho_{ta} - \rho_b}{\frac{aLN_s^b}{100} \left( \frac{P_d}{A\sigma_{s_s}} \right)^m} \quad (4)$$

191

192 where,  $\rho_{ta}$  is the allowable track deformation;  $\rho_b$  is the contribution to track deformation by  
 193 the ballast layer;  $N_s$  is the total equivalent number of load repetitions in the subgrade for the  
 194 design traffic tonnage;  $P_d$  is the design dynamic wheel load;  $\sigma_{s_s}$  is the unconfined  
 195 compressive strength of the soil;  $a$ ,  $b$  and  $m$  are material parameters dependent on the soil  
 196 type (see Table 2 of the companion paper, i.e., Part I: Development);  $A$  is the area coefficient  
 197 ( $= 1 \text{ m}^2$ ); and  $L$  is the length coefficient ( $= 1 \text{ m}$ ).

198 Step 4: Finally, determine the required granular layer thickness,  $H_b$ , needed to prevent the  
199 excessive plastic deformation of the track, as follows:

- 200 • Select a suitable design chart from *Appendix C* (e.g., Fig. 5) that best corresponds to  
201 the ballast modulus, existing subgrade soil type, and modulus.
- 202 • Calculate the granular layer thickness,  $H_b$ , corresponding to the allowable deformation  
203 influence factor of subgrade and thickness of deformable subgrade layer using the  
204 selected design charts.
- 205 • Compare the design thickness obtained in this step with the thickness assumed in the  
206 calculation of the granular layer deformation in Step 2. If the obtained thickness from  
207 Step 4 is not equal to the assumed thickness, then repeat Steps 2-4 until the assumed  
208  $H_b$  converges with the design thickness obtained in Step 4. In each iteration, the  
209 calculated thickness can be assumed for the next iteration to achieve faster  
210 convergence.

## 211 **Design applications**

212 To validate the proposed design method, it is applied to four well-documented case studies  
213 found in the literature and the results obtained are compared with field measurements. These  
214 two case studies are for test tracks reported by Li and Selig (1998b), including the  
215 Association of American Railroads (AAR) low track modulus (LTM) and trial low track  
216 modulus (TLTM). Another two case studies of real track sites at the Northeast Corridor (NC)  
217 between Baltimore and Philadelphia are also considered for additional validation of the  
218 proposed design method, and results obtained are again compared with field measurements  
219 and found to be in good agreement.

**220 LTM and TLTM tracks**

221 In 1991, a 183 m long low track modulus (LTM) test track was built on a fat clay type  
222 subgrade at the Association of American Railroads (AAR) Heavy Tonnage Loop (HTL) in  
223 Pueblo, Colorado. The information needed for design of these ballasted tracks are given in  
224 Table 1. Prior to the construction of the LTM, a 30 m long trial low track modulus (TLTM)  
225 test track was constructed to examine the practicality of building a longer LTM track. The  
226 key objective of constructing the LTM test track was to investigate the impact of soft  
227 subgrade on track performance under repeated heavy axle train (HAT) moving loads (Li and  
228 Selig 1996). The subgrade soil at the Pueblo test track site was originally silty sand, which  
229 does not represent a soft subgrade soil. To construct a track on soft subgrade soil, a 3.66 m  
230 wide and 1.5 m deep trench was dug in the natural subgrade and filled with the Mississippi  
231 buckshot clay of liquid limit ( $LL = 60\sim 70$ ) and plasticity index ( $PI = 40\sim 45$ ). To achieve a  
232 subgrade of low stiffness, the filled material within the trench was compacted with the water  
233 content (30%) and dry density at 90% of its maximum dry density, which according to the  
234 ASTM D698 was found to be  $14.91 \text{ kN/m}^3$ . Although the water content for both the LTM and  
235 TLTM subgrades was targeted to be 30%, the average water contents in the LTM and TLTM  
236 subgrades were actually 33% and 29%, respectively (Li and Selig 1996). Hence, the  
237 corresponding unconfined compressive strength of subgrade soil was about 90 kPa for the  
238 LTM track and 166 kPa for the TLTM track. The relevant soil modulus of the LTM track  
239 subgrade varied from 14 MPa to 21 MPa, while it was in the range of 41MPa to 55 MPa for  
240 the TLTM track. The difference between these two track sites was in their subgrade modulus  
241 and unconfined compressive strength (see Table 1). Accordingly, the design thickness for  
242 each track is expected to be different.

243 *Step-by-step calculation for preventing progressive shear failure*

244 Step 1: At first, the information needed for design of ballasted railway track foundations  
 245 (i.e., loading condition, design criteria, ballast and subgrade material characteristics) are  
 246 specified and listed in Table 1. For train geometry, the value of wheel spacing factor (WSF)  
 247 corresponding to a wheel spacing of 1.8 m is found to be 1.38 (obtained from Fig. 12b of the  
 248 companion paper, i.e., Part I: Development). Also, for this particular track-ground condition,  
 249 the value of dynamic amplification factor (DAF) corresponding to the train speed is obtained  
 250 to be 1.04, using Fig. 13 of the companion paper (i.e., Part I: Development). Afterwards, the  
 251 design dynamic wheel load,  $P_d$ , is calculated to be 250 kN using Equation (7) of the  
 252 companion paper (i.e., Part I: Development). The equivalent number of load repetitions in the  
 253 subgrade layer is determined using Equation (9) of the companion paper (i.e., Part I:  
 254 Development) to be  $N_s = 386,000$ .

255 Step 2: Considering the appropriate respective design parameters and number of load  
 256 repetitions,  $N_s$ , obtained in Step 1, the allowable deviatoric stress at the subgrade surface,  
 257  $\sigma_{(d_s)a}$ , is calculated using Equation (1) to be 41 kPa and 76 kPa for the LTM and TLTM  
 258 tracks, respectively.

259 Step 3: The allowable strain influence factors corresponding to the allowable deviatoric  
 260 stresses,  $\sigma_{(d_s)a}$ , and design dynamic wheel load,  $P_d$ , are determined using Equation (2) to be  
 261  $I_{(\varepsilon_s)a} = 0.16$  for the LTM track and 0.31 for the TLTM track.

262 Step 4: The design chart A2 of *Appendix A* is selected as it corresponds to ballast modulus  $E_b$   
 263 = 270 MPa, for both the LTM and TLTM tracks (see Fig. 2). The required granular layer  
 264 thickness for the LTM track needed to prevent the progressive shear failure is determined for  
 265  $I_{\varepsilon_s} = 0.16$  and  $E_s = 15$  MPa, and is found to be  $H_b = 0.53$  m. Similarly, using the same design

266 chart, the required granular layer thickness for the TLTM track is found to be  $H_b = 0.40$  m  
 267 considering  $I_{\varepsilon_s} = 0.31$ ,  $E_s = 41$  MPa and  $E_b = 270$  MPa.

268 *Step-by-step calculation for preventing excessive plastic deformation*

269 Step 1: This step is similar to Step 1 in the design procedure for preventing the progressive  
 270 shear failure. Therefore, the design dynamic wheel load is obtained to be  $P_d = 250$  kN and the  
 271 equivalent number of load repetitions in the subgrade to be  $N_s = 386,000$ . Moreover, the  
 272 number of load repetitions in the ballast layer is determined using Equation (8) of the  
 273 companion paper (i.e., Part I: Development) to be  $N_b = 772000$ .

274 Step 2: At first, the granular layer thickness is assumed to be equal to the thickness obtained  
 275 from the design procedure for preventing the progressive shear failure (i.e.,  $H_b = 0.53$  m for  
 276 the LTM track and  $H_b = 0.40$  m for the TLTM track). For the LTM track with  $E_b = 270$  MPa,  
 277  $H_b = 0.53$  m and  $E_s = 15$  MPa, the distribution of the dimensionless strain influence factor,  
 278  $I_{\varepsilon_b}$ , with the ballast depth is obtained from *Appendix B* (Charts B7 and B8). Afterwards, for  
 279 the granite ballast (assumed), the deformation of the ballast layer,  $\rho_b$ , is determined using  
 280 Equation (3) to be 0.011 m, considering  $\sigma_{s_b} = 307$  kPa,  $P_d = 250$  kN and  $N_b = 772,000$ .  
 281 Similarly, for the TLTM track with  $E_b = 270$  MPa,  $H_b = 0.40$  m and  $E_s = 41$  MPa, the  
 282 distribution of dimensionless strain influence factor,  $I_{\varepsilon_b}$ , with ballast depth is obtained from  
 283 *Appendix B* (Charts B6 and B7). Afterwards, the deformation of the ballast layer is  
 284 determined using Equation (3) to be 0.006 m.

285 Step 3: For the LTM track loading and subgrade conditions (i.e.,  $P_d = 250$  kN,  $N_s = 386000$ ,  
 286 CH type subgrade and  $\sigma_{s_s} = 90$  kPa) and the design criterion of  $\rho_{ta} = 0.025$  m, the  
 287 allowable subgrade deformation influence factor,  $I_{(\rho_s)a}$ , is obtained to be 0.01 using

288 Equation (4). Likewise, for the TLTM track, the allowable subgrade deformation influence  
 289 factor is obtained using Equation (4) to be  $I_{(\rho_s)_a} = 0.06$  for  $P_d = 250$  kN,  $N_s = 386000$ , CH  
 290 type subgrade, and  $\sigma_{s_s} = 165$  kPa.

291 Step 4: To determine the design thickness, chart C21 from *Appendix C* [see Fig. 5(a)] is  
 292 selected which best corresponds to the LTM track substructure conditions (i.e.,  $E_b = 270$   
 293 MPa,  $E_s = 15$  MPa, and CH soil). From this chart, the required granular layer thickness  
 294 corresponding to the deformable subgrade layer (i.e.,  $H_s = 1.5$  m and  $I_{(\rho_s)_a} = 0.01$  obtained  
 295 in Step 3), is found to be  $H_b = 0.66$  m. As the obtained thickness is not equal to assumed  
 296 thickness (i.e. obtained  $H_b \neq H_b$  of Step 1), Step 2 (i.e., calculation of granular ballast  
 297 deformation,  $\rho_b$ ) is repeated considering the granular ballast thickness obtained in Step 4  
 298 (i.e.,  $H_b = 0.66$  m). After several repetitions of Steps 2–4, the granular layer thickness for the  
 299 LTM track is obtained to be  $H_b = 0.70$  m. Similarly, for the TLTM track with  $E_b = 270$  MPa,  
 300  $E_s = 41$  MPa, and CH soil, Fig. 5(b) is selected from *Appendix C*. Employing the selected  
 301 design chart, the required granular layer thickness is determined corresponding to the  
 302 deformable subgrade layer (i.e.,  $H_s = 1.5$  m and  $I_{(\rho_s)_a} = 0.06$ ) to be  $H_b = 0.25$  m. Again, as  
 303 the obtained  $H_b \neq H_b$  of Step 1, Steps 2-4 are repeated. Finally, the required granular layer  
 304 thickness needed to prevent the excessive plastic deformation is calculated to be  $H_b = 0.30$  m.

### 305 *Design thickness*

306 As presented above, the granular layer thickness required to prevent the excessive plastic  
 307 deformation (i.e.,  $H_b = 0.70$  m) for the LTM track is higher than that needed to prevent the  
 308 progressive shear failure (i.e.,  $H_b = 0.53$  m). Thus, the design thickness is the maximum of  
 309 the two obtained results (i.e.,  $H_b = 0.70$  m). On the other hand, for the TLTM track, the  
 310 granular layer thickness required to prevent the excessive plastic deformation (i.e.,  $H_b = 0.30$

311 m) is less than that needed to prevent the progressive shear failure (i.e.,  $H_b = 0.40$  m). Hence,  
312 the design thickness to be used is  $H_b = 0.40$  m.

### 313 *Comparisons between proposed design method and field measurements*

314 Based on the design criteria for preventing the progressive shear failure (i.e.,  $\varepsilon_{(p-s)a} \leq 2\%$ )  
315 and for preventing the excessive plastic deformation (i.e.,  $\rho_{ta} \leq 0.025$  m), the required  
316 granular layer thickness for the LTM and TLTM tracks are determined to be  $H_b = 0.70$  m and  
317  $0.40$  m, respectively, as calculated in the earlier section. In reality, during the construction of  
318 both the LTM and TLTM tracks, a granular layer of  $0.45$  m thickness ( $0.30$  m ballast and  
319  $0.15$  m sub-ballast) was adopted based on an assumption of  $30\%$  water content in the  
320 subgrade soil and minimum density of  $90\%$  of the standard maximum dry density.  
321 Afterwards, the track response in these sites was measured and the subgrade conditions were  
322 evaluated experimentally, which provide an excellent opportunity to assess the proposed  
323 design method. From the field measurements, it was found that the LTM track with the  
324 adopted granular layer thickness of  $0.45$  m was unable to bear the HAL for design traffic of  
325  $60$  MGT, and thus had difficulty in sustaining the required track surface geometry. The LTM  
326 track subgrade suffered rapid progressive shear failure and excessive plastic deformation.  
327 Therefore, the test track needed frequent rail lifting by ballast tamping. Fig. 6 shows the  
328 cumulative track settlement with the traffic loading for the LTM track (Li 1994). It can be  
329 seen that the track actually required frequent ballast tamping and surfacing (rail lift up)  
330 following  $12.4$  MGT, and finally, the traffic along the track had to be stopped after  
331 approximately  $62.3$  MGT and the test track was then rebuilt. On the other hand, the TLTM  
332 track with the same granular layer thickness of  $0.45$  m was able to carry the HAL for design  
333 traffic of  $60$  MGT without any track failure. Consequently, no major track maintenance was  
334 invoked during the design life of this track.

335 A comparison between the originally adopted  $H_b$  and that obtained from design (see Table 2)  
 336 indicates that the adopted thickness for the LTM track of 0.45 m was much less than the  
 337 required thickness of 0.70 m, but the adopted thickness for the TLTM track of 0.45 m was  
 338 higher than the required thickness of 0.40 m. Therefore, the LTM track was unable to  
 339 maintain the track geometry and invoked maintenance, whereas the TLTM track was able to  
 340 sustain the required track geometry without any maintenance. In other words, the proposed  
 341 design method was successful in predicting the failure of the LTM track and the proper  
 342 thickness of the TLTM track. These results are extremely encouraging for the proposed  
 343 design method.

344 As an additional validation tool, the actual LTM track-subgrade condition with the adopted  
 345 0.45 m granular layer thickness is simulated using the 3D FE modelling and the distribution  
 346 of the strain influence factor with depth in the ballast and subgrade layers is obtained. Then,  
 347 the cumulative vertical track deflections for the ballast and subgrade layers at different traffic  
 348 loads are computed using the results obtained from the 3D FE modelling as well as the  
 349 following equation developed in the companion paper (i.e., Part I: Development):

350

$$\begin{aligned}
 \rho_t = & \frac{x[1 + \ln(N_b)]^z}{100} \left( \frac{P_d}{A\sigma_{s_b}} \right)^y \int_0^{H_b} (I_{\varepsilon_b})^y dh + \\
 & \frac{aLN_s^b}{100} \left( \frac{P_d}{A\sigma_{s_s}} \right)^m \int_0^{H_s} (I_{\varepsilon_s})^m \frac{dh}{L}
 \end{aligned} \tag{5}$$

352

353 The cumulative track deflections are then plotted against the traffic load in MGT and  
 354 compared with the field measurements available in the literature (Li 1994), as shown in Fig.  
 355 7. It can be clearly seen that good agreement exists between the FE predictions and field  
 356 measurements, which confirms that the validity of the FE modelling process and improved



357 empirical models for predicting the cumulative plastic deformation of ballast and subgrade  
358 adopted in this study. This indicates that the design method developed in this study based on  
359 the combined FE modelling and improved empirical models is reliable and can be used with  
360 confidence to predict the railway track behavior.

### 361 **Northeast Corridor track**

362 In this section, two more case studies of real track sites at the Northeast Corridor (NC)  
363 between Baltimore and Philadelphia are used for further validation of the proposed method.  
364 One of the two sites is located at Edgewood, Maryland, and the other site is located at  
365 Aberdeen, Maryland, some 16 km apart from the Edgewood site. The track in Edgewood site  
366 suffered frequent bouts of differential settlements over a distance of approximately 10 km.  
367 This track site needed frequent maintenance by ballast tamping at least twice a year.  
368 Moreover, remedy measures such as application of geotextiles and lime slurry injection were  
369 taken since 1984; however, such remedies were not fruitful. For the other site at Aberdeen,  
370 only a small portion of the track (about 60 m long) suffered a problem of mud pumping;  
371 however, the geometry deterioration was not a concern (Li and Selig 1998b).

372 To investigate the key reasons for track failures at both sites, the loading characteristics and  
373 material properties were studied by Li and Selig (1994). Based on the information available  
374 in the literature, the minimum required granular layer thickness for both sites are determined  
375 using the current proposed design method. At the Edgewood site, the subgrade soil was lean  
376 clay (LC) with unconfined compressive strength of approximately 48-83 kPa. On the other  
377 hand, the subgrade soil at the Aberdeen site was also lean clay but its unconfined  
378 compressive strength was in the range of 97–290 kPa. The subgrade soil properties and other  
379 information required for design of tracks at both sites are given in Table 3. As both sites were  
380 parts of the NC and not far away from each other, the traffic was the same. The traffic along

381 the NC track was mixed (50% passenger trains and 50% freight trains). Table 4 gives the  
382 loading characteristics used for design of these two tracks. As the traffic was mixed, the  
383 number of equivalent load applications in the ballast and subgrade layers is determined using  
384 Equations (7-13) of the companion paper (i.e., Part I: Development).

385 Based on the design criteria of preventing the progressive shear failure (i.e.  $\varepsilon_{pa} \leq 2\%$ ) and for  
386 preventing the excessive plastic deformation (i.e.  $\rho_{ta} \leq 0.025$  m), the required granular layer  
387 thicknesses,  $H_b$ , for the Edgewood site are determined to be 1.08 m and 1.16 m, respectively.  
388 Consequently, the design thickness for this site should be taken as 1.20 m. However, the  
389 actual granular layer thickness at the Edgewood site was varied from 0.30 to 0.50 m (from the  
390 cone penetration tests and cross trench measurements of the track site), as reported by Li and  
391 Selig (1994). This thickness is significantly less than the obtained design thickness of 1.20 m  
392 required to reduce the dynamic train induced stresses transmitted to the subgrade to prevent  
393 the progressive shear failure and excessive plastic deformation. As a result, it is not surprising  
394 that the track of this site has suffered a significant progressive shear failure at the subgrade  
395 surface, and deep ballast pockets have also occurred. Moreover, the non-uniform compressive  
396 strength of the subgrade (48 kPa to 83 kPa) caused excessive differential track settlement.

397 For the Aberdeen site, the required granular layer thickness calculated from the proposed  
398 design method is  $H_b = 0.66$  m for preventing the progressive shear failure and  $H_b = 0.60$  m  
399 for preventing the excessive plastic deformation. Therefore, the design thickness of this site  
400 should be  $H_b \approx 0.70$  m. From the field measurements reported by Li and Selig (1994), the  
401 actual granular layer thickness at this site was varied between 0.70 and 1.0 m, which is equal  
402 or larger than the required design thickness. As the dynamic train induced stresses in the  
403 subgrade were lower than the allowable value, this track was able to carry the design load  
404 without any geometry deterioration. Comparison of the design thickness obtained from the

405 proposed design method and actual thickness at both the Edgewood and Aberdeen sites is  
406 summarized in Table 5, which also includes the track conditions for both sites. Evidently, the  
407 results of the proposed design method are consistent with the field measurements.

## 408 **Summary and conclusions**

409 In this paper, step-by-step design procedures were presented for a new design method of  
410 ballasted railway track foundations. The new proposed method has substantial benefits over  
411 the existing methods in the way at which the railway traffic was characterized and stress was  
412 analyzed. In addition, the new method has taken into account the deformation of both the  
413 ballast and subgrade layers. The main parameters considered in design include the train  
414 speed, track-ground condition, geometry and magnitude of train wheel loads, number of load  
415 repetition, as well as modulus, thickness and type of ballast and subgrade. All these  
416 parameters considerably affect a safe design for preventing track failures. Design predictions  
417 obtained from the developed design method were examined against field measurements for  
418 four different case studies and the results were found to be in good agreement. Consequently,  
419 the proposed design method can be used with confidence and it is expected to provide a  
420 significant contribution to the current railway track code of practice. To facilitate the use of  
421 the new design method by practitioners, a user friendly software will be developed in the near  
422 future and will be made available upon request.

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449 Massachusetts, USA.

## List of symbols

$a$	material parameter pertinent to the subgrade soil type
$b$	material parameter pertinent to the subgrade soil type
$m$	material parameter pertinent to the subgrade soil type
$x$	material parameter dependent on ballast type
$y$	material parameter dependent on ballast type
$z$	material parameter dependent on ballast type
$A$	area coefficient
$E_b$	ballast modulus
$E_s$	subgrade soil modulus
$H_b$	granular layer thickness
$H_s$	subgrade thicknesses
$L$	length coefficient
$L_a$	wheel spacing
$N_b$	number of load applications in the ballast layer
$N_s$	number of load applications in the subgrade layer
$P_d$	design dynamic wheel load
$P_s$	maximum static wheel load
$\varepsilon_{(p\_s)a}$	allowable subgrade surface cumulative plastic strain
$\sigma_{(d\_s)a}$	allowable deviatoric stress at the subgrade surface
$\sigma_{s\_b}$	compressive strength of ballast at 50 kPa confining pressure
$\sigma_{s\_s}$	unconfined compressive strength of the soil
$\rho_b$	deformation of granular ballast layer
$\rho_{ta}$	allowable total plastic deformation of the track
$I_{\varepsilon\_b}$	strain influence factor in the granular layer
$I_{(\varepsilon\_s)a}$	allowable subgrade surface strain influence factor
$I_{(\rho\_s)a}$	allowable subgrade deformation influence factor

## Figure captions

**Fig. 1.** Flowchart of design of railway track foundations for preventing the progressive shear failure of track subgrade.

**Fig. 2.** Typical example of design chart to calculate the granular layer thickness for preventing the progressive shear failure of track subgrade (obtained from *Appendix A*, Chart A2).

**Fig. 3.** Flowchart of design of railway track foundations for preventing the excessive track deformation.

**Fig. 4.** Distribution of strain influence factor with depth for the ballast layer.

**Fig. 5.** Typical examples of design charts to calculate the granular layer thickness for preventing the excessive track deformation (obtained from *Appendix C*, Charts C21 and C25).

**Fig. 6.** Field measurements of average settlement and lift-up of rail with traffic load for the LTM test track (redrawn from Li 1994).

**Fig. 7.** Comparison between new design method and field measurements.

## Table captions

**Table 1.** Design parameters for the LTM and TLTM test tracks (adapted from Li et al. 1996).

**Table 2.** Design results and track conditions for the LTM and TLTM test tracks.

**Table 3.** Design parameters for tracks at Edgewood and Aberdeen sites (adapted from Li and Selig 1998b).

**Table 4.** Traffic characteristics at the Northeast Corridor between Baltimore and Philadelphia (adapted from Li and Selig 1998b).

**Table 5.** Comparison of results between new design method and site conditions for tracks at Edgewood and Aberdeen sites.



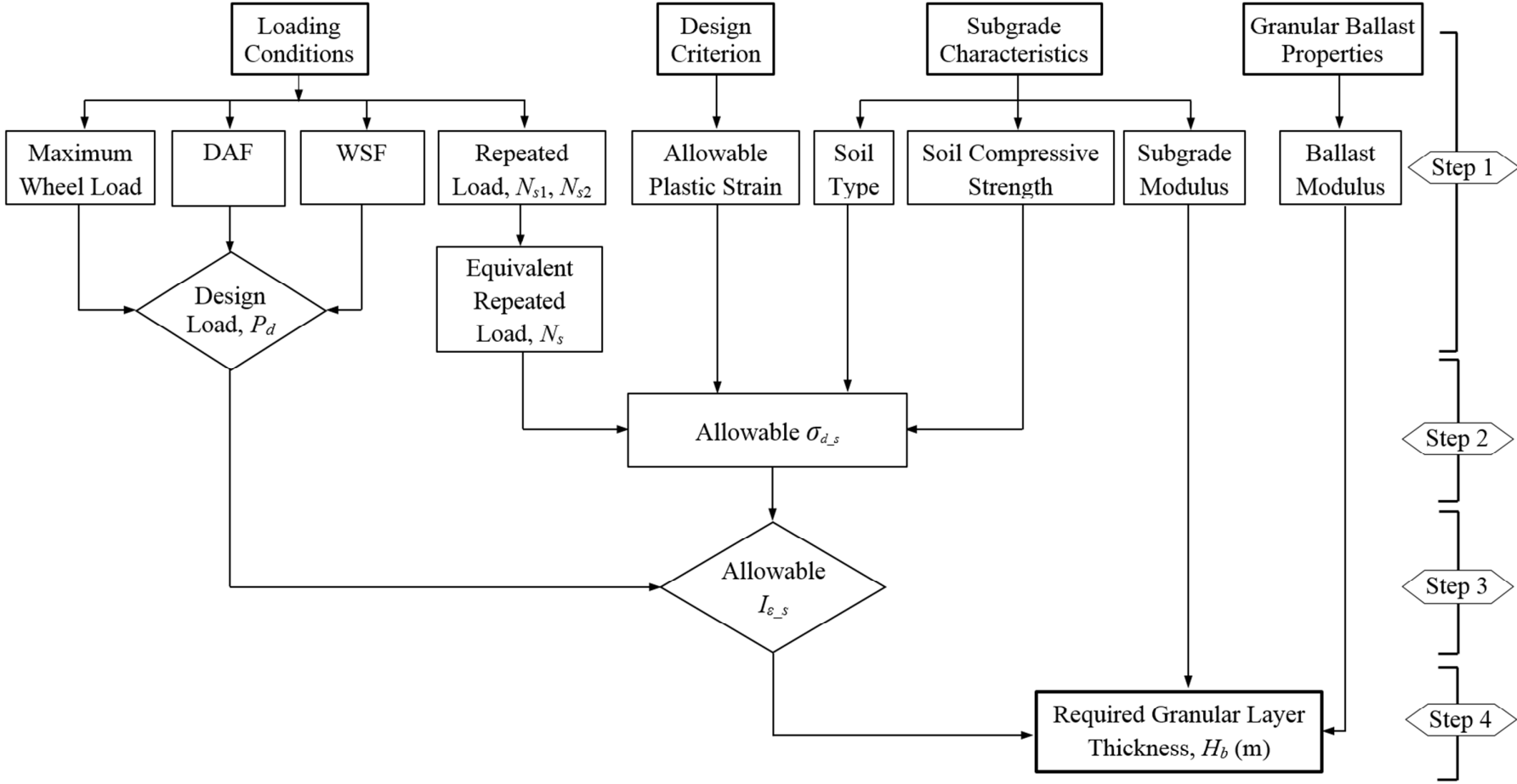
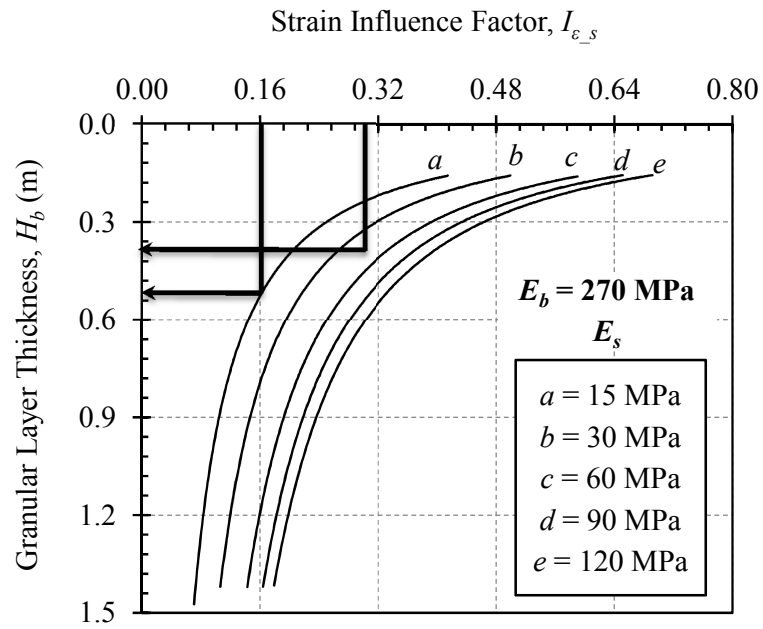


Fig. 1. Flowchart of design of railway track foundations for preventing the progressive shear failure of track subgrade.



**Fig. 2.** Typical example of design chart to calculate the granular layer thickness for preventing the progressive shear failure of track subgrade (obtained from *Appendix A*, Chart A2).

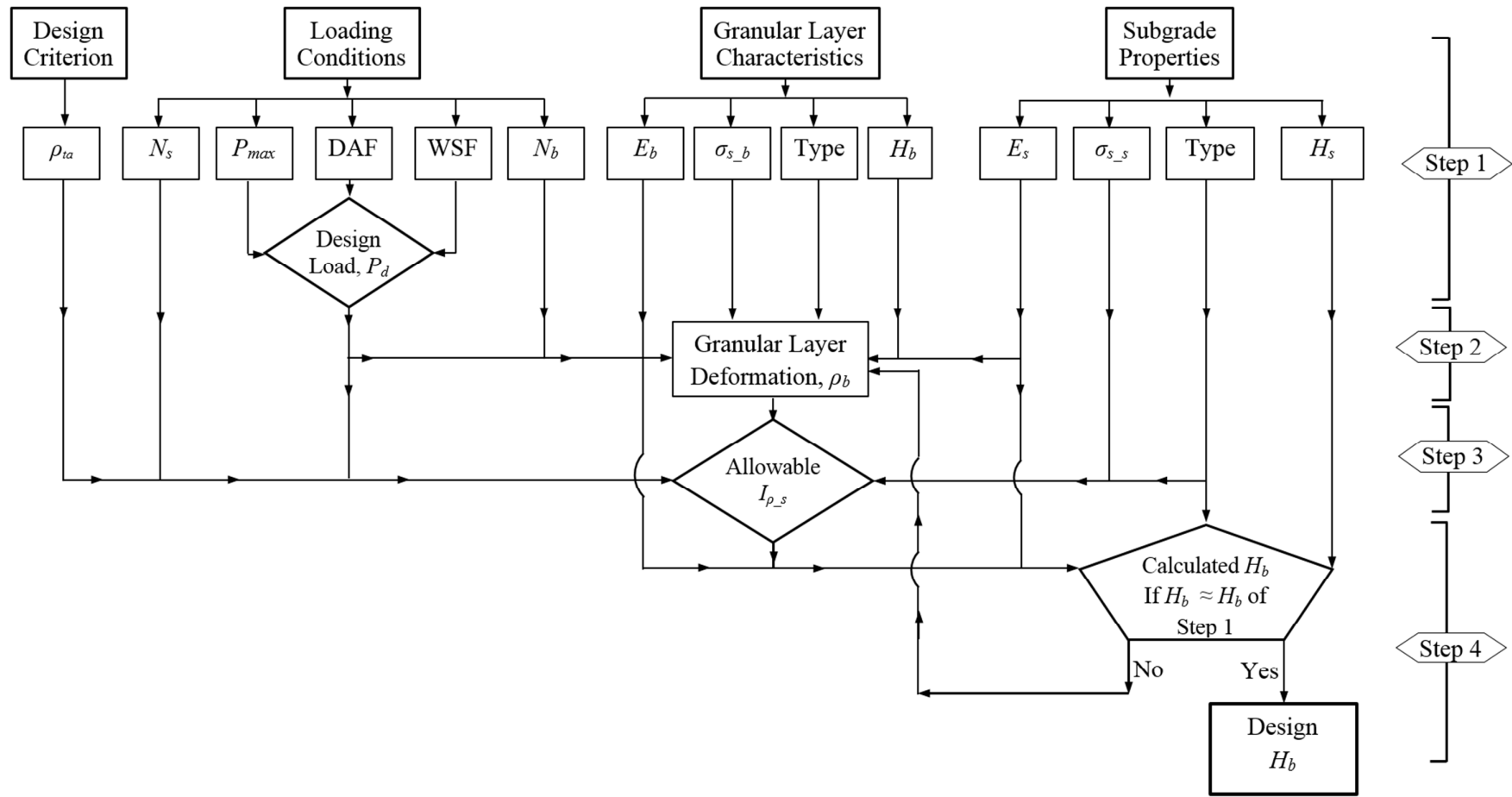
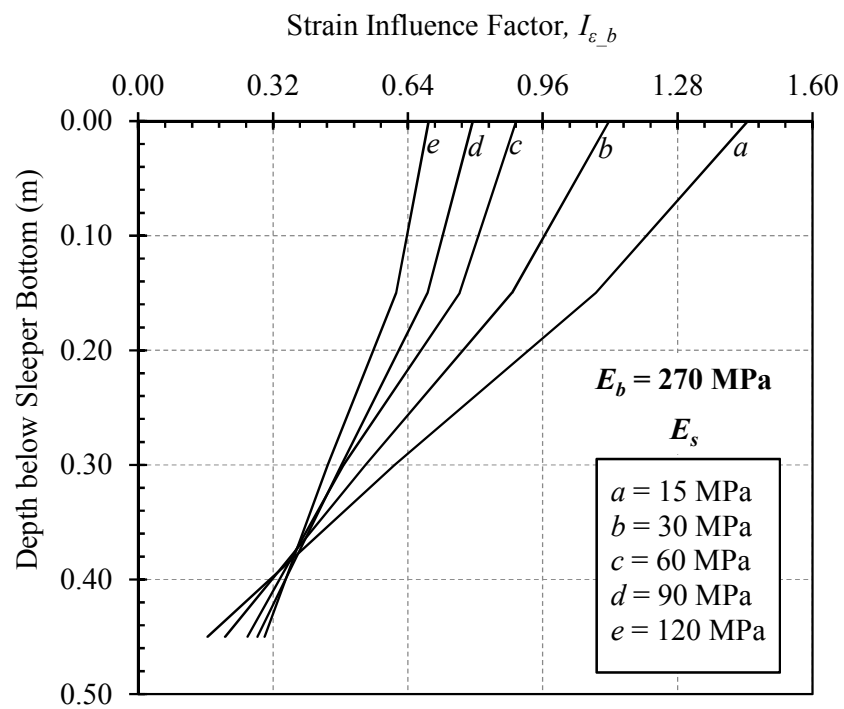
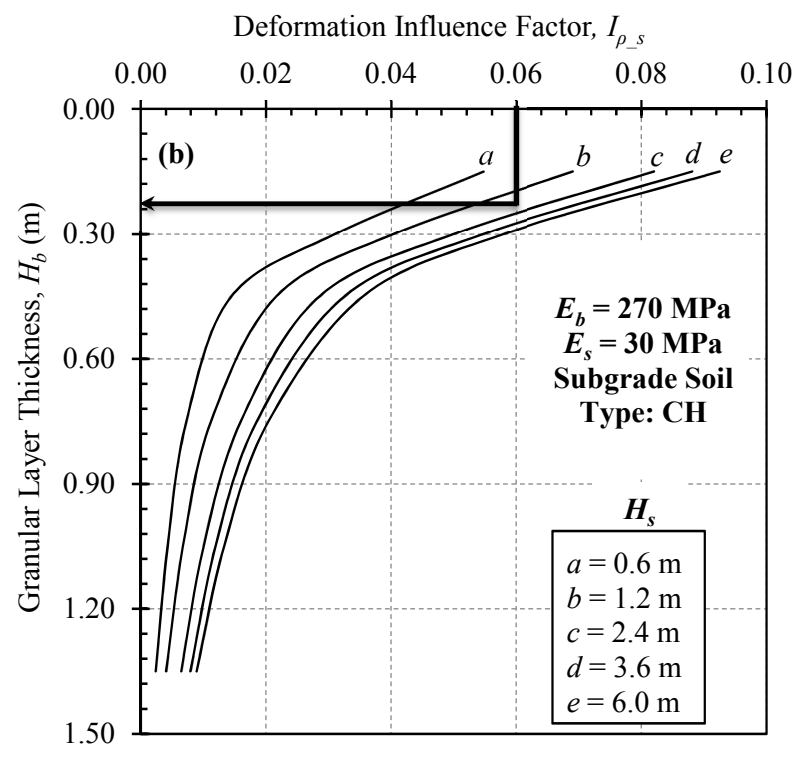
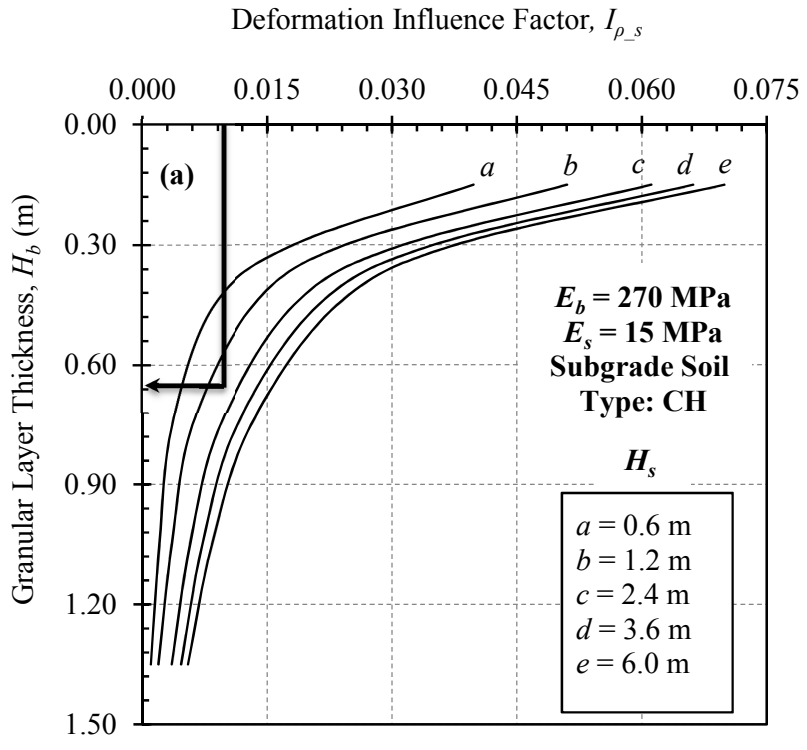


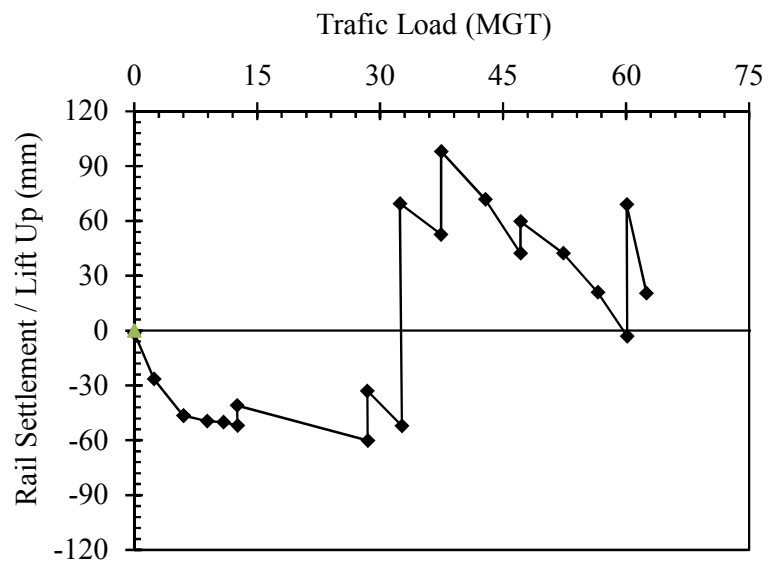
Fig. 3. Flowchart of design of railway track foundations for preventing the excessive track deformation.



**Fig. 4.** Distribution of strain influence factor with depth for the ballast layer.



**Fig. 5.** Typical examples of design charts to calculate the granular layer thickness for preventing the excessive track deformation (obtained from *Appendix C*, Charts C21 and C25).



**Fig. 6.** Field measurements of average settlement and lift-up of rail with traffic load for the LTM test track (redrawn from Li 1994).

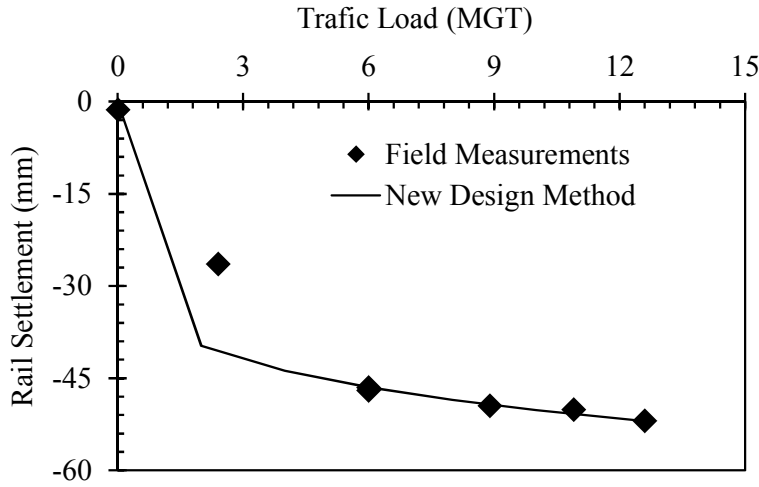


Fig. 7. Comparison between new design method and field measurements.

**Table 1.** Design parameters for the LTM and TLTM test tracks (adapted from Li et al. 1996).

Design parameters	LTM	TLTM
Loading condition		
Static wheel load, $P_s$ (kN)	173	173
Wheel spacing, $L_a$ (m)	1.8	1.8
Train speed, (m/s)	18	18
Design tonnage (MGT)	60	60
Design criteria		
Cumulative plastic strain, $\varepsilon_{(p_s)a}$ (%)	2%	2%
Cumulative plastic deformation, $\rho_{ta}$ (mm)	25	25
Subgrade characteristics		
Soil type	Fat clay (CH)	Fat clay (CH)
Thickness, $H_s$ (m)	1.50	1.50
Subgrade modulus, $E_s$ (MPa)	15	41
Unconfined compressive strength, $\sigma_{s_s}$ (kPa)	90	165
Ballast characteristics		
Ballast type (assumed)	Granite (G)	Granite (G)
Ballast modulus, $E_b$ (MPa)	270	270
Compressive strength, $\sigma_{s_b}$ (kPa)	307	307



**Table 2.** Design results and track conditions for the LTM and TLTM test tracks.

Comparison parameters	LTM	TLTM
Unconfined compressive strength, $\sigma_{s_s}$ (kPa)	90	165
Subgrade modulus, $E_s$ (MPa)	14	41
Adopted granular layer thickness, $H_b$ (m)	0.45	0.45
Required granular layer thickness, $H_b$ (m)	0.70	0.40
Track condition with the adopted granular layer thickness	Track excessive plastic deformation and progressive shear failure	No track failures

**Table 3.** Design parameters for tracks at Edgewood and Aberdeen sites (adapted from Li and Selig 1998b).

Design parameters	Edgewood site	Aberdeen site
Subgrade characteristics		
Soil type	Lean clay (CL)	Lean clay (CL)
Thickness, $H_s$ (m)	1.5	1.5
Subgrade modulus, $E_s$ (MPa)	15	30
Unconfined compressive strength, $\sigma_{s_s}$ (kPa)	48-83	97-290
Ballast characteristics		
Ballast type (assumed)	Granite (G)	Granite (G)
Ballast modulus, $E_b$ (MPa)	270	270
Compressive strength, $\sigma_{s_b}$ (kPa)	307	307
Design criteria (for 10 years)		
Cumulative plastic strain, $\varepsilon_{(p_s)a}$ (%)	2%	2%
Cumulative plastic deformation, $\rho_{ta}$ (mm)	25	25

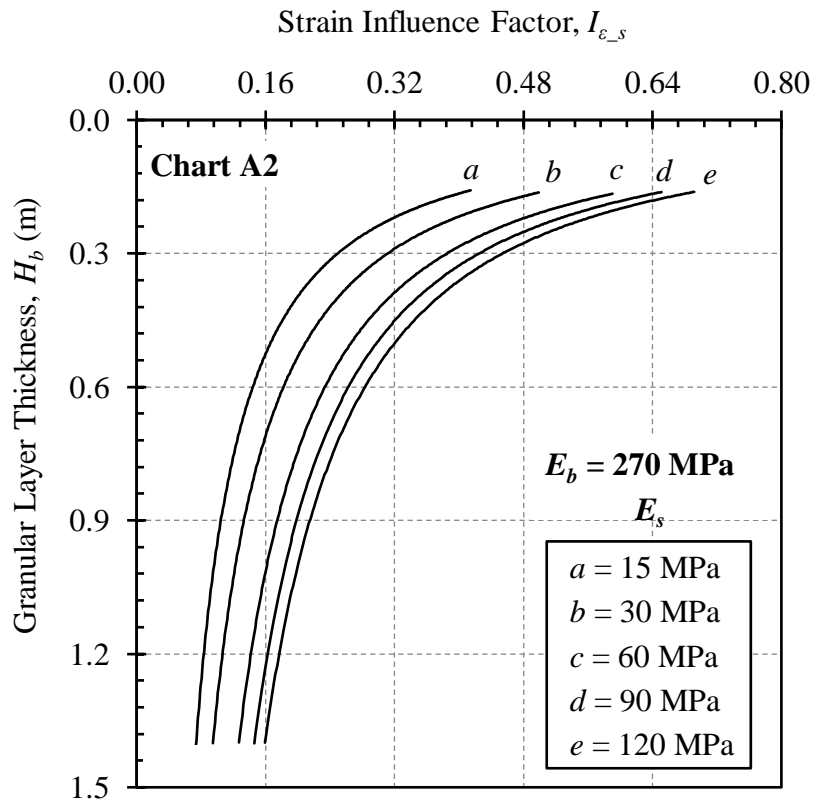
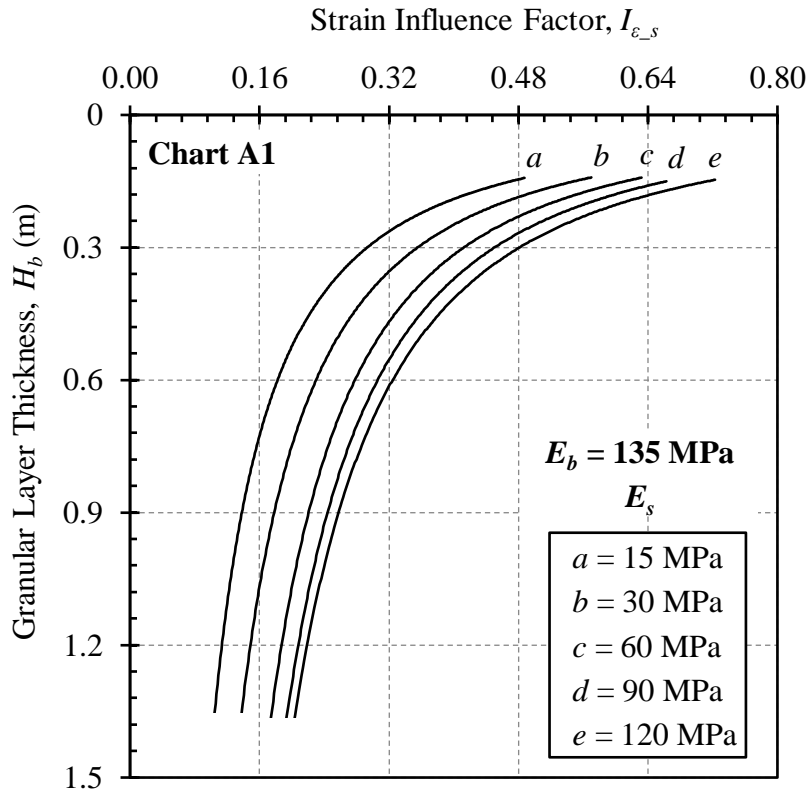
**Table 4.** Traffic characteristics at the Northeast Corridor between Baltimore and Philadelphia (adapted from Li and Selig 1998b).

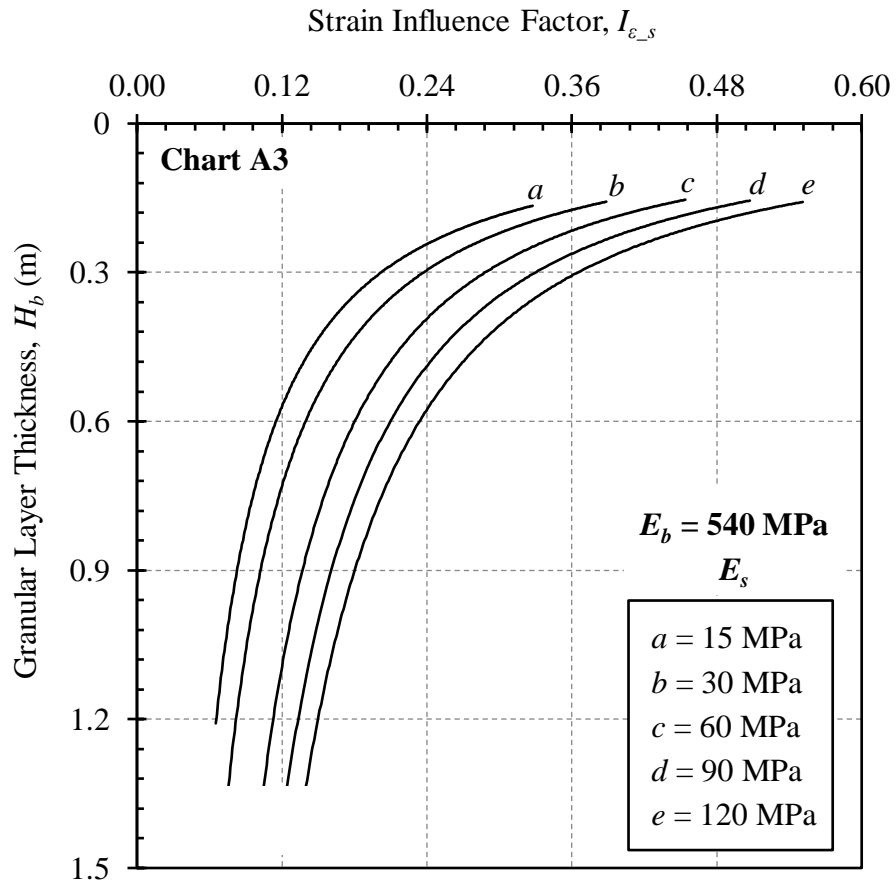
Loading condition	Annual traffic tonnage (MGT)	Static wheel load (kN)	Speed (km/h)	Wheel spacing (m)
Freight train				
Wheel 1	15	156	60	2.2
Wheel 2	22	44	60	2.2
Passenger train				
Wheel 1	15	70	190	2.9

**Table 5.** Comparison of results between new design method and site conditions for tracks at Edgewood and Aberdeen sites.

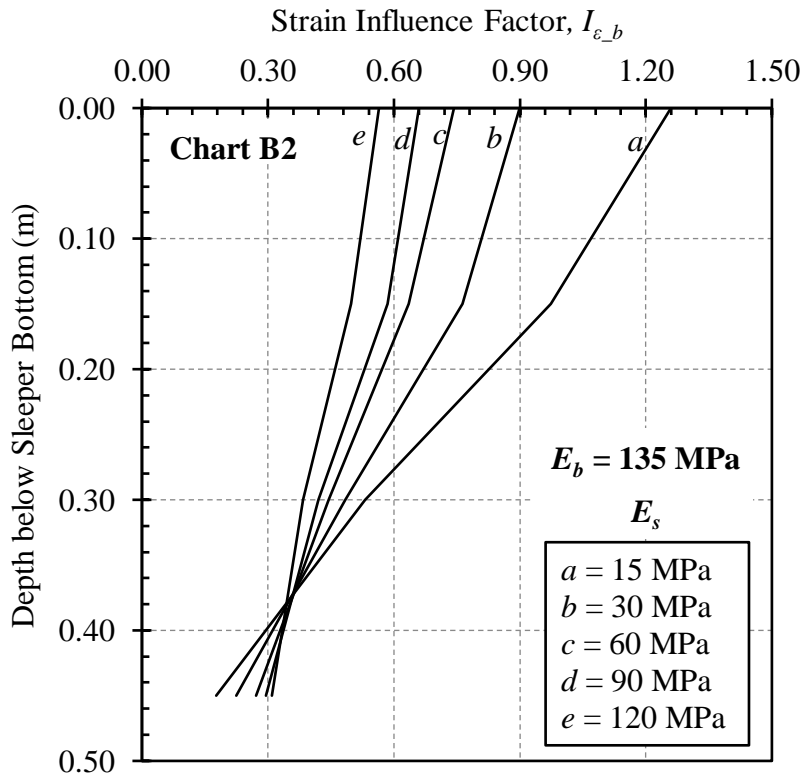
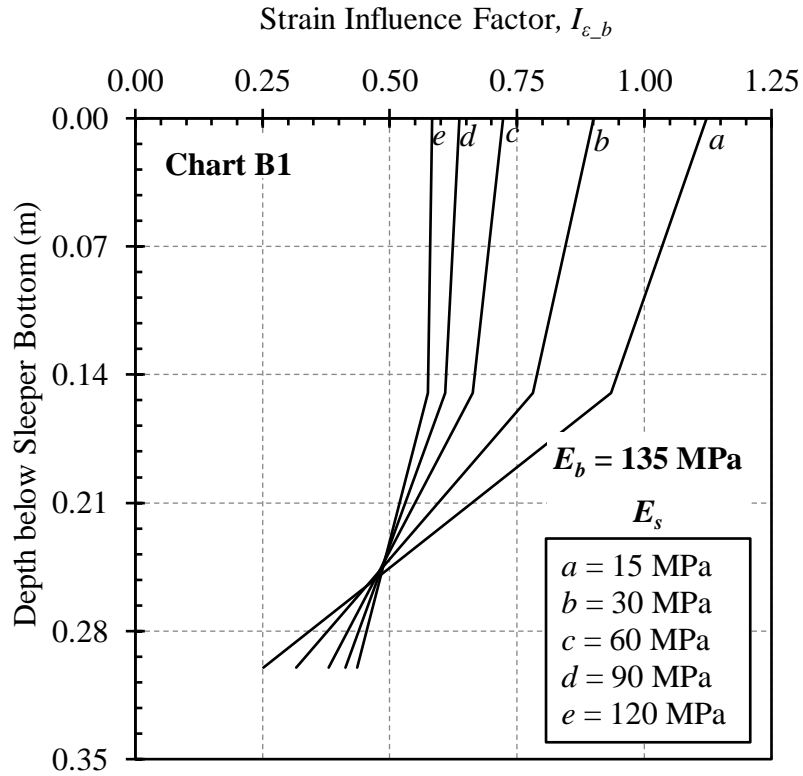
<b>Comparison parameters</b>	<b>Edgewood site</b>	<b>Aberdeen site</b>
Design thickness, $H_b$ (m)	1.20	0.70
Existing thickness, $H_b$ (m)	0.3-0.5	0.70-1.0
Remark	Existing thickness is less than design thickness.	Existing thickness is more than design thickness.
Track failure condition for the adopted thickness	Subgrade progressive shear failure, deep ballast pocket and differential settlement	No track failures

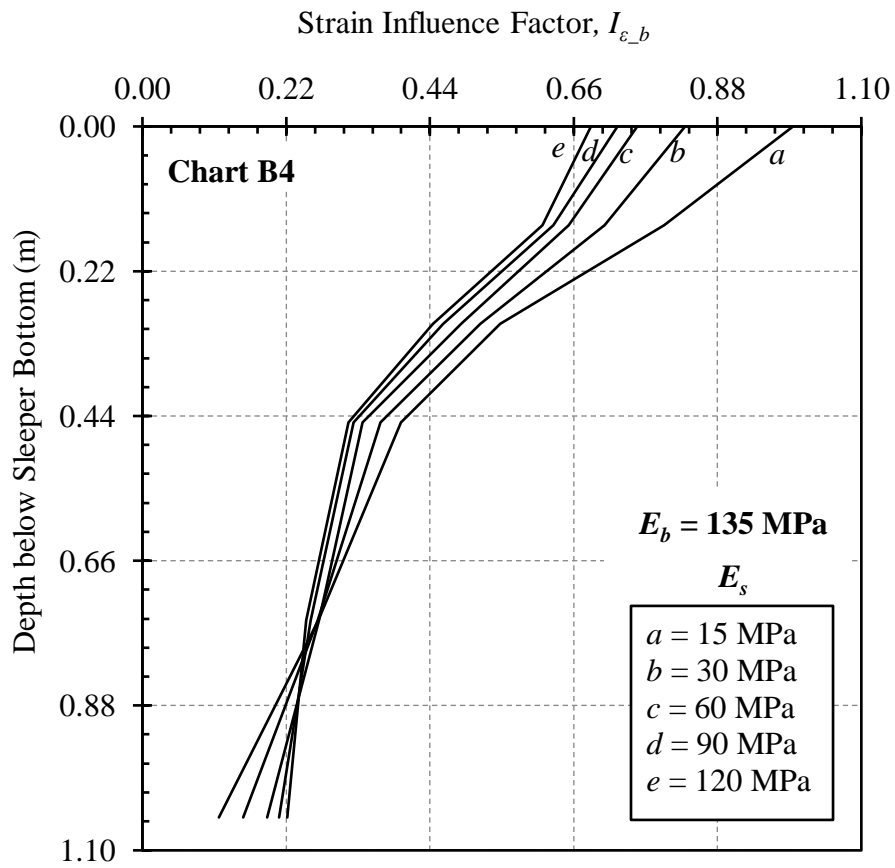
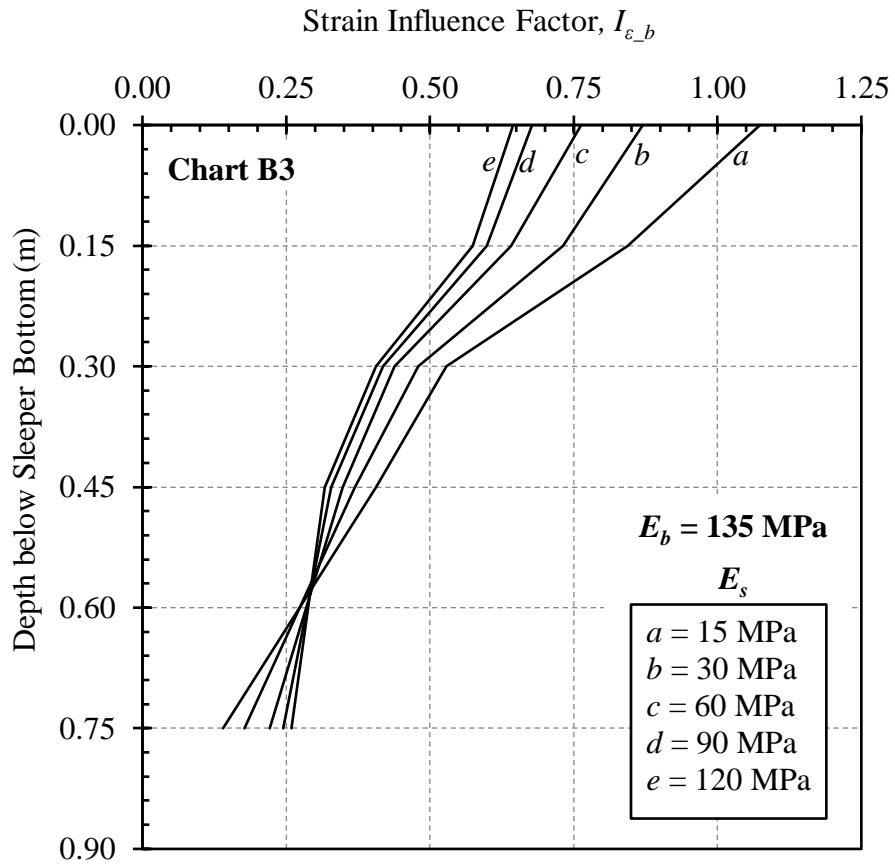
**Appendix A: Design Charts to Calculate the Granular Layer Thickness for Preventing the Progressive Shear Failure.**



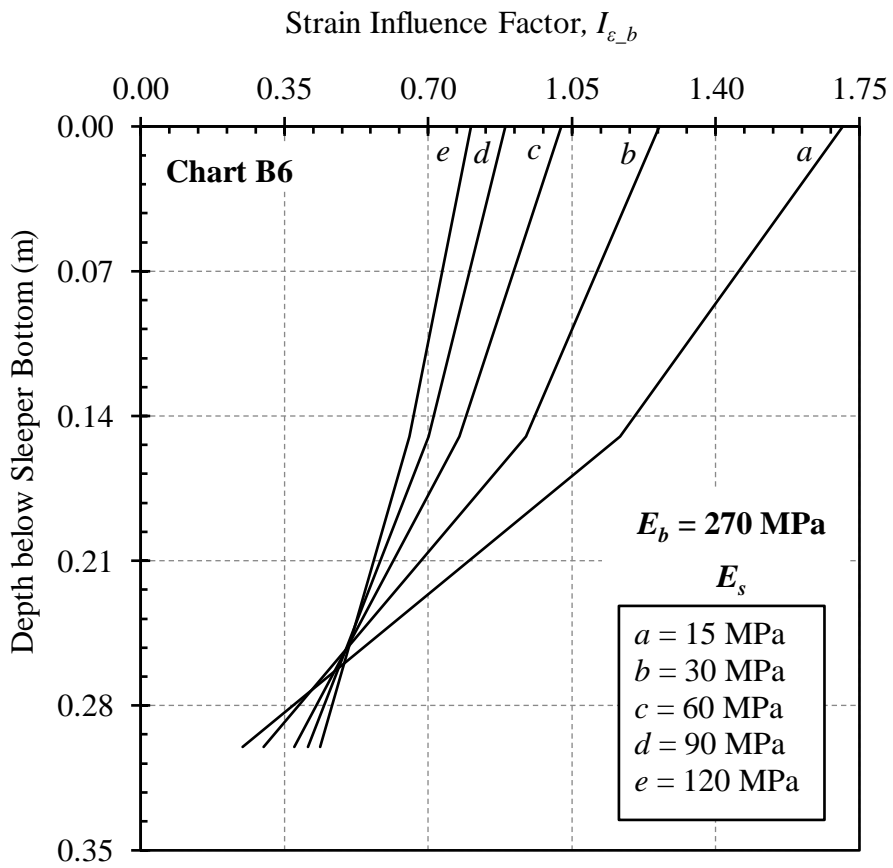
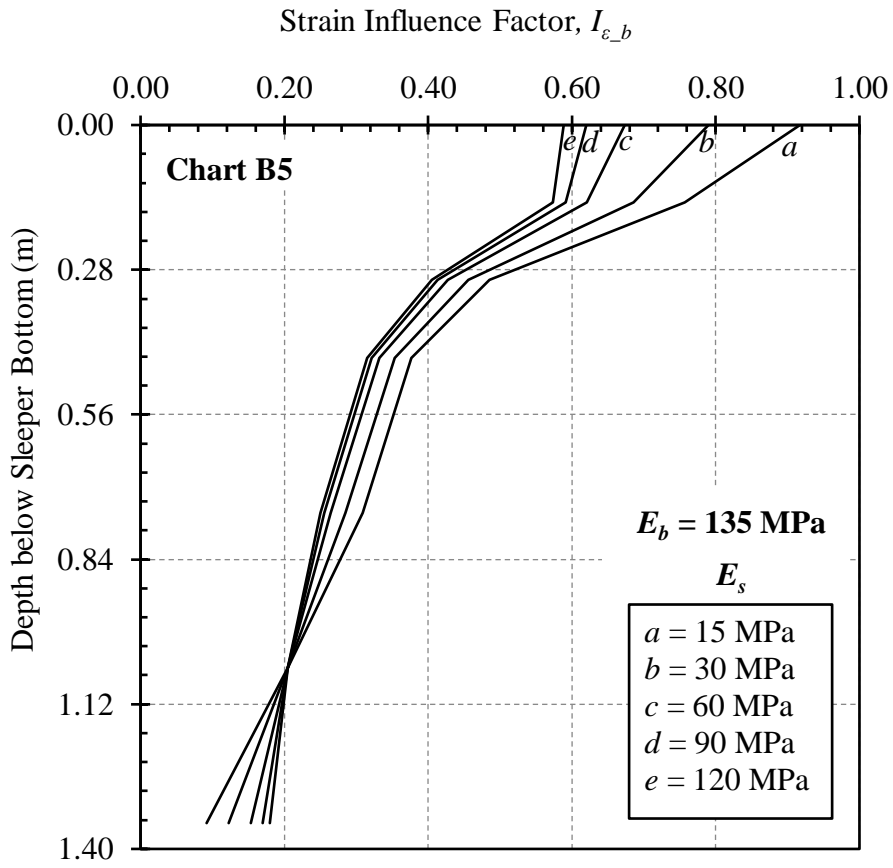


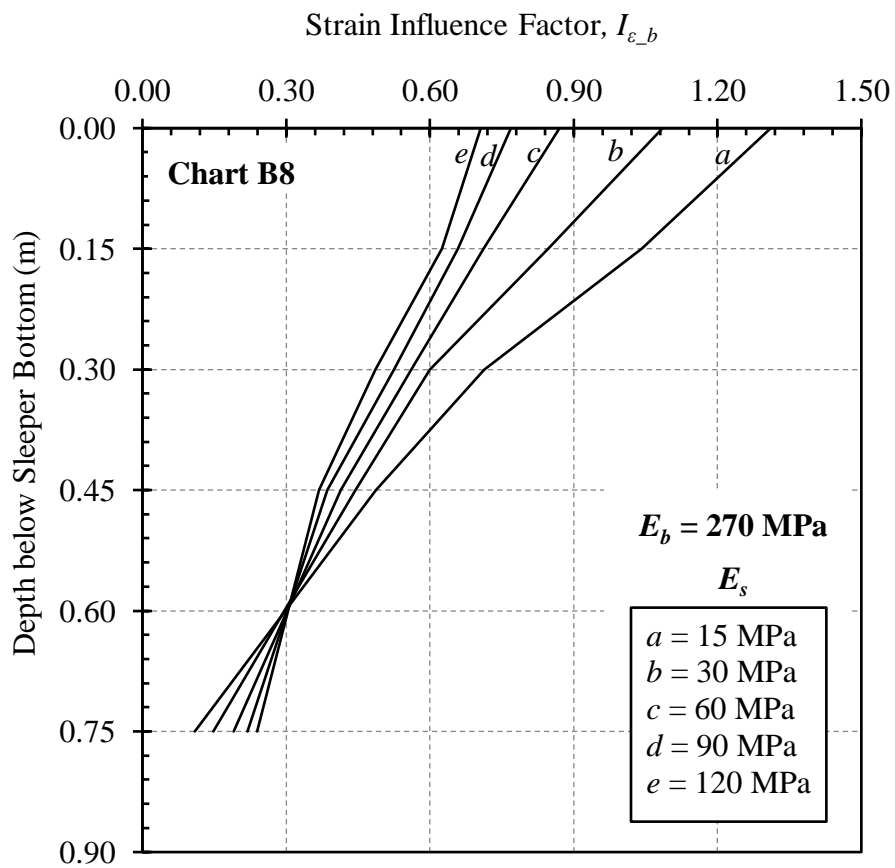
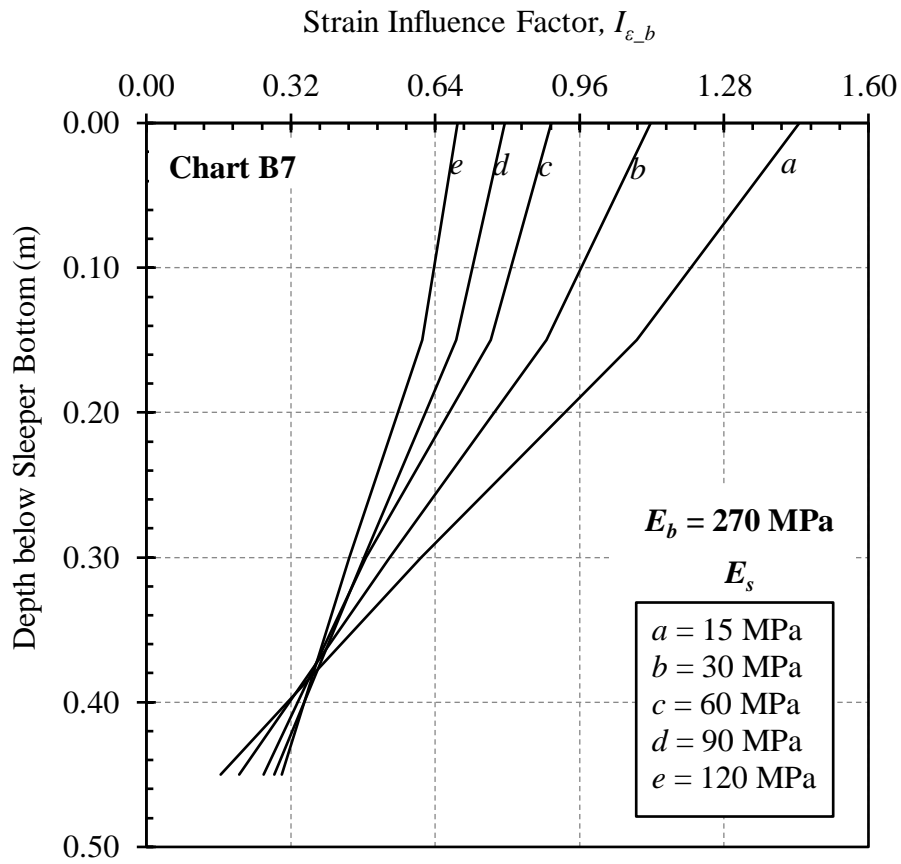
**Appendix B: Distribution of Strain Influence Factor with Depth for the Ballast Layer.**

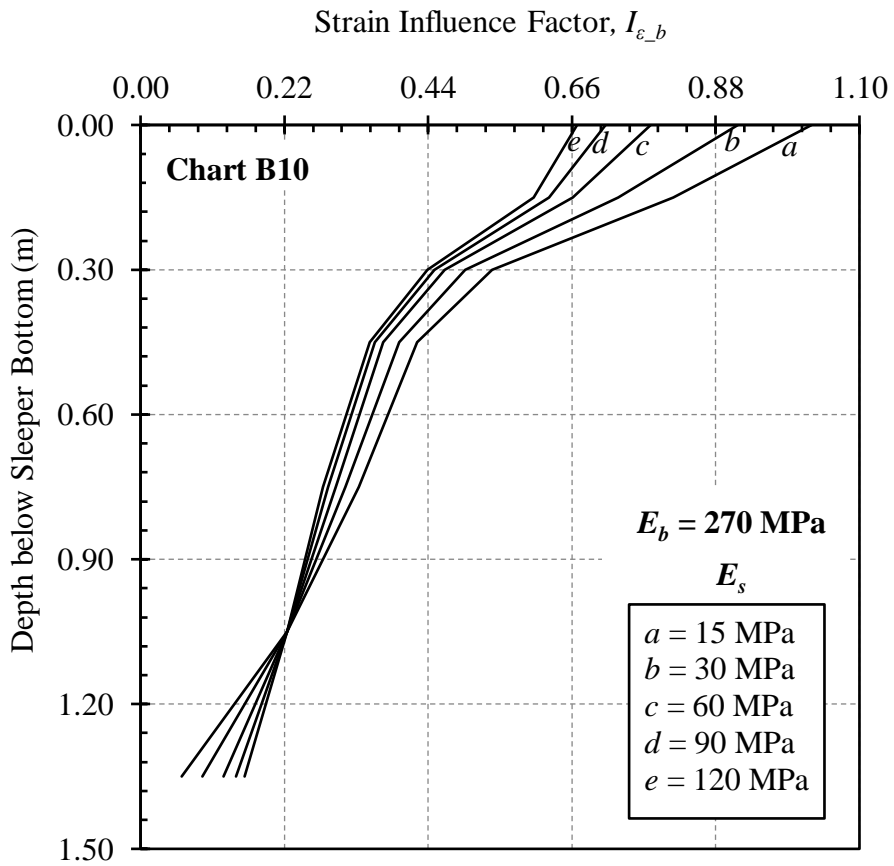
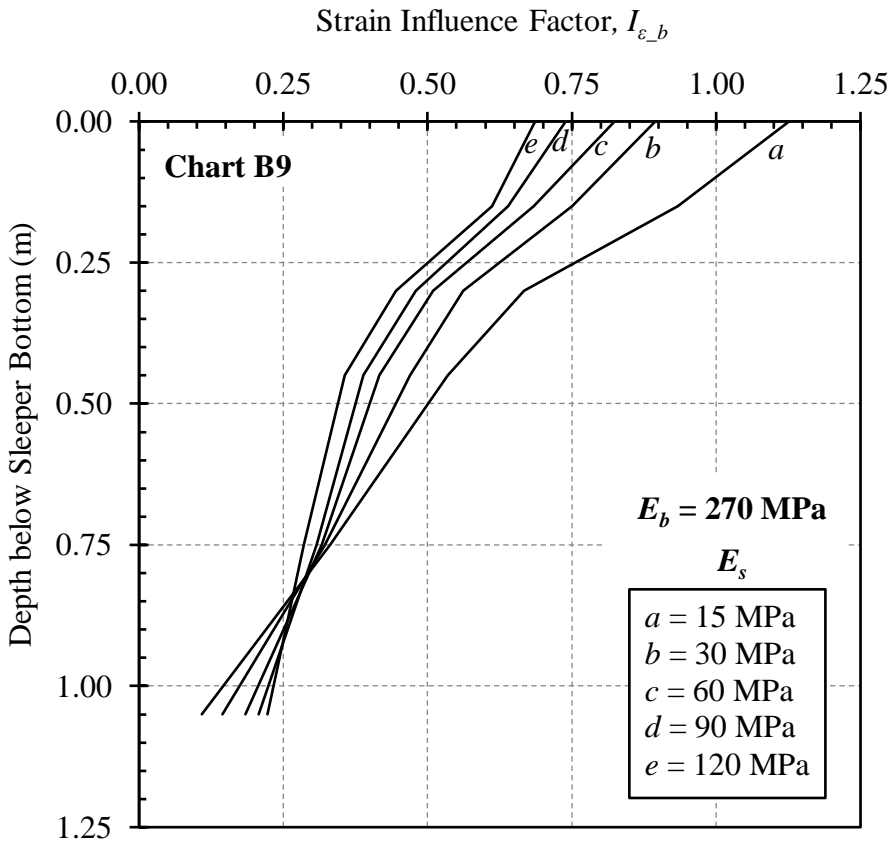


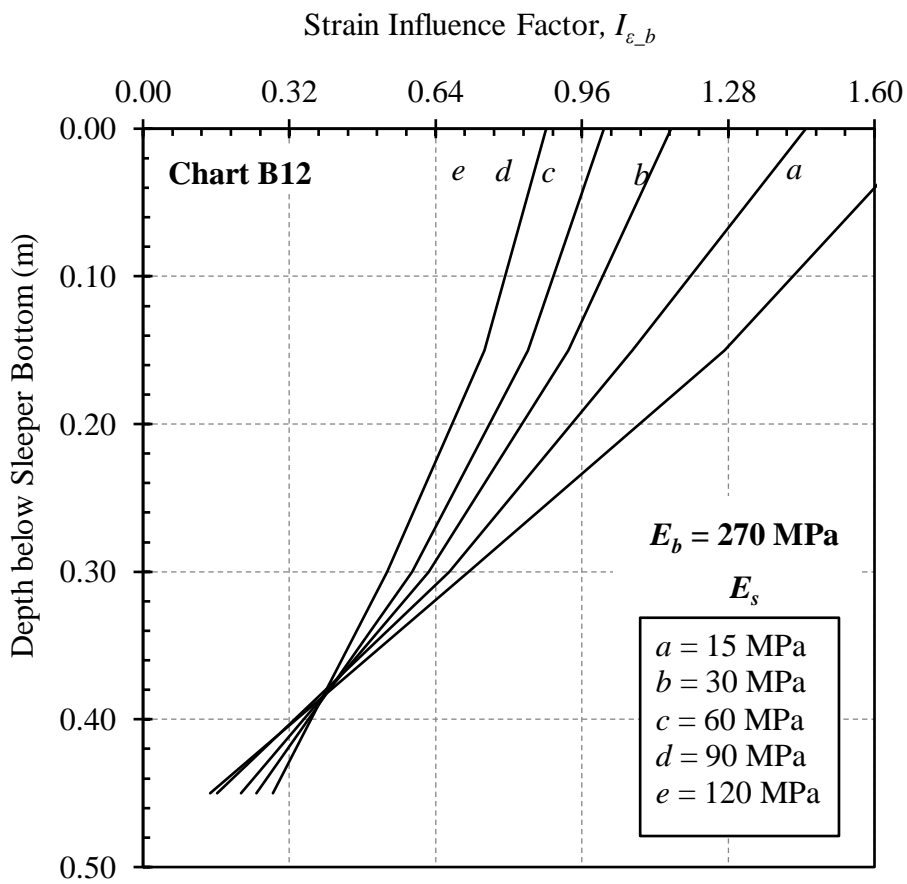
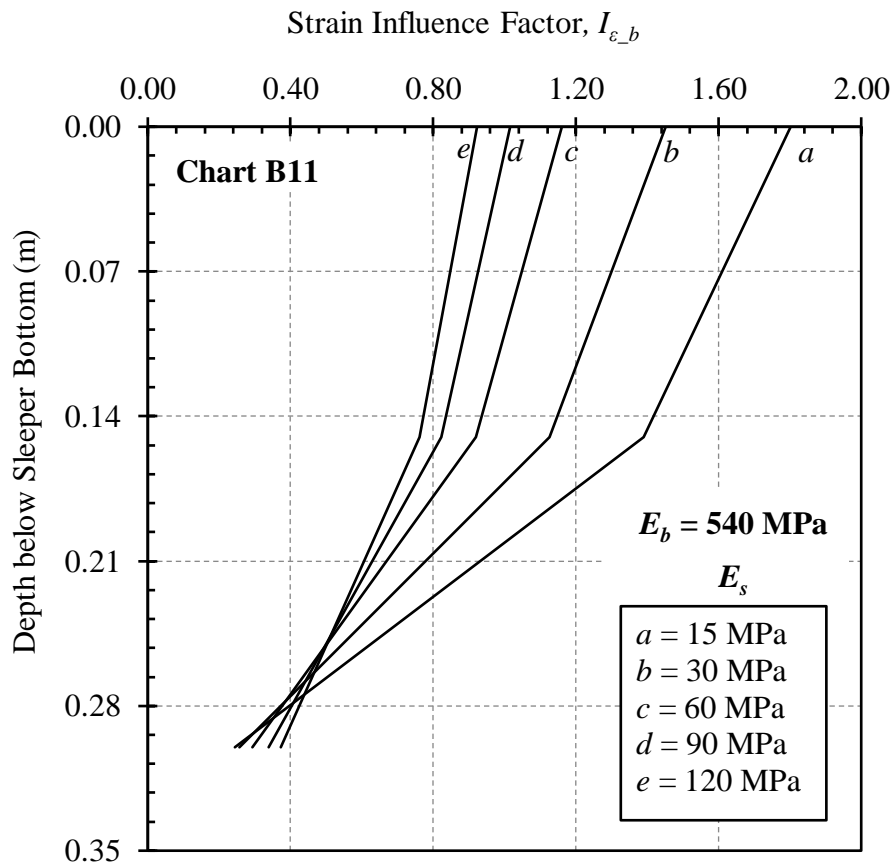


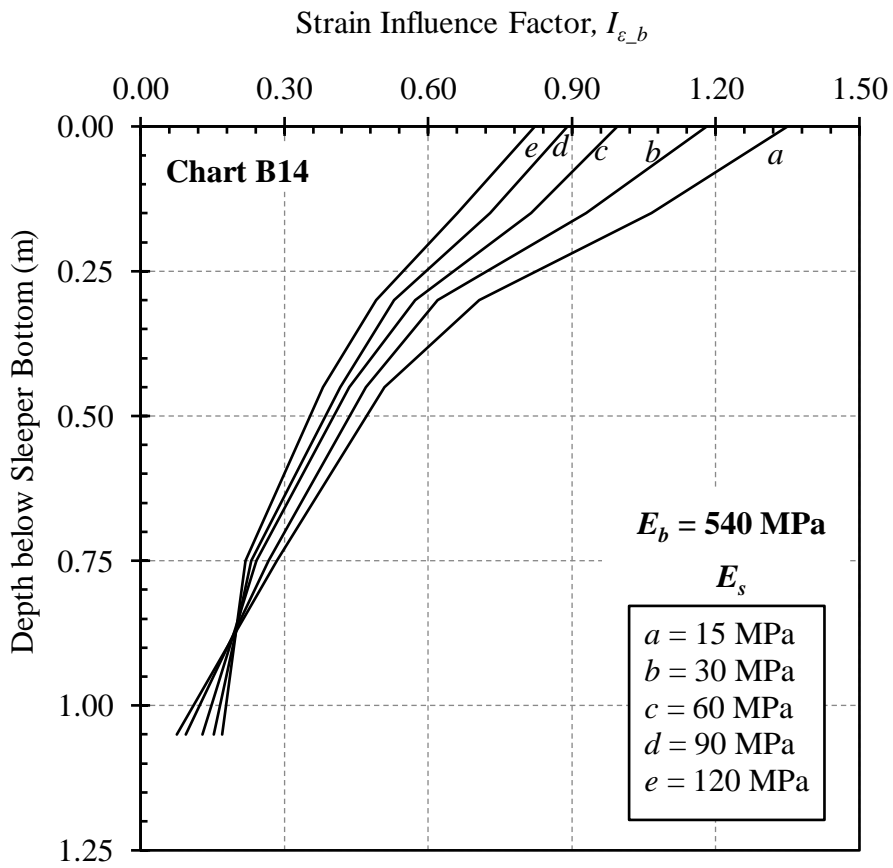
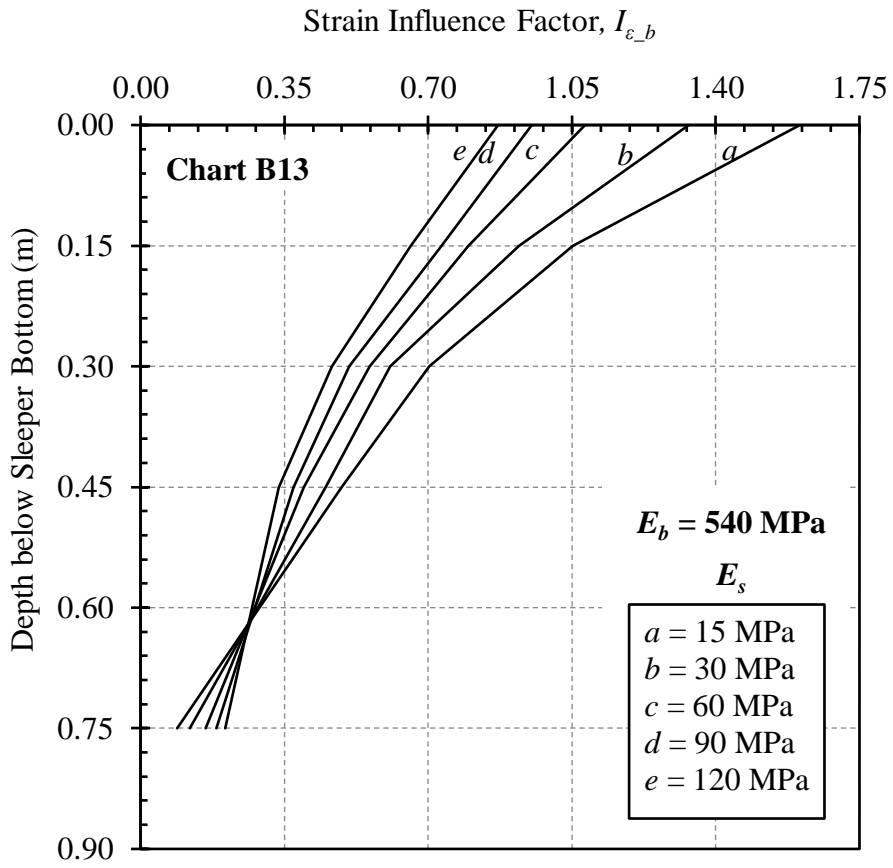


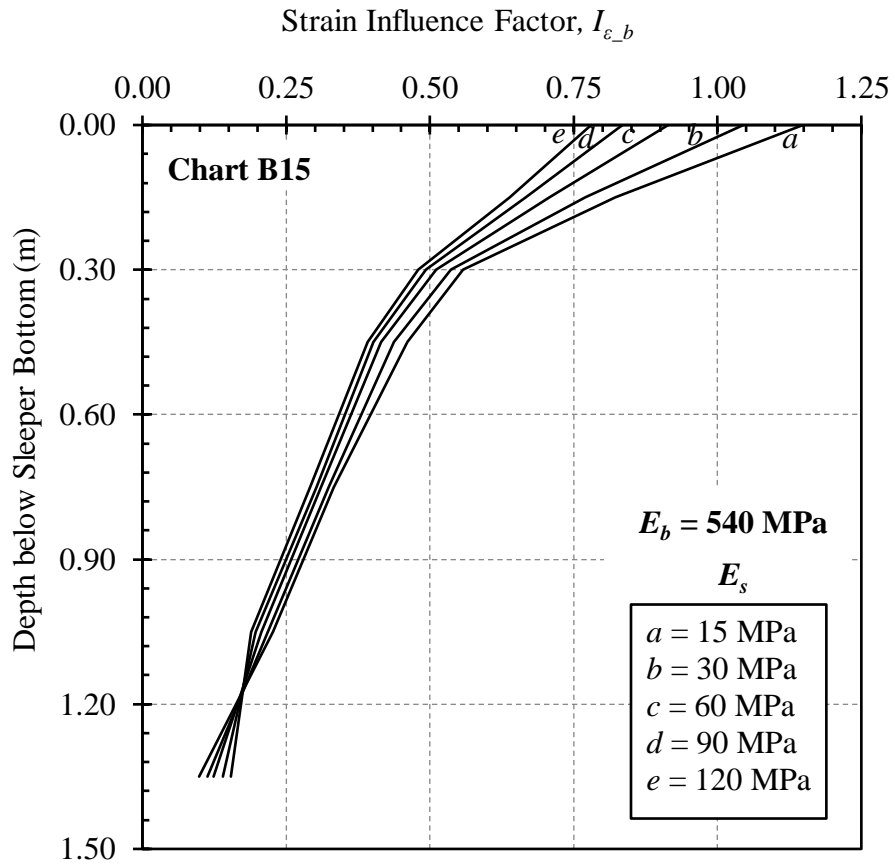












**Appendix C: Design Charts to Calculate the Granular Layer Thickness for Preventing the Excessive Plastic Deformation.**

