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Positive solutions for a system of nonlinear fractional nonlocal boundary value problems with parameters and p -Laplacian operator

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Abstract

In this paper, we investigate the existence of positive solutions for a system of nonlinear fractional differential equations nonlocal boundary value problems with parameters and p -Laplacian operator. Under different combinations of superlinearity and sublinearity of the nonlinearities, various existence results for positive solutions are derived in terms of different values of parameters via the Guo-Krasnosel'skii fixed point theorem.

Keywords: positive solution; p -Laplacian operator; fractional differential system; nonlocal boundary value problem

1 Introduction

In this paper, we investigate the following system of nonlinear fractional differential equations nonlocal boundary value problems with parameters and p -Laplacian operator:

$$\begin{cases} -D_{0^+}^{\alpha_1}(\varphi_{p_1}(D_{0^+}^{\beta_1}u(t))) = \lambda f(t, u(t), v(t)), & 0 < t < 1, \\ -D_{0^+}^{\alpha_2}(\varphi_{p_2}(D_{0^+}^{\beta_2}v(t))) = \mu g(t, u(t), v(t)), & 0 < t < 1, \\ u(0) = u(1) = u'(0) = u'(1) = 0, & D_{0^+}^{\beta_1}u(0) = 0, & D_{0^+}^{\beta_1}u(1) = b_1 D_{0^+}^{\beta_1}u(\eta_1), \\ v(0) = v(1) = v'(0) = v'(1) = 0, & D_{0^+}^{\beta_2}v(0) = 0, & D_{0^+}^{\beta_2}v(1) = b_2 D_{0^+}^{\beta_2}v(\eta_2), \end{cases} \quad (1.1)$$

where $\alpha_i \in (1, 2]$, $\beta_i \in (3, 4]$, $D_{0^+}^{\alpha_i}$ and $D_{0^+}^{\beta_i}$ are the standard Riemann-Liouville derivatives, $\varphi_{p_i}(s) = |s|^{p_i-2}s$, $p_i > 1$, $\varphi_{p_i}^{-1} = \varphi_{q_i}$, $\frac{1}{p_i} + \frac{1}{q_i} = 1$, $\eta_i \in (0, 1)$, $b_i \in (0, \eta_i^{\frac{1-\alpha_i}{p_i-1}})$, $i = 1, 2$, $f, g \in C([0, 1] \times [0, +\infty)^2, [0, +\infty))$, λ and μ are positive parameters.

Fractional differential equation models are proved to be more adequate than integer order models for some problems in science and engineering. Fractional differential equations play a very important role in various fields due to their deep real world background. For an introduction of fractional calculus and fractional differential equations, we refer the reader to [1–3] and the references therein.

Turbulent flow in a porous medium is a fundamental mechanics problem. For studying this type of problem, Leibenson [4] introduced differential equations with p -Laplacian

operator

$$(\varphi_p(u'(t)))' = f(t, u(t)). \tag{1.2}$$

The study of differential equation with p -Laplacian operator is of significance theoretically and practically. It is quite natural to study fractional differential equation relative to equation (1.2).

Recently, many scholars have paid more attention to the fractional order differential equation boundary value problems with p -Laplacian operator, see [5–23]. In [15], Lu et al. investigated a class of boundary value problems for fractional differential equations with p -Laplacian

$$\begin{cases} D_{0+}^\alpha (\varphi_p(D_{0+}^\beta u(t))) = f(t, u(t)), & 0 < t < 1, \\ u(0) = u'(0) = u'(1) = 0, & D_{0+}^\beta u(0) = D_{0+}^\beta u(1) = 0, \end{cases}$$

where $\alpha \in (1, 2]$, $\beta \in (2, 3]$, D_{0+}^α and D_{0+}^β are the standard Riemann-Liouville derivatives, $f \in C([0, 1] \times [0, +\infty), [0, +\infty))$. The existence and multiplicity results of positive solutions were obtained by using the Guo-Krasnosel'skii fixed point theorem, the Leggett-Williams fixed point theorem and the upper and lower solutions method. Xu and Dong [17] considered the following three point boundary value problem of fractional differential equation with p -Laplacian operator:

$$\begin{cases} -D_{0+}^\alpha (\varphi_p(D_{0+}^\beta u(t))) = f(t, u(t)), & 0 < t < 1, \\ u(0) = u(1) = u'(0) = u'(1) = 0, & D_{0+}^\beta u(0) = 0, & D_{0+}^\beta u(1) = bD_{0+}^\beta u(\eta), \end{cases}$$

where $\alpha \in (1, 2]$, $\beta \in (3, 4]$, D_{0+}^α and D_{0+}^β are the standard Riemann-Liouville derivatives, $\eta \in (0, 1)$, $b \in (0, \eta^{\frac{1-\alpha}{p-1}})$, $f \in C([0, 1] \times [0, +\infty), [0, +\infty))$. The existence and uniqueness of positive solutions were obtained by using the upper and lower solutions method and Schauder's fixed point theorem, the iterative sequences for the unique solution were also given.

In [18], Zhang et al. considered the eigenvalue problems of fractional differential equations with integral boundary conditions and p -Laplacian operator

$$\begin{cases} -D_{0+}^\alpha (\varphi_p(D_{0+}^\beta u(t))) = \lambda f(t, u(t)), & 0 < t < 1, \\ u(0) = 0, & D_{0+}^\beta u(0) = 0, & u(1) = \int_0^1 u(s) dA(s), \end{cases} \tag{1.3}$$

where $\alpha \in (0, 1]$, $\beta \in (1, 2]$, D_{0+}^α and D_{0+}^β are the standard Riemann-Liouville derivatives, A is a function of bounded variation, $\int_0^1 u(s) dA(s)$ is the Riemann-Stieltjes integral, $f(t, u) : (0, 1) \times (0, +\infty) \rightarrow [0, +\infty)$ is a continuous function that may be singular at $t = 0, 1$ and $u = 0$. The existence of positive solutions of problem (1.3) was established by using the upper and lower solutions method and Schauder's fixed point theorem. Lv [20] discussed an m -point boundary value problem of fractional differential equation with p -Laplacian operator

$$\begin{cases} D_{0+}^\alpha (\varphi_p(D_{0+}^\beta u(t))) + \varphi_p(\lambda) f(t, u(t)) = 0, & 0 < t < 1, \\ u(0) = 0, & D_{0+}^\gamma u(1) = \sum_{i=1}^{m-2} \xi_i D_{0+}^\gamma u(\eta_i), & D_{0+}^\beta u(0) = 0, \end{cases} \tag{1.4}$$

where $\alpha, \gamma \in (0, 1], \beta \in (1, 2], \beta - \alpha - 1 \geq 0, \beta - \gamma - 1 \geq 0, \lambda$ is a positive parameter, $0 < \xi_i, \eta_i < 1, \sum_{i=1}^{m-2} \xi_i \eta_i^{\beta-\alpha-1} < 1, f \in C([0, 1] \times [0, +\infty), [0, +\infty))$. The existence and multiplicity of positive solutions for system (1.4) were established via the monotone iterative method and the fixed point index theory.

The system of fractional differential equations boundary value problems with p -Laplacian operator have also received much attention and have developed very rapidly, see [24–32]. In [24], Li et al. studied the following fractional differential system involving the p -Laplacian operator and nonlocal boundary conditions:

$$\begin{cases} D_{0^+}^{\alpha_1}(\varphi_{p_1}(D_{0^+}^{\beta_1}u(t))) = f(t, v(t)), & 0 < t < 1, \\ D_{0^+}^{\alpha_2}(\varphi_{p_2}(D_{0^+}^{\beta_2}v(t))) = g(t, u(t)), & 0 < t < 1, \\ u(0) = D_{0^+}^{\beta_1}u(0) = 0, & D_{0^+}^{\gamma_1}u(1) = \sum_{j=1}^{m-2} a_{1j}D_{0^+}^{\gamma_1}u(\eta_j), \\ v(0) = D_{0^+}^{\beta_2}v(0) = 0, & D_{0^+}^{\gamma_2}v(1) = \sum_{j=1}^{m-2} a_{2j}D_{0^+}^{\gamma_2}v(\eta_j), \end{cases}$$

where $\alpha_i, \gamma_i \in (0, 1], \beta_i \in (1, 2], D_{0^+}^{\alpha_i}, D_{0^+}^{\beta_i}$ and $D_{0^+}^{\gamma_i}$ are the standard Riemann-Liouville derivatives, $i = 1, 2$. The conditions for the existence of the maximal and minimal solutions to the system were established.

Ren et al. [25] considered the following nonlocal fractional differential system:

$$\begin{cases} -D_{0^+}^{\alpha_1}(\varphi_{p_1}(-D_{0^+}^{\beta_1}u(t))) = f(u(t), v(t)), & 0 < t < 1, \\ -D_{0^+}^{\alpha_2}(\varphi_{p_2}(-D_{0^+}^{\beta_2}v(t))) = g(u(t), v(t)), & 0 < t < 1, \\ u(0) = D_{0^+}^{\beta_1}u(0) = D_{0^+}^{\beta_1}u(1) = 0, & u(1) = \int_0^1 u(s) dA(s), \\ v(0) = D_{0^+}^{\beta_2}v(0) = D_{0^+}^{\beta_2}v(1) = 0, & v(1) = \int_0^1 v(s) dB(s), \end{cases}$$

where $\alpha_i, \beta_i \in (1, 2], D_{0^+}^{\alpha_i}$ and $D_{0^+}^{\beta_i}$ are the standard Riemann-Liouville derivatives. A and B are functions of bounded variations, $\int_0^1 u(s) dA(s)$ and $\int_0^1 v(s) dB(s)$ are Riemann-Stieltjes integrals. By introducing a new type of growth conditions and using the monotone iterative technique, some new results about the existence of maximal and minimal solutions were established, and the estimation of the lower and upper bounds of the maximum and minimum solutions was also derived.

By means of the Avery-Henderson fixed point theorem and six functionals fixed point theorem, Rao [26] investigated the existence of multiple positive solutions for a coupled system of p -Laplacian fractional order two point boundary value problems

$$\begin{cases} -D_{a^+}^{\alpha_1}(\varphi_p(D_{a^+}^{\beta_1}u(t))) = f(t, u(t), v(t)), & a < t < b, \\ -D_{a^+}^{\alpha_2}(\varphi_p(D_{a^+}^{\beta_2}v(t))) = g(t, u(t), v(t)), & a < t < b, \\ \xi u(a) - \eta u'(a) = 0, & \gamma u(b) + \delta u'(b) = 0, & D_{a^+}^{\beta_1}u(a) = 0, \\ \xi v(a) - \eta v'(a) = 0, & \gamma v(b) + \delta v'(b) = 0, & D_{a^+}^{\beta_2}v(a) = 0, \end{cases}$$

where $\alpha_i \in (0, 1], \beta_i \in (1, 2], D_{a^+}^{\alpha_i}$ and $D_{a^+}^{\beta_i}$ are the standard Riemann-Liouville derivatives.

He and Song [29] discussed the following fractional order differential system with p -Laplacian operator:

$$\begin{cases} D_{0^+}^{\alpha_1}(\varphi_{p_1}(D_{0^+}^{\beta_1}u(t))) = \lambda f(t, v(t)), & 0 < t < 1, \\ D_{0^+}^{\alpha_2}(\varphi_{p_2}(D_{0^+}^{\beta_2}v(t))) = \mu g(t, u(t)), & 0 < t < 1, \\ u(0) = 0, u(1) = a_1u(\xi_1), & D_{0^+}^{\beta_1}u(0) = 0, & D_{0^+}^{\beta_1}u(1) = b_1D_{0^+}^{\beta_1}u(\eta_1), \\ v(0) = 0, v(1) = a_2v(\xi_2), & D_{0^+}^{\beta_2}v(0) = 0, & D_{0^+}^{\beta_2}v(1) = b_2D_{0^+}^{\beta_2}v(\eta_2), \end{cases}$$

where $\alpha_i, \beta_i \in (1, 2], D_{0^+}^{\alpha_i}$ and $D_{0^+}^{\beta_i}$ are the standard Riemann-Liouville derivatives, $\xi_i, \eta_i \in (0, 1), a_i, b_i \in [0, 1], i = 1, 2$. λ and μ are positive parameters. The uniqueness of solution was established by using the Banach contraction mapping principle. Khan et al. [31] considered the existence and uniqueness of solutions to a coupled system of fractional differential equations with p -Laplacian operator. The functions involved in the proposed coupled system were continuous and satisfied certain growth conditions. By using topological degree theory, some conditions were established which ensured the existence and uniqueness of solution to the proposed problem.

Motivated by the papers mentioned above, in this paper, we study the existence of positive solutions for a system of nonlinear fractional differential equations nonlocal boundary value problems with parameters and p -Laplacian operator. Under different combinations of superlinearity and sublinearity of the functions f and g , various existence results for positive solutions are derived in terms of different values of λ and μ via the Guo-Krasnosel'skii fixed point theorem. Moreover, in this paper it is possible to replace the four point boundary conditions by multi-point boundary conditions or integral boundary conditions with minor modifications.

2 Preliminaries and lemmas

We present here the definitions, some lemmas from the theory of fractional calculus and some auxiliary results that will be used to prove our main theorems.

Definition 2.1 ([1–3]) The Riemann-Liouville fractional integral of order $\alpha > 0$ of a function $f : (0, +\infty) \rightarrow (-\infty, +\infty)$ is given by

$$I_{0^+}^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds$$

provided the right-hand side is pointwise defined on $(0, +\infty)$.

Definition 2.2 ([1–3]) The Riemann-Liouville fractional derivative of order $\alpha > 0$ of a continuous function $f : (0, +\infty) \rightarrow (-\infty, +\infty)$ is given by

$$D_{0^+}^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\alpha-1} f(s) ds,$$

where n is the smallest integer not less than α , provided the right-hand side is pointwise defined on $(0, +\infty)$.

Lemma 2.1 ([1–3]) *Let $\alpha > 0$. Then the following equality holds for $u \in L(0, 1)$ and $D_{0^+}^\alpha u(t) \in L(0, 1)$:*

$$I_{0^+}^\alpha D_{0^+}^\alpha u(t) = u(t) + c_1 t^{\alpha-1} + c_2 t^{\alpha-2} + \dots + c_n t^{\alpha-n},$$

where $c_1, c_2, \dots, c_n \in (-\infty, +\infty), n - 1 < \alpha \leq n$.

We transform problem (1.1) to its equivalent integral equations. Denote $\varphi_{p_1}(D_{0^+}^{\beta_1} u(t)) = z(t)$, then

$$z(0) = 0, \quad z(1) = b_1^{p_1-1} z(\eta_1).$$

We now consider the following fractional differential equation:

$$\begin{cases} -D_{0^+}^{\alpha_1} z(t) = y(t), & 0 < t < 1, \\ z(0) = 0, & z(1) = b_1^{p_1-1} z(\eta_1). \end{cases} \tag{2.1}$$

Lemma 2.2 ([17]) *If $y \in C[0, 1]$, then problem (2.1) has a unique solution*

$$z(t) = \int_0^1 H_1(t, s) y(s) ds,$$

where

$$H_1(t, s) = h_1(t, s) + \frac{b_1^{p_1-1} t^{\alpha_1-1}}{1 - b_1^{p_1-1} \eta_1^{\alpha_1-1}} h_1(\eta_1, s),$$

$$h_1(t, s) = \frac{1}{\Gamma(\alpha_1)} \begin{cases} [t(1-s)]^{\alpha_1-1}, & 0 \leq t \leq s \leq 1, \\ [t(1-s)]^{\alpha_1-1} - (t-s)^{\alpha_1-1}, & 0 \leq s \leq t \leq 1. \end{cases}$$

From the above analysis, the boundary value problem

$$\begin{cases} -D_{0^+}^{\alpha_1}(\varphi_{p_1}(D_{0^+}^{\beta_1} u(t))) = y(t), & 0 < t < 1, \\ u(0) = u(1) = u'(0) = u'(1) = 0, & D_{0^+}^{\beta_1} u(0) = 0, \quad D_{0^+}^{\beta_1} u(1) = b_1 D_{0^+}^{\beta_1} u(\eta_1), \end{cases} \tag{2.2}$$

is equal to

$$\begin{cases} D_{0^+}^{\beta_1} u(t) = \varphi_{q_1}(\int_0^1 H_1(t, s) y(s) ds), \\ u(0) = u(1) = u'(0) = u'(1) = 0. \end{cases}$$

Lemma 2.3 ([17]) *If $y \in C[0, 1]$, then problem (2.2) has a unique solution*

$$u(t) = \int_0^1 G_1(t, s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) y(\tau) d\tau \right) ds,$$

where

$$G_1(t, s) = \frac{1}{\Gamma(\beta_1)} \begin{cases} t^{\beta_1-2}(1-s)^{\beta_1-2}[(s-t) + (\beta_1-2)(1-t)s], & 0 \leq t \leq s \leq 1, \\ t^{\beta_1-2}(1-s)^{\beta_1-2}[(s-t) + (\beta_1-2)(1-t)s] + (t-s)^{\beta_1-1}, & 0 \leq s \leq t \leq 1. \end{cases} \tag{2.3}$$

Lemma 2.4 ([17, 33]) *The function $G_1(t, s)$ defined by (2.3) is continuous on $[0, 1] \times [0, 1]$ and has the following properties:*

- (a) $G_1(t, s) > 0$ for all $(t, s) \in (0, 1) \times (0, 1)$;
- (b) $(\beta_1 - 2)k_1(t)l_1(s) \leq \Gamma(\beta_1)G_1(t, s) \leq M_1l_1(s)$, $(t, s) \in (0, 1) \times (0, 1)$;
- (c) $(\beta_1 - 2)k_1(t)l_1(s) \leq \Gamma(\beta_1)G_1(t, s) \leq M_1k_1(t)$, $(t, s) \in (0, 1) \times (0, 1)$;

where

$$k_1(t) = t^{\beta_1-2}(1-t)^2, \quad l_1(s) = s^2(1-s)^{\beta_1-2},$$

$$M_1 = \max\{\beta_1 - 1, (\beta_1 - 2)^2\}.$$

Similarly, we can obtain the following Lemmas 2.5 and 2.6 for the following boundary value problem:

$$\begin{cases} -D_{0^+}^{\alpha_2}(\varphi_{p_2}(D_{0^+}^{\beta_2}v(t))) = y(t), & 0 < t < 1, \\ v(0) = v(1) = v'(0) = v'(1) = 0, & D_{0^+}^{\beta_2}v(0) = 0, & D_{0^+}^{\beta_2}v(1) = b_2D_{0^+}^{\beta_2}u(\eta_2). \end{cases} \tag{2.4}$$

Lemma 2.5 *If $y \in C[0, 1]$, then problem (2.4) has a unique solution*

$$v(t) = \int_0^1 G_2(t, s)\varphi_{q_2}\left(\int_0^1 H_2(s, \tau)y(\tau) d\tau\right) ds,$$

where

$$G_2(t, s) = \frac{1}{\Gamma(\beta_2)} \begin{cases} t^{\beta_2-2}(1-s)^{\beta_2-2}[(s-t) + (\beta_2-2)(1-t)s], & 0 \leq t \leq s \leq 1, \\ t^{\beta_2-2}(1-s)^{\beta_2-2}[(s-t) + (\beta_2-2)(1-t)s] + (t-s)^{\beta_2-1}, & 0 \leq s \leq t \leq 1, \end{cases} \tag{2.5}$$

$$H_2(t, s) = h_2(t, s) + \frac{b_2^{p_2-1}t^{\alpha_2-1}}{1 - b_2^{p_2-1}\eta_2^{\alpha_2-1}}h_1(\eta_2, s),$$

$$h_2(t, s) = \frac{1}{\Gamma(\alpha_2)} \begin{cases} [t(1-s)]^{\alpha_2-1}, & 0 \leq t \leq s \leq 1, \\ [t(1-s)]^{\alpha_2-1} - (t-s)^{\alpha_2-1}, & 0 \leq s \leq t \leq 1. \end{cases}$$

Lemma 2.6 *The function $G_2(t, s)$ given by (2.5) has the properties:*

- (a) $G_2(t, s) > 0$, $(t, s) \in (0, 1) \times (0, 1)$;
- (b) $(\beta_2 - 2)k_2(t)l_2(s) \leq \Gamma(\beta_2)G_2(t, s) \leq M_2l_2(s)$, $(t, s) \in (0, 1) \times (0, 1)$;
- (c) $(\beta_2 - 2)k_2(t)l_2(s) \leq \Gamma(\beta_2)G_2(t, s) \leq M_2k_2(t)$, $(t, s) \in (0, 1) \times (0, 1)$;

where

$$k_2(t) = t^{\beta_2-2}(1-t)^2, \quad l_2(s) = s^2(1-s)^{\beta_2-2}, \quad M_2 = \max\{\beta_2 - 1, (\beta_2 - 2)^2\}.$$

Let $X = C[0, 1]$, then X is a Banach space with the norm $\|u\| = \sup_{t \in [0,1]} |u(t)|$. Let $Y = X \times X$, then Y is a Banach space with the norm $\|(u, v)\|_Y = \|u\| + \|v\|$. For $\theta_1, \theta_2 \in (0, 1)$ and $\theta_1 < \theta_2$, denote

$$P = \left\{ (u, v) \in Y : u(t) \geq 0, v(t) \geq 0, \forall t \in [0, 1], \min_{t \in [\theta_1, \theta_2]} (u(t) + v(t)) \geq \gamma \|(u, v)\|_Y \right\},$$

where $\gamma = \min\{\gamma_1, \gamma_2\}$ and $\gamma_i = \frac{(\beta_i-2)}{M_i} \min_{t \in [\theta_1, \theta_2]} k_i(t), i = 1, 2$, then P is a cone of Y . Define operators $T_1, T_2 : Y \rightarrow X$ and $Q : Y \rightarrow Y$ as follows:

$$T_1(u, v)(t) = \varphi_{q_1}(\lambda) \int_0^1 G_1(t, s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds, \quad t \in [0, 1],$$

$$T_2(u, v)(t) = \varphi_{q_2}(\mu) \int_0^1 G_2(t, s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g(\tau, u(\tau), v(\tau)) d\tau \right) ds, \quad t \in [0, 1],$$

$$Q(u, v) = (T_1(u, v), T_2(u, v)), \quad (u, v) \in Y.$$

It is well known that if (u, v) is a fixed point of the operator Q in P , then (u, v) is a positive solution of system (1.1).

Lemma 2.7 $Q : P \rightarrow P$ is a completely continuous operator.

Proof For $(u, v) \in P$ and $t \in [0, 1]$, obviously, $T_1(u, v)(t) \geq 0, T_2(u, v)(t) \geq 0$. It follows from Lemmas 2.4 and 2.6 that

$$\begin{aligned} T_1(u, v)(t) &= \varphi_{q_1}(\lambda) \int_0^1 G_1(t, s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\leq \varphi_{q_1}(\lambda) \int_0^1 \frac{M_1 l_1(s)}{\Gamma(\beta_1)} \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds, \end{aligned}$$

then

$$\|T_1(u, v)\| \leq \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda) \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds.$$

Similarly,

$$\|T_2(u, v)\| \leq \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu) \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g(\tau, u(\tau), v(\tau)) d\tau \right) ds.$$

Therefore

$$\begin{aligned} \|Q(u, v)\|_Y &\leq \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda) \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\quad + \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu) \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g(\tau, u(\tau), v(\tau)) d\tau \right) ds. \end{aligned}$$

On the other hand, for any $t \in [\theta_1, \theta_2]$, by Lemma 2.4, we have

$$\begin{aligned} T_1(u, v)(t) &\geq \varphi_{q_1}(\lambda) \int_0^1 \frac{\beta_1 - 2}{\Gamma(\beta_1)} k_1(t) l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &= \frac{\beta_1 - 2}{\Gamma(\beta_1)} k_1(t) \varphi_{q_1}(\lambda) \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{M_1} k_1(t) \|T_1(u, v)\|, \end{aligned}$$

then

$$\min_{t \in [\theta_1, \theta_2]} T_1(u, v)(t) \geq \gamma_1 \|T_1(u, v)\|.$$

Similarly,

$$\min_{t \in [\theta_1, \theta_2]} T_2(u, v)(t) \geq \gamma_2 \|T_2(u, v)\|.$$

Hence

$$\begin{aligned} &\min_{t \in [\theta_1, \theta_2]} (T_1(u, v)(t) + T_2(u, v)(t)) \\ &\geq \min_{t \in [\theta_1, \theta_2]} T_1(u, v)(t) + \min_{t \in [\theta_1, \theta_2]} T_2(u, v)(t) \\ &\geq \gamma_1 \|T_1(u, v)\| + \gamma_2 \|T_2(u, v)\| \\ &\geq \gamma (\|T_1(u, v)\| + \|T_2(u, v)\|) = \gamma \|Q(u, v)\|_Y, \end{aligned}$$

that is, $Q(P) \subset P$.

By the Ascoli-Arzelà theorem and the continuity of f, g, G_i and H_i , we deduce that T_1 and T_2 are completely continuous operators, then Q is a completely continuous operator. This completes the proof. \square

Lemma 2.8 ([34]) *Let P be a positive cone in a Banach space E , Ω_1 and Ω_2 are bounded open sets in E , $\theta \in \Omega_1$, $\overline{\Omega_1} \subset \Omega_2$, $A : P \cap (\overline{\Omega_2} \setminus \Omega_1) \rightarrow P$ is a completely continuous operator. If the following conditions are satisfied:*

- (i) $\|Ax\| \leq \|x\|, \forall x \in P \cap \partial\Omega_1, \|Ax\| \geq \|x\|, \forall x \in P \cap \partial\Omega_2$, or
- (ii) $\|Ax\| \geq \|x\|, \forall x \in P \cap \partial\Omega_1, \|Ax\| \leq \|x\|, \forall x \in P \cap \partial\Omega_2$, then A has at least one fixed point in $P \cap (\overline{\Omega_2} \setminus \Omega_1)$.

3 Main results

Denote

$$\begin{aligned} f_0^s &= \limsup_{u+v \rightarrow 0} \max_{t \in [0,1]} \frac{f(t, u, v)}{\varphi_{p_1}(u+v)}, & g_0^s &= \limsup_{u+v \rightarrow 0} \max_{t \in [0,1]} \frac{g(t, u, v)}{\varphi_{p_2}(u+v)}, \\ f_0^i &= \liminf_{u+v \rightarrow 0} \min_{t \in [\theta_1, \theta_2]} \frac{f(t, u, v)}{\varphi_{p_1}(u+v)}, & g_0^i &= \liminf_{u+v \rightarrow 0} \min_{t \in [\theta_1, \theta_2]} \frac{g(t, u, v)}{\varphi_{p_2}(u+v)}, \\ f_\infty^s &= \limsup_{u+v \rightarrow \infty} \max_{t \in [0,1]} \frac{f(t, u, v)}{\varphi_{p_1}(u+v)}, & g_\infty^s &= \limsup_{u+v \rightarrow \infty} \max_{t \in [0,1]} \frac{g(t, u, v)}{\varphi_{p_2}(u+v)}, \end{aligned}$$

$$\begin{aligned}
 f_\infty^i &= \liminf_{u+v \rightarrow \infty} \min_{t \in [\theta_1, \theta_2]} \frac{f(t, u, v)}{\varphi_{p_1}(u+v)}, & g_\infty^i &= \liminf_{u+v \rightarrow \infty} \min_{t \in [\theta_1, \theta_2]} \frac{g(t, u, v)}{\varphi_{p_2}(u+v)}. \\
 A &= \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) d\tau \right) ds, & B &= \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) d\tau \right) ds, \\
 C &= \int_{\theta_1}^{\theta_2} l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) d\tau \right) ds, & D &= \int_{\theta_1}^{\theta_2} l_2(s) \varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) d\tau \right) ds.
 \end{aligned}$$

For $f_0^s, g_0^s, f_\infty^i, g_\infty^i \in (0, \infty)$, we define the symbols L_1, L_2, L_3 and L_4 as follows:

$$\begin{aligned}
 L_1 &= \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2(\beta_1 - 2)C\gamma_1\gamma} \right) \frac{1}{f_\infty^i}, & L_2 &= \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{f_0^s}, \\
 L_3 &= \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2(\beta_2 - 2)D\gamma_2\gamma} \right) \frac{1}{g_\infty^i}, & L_4 &= \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{g_0^s}.
 \end{aligned}$$

Theorem 3.1 (1) *If $f_0^s, g_0^s, f_\infty^i, g_\infty^i \in (0, \infty), L_1 < L_2, L_3 < L_4$, then for each $\lambda \in (L_1, L_2)$ and $\mu \in (L_3, L_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(2) *If $f_0^s = 0, g_0^s, f_\infty^i, g_\infty^i \in (0, \infty), L_3 < L_4$, then for each $\lambda \in (L_1, \infty)$ and $\mu \in (L_3, L_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(3) *If $g_0^s = 0, f_0^s, f_\infty^i, g_\infty^i \in (0, \infty), L_1 < L_2$, then for each $\lambda \in (L_1, L_2)$ and $\mu \in (L_3, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(4) *If $f_0^s = g_0^s = 0, f_\infty^i, g_\infty^i \in (0, \infty)$, then for each $\lambda \in (L_1, \infty)$ and $\mu \in (L_3, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(5) *If $\{f_0^s, g_0^s \in (0, \infty), f_\infty^i = \infty\}$ or $\{f_0^s, g_0^s \in (0, \infty), g_\infty^i = \infty\}$, then for each $\lambda \in (0, L_2)$ and $\mu \in (0, L_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(6) *If $\{f_0^s = 0, g_0^s \in (0, \infty), g_\infty^i = \infty\}$ or $\{f_0^s = 0, g_0^s \in (0, \infty), f_\infty^i = \infty\}$, then for each $\lambda \in (0, \infty)$ and $\mu \in (0, L_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(7) *If $\{f_0^s \in (0, \infty), g_0^s = 0, g_\infty^i = \infty\}$ or $\{f_0^s \in (0, \infty), g_0^s = 0, f_\infty^i = \infty\}$, then for each $\lambda \in (0, L_2)$ and $\mu \in (0, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

(8) *If $\{f_0^s = g_0^s = 0, g_\infty^i = \infty\}$ or $\{f_0^s = g_0^s = 0, f_\infty^i = \infty\}$, then for each $\lambda \in (0, \infty)$ and $\mu \in (0, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.*

Proof Because the proofs of the above cases are similar, in what follows we will prove two of them, namely cases (1) and (6).

(1) For any $\lambda \in (L_1, L_2)$ and $\mu \in (L_3, L_4)$, there exists $0 < \varepsilon < \min\{f_\infty^i, g_\infty^i\}$ such that

$$\begin{aligned}
 \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2(\beta_1 - 2)C\gamma_1\gamma} \right) \frac{1}{f_\infty^i - \varepsilon} &\leq \lambda \leq \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{f_0^s + \varepsilon}, \\
 \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2(\beta_2 - 2)D\gamma_2\gamma} \right) \frac{1}{g_\infty^i - \varepsilon} &\leq \mu \leq \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{g_0^s + \varepsilon}.
 \end{aligned}$$

By the definitions of f_0^s and g_0^s , there exists $R_1 > 0$ such that

$$\begin{aligned}
 f(t, u, v) &< (f_0^s + \varepsilon) \varphi_{p_1}(u+v), & t \in [0, 1], 0 \leq u+v \leq R_1, \\
 g(t, u, v) &< (g_0^s + \varepsilon) \varphi_{p_2}(u+v), & t \in [0, 1], 0 \leq u+v \leq R_1.
 \end{aligned}$$

Denote $\Omega_1 = \{(u, v) \in Y : \|(u, v)\|_Y < R_1\}$, for any $(u, v) \in P \cap \partial\Omega_1$ and $t \in [0, 1]$, we have $0 \leq u(t) + v(t) \leq \|u\| + \|v\| = \|(u, v)\|_Y = R_1$, then

$$\begin{aligned} T_1(u, v)(t) &= \varphi_{q_1}(\lambda) \int_0^1 G_1(t, s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\leq \varphi_{q_1}(\lambda) \int_0^1 \frac{M_1}{\Gamma(\beta_1)} l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) (f_0^s + \varepsilon) \varphi_{p_1}(u(\tau) + v(\tau)) \, d\tau \right) ds \\ &\leq \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda) \varphi_{q_1}(f_0^s + \varepsilon) \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) \varphi_{p_1}(\|u\| + \|v\|) \, d\tau \right) ds \\ &= \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda (f_0^s + \varepsilon)) A \|(u, v)\|_Y \\ &\leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

so $\|T_1(u, v)\| \leq \frac{1}{2} \|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_1$. In a similar manner, we deduce

$$\begin{aligned} T_2(u, v)(t) &= \varphi_{q_2}(\mu) \int_0^1 G_2(t, s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\leq \varphi_{q_2}(\mu) \int_0^1 \frac{M_2}{\Gamma(\beta_2)} l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) (g_0^s + \varepsilon) \varphi_{p_2}(u(\tau) + v(\tau)) \, d\tau \right) ds \\ &\leq \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu (g_0^s + \varepsilon)) \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) \, d\tau \right) ds \|(u, v)\|_Y \\ &= \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu (g_0^s + \varepsilon)) B \|(u, v)\|_Y \leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned} \tag{3.1}$$

then $\|T_2(u, v)\| \leq \frac{1}{2} \|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_1$. Hence

$$\|Q(u, v)\|_Y = \|T_1(u, v)\| + \|T_2(u, v)\| \leq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_1. \tag{3.2}$$

On the other hand, by the definitions of f_∞^i and g_∞^i , there exists $\overline{R}_2 > 0$ such that

$$\begin{aligned} f(t, u, v) &\geq (f_\infty^i - \varepsilon) \varphi_{p_1}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, u + v \geq \overline{R}_2, \\ g(t, u, v) &\geq (g_\infty^i - \varepsilon) \varphi_{p_2}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, u + v \geq \overline{R}_2. \end{aligned}$$

Denote $R_2 = \max\{2R_1, \frac{\overline{R}_2}{\gamma}\}$ and $\Omega_2 = \{(u, v) \in Y : \|(u, v)\|_Y < R_2\}$. For any $(u, v) \in P \cap \partial\Omega_2$, we have $\min_{t \in [\theta_1, \theta_2]} (u(t) + v(t)) \geq \gamma \|(u, v)\|_Y = \gamma R_2 \geq \overline{R}_2$, then

$$\begin{aligned} T_1(u, v)(\theta_1) &= \varphi_{q_1}(\lambda) \int_0^1 G_1(\theta_1, s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\geq \varphi_{q_1}(\lambda) \int_0^1 \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) f(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \varphi_{q_1}(\lambda) \int_{\theta_1}^{\theta_2} l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) (f_\infty^i - \varepsilon) \varphi_{p_1}(u(\tau) + v(\tau)) \, d\tau \right) ds \end{aligned}$$

$$\begin{aligned} &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \varphi_{q_1}(\lambda(f_\infty^i - \varepsilon)) \int_{\theta_1}^{\theta_2} l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) \varphi_{p_1}(\gamma \| (u, v) \|_Y) d\tau \right) ds \\ &= \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \gamma \varphi_{q_1}(\lambda(f_\infty^i - \varepsilon)) C \| (u, v) \|_Y \geq \frac{1}{2} \| (u, v) \|_Y, \end{aligned}$$

and $\|T_1(u, v)\| \geq \frac{1}{2} \| (u, v) \|_Y, (u, v) \in P \cap \partial\Omega_2$. Similarly, we have

$$\begin{aligned} &T_2(u, v)(\theta_2) \\ &= \varphi_{q_2}(\mu) \int_0^1 G_2(\theta_2, s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \varphi_{q_2}(\mu) \int_0^1 \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 l_2(s) \varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) g(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \varphi_{q_2}(\mu) \int_0^1 l_2(s) \varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) (g_\infty^i - \varepsilon) \varphi_{p_2}(u(\tau) + v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \varphi_{q_2}(\mu) \int_{\theta_1}^{\theta_2} l_2(s) \varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) (g_\infty^i - \varepsilon) \varphi_{p_2}(\gamma \| (u, v) \|_Y) d\tau \right) ds \\ &= \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \gamma \varphi_{q_2}(\mu (g_\infty^i - \varepsilon)) D \| (u, v) \|_Y \geq \frac{1}{2} \| (u, v) \|_Y, \end{aligned}$$

then $\|T_2(u, v)\| \geq \frac{1}{2} \| (u, v) \|_Y, (u, v) \in P \cap \partial\Omega_2$. So,

$$\|Q(u, v)\|_Y \geq \| (u, v) \|_Y, \quad (u, v) \in P \cap \partial\Omega_2. \tag{3.3}$$

Therefore, by (3.2), (3.3) and Lemma 2.8, we conclude that Q has at least one fixed point $(u, v) \in P \cap (\overline{\Omega_2} \setminus \Omega_1)$ with $R_1 \leq \| (u, v) \|_Y \leq R_2$.

(6) Suppose $f_0^s = 0, g_0^s \in (0, \infty), g_\infty^i = \infty$, then for any $\lambda \in (0, \infty)$ and $\mu \in (0, L_4)$, there exists $\varepsilon > 0$ such that

$$0 < \lambda < \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{\varepsilon}, \quad \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{(\beta_2 - 2)D\gamma_2\gamma} \right) \varepsilon < \mu < \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{g_0^s + \varepsilon}.$$

By the definitions of f_0^s and g_0^s , there exists $R_3 > 0$ such that

$$\begin{aligned} &f(t, u, v) < \varepsilon \varphi_{p_1}(u + v), \quad t \in [0, 1], 0 \leq u + v \leq R_3, \\ &g(t, u, v) < (g_0^s + \varepsilon) \varphi_{p_2}(u + v), \quad t \in [0, 1], 0 \leq u + v \leq R_3. \end{aligned}$$

Denote $\Omega_3 = \{(u, v) \in Y : \| (u, v) \|_Y < R_3\}$. For any $(u, v) \in P \cap \partial\Omega_3$ and $t \in [0, 1]$, we have

$$\begin{aligned} T_1(u, v)(t) &\leq \varphi_{q_1}(\lambda) \int_0^1 \frac{M_1}{\Gamma(\beta_1)} l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) \varepsilon \varphi_{p_1}(u(\tau) + v(\tau)) d\tau \right) ds \\ &\leq \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda \varepsilon) \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) d\tau \right) ds \| (u, v) \|_Y \\ &= \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda \varepsilon) A \| (u, v) \|_Y < \frac{1}{2} \| (u, v) \|_Y, \end{aligned}$$

then $\|T_1(u, v)\| \leq \frac{1}{2} \| (u, v) \|_Y, (u, v) \in P \cap \partial\Omega_3$.

Similar to (3.1) of (1), we get $\|T_2(u, v)\| \leq \frac{1}{2}\|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_3$, then

$$\|Q(u, v)\|_Y \leq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_3. \tag{3.4}$$

On the other hand, by $g_\infty^i = \infty$, there exists $\overline{R}_4 > 0$ such that

$$g(t, u, v) \geq \frac{1}{\varepsilon}\varphi_{p_2}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, u + v \geq \overline{R}_4.$$

Let $R_4 = \max\{2R_3, \frac{\overline{R}_4}{\gamma}\}$ and $\Omega_4 = \{(u, v) \in Y : \|(u, v)\|_Y < R_4\}$. For any $(u, v) \in P \cap \partial\Omega_4$, we have $\min_{t \in [\theta_1, \theta_2]}(u(t) + v(t)) \geq \gamma\|(u, v)\|_Y = \gamma R_4 \geq \overline{R}_4$, then

$$\begin{aligned} T_2(u, v)(\theta_2) &= \varphi_{q_2}(\mu) \int_0^1 G_2(\theta_2, s)\varphi_{q_2} \left(\int_0^1 H_2(s, \tau)g(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\geq \varphi_{q_2}(\mu) \int_0^1 \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 l_2(s)\varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau)g(\tau, u(\tau), v(\tau)) \, d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \varphi_{q_2}(\mu) \int_0^1 l_2(s)\varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) \frac{1}{\varepsilon} \varphi_{p_2}(u(\tau) + v(\tau)) \, d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \varphi_{q_2}(\mu) \int_{\theta_1}^{\theta_2} l_2(s)\varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau) \frac{1}{\varepsilon} \varphi_{p_2}(\gamma\|(u, v)\|_Y) \, d\tau \right) ds \\ &= \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \gamma \varphi_{q_2} \left(\frac{\mu}{\varepsilon} \right) D \|(u, v)\|_Y > \|(u, v)\|_Y. \end{aligned}$$

Therefore

$$\|Q(u, v)\|_Y \geq \|T_2(u, v)\| \geq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_4. \tag{3.5}$$

By (3.4), (3.5) and Lemma 2.8, we conclude that Q has at least one fixed point $(u, v) \in P \cap (\overline{\Omega}_4 \setminus \Omega_3)$ with $R_3 \leq \|(u, v)\|_Y \leq R_4$. This completes the proof. \square

For $f_0^i, g_0^i, f_\infty^s, g_\infty^s \in (0, \infty)$, we define the symbols $\tilde{L}_1, \tilde{L}_2, \tilde{L}_3, \tilde{L}_4$ as follows:

$$\begin{aligned} \tilde{L}_1 &= \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2(\beta_1 - 2)C\gamma_1\gamma} \right) \frac{1}{f_0^i}, & \tilde{L}_2 &= \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{f_\infty^s}, \\ \tilde{L}_3 &= \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2(\beta_2 - 2)D\gamma_2\gamma} \right) \frac{1}{g_0^i}, & \tilde{L}_4 &= \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{g_\infty^s}. \end{aligned}$$

Theorem 3.2 (1) If $f_\infty^s, g_\infty^s, f_0^i, g_0^i \in (0, \infty)$, and $\tilde{L}_1 < \tilde{L}_2, \tilde{L}_3 < \tilde{L}_4$, then for each $\lambda \in (\tilde{L}_1, \tilde{L}_2)$ and $\mu \in (\tilde{L}_3, \tilde{L}_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(2) If $f_\infty^s, f_0^i, g_0^i \in (0, \infty), g_\infty^s = 0$, and $\tilde{L}_1 < \tilde{L}_2$, then for each $\lambda \in (\tilde{L}_1, \tilde{L}_2)$ and $\mu \in (\tilde{L}_3, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(3) If $g_\infty^s, f_0^i, g_0^i \in (0, \infty), f_\infty^s = 0, \tilde{L}_3 < \tilde{L}_4$, then for each $\lambda \in (\tilde{L}_1, \infty)$ and $\mu \in (\tilde{L}_3, \tilde{L}_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(4) If $f_0^i, g_0^i \in (0, \infty), f_\infty^s = g_\infty^s = 0$, then for each $\lambda \in (\tilde{L}_1, \infty)$ and $\mu \in (\tilde{L}_3, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(5) If $\{f_\infty^s, g_\infty^s \in (0, \infty), f_0^i = \infty\}$ or $\{f_\infty^s, g_\infty^s \in (0, \infty), g_0^i = \infty\}$, then for each $\lambda \in (0, \tilde{L}_2)$ and $\mu \in (0, \tilde{L}_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(6) If $\{f_0^i = \infty, f_\infty^s \in (0, \infty), g_\infty^s = 0\}$ or $\{f_\infty^s \in (0, \infty), g_\infty^s = 0, g_0^i = \infty\}$, then for each $\lambda \in (0, \tilde{L}_2)$ and $\mu \in (0, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(7) If $\{f_0^i = \infty, g_\infty^s \in (0, \infty), f_\infty^s = 0\}$ or $\{g_\infty^s \in (0, \infty), g_0^i = \infty, f_\infty^s = 0\}$, then for each $\lambda \in (0, \infty)$ and $\mu \in (0, \tilde{L}_4)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

(8) If $\{f_\infty^s = g_\infty^s = 0, f_0^i = \infty\}$ or $\{f_\infty^s = g_\infty^s = 0, g_0^i = \infty\}$, then for each $\lambda \in (0, \infty)$ and $\mu \in (0, \infty)$, system (1.1) has at least one positive solution $(u(t), v(t)), t \in (0, 1)$.

Proof Because the proofs of the above cases are similar, in what follows we will prove two of them, namely cases (1) and (6).

(1) For any $\lambda \in (\tilde{L}_1, \tilde{L}_2)$ and $\mu \in (\tilde{L}_3, \tilde{L}_4)$, there exists $0 < \varepsilon < \min\{f_0^i, g_0^i\}$ such that

$$\begin{aligned} \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2(\beta_1 - 2)C\gamma_1\gamma} \right) \frac{1}{f_0^i - \varepsilon} &\leq \lambda \leq \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{f_\infty^s + \varepsilon}, \\ \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2(\beta_2 - 2)D\gamma_2\gamma} \right) \frac{1}{g_0^i - \varepsilon} &\leq \mu \leq \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{g_\infty^s + \varepsilon}. \end{aligned}$$

By the definitions of f_0^i and g_0^i , there exists $R_1 > 0$ such that

$$\begin{aligned} f(t, u, v) &\geq (f_0^i - \varepsilon)\varphi_{p_1}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, u + v \leq R_1, \\ g(t, u, v) &\geq (g_0^i - \varepsilon)\varphi_{p_2}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, u + v \leq R_1. \end{aligned}$$

Denote $\Omega_1 = \{(u, v) \in Y : \|(u, v)\| < R_1\}$, for any $(u, v) \in P \cap \partial\Omega_1$, we have

$$\begin{aligned} T_1(u, v)(\theta_1) &= \varphi_{q_1}(\lambda) \int_0^1 G_1(\theta_1, s)\varphi_{q_1} \left(\int_0^1 H_1(s, \tau)f(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \varphi_{q_1}(\lambda) \int_{\theta_1}^{\theta_2} \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 l_1(s)\varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau)f(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \varphi_{q_1}(\lambda) \int_{\theta_1}^{\theta_2} l_1(s)\varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau)(f_0^i - \varepsilon)\varphi_{p_1}(u(\tau) + v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \gamma \varphi_{q_1}(\lambda)(f_0^i - \varepsilon)C \|(u, v)\|_Y \geq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

then $\|T_1(u, v)\| \geq \frac{1}{2} \|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_1$. Similarly, we have

$$\begin{aligned} T_2(u, v)(\theta_2) &= \varphi_{q_2}(\mu) \int_0^1 G_2(\theta_2, s)\varphi_{q_2} \left(\int_0^1 H_2(s, \tau)g(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \varphi_{q_2}(\mu) \int_{\theta_1}^{\theta_2} \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 l_2(s)\varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau)g(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \varphi_{q_2}(\mu) \int_{\theta_1}^{\theta_2} l_2(s)\varphi_{q_2} \left(\int_{\theta_1}^{\theta_2} H_2(s, \tau)(g_0^i - \varepsilon)\varphi_{p_2}(u(\tau) + v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_2 - 2}{\Gamma(\beta_2)} \gamma_2 \gamma \varphi_{q_2}(\mu)(g_0^i - \varepsilon)D \|(u, v)\|_Y \geq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

then $\|T_2(u, v)\| \geq \frac{1}{2}\|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_1$. Therefore

$$\|Q(u, v)\|_Y \geq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_1. \tag{3.6}$$

On the other hand, we define $f^*, g^* : [0, 1] \times [0, +\infty) \rightarrow [0, +\infty)$ as follows:

$$f^*(t, x) = \max_{0 \leq u+v \leq x} f(t, u, v), \quad g^*(t, x) = \max_{0 \leq u+v \leq x} g(t, u, v),$$

then

$$\begin{aligned} f(t, u, v) &\leq f^*(t, x), \quad t \in [0, 1], u, v \geq 0, u + v \leq x, \\ g(t, u, v) &\leq g^*(t, x), \quad t \in [0, 1], u, v \geq 0, u + v \leq x. \end{aligned}$$

Clearly, $f^*(t, x)$ and $g^*(t, x)$ are nondecreasing on x , by the proof of [35], we have

$$\limsup_{x \rightarrow +\infty} \max_{t \in [0, 1]} \frac{f^*(t, x)}{\varphi_{p_1}(x)} \leq f_\infty^s, \quad \limsup_{x \rightarrow +\infty} \max_{t \in [0, 1]} \frac{g^*(t, x)}{\varphi_{p_2}(x)} \leq g_\infty^s.$$

From the above inequalities, there exists $\overline{R}_2 > 0$ such that

$$\begin{aligned} \frac{f^*(t, x)}{\varphi_{p_1}(x)} &\leq \limsup_{x \rightarrow +\infty} \max_{t \in [0, 1]} \frac{f^*(t, x)}{\varphi_{p_1}(x)} + \varepsilon \leq f_\infty^s + \varepsilon, \quad t \in [0, 1], x \geq \overline{R}_2, \\ \frac{g^*(t, x)}{\varphi_{p_2}(x)} &\leq \limsup_{x \rightarrow +\infty} \max_{t \in [0, 1]} \frac{g^*(t, x)}{\varphi_{p_2}(x)} + \varepsilon \leq g_\infty^s + \varepsilon, \quad t \in [0, 1], x \geq \overline{R}_2. \end{aligned}$$

Then $f^*(t, x) \leq (f_\infty^s + \varepsilon)\varphi_{p_1}(x), g^*(t, x) \leq (g_\infty^s + \varepsilon)\varphi_{p_2}(x), t \in [0, 1], x \geq \overline{R}_2$.

Denote $R_2 = \max\{2R_1, \overline{R}_2\}, \Omega_2 = \{(u, v) \in Y : \|(u, v)\|_Y < R_2\}$. For any $(u, v) \in P \cap \partial\Omega_2$, by the definitions of f^* and g^* , we have

$$f(t, u(t), v(t)) \leq f^*(t, \|(u, v)\|_Y), \quad g(t, u(t), v(t)) \leq g^*(t, \|(u, v)\|_Y), \quad t \in [0, 1],$$

so

$$\begin{aligned} T_1(u, v)(t) &\leq \varphi_{q_1}(\lambda) \int_0^1 \frac{M_1}{\Gamma(\beta_1)} l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f^*(\tau, \|(u, v)\|_Y) d\tau \right) ds \\ &\leq \varphi_{q_1}(\lambda) \frac{M_1}{\Gamma(\beta_1)} \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) (f_\infty^s + \varepsilon) \varphi_{p_1}(\|(u, v)\|_Y) d\tau \right) ds \\ &= \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda (f_\infty^s + \varepsilon)) A \|(u, v)\|_Y \leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

and $\|T_1(u, v)\| \leq \frac{1}{2}\|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_2$. Similarly, we have

$$\begin{aligned} T_2(u, v)(t) &\leq \varphi_{q_2}(\mu) \int_0^1 \frac{M_2}{\Gamma(\beta_2)} l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g^*(\tau, \|(u, v)\|_Y) d\tau \right) ds \\ &\leq \varphi_{q_2}(\mu) \frac{M_2}{\Gamma(\beta_2)} \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) (g_\infty^s + \varepsilon) \varphi_{p_2}(\|(u, v)\|_Y) d\tau \right) ds \\ &= \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu (g_\infty^s + \varepsilon)) B \|(u, v)\|_Y \leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

so $\|T_2(u, v)\| \leq \frac{1}{2}\|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_2$. Therefore

$$\|Q(u, v)\|_Y = \|T_1(u, v)\| + \|T_2(u, v)\| \leq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_2. \tag{3.7}$$

From (3.6), (3.7) and Lemma 2.8, we get that Q has at least one fixed point $(u, v) \in P \cap (\Omega_2 \setminus \Omega_1)$ with $R_1 \leq \|(u, v)\|_Y \leq R_2$.

(6) Suppose $f_0^i = \infty, f_\infty^s \in (0, \infty), g_\infty^s = 0$, for any $\lambda \in (0, \tilde{L}_2)$ and $\mu \in (0, \infty)$, there exists $\varepsilon > 0$ such that

$$\varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{(\beta_1 - 2)C\gamma_1\gamma} \right) \varepsilon < \lambda < \varphi_{p_1} \left(\frac{\Gamma(\beta_1)}{2AM_1} \right) \frac{1}{f_\infty^0 + \varepsilon}, \quad 0 < \mu < \varphi_{p_2} \left(\frac{\Gamma(\beta_2)}{2BM_2} \right) \frac{1}{\varepsilon}.$$

By $f_0^i = \infty$, there exists $R_3 > 0$ such that

$$f(t, u, v) \geq \frac{1}{\varepsilon} \varphi_{p_1}(u + v), \quad t \in [\theta_1, \theta_2], u, v \geq 0, 0 \leq u + v \leq R_3.$$

Choose $\Omega_3 = \{(u, v) \in Y : \|(u, v)\|_Y < R_3\}$, then for any $(u, v) \in P \cap \partial\Omega_3$, we have

$$\begin{aligned} T_1(u, v)(\theta_1) &\geq \varphi_{q_1}(\lambda) \int_{\theta_1}^{\theta_2} \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) f(\tau, u(\tau), v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \varphi_{q_1}(\lambda) \int_{\theta_1}^{\theta_2} l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) \frac{1}{\varepsilon} \varphi_{p_1}(u(\tau) + v(\tau)) d\tau \right) ds \\ &\geq \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \gamma \varphi_{q_1} \left(\frac{\lambda}{\varepsilon} \right) \int_{\theta_1}^{\theta_2} l_1(s) \varphi_{q_1} \left(\int_{\theta_1}^{\theta_2} H_1(s, \tau) d\tau \right) ds \|(u, v)\|_Y \\ &= \frac{\beta_1 - 2}{\Gamma(\beta_1)} \gamma_1 \gamma \varphi_{q_1} \left(\frac{\lambda}{\varepsilon} \right) C \|(u, v)\|_Y \geq \|(u, v)\|_Y. \end{aligned}$$

Thus,

$$\|Q(u, v)\|_Y \geq \|T_1(u, v)\| \geq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_3. \tag{3.8}$$

On the other hand, we define $f^*, g^* : [0, 1] \times [0, +\infty) \rightarrow [0, +\infty)$ as follows:

$$f^*(t, x) = \max_{0 \leq u+v \leq x} f(t, u, v), \quad g^*(t, x) = \max_{0 \leq u+v \leq x} g(t, u, v).$$

By the proof of [35], we have

$$\limsup_{x \rightarrow +\infty} \max_{t \in [0,1]} \frac{f^*(t, x)}{\varphi_{p_1}(x)} \leq f_\infty^s, \quad \lim_{x \rightarrow +\infty} \max_{t \in [0,1]} \frac{g^*(t, x)}{\varphi_{p_2}(x)} = 0.$$

For above $\varepsilon > 0$, there exists $\overline{R}_4 > 0$ such that, for any $t \in [0, 1], x \geq \overline{R}_4$, we have

$$\begin{aligned} \frac{f^*(t, x)}{\varphi_{p_1}(x)} &\leq \limsup_{x \rightarrow +\infty} \max_{t \in [0,1]} \frac{f^*(t, x)}{\varphi_{p_1}(x)} + \varepsilon \leq f_\infty^s + \varepsilon, \\ \frac{g^*(t, x)}{\varphi_{p_2}(x)} &\leq \lim_{x \rightarrow +\infty} \max_{t \in [0,1]} \frac{g^*(t, x)}{\varphi_{p_2}(x)} + \varepsilon = \varepsilon, \end{aligned}$$

hence $f^*(t, x) \leq (f_\infty^s + \varepsilon)\varphi_{p_1}(x), g^*(t, x) \leq \varepsilon\varphi_{p_2}(x)$.

Let $R_4 = \max\{2R_3, \overline{R_4}\}$ and $\Omega_4 = \{(u, v) \in Y : \|(u, v)\|_Y < R_4\}$. For any $(u, v) \in P \cap \partial\Omega_4$ and $t \in [0, 1]$, we have

$$f(t, u(t), v(t)) \leq f^*(t, \|(u, v)\|_Y), \quad g(t, u(t), v(t)) \leq g^*(t, \|(u, v)\|_Y),$$

then

$$\begin{aligned} T_1(u, v)(t) &\leq \varphi_{q_1}(\lambda) \int_0^1 \frac{M_1}{\Gamma(\beta_1)} l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) f^*(\tau, \|(u, v)\|_Y) d\tau \right) ds \\ &\leq \varphi_{q_1}(\lambda) \frac{M_1}{\Gamma(\beta_1)} \int_0^1 l_1(s) \varphi_{q_1} \left(\int_0^1 H_1(s, \tau) (f_\infty^s + \varepsilon) \varphi_{p_1}(\|(u, v)\|_Y) d\tau \right) ds \\ &= \frac{M_1}{\Gamma(\beta_1)} \varphi_{q_1}(\lambda (f_\infty^s + \varepsilon)) A \|(u, v)\|_Y \leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

so $\|T_1(u, v)\| \leq \frac{1}{2} \|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_4$. In a similar manner, we deduce

$$\begin{aligned} T_2(u, v)(t) &\leq \varphi_{q_2}(\mu) \int_0^1 \frac{M_2}{\Gamma(\beta_2)} l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) g^*(\tau, \|(u, v)\|_Y) d\tau \right) ds \\ &\leq \varphi_{q_2}(\mu) \frac{M_2}{\Gamma(\beta_2)} \int_0^1 l_2(s) \varphi_{q_2} \left(\int_0^1 H_2(s, \tau) \varepsilon \varphi_{p_2}(\|(u, v)\|_Y) d\tau \right) ds \\ &= \frac{M_2}{\Gamma(\beta_2)} \varphi_{q_2}(\mu \varepsilon) B \|(u, v)\|_Y \leq \frac{1}{2} \|(u, v)\|_Y, \end{aligned}$$

so $\|T_2(u, v)\| \leq \frac{1}{2} \|(u, v)\|_Y, (u, v) \in P \cap \partial\Omega_4$. Therefore

$$\|Q(u, v)\|_Y = \|T_1(u, v)\| + \|T_2(u, v)\| \leq \|(u, v)\|_Y, \quad (u, v) \in P \cap \partial\Omega_4. \tag{3.9}$$

From (3.8), (3.9) and Lemma 2.8, we conclude that Q has at least one fixed point $(u, v) \in P \cap (\overline{\Omega_4} \setminus \Omega_3)$ with $R_3 \leq \|(u, v)\|_Y \leq R_4$. This completes the proof. \square

4 Conclusion

In this paper, we study the existence of positive solutions for a system of nonlinear fractional differential equations nonlocal boundary value problems with parameters and p -Laplacian operator. Under different combinations of superlinearity and sublinearity of the functions f and g , various existence results for positive solutions are derived in terms of different values of λ and μ via the Guo-Krasnosel'skii fixed point theorem.

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Abbreviations

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The authors declare that they have no competing interests.

Authors' contributions

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